Modeling Power Amplifiers using Memory Polynomials

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In this paper we present measured in- and output data of a power amplifier (PA). We compare this data with an AM-AM and AM-PM model. We conclude that a more sophisticated PA model is needed to cope with severe memory effects. We suggest to use memory polynomials and introduce two approaches to deduce the polynomial coefficients from the measured data: the Least-Squares and Crosscorrelation approaches. We construct PA models according to both approaches, using the measured data. We compare the two PA models with the original AM-AM and AM-PM model.

Introduction

Power Amplifiers (PAs) are inherently non-linear. There are several techniques to linearize PAs in both the analog and digital domain. One of the techniques is digital predistortion at baseband (see [1]), where the general principle is to determine the baseband-equivalent input-output behavior of the PA and to apply the inverse of this relation to the baseband signal before it is converted from digital to analog.

Because the input-output relation of the PA changes in time due to temperature changes and aging of components, a control mechanism constantly adapts the predistortion. For that purpose, the behavior of the PA has to be monitored. At fixed time intervals, a baseband-equivalent model of the PA behavior is deduced using measured data. The model obtained is used to adapt the predistortion. In for example UMTS basestations, the behavior of the PA can change relatively fast, in the order of milliseconds and the creation of a PA model has to be of low computational complexity. However, for digital predistortion to be effective, the PA model has to be very accurate.

This paper starts with the presentation of measured baseband-equivalent input- and output data of a real PA and its corresponding AM-AM and AM-PM PA model. Both are provided by Philips Semiconductors in Nijmegen. Second, we present two approaches to generate PA models: the Least-Squares- and Crosscorrelation PA modeling approaches. Third, we compare the PA models obtained via these two approaches with the original data and the AM-AM and AM-PM PA model. Finally, we draw some conclusions.

PA data and model

The data provided by Philips Semiconductors Nijmegen consisted of:

- Results of measurements on a PA, consisting of 48k samples of a stimulus signal and the corresponding response signal. The specific PA is dedicated to 2.11-2.17 GHz WCDMA operation, biased in class AB. The typenumber of the device is BLF4G22-100 and it is realized in the Philips fourth generation LDMOST technology.

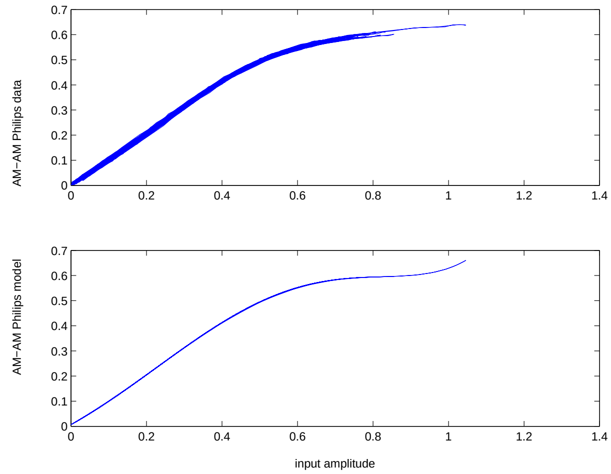

Figure 1: AM-AM distortion of the Philips data and the Philips model

- A model of the PA, based on the measurements. The specific PA model, provided by Philips, is described by the following expression:

$$y(n) = f(|x(n)|) \cdot e^{i \cdot g(|x(n)|)} \cdot \frac{x(n)}{|x(n)|} \tag{1}$$

  where

$$\begin{aligned} f(x) &= 0.0204 + 0.8445\,x + 0.2918\,x^2 - \\ &\quad 0.0588\,x^3 - 0.0729\,x^4 + 0.0175\,x^5 \\ g(x) &= 0.0660 - 0.0388\,x + 0.1963\,x^2 - \\ &\quad 0.2777\,x^3 + 0.1140x^4 - 0.0146x^5 \end{aligned}$$

  The polynomials $f(x)$ and $g(x)$ describe the so-called AM-AM or Amplitude-Amplitude distortion and AM-PM or Amplitude-Phase distortion respectively.

Both the measured in- and output samples are used to generate AM-AM and AM-PM plots. Using the PA model described above, measured input samples are used to determine corresponding output samples. In figure 1, the AM-AM distortion for the original data and the model provided by Philips are presented.
In figure 2, the AM-PM distortion for the original data and the model provided by Philips are presented.
From both figures we see that the original data is rather scattered, in contrast with the results obtained via the PA model. The scattering is due to severe memory effects, introduced by the PA, which are not covered by the AM-AM and AM-PM PA model. A very general way to model a non-linearity including memory effects is a Volterra model. A general Volterra model consists of many parameters and the complexity of an algorithm to determine these parameters is high. A simplified Volterra model is a model consisting of memory polynomials. Modeling the behavior of a PA, for the purpose of digital predistortion by means of a memory polynomial, was introduced by [2]. A general description of a memory polynomial is given in expression 2.

$$y(t) = D_0(x(t)) + D_1(x(t-1)) + D_2(x(t-2)) + \ldots \tag{2}$$

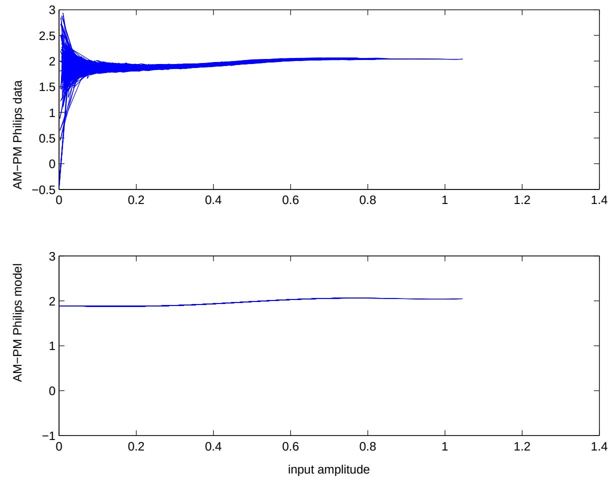

Figure 2: AM-PM distortion of the Philips data and the Philips model

where

$$D_i(x) = a_{0i}\psi_0(x) + a_{1i}\psi_1(x) + a_{2i}\psi_2(x)... \tag{3}$$

In this expression x indicates the sampled baseband equivalent input signal of the PA and y indicates the output. The functions ψ_i are polynomials. In our approach we use orthogonal polynomials because they yield more stable behavior when applied in a digital predistortion system compared to 'normal' polynomials (see [3]). The orthogonal polynomials are:

$$\begin{aligned}
\psi_0(x) &= x \mid x \mid^{-1} \\
\psi_1(x) &= x \\
\psi_2(x) &= 4 \mid x \mid x - 3x \\
\psi_3(x) &= 15 \mid x \mid^2 x - 20 \mid x \mid x + 6x \\
\psi_4(x) &= 56 \mid x \mid^3 x - 105 \mid x \mid^2 x + 60 \mid x \mid x - 10x \\
\psi_5(x) &= 210 \mid x \mid^4 x - 504 \mid x \mid^3 x + 420 \mid x \mid^2 x \\
&\quad - 140 \mid x \mid x + 15x
\end{aligned} \tag{4}$$

We limit ourselves to polynomials up to the fifth degree which give satisfactory results in practical situations.

When using memory polynomials, the behavior of the PA is deduced by estimating the polymial coefficients $a_{k\tau}$ given the samples of the input x and the output y. A relatively straightforward way to estimate the polynomial coefficients is by using the Least Squares criterion.

Least-Squares PA modeling

In the Least Squares (LS) PA modeling approach, we try to find the coefficients $a_{k\tau}$ in such way that if we use these coeficients, together with the input samples x, we obtain an estimate $\hat{y}$ of the output signal y in such way that $\mid \hat{y}(t) - y(t) \mid^2$, summed over all samples, is minimized. This approach is elaborated in [3]. The number of full-precision multiplcations that have to be executed within the LS PA modeling approach is at least $O(T)$; it scales linearly with the number of samples used to estimate the polynomial coefficients $\hat{\mathbf{a}}$. Since the number samples can be relatively large, the complexity is relatively high. An alternative with reduced complexity is the Crosscorrelation PA modeling approach.

Crosscorrelation PA modeling

To reduce the digital complexity for PA modeling, we developed the Crosscorrelation PA modeling algorithm. The algorithm consists of two parts. In the first part, crosscorrelation functions are generated without full-precision multiplications being involved. The generated crosscorrelation functions consist of a fixed number of elements (lags), independent of the number of samples. In the second part we estimate the polynomial coefficients using the least-squares solution based on the fourier transforms of the crosscorrelation functions. In this part, full precision multiplications are involved but the number of elements of the crosscorrelation functions is fixed. This approach is explained in more detail in [4].

In the Crosscorrelation PA modeling approach, the number of spectral points equals N. If an FFT is used to transform the vectors from the time domain to the frequency domain, the complexity is $O(N \log_2 N)$. The complexity of the LS PA modeling approach is $O(T)$. In general N is much smaller than T so the Crosscorrelation approach has lower complexity than the LS approach. The reduction of the complexity is due to the crosscorrelation which does not involve full-precision multiplications are required.

Results

We used both approaches, LS and Crosscorrelation, to determine PA models using the measured data. Using these PA models, the measured input signal x is used to determine a corresponding output signal $\hat{y}$. The pairs of signals $(x, \hat{y})$ are used to generate AM-AM and AM-PM plots. In figure 3, the AM-AM distortion of the memory polynomials, determined by the LS- and Crosscorrelation approach, are given.

In figure 4, the AM-AM distortion of the memory polynomials, determined by the LS- and Crosscorrelation approach, are given.

We see that the PA models result in scattered AM-AM and AM-PM plots. However, using these plots it is difficult to determine the quality of the PA models. We therefore determined the spectra of the original measured signal y and of the estimated outputs $\hat{y}$ using the different PA models. The spectra of the Philips response data, the response of the model provided by Philips, the response of the PA model using the LS approach and the response of the PA model using the Crosscorrelation approach are given in figure 5.

The input signal is an oversampled Wideband CDMA signal and because of the PA nonlinearity, there is significant power outside the primary channel as well. If we concentrate on these Adjacent Channels, we see that the estimates of the power of the response of the Crosscorrelation-based PA model is closest to the power of the actual Philips response data.

Besides an analysis of the responses in the frequency domain, we analyzed the responses in the time domain as well. We determined the Mean Square Error (MSE) between the

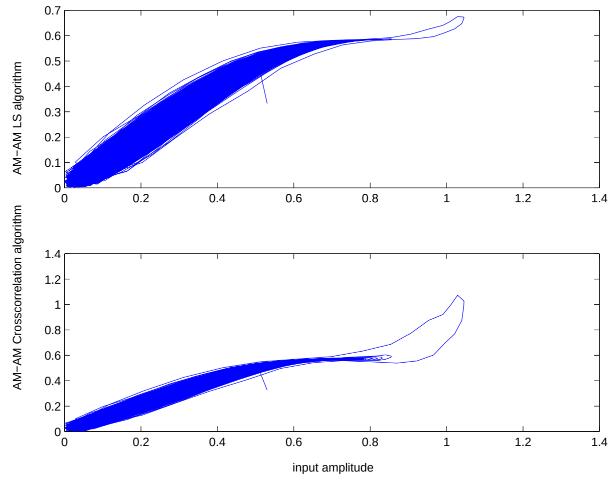

Figure 3: AM-AM for the PA models based on the LS- and Crosscorrelation approach

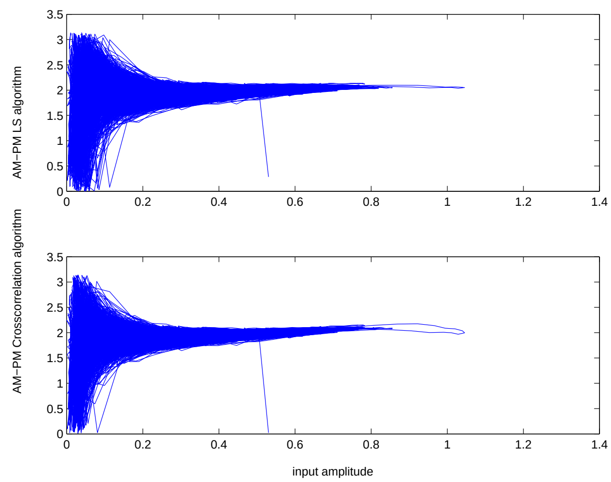

Figure 4: AM-PM for the PA models based on the LS- and Crosscorrelation approach

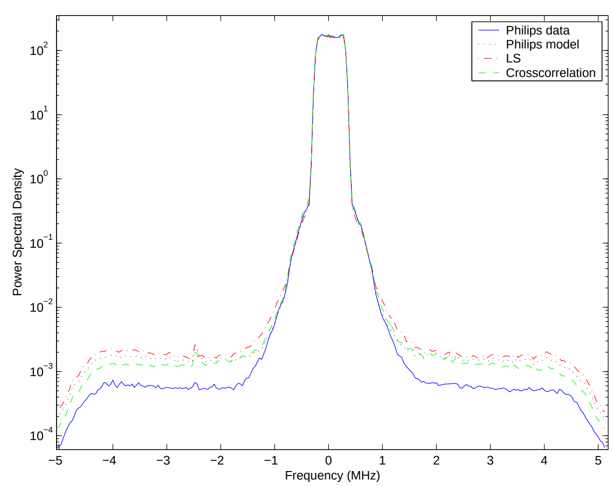

Figure 5: Spectra Philips data, Philips model, LS- and Crosscorrelation model

Philips response data and the responses based on the three PA models. The MSEs are given in table 1.

PA model	MSE
Philips	$4.0231\ 10^{-5}$
LS based	$1.1\ 10^{-3}$
Crosscorrelation based	$8.1758\ 10^{-4}$

Table 1: Mean Square Errors of the PA models

We see that the model provided by Philips Semiconductors gives the best estimate. We also see that the model which gives the smallest MSE in the time domain does not automatically generate a signal with a spectrum that fits the spectrum of the original response data best.

Conclusion

In this paper we presented measured stimulus- and response data to construct a model of a PA. The PA model is based on memory polynomials. We used a straightforward Least Squares approach toe determine the polynomial coefficients. To reduce the complexity, we developed the Crosscorrelation approach. Both approaches are used to generate a PA model and together with the PA model provided by Philips, they are used to generate responses using the measured stimulus data as input. These responses are compared with the original response data to determine the quality of the PA models. The quality is determined in the time domain and in the frequency domain. In the time domain, we use the Mean Square Error between the measured response and the generated responses as a measure of the quality. In this case, the PA model provided by Philips resembles the behavior of the real PA best. If we use the power in the adjacent channels as an estimate of the quality of the PA model in the frequency domain, the PA model obtained via the Crosscorrelation approach resembles the real PA best. For digital predistortion of PAs, it is important that an accurate model of PA can be constructed using an algorithm with low complexity. We have shown that the Crosscorrelation approach effectively reduces the complexity.

References

[1] F. Zavosh, M. Thomas, C. Thron, T. Hall, D. Artusi, D. Anderson, D. Ngo, and D. Runton, “Digital predistortion techniques for RF power amplifiers with CDMA applications,” *Microwave Journal*, Oct. 1999.

[2] J. Kim and K. Konstantinou, “Digital predistortion of wideband signals based on power amplifier model with memory,” *Electronics Letters*, vol. 37, no. 23, Nov. 2001, pp. 1417-1418.

[3] L. Ding, G.T. Zhou, D.R. Morgan, Z. Ma, J.S. Kenney, J. Kim, and C.R. Giardina, “A robust digital baseband predistorter constructed using memory polynomials,” *IEEE Trans. on Communications*, vol. 52, no. 1, Jan 2004, pp. 159-165.

[4] A.B.J. Kokkeler, “A Crosscorrelation Predistorter using Memory Polynomials,” *Proceedings of the IEEE International Symposium on Circuits and Systems*, May 2004.