OPTIMISED FREQUENCY RANGE OF ACTIVE JOINTS FOR NANOMETRE RANGE STROKE

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Abstract *— This paper describes the modelling of a micro bimorph cantilever which is composed of a Silicon Nitride cantilever beam coated on top with a thin Chromium layer. The structure functions as a vertical electrostatic actuator for nanometre displacements with stress induced upward curvature in the off-state. A detailed description of the optimisation of the frequency of the cantilever as a function of the thickness of the chromium layer and the deflection of the cantilever is presented. The developed model suggests that resonance frequencies of several MHz can be obtained for structures providing up to 300 nm stroke.*

Key Words: *bimorph cantilever, active joints, electrostatic actuator*

I INTRODUCTION

Bimorph cantilevers, (Figure 1) electrostatically actuated to function as active joints, [1] are widely used for various applications. In this paper, active joints for nanometric displacements, useful e.g. for probing purposes, are investigated and optimisation for high frequency operation is discussed. By optimising the thickness of various layers, the length of the cantilever and the voltage applied for the deflection, the frequency of vibration of the cantilever can be optimised for the required application. To analyse the transverse vibration of the bimorph structure, the position of the neutral axis, the stress in each layer and the stress induced moment of the structure are determined. Finally, the relation of the resonance frequency as a function of the thickness of the Chromium layer allows for a frequency optimised design of the active joint at a given stroke.

II THEORY

The active joint studied here consists of two layers of different materials in which a deformable beam is bent (in the linearly elastic region) by the bimorph effect as well as by applying an electrostatic force [1]. In this contribution, for fabricating the bimorph structure, we propose to use a thin layer of Chromium (Cr) on top of a thick layer of Silicon Nitride (SiN). This bimorph has to be optimised for a maximum first order mode resonance frequency at a given off-state deflection (50 - 300 nm). Figure 1 shows the bimorph cantilever being analysed.

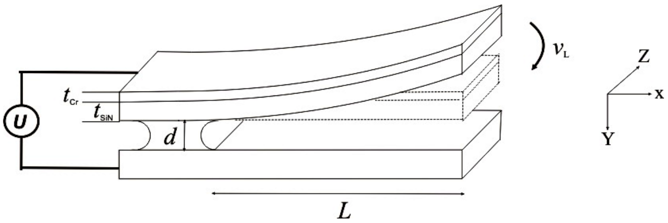

Figure 1. Bimorph cantilever beam. In the off-state, the beam is deflected by a stress induced moment. On electrostatic actuation the beam is pulled downward.

The shifted position of the neutral axis after the deposition of the Cr layer is obtained by taking the condition that the resultant axial force acting on the cross section is zero. It is given relative to the Cr/SiN layer interface as:

$$y_0 = \frac{1}{2} \cdot \frac{E_{SiN} t_{SiN}^2 - E_{Cr} t_{Cr}^2}{E_{SiN} t_{SiN} + E_{Cr} t_{Cr}} \quad (1)$$

where t_{SiN} is the thickness of the SiN layer, t_{Cr} is the thickness of the Cr layer, E_{SiN} is the modulus of elasticity of the SiN layer which is $380 \times 10^9 \, Pa$ [2], E_{Cr} is the modulus of elasticity of the Cr layer which is $140 \times 10^9 \, Pa$ [2]. In equation (1), a positive y_0 means that the neutral axis is in the SiN layer.

For the initial design purposes we assume that the bimorph is curved upward by thermal-mismatch induced stresses causing the Cr layer to be in tensile and the SiN layer to be in compressive stress. Hence we do neglect any deposition and morphologically induced stresses. Without deflection, stress in each layer is given by:

$$\sigma_{SiN} = -E_{SiN} \cdot \left(\frac{\Delta T \cdot E_{Cr} A_{Cr} (\alpha_{Cr} - \alpha_{SiN})}{E_{Cr} A_{Cr} + E_{SiN} A_{SiN}} \right) \quad (2)$$

$$\sigma_{Cr} = E_{Cr} \cdot \left(\frac{\Delta T \cdot E_{SiN} A_{SiN} (\alpha_{Cr} - \alpha_{SiN})}{E_{Cr} A_{Cr} + E_{SiN} A_{SiN}} \right) \quad (3)$$

where ΔT is the temperature difference between the Cr deposition temperature and the room temperature which is assumed as 380 K, α_{Cr} is the coefficient of thermal expansion for Cr layer, α_{SiN} is the coefficient of thermal expansion for SiN layer, A_{Cr} is the cross sectional area of Cr layer and A_{SiN} is the cross sectional area of SiN layer.

The moment (M) resulting from the stresses and acting at the cross section is independent of x:

$$\mathrm{M} = W\sigma_{Cr}\left(\frac{t_{Cr}^2}{2} + y_0 t_{Cr}\right) - W\sigma_{SiN}\left(\frac{t_{SiN}^2}{2} - y_0 t_{SiN}\right) \quad (4)$$

This moment is counteracted by the bending moment in the beam. The flexural rigidity (EI) with respect to the neutral axis y_0 is:

$$EI = E_{Cr} W\left(\frac{t_{Cr}^3}{3} + y_0 t_{Cr}^2 + y_0^2 t_{Cr}\right) + E_{SiN} W\left(\frac{t_{SiN}^3}{3} - y_0 t_{SiN}^2 + y_0^2 t_{SiN}\right) \quad (5)$$

In the differential equation of the deflection curve of the beam, the boundary conditions [3] that at the clamped edge ($x=0$) the deflection $v = 0$ & $v' = 0$ and at the free edge ($x=L$) $v'' = 0$, is applied to get the deflection at the free end as:

$$v_L = v(L) = \frac{M}{2EI} L^2 \quad (6)$$

Solving the general equation for the transverse free vibration of a beam

$$EI\frac{\partial^4 v(x,t)}{\partial x^4} = -\rho A \frac{\partial^2 v(x,t)}{\partial t^2}, \quad (7)$$

and applying the boundary conditions, yield the frequency equation for the structure as [4]:

$$\cos(k_n L)\cosh(k_n L) = -1 \quad (8)$$

where $$k_n^4 = \frac{\omega_n^2 \rho A}{EI} \quad (9)$$

Here L is the length of the bimorph, ρA is the linear mass density of the beam, EI is the (compound) flexural rigidity of the beam, ω_n is the frequency of the n^{th} order vibration mode of the beam and $v(x,t)$ is the deflection of the bimorph from its rest position. For the first mode (n=1) of vibration, $k_1 L = C_1 = 1.8751041$ [4] and so from equation (9) the frequency is obtained as:

$$f_1 = \frac{1}{2\pi}\left(\frac{C_1}{L}\right)^2 \sqrt{\frac{EI}{W(t_{Cr}\rho_{Cr} + t_{SiN}\rho_{SiN})}} \quad (10)$$

Here the breadth of the bimorph (W) is taken as 10 µm, the density of the SiN layer (ρ_{SiN}) is 2887 Kg/m^3 [2] and the density of the Cr layer (ρ_{Cr}) is 7190 Kg/m^3 [2].

In order to derive an approximation for the voltage needed to straighten the bimorph from its initial curved shape, the electrostatic (E_{es}) and the elastic (E_{el}) energies need have to be calculated. When straightening the bimorph, an increase in E_{el}

$$E_{el} = \frac{EI}{2R^2} L \quad (11)$$

is required [5], where R is the radius of the initial stress induced curvature. For small deflection it can be approximated by:

$$R \approx \frac{L^2}{2v_L} = \frac{EI}{M} \quad (12)$$

This energy needs to be supplied by the electrostatic field which is:

$$E_{es} = \frac{1}{2} \cdot \frac{d'}{\varepsilon_0 WL} \cdot Q^2 \quad (13)$$

where $d' = d + (t_{SiN}/\varepsilon_{SiN})$ is the dielectric thickness, d is the gap between the bimorph and the substrate and Q is the charge on the electrodes. The total energy, taking into account small departures (δ) from the straightened position of the bimorph and using equation (12), is given by:

$$E_{tot} = 2\frac{EI}{L^3}(v_L + \delta)^2 + \frac{1}{2} \cdot \frac{(d' - \delta)}{\varepsilon_0 WL} \cdot Q^2 \quad (14)$$

To calculate the voltage needed to straighten the bimorph (U_s), we impose the equilibrium condition that $dE_{tot}/d\delta = 0$ (minimum energy) for $\delta = 0$, and then solve for Q. Finally, we use $Q = CU$ yielding:

$$U_s = 2\left(\frac{d + t_{SiN}/\varepsilon_{SiN}}{L^2}\right)\sqrt{\frac{2EIv_L}{\varepsilon_0 W}} \quad (15)$$

where ε_0 and ε_{SiN} are the relative and absolute dielectric constants, whose values are 8.85×10^{-12} F/m and 7.5 respectively [6] and C is the capacitance between the electrodes. Obviously, for a straightened beam the electrostatic field would cause a position dependent distributed moment on the beam which cannot be compensated by the stress-induced (equation 4) and bending moment [5]. Hence, when the voltage U equals U_s, the beam will certainly be pulled in and U_s can be considered as an overestimation of the pull-in voltage.

III CALCULATIONS

In the optimisation, Cr thickness is the variable to which the design is optimised. All material parameters (Young's moduli and thermal expansion coefficients) are taken from literatures. In the design, the SiN thickness can be chosen, but for this study we fixed it at a thickness of 1 µm. The off-state displacement (v_L) was chosen as a parameter and the designs were evaluated for 6 values of it ranging between 50 and 300 nm. The approach is graphically illustrated in the figure below.

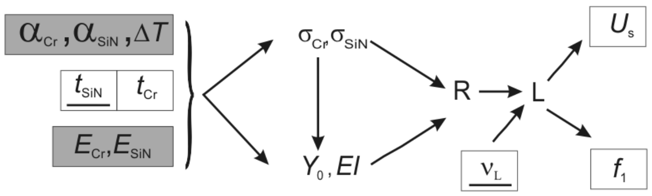

Figure 2. Calculation strategy: Grey boxed values are fixed material properties. Underlined boxed values were chosen. Cr thickness was varied to find an optimum design.

Using the material parameters, the layer thicknesses and equations (2) and (3), the stresses σ_{Cr} and σ_{SiN} in the bimorph are calculated. From the same input plus the stress, the neutral axis (1) and the flexural rigidity (5) are calculated. From these results, the radius of curvature is determined according to (12). Making use of (6) and (12) and taking a required off-state deflection as input, the bimorph length is determined. Finally, from (9) the resonance frequency and from (15) the switching voltage are derived as functions of t_{Cr}. The model was implemented in a MATLAB script and the results are presented in the next section.

IV OPTIMISATION

Starting from t_{Cr}=0 there is an increasing stress in the bimorph. However, the flexural rigidity is not significantly changing up to Cr thickness comparable to the SiN thickness. Hence, for t_{Cr} much smaller than t_{SiN} the radius of curvature is almost proportional to $1/t_{Cr}$ (see Figure 3).

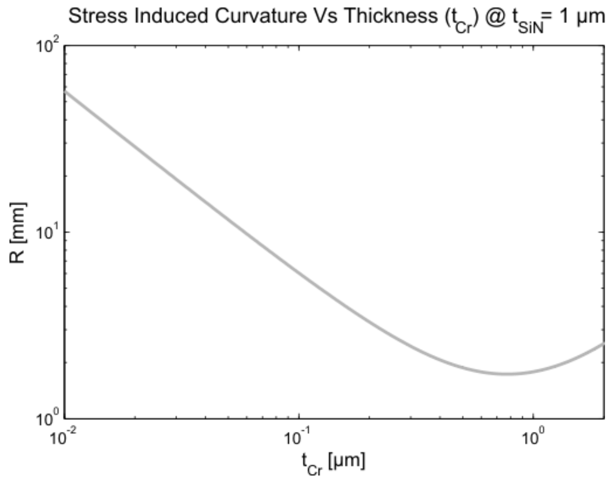

Figure 3. Radius of the stress induced curvature of the bimorph as a function of the Cr thickness.

When the Cr thickness is comparable to the SiN thickness, the stress induced moment becomes maximum and hence the radius of curvature is minimum at about 2 mm. Here, the shortest required bimorph lengths are obtained as well (Figure 4).

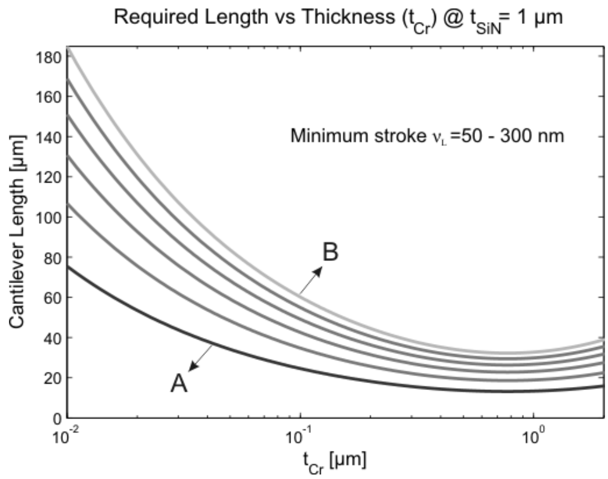

Figure 4. Required length of the bimorph as a function of the Cr thickness for v_L=50 nm (curve A) to v_L=300 nm (curve B).

The length of the bimorph structure not only depends on the radius of curvature but also on the requirements of the off-state upward deflection v_L. Figure 4 shows the required length as a function of

t_{Cr} with v_L as the parameter. Clearly, for larger v_L the required bimorph length increases, but this increase is modest since the displacement depends quadratically on the length (equation 6). Minimum required length of the bimorph structures is obtained between 800 and 900 nm Cr thicknesses.

At very thin Cr layer thickness, the radius of curvature is large and hence the required bimorph length to get a set off-state deflection becomes large as well. However, if the length of the cantilever is increased beyond a certain length then during the release of the bimorph using wet sacrificial etching, the cantilever will have the tendency to stick to the substrate and stay there. So, the length of the cantilever has to be less than its critical length of sticking. Using dry etching or vapour etching methods, these restrictions may be relaxed.

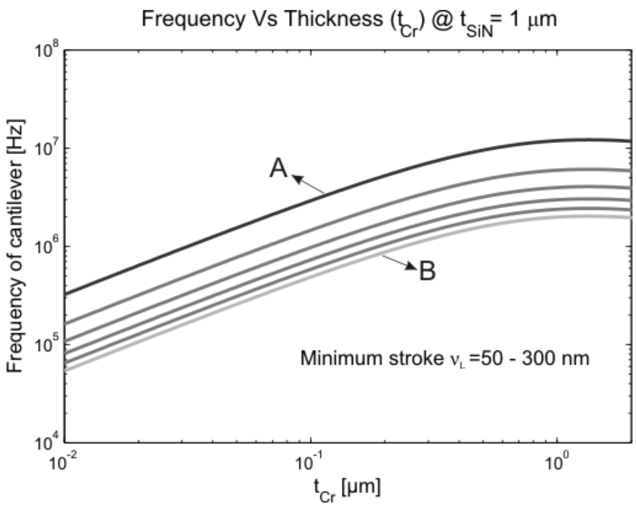

Figure 5. Achievable first order mode resonance frequency as a function of the Cr thickness for v_L=50 nm (curve A) to v_L=300 nm (curve B).

In Figure 5, the frequency of the cantilever in its first mode of vibration is plotted as a function of the thickness of the Cr layer, keeping the thickness of SiN as 1 μm. The plots are for different off-state deflections ranging from 50 nm to 300 nm. It is seen that the frequency of vibration is high for a lower deflection and vice versa. As the thickness of Cr increases up to about 1 μm, the frequency of vibration increases where the cantilever length is a minimum. Figure 5 shows that the resonance frequencies as high as 10 MHz are feasible for 50 nm off-state deflection, dropping to about 1 MHz for 300 nm off-state deflection. Figure 6 shows that high resonance frequencies come at the price of large switching voltage; U_s peaks at the values were f_1 is maximum. The graph also shows that at $t_{Cr} << t_{SiN}$, U_s is proportional to t_{Cr}.

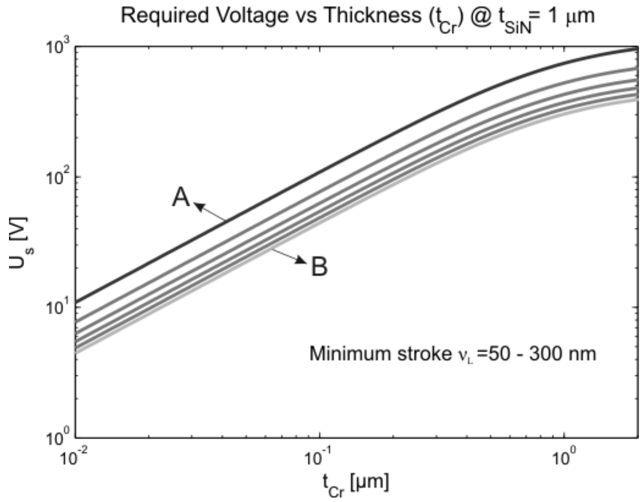

Figure 6. Required switching voltage as a function of the Cr thickness for v_L=50 nm (curve A) to v_L=300 nm (curve B).

V CONCLUSIONS

The optimisation for the frequency range for the active joints is analysed for different thicknesses of the upper (Cr) layer of a bimorph structure. It is observed that the resonance frequency has a maximum value when Cr thickness is comparable to the lower SiN thickness. Peak values can be 1 – 10 MHz, depending on off-state deflection requirement. Here, the analysis is done only for the first mode of vibration, but is applicable for higher modes as well.

ACKNOWLEDGEMENTS

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