

A Bayesian Network Based Risk Model for Volume Loss in Soft Soils in Mechanized Bored Tunnels

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ABSTRACT

Volume loss is one of the most important risks when boring a tunnel. This is particularly true when a tunnel is being constructed in soft soils. The risk of excessive volume loss, if materialised can lead to large consequences such as damage in buildings on the surface. This paper describes the development and use of a Bayesian based risk model containing more than forty relevant risk factors associated with the occurrence of volume loss in soft soils in mechanized bored tunnels. The developed risk model takes into account additional factors other than those normally used in analytical methods to estimate volume loss. The considered risk factors involve issues related to the excavation process, design, monitoring, human factors as well as variables associated with ground conditions. By means of data elicited from tunnel experts and the analysis of the importance of the various factors using sensitivity analysis, the model is evaluated and its ability to provide information to derive specific risk measures is verified.

1 INTRODUCTION

The construction of bored tunnels in soft ground inevitably causes ground movements. In urban areas such movements may affect adjacent buildings. Assessment of the most important risk factors influencing ground movements is therefore an essential part of planning, design and construction of a tunnelling project in urban environment (Mair et al., 1996). The problem of ground movement assessment has interested many researchers, among others, Peck (1969), Cording and Hansmire (1975), Attewell and Woodman (1982), O'Reilly and New (1982), Rankin (1988), Mair et al (1996), Macklin (1999). As a result they have provided semi-empirical approaches to calculate volume loss and ground movements. In the specific case of mechanized bored tunnelling, ground movements will depend mainly on ground conditions, tunnel geometry and depth, rate of tunnel advance, variability of internal support pressure and time taken to install support to the ground (Guglielmetti et al. 2008). Nevertheless, other factors leading to unexpected excessive ground deformations might arise during construction stage as a consequence of uncertainties in the ground, extraordinary events and shortcomings in the boring operations and flaws in design. Identifying and modelling these risk factors is highly desirable in order to reduce the chance of occurrence of an excessive ground movement event.

Identifying and modelling these additional risk factors is not straightforward. Available sources of information often lack details, for instance, about how risk factors could unfold over time, how they interact with each other and the conditions influencing the factors occurring. To partly overcome this situation, expert judgements could be used as alternative source of information as demonstrated by Bles et al. (2003) and Choi and Mahadevan

(2008). Even when this approach is adopted, modelling of incomplete information and expert judgements together will still require special tools. It has been shown that Bayesian Belief Networks are a suitable tool for integrating and representing incomplete knowledge and modelling of high dimensional phenomena (Sigurdsson et al. 2001, Weber et al. 2010). Sousa (2010) was the first to demonstrate the application of Bayesian Networks for tunnelling. Based on information from a tunnelling project in Porto, the author developed a geologic prediction model. BBNs were also used by Špačková and Straub (2011) to model the excavation performance of a road tunnel built using the New Austrian Tunnelling Method.

This paper describes the development and use of a Bayesian based risk model containing more than forty relevant risk factors associated with the occurrence of volume loss in soft soils for mechanized bored tunnels. The developed risk model takes into account additional factors other than those normally used in analytical methods to estimate volume loss. The considered risk factors involve issues related to the excavation process, design, monitoring, human factors as well as variables associated with ground conditions. By means of data elicited from tunnel experts and through analysis of the importance of the various factors using sensitivity analysis, the model provides information to derive specific risk measures.

The remainder of the paper is divided into four sections. Section 2 explains how the risk model was developed. Section 3 discusses the methods used to identify critical risk factors. The final two sections then respectively discuss results and draw conclusions based on this work.

2 RISK MODEL DEVELOPMENT AND VALIDATION

In this section the steps required to develop the risk model for volume loss in soft soils in mechanized bored tunnels is described briefly. In developing the risk model, the data acquired is structured using Bayesian networks as will be explained. This section also describes how the model was validated based on an iterative process involving discrepancy analysis, experts review and other evaluation methods.

2.1 Information gathering

This step is intended to obtain information on the:

- relevant risk factors associated with volume loss risks,
- relationships among the risk factors,
- the chance of risk factors occurring.

The relevant risk factors are assumed to be those identified as such by experts and are defined in terms of deviations from project requirements (assumptions, expectations, specifications, tolerances, and thresholds) that could lead to the occurrence of volume loss.

Information on the chance of the relevant risk factors naturally depends on the particular setting of each project. However, for test purposes, experts were asked to give their best estimation of the likelihood of the risk factors occurring based on the standard construction practice.

The step of information gathering started off by a literature study, including risk databases, failure case reports, and specialised treatises on tunnel works that provided, as an output, an inventory of risks factors associated with volume loss.

The experts consulted were then presented with the inventory of potential risk factors as derived from the literature study and were asked to add possible additional factors to this list. The experts also suggested plausible relationships and provided the probabilistic information regarding the chance of occurrence of the risk factors and the conditional probabilities measuring the relationships amongst the factors.

This study used the procedures described mainly by Goossens et al. (2008) and Hallowell and Gambatese (2010) to gathering data from experts. To ensure the reliability of this data, these procedures employed encompass a number of steps including the identification and selection of experts, pilot elicitation sessions, discrepancy and sensitivity analysis of data gathered, feedback to experts.

Thirty-one experts involved in ongoing or past underground construction projects, such as bored tunnels and deep shaft excavations, in the Netherlands participated in this investigation. All participants are from the Netherlands or Germany. The experts all had a minimum of ten years of tunnelling experience. They are all working, or had worked, for organizations such as research institutes, governmental agencies, or engineering companies providing tunnel design services and/or construction and/or supervision of tunnel works.

2.2 Knowledge representation using Bayesian Belief Networks

The Bayesian Belief Networks, BBN, approach is essentially a framework for modelling the relationships between a set of variables, and for capturing the uncertainty in the dependencies between these variables using conditional probabilities (van der Gaag, 1996). The probability of a value of a factor occurring in the BBN is determined by the occurrence of change in other interrelated factors (Onisko et al. 2001). The inference mechanism used in a BBN is the Bayes theorem which makes it possible to compute the probability of an effect on any variable in the model from the probability of a given cause. With two directly related variables, the probabilities can be computed as follows (Vick, 2002):

$$P[\text{effect}] = [P[\text{effect/cause}] \cdot P[\text{cause}]] / P[\text{cause/effect}] \quad (1)$$

where:

$P[\text{cause}]$ = probability that the cause occurs,

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$P[\text{effect/cause}]$ = conditional probability of the effect, given the cause,

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The posterior probability of the cause from the effect can similarly be derived as:

$$P[\text{cause/effect}] = [P[\text{effect/cause}] \cdot P[\text{cause}]] / P[\text{effect}] \quad (2)$$

In the context of risks, BBNs can be used to construct risk models composed of scenarios based on a set of known possible risk factors associated with the risks being analysed. These possible scenarios must be structured as a set of mutually exclusive and collectively exhaustive elements to which a probability distribution can be attributed. The probabilities of the risk factors are usually encoded based on expert judgement.

In a BBN, the interrelationships between variables are expressed graphically in the form of diagrams. Variables are represented by nodes. Diagram nodes that have interdependencies are connected by arcs, whereas independent nodes are not connected. The direction attached to an arc reflects the direction of causal influence, which might be indicated by an expert, or is scientifically proven. Figure 1 shows a BBN model produced in this research to represent interactions between the risk factors identified leading to the event "excessive volume loss" for bored tunnels in soft soils. Information on conditional probabilities attached to the causal influences of the risk factors is not indicated on the diagram but is stored in the model and is accessible to the user.

Each variable in the model is regarded as an event or a condition representing a fault event, state of failure, or an unfavourable condition. Fault events or states of failure associated with a variable can be events in which a risk factor exceeds a predefined threshold. Accordingly, most of the variables have two possible states: 'absent' or 'present'. A variable is regarded as having the status 'absent' when it is not active under the particular conditions being analysed. Although variables can have more than two possible states, few variables in the model required to be expressed in such a way. The 'present' status was further discretized into five chance categories. In line with this, experts were asked to provide estimates of chance regarding the occurrence of risk factors in terms of qualitative probabilities using a scale of five categories.

For this research, both the Netica (Norsys Corporation, Canada) and the Genie (Decisions Systems Laboratory, University of Pittsburgh) software packages were used to construct the networks and to perform the analyses. The model developed was compiled in both software packages in order to verify the correctness of the computations when propagating the data incorporated into the model.

2.3 Risk model validation

Models can be validated by testing how they behave when analysing well-known scenarios (Langseth and Portinale, 2007). This option is challenging in this study because information on well-known scenarios is not available. The use of information from historical data is constrained by the fact the only partial information is available, making validation unreliable and impracticable. Therefore, to verify the model's reliability, different evaluations have been employed as explained below.

To eliminate errors and inaccuracies in data, and to ensure that the probability statements reliably represent expert knowledge, a discrepancy analysis was conducted. Discrepancy analysis aims to identify those pieces of data where the experts' assessments differ the most. These data should be reviewed to see if there are avoidable causes of the discrepancy (Cooke and Goossens, 2000) or for the purpose of adopting values based on established confidence bands (Ayyub, 2001). In our case, discrepancy analysis provided information on which pieces of information were suitable for incorporation into the model, which needed to be revisited by its provider, and which had to be rejected or retained for further analysis to assess the effect of discrepant information in the model.

In addition, model's structure was reviewed by various experts during the elicitation sessions. By considering the diagrams depicting the risk being studied, each expert consulted had the opportunity to review the relationships amongst the variables in the model and provide estimates of the strength of the influence of these relationships. This can be seen as an internal validation of the model. Few divergences arose among the experts on the existence of some relationships and their impact was investigated. It was verified that the impact of these divergences on model performance was of little significance.

After this validation process, any remaining inaccuracies were investigated by computing entropy and mutual information measures. This analysis is described in more detail in Woodberry et al. (2004).

3 MOST RELEVANT FACTORS IDENTIFICATION

The model development process has been described in the previous section. Capturing and representing risk-related information in the model required a careful process of data collection and refinement that provided as output a model. The model is intended to provide information that supports risk management decisions; more specifically, supportive information to derive appropriate risk mediation measures. Appropriate measures are those that successfully either avoid or mitigate a risk, or respond satisfactorily to the materialised

risks given constrained resources. In principle, and as part of a cause-reduction approach to risk management, these measures should act upon those dominant risk factors that most influence the occurrence of a given risk. This section provides a brief description of the approach adopted in this study to analyse the model in order to identify these relevant risk factors.

Borgonovo (2006) distinguished three families of importance measures that allow relevant variables to be identified. The first is based on the correlation between input variables and the output. The variable with the highest correlation is ranked as the most influential component and so on. The second categorization of importance measures is based on the contribution to output variance of their probability distributions. The underpinning idea is to compute the change in the output variance obtained by eliminating the uncertainties in a variable under study. The variable that produces most change in the output variance is regarded as the most important. The third importance measure involves moment-independent sensitivity indicators. The idea of using this measure is to assess the change in uncertainty in the entire distribution of the output Y when the uncertainties attached to an input variable X_i are reduced. The variable that produces most change in the output distribution is acknowledged as the most critical (Borgonovo, 2006). The method adopted in this study relies on both correlation and uncertainty measures.

The information used to rank risk factors for the input-output correlation approach is computed using the Bayesian theorem explained above, which is powered and automated by Bayesian Networks. The necessary data was directly elicited from experts.

In using the uncertainty measures the analysis needs to be based on a sensitivity analysis. Saltelli (2002) defines sensitivity analysis as the determination of how uncertainty in the output of a model can be apportioned to different sources of uncertainty in the model's inputs.

In the standard approaches to sensitivity analysis, one variable is removed from the original set of input variables and the sensitivity value related to the remaining input variables then calculated and compared with those of other subsets of input variables (Deng et al. 2010). In our study, Borgonovo's measure is used as a sensitivity indicator. This is an alternative approach that examines the global response of a model's output by looking at the whole output distribution changes while assessing the influence of uncertainty. The details of the computation of this measures are available from Borgonovo (2006).

4 RESULTS

In Figure 1, the components of the developed risk model for excessive volume loss are displayed (variable states are not shown for reasons of clarity). About forty risk factors were identified as relevant to the occurrence of excessive volume loss leading to ground movement in soft soils for bored tunnels.

In order to verify the ability of the model and the analysis approach to support decisions, experimental data was used. This experimental data consist of information of the chance of occurrence of the risk factors. To obtain that information, the experts consulted were explicitly asked to give their judgements on the chances of the risk factors based on the real situation found in practice and not on ideal situations. For the case of the variables related to ground conditions, which depend entirely on the particular setting of each project; they were modelled using uniform distributions. Once information on the chance of risk factors is incorporated into the model, the importance measures can be computed for each factor in relation to other as shown in Table 1.

Table 1 summarises the results obtained from the computation of input-output correlation and Borgonovo's (δI) importance measures for risk factors directly related to 'excessive volume loss' event. An additional measure (likelihood measure) is also reported and indirectly accounts for the unconditional probability of occurrence of each risk factor. The

numbers without parenthesis in Table 1 reflect the results of the computation of each importance measure for each risk factor (sensitivity indicators). The larger the sensitivity indicator, the more important a variable is. The numbers in all parentheses indicate the relative importance of the risk factors based on the estimated values of the sensitivity indicators. This provides an indication about how resources can be apportioned to control the occurrence of excessive volume loss event.

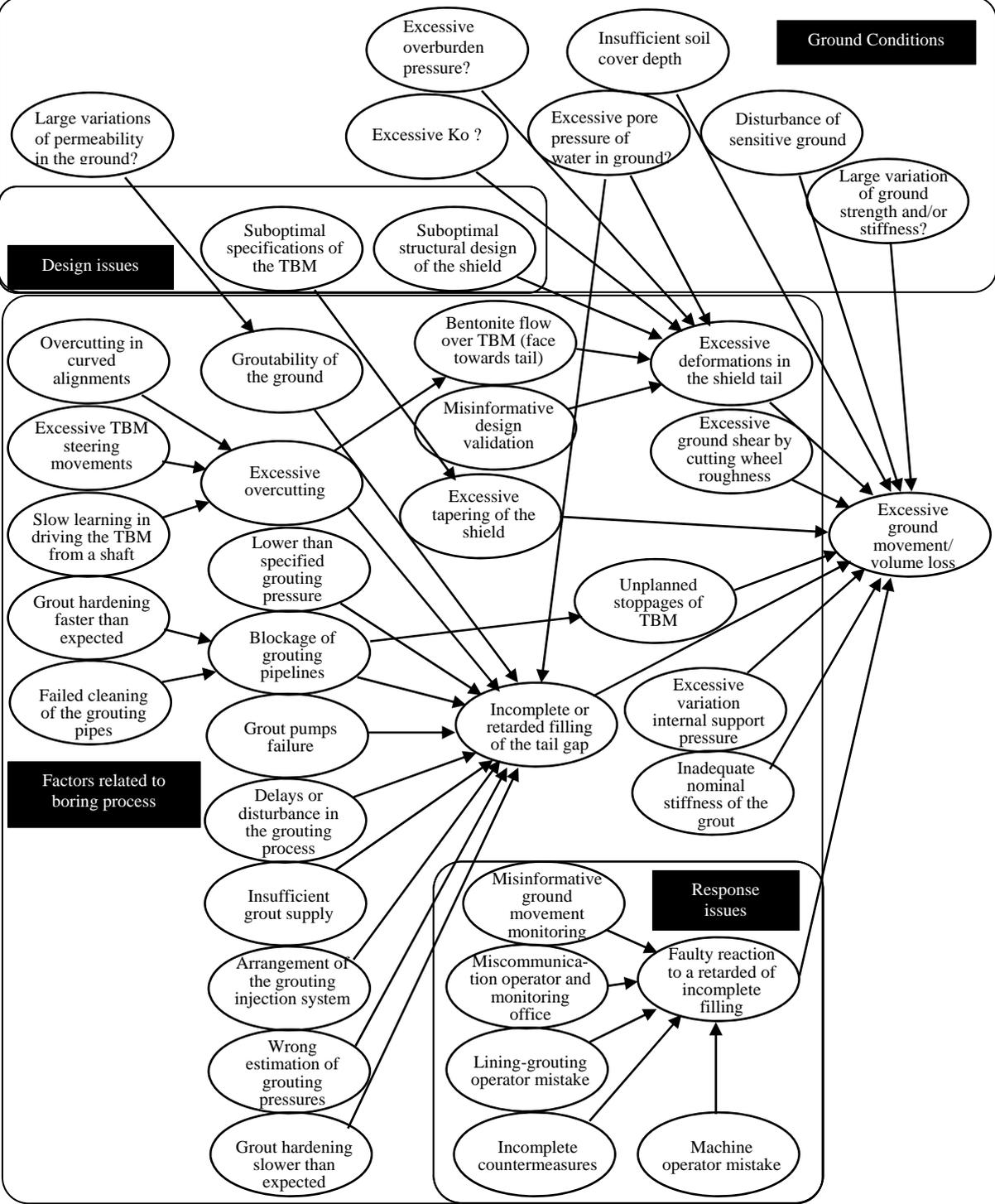


Figure 1 Excessive volume loss risk model for bored tunnels in soft soils

The results presented in Table 1 depend on the experimental data employed, and therefore the ranking order could change with different conditions and specific project information used as input to the model. For instance, the sensitivity analysis output would change with different ground conditions or different combinations of construction procedures.

As expected, each importance measure provides different ranks, this is because each importance measure relies on different criteria associated with the decision-makers preferences. Providing information on the most relevant factors according to different criteria is intentional in order to allow decision-makers to use this information according to their own preferences and the particularities of the specific problem at hand.

Table 1 Ranking of risk factors according to likelihood, Correlation and Borgonovo's measures for the target risk factor 'volume loss' event.

	Criteria/measure		
	Likelihood	Correlation	δI
Target risk: Excessive volume loss			
Directly related risk factors			
Disturbance of sensitivity ground caused by stress changes	0.600 (4)	1.000 (1)	0.140 (2)
Incomplete or retarded filling of the tail gap	1.000 (1)	1.000 (1)	0.048 (8)
Excessive variation of internal support pressure	0.667 (3)	0.800 (2)	0.083 (6)
Excessive deformations in the shield tail	0.010 (5)	0.720 (3)	0.167 (1)
Large variation of strength and/or stiffness in the ground	0.600 (4)	0.650 (4)	0.100 (4)
Faulty reaction to a retarded of incomplete filling	1.000 (1)	0.600 (5)	0.089 (5)
Insufficient soil cover depth	0.600 (4)	0.600 (5)	0.072 (7)
Excessive ground shear caused by roughness of the cutting wheel	0.010 (5)	0.580 (6)	0.007 (9)
Inadequate nominal stiffness of the grout	0.010 (5)	0.500 (7)	0.002 (10)
Excessive tapering of the shield	0.286 (8)	0.300 (8)	0.107 (3)
Unplanned stoppages of TBM	0.990 (2)	0.300 (8)	0.010 (11)

From Table 1, it is particularly important to notice that some issues having a low likelihood measure (column 2) might result as being very important according to the correlation (column 3) and uncertainty (column 4) measures. For instance, the result on the low probability risk factor 'Excessive deformations in the shield tail' whose likelihood indicator is 0.010 shows that this risk factor is relatively quite important in terms of the degree of influence (indicator = 0.720) and uncertainty contribution on the model's output (indicator = 0.167). For the case of the very likely risk factor 'Unplanned stoppages of TBM', it resulted to be of relative little influence in terms of correlation (indicator = 0.300) and uncertainty (indicator = 0.010) on the occurrence of an excessive volume loss event. These facts indicate that using merely a single measure to decide on the allocation of resources to control the risk under consideration likely misinform decision making. A more comprehensive approach might be to use the measures all together. To illustrate this, in Table 1, risk factors were ordered according the correlation measure and then Borgonovo's measure are used to order issues having the same position according to the correlation criteria. This approach was useful for example in establishing the undetermined position of the risk factor 'Incomplete or retarded filling of the tail gap' that ranked at equal position with the factor 'Disturbance of sensitivity ground caused by stress changes' according to the correlation criteria. A similar evaluation can be made combining the correlation and likelihood measures.

This analysis was repeated for every variable in the model which delivered information on the most relevant factors, thereby the ability of the model to provide information to support risk management decisions was verified. The information provided by the model can also be combined with other criteria, such as the cost of the risk measures or the controllability of risk factors enabling better informed decision making.

It is also verifiable that the above analysis can be conducted for any variable or sets of variables in the model in order to identify relevant risk factors using project specific information and on a case-by-case basis. This includes the analysis of different risk factors particular to a project in conjunction with those in the model.

5 DISCUSSION

This paper has reported on a model representing the risks factors associated with excessive volume loss event in soft soils in bored tunnels. After an intensive and careful elicitation process with experts, the model containing an exhaustive collection of risk factors, their interactions, and the associated probabilistic information was obtained. The information delivered by the model extends the knowledge normally provided by the standard approaches used to characterise construction risks in terms of the following aspects:

- the relevant risk factors associated with volume loss event,
- the chance of relevant risk factors occurring,
- the plausible relationships among risk factors,
- the strength of the relationships in terms of conditional probabilities.

This study further advance risk assessment of tunnel works by addressing the following complex issues related to:

- the acquisition of information on uncertain risk factors that could lead to undesirable failures with large consequences: the approach was designed to capture low probability risk factors viewed also as relevant by experts;
- capturing and modelling interactions among risks factors to provide a more optimal determination of risk mitigation actions by understanding whether risk is apportioned by individual factors or by the joint action of a set of them: verified correlations among risk factors directly related to a variable were modelled;
- the need to make more rational and informed risk management decisions: the developed model renders information relying on sensitivity analysis to identify critical factors allowing decision-maker to use, afterwards, her/his preferences to make choices;
- the need for interpretable and useful uncertainty-based risk models: the model's output consists of rankings of relevant factors providing clear-cut information to derive a portfolio of risk remediation measures.

The work however requires further evaluation and this would be an appropriate subject for additional research. Future studies are required to determine whether there is a need for improvements and refinements concerning the applicability of the approach and the use of knowledge rendered by the model in real projects. This is challenging given the need to provide information on construction risks that are both project- and context-dependent.

Although the developed tunnelling risk model provides relevant information on interactions amongst causes involving various factors, the model is inherently incomprehensive because it is unlikely that it encompasses the complete range of possible risk factors. This might be due to the experts' combined experiences not being sufficiently inclusive, leading to a limited understanding of the risk, or to undetected flaws in the elicitation procedure. However, as new information becomes available from documented project experiences or from further research, it will be possible to update the model to reflect this new information.

The knowledge provided by the model in its application for bored tunnels as described in this paper is limited to soil conditions similar to Dutch ground conditions and construction practices. Dutch ground conditions are characterised by saturated, low-stiffness sandy soils with medium-fine sized particles and a high groundwater table. The developed risk model also focuses on tunnels boring using closed shields (such as slurry and earth pressure balance shields) and supported by concrete linings.

6 CONCLUSIONS

This paper has reported on a model representing the risks factors associated with excessive volume loss event in soft soils in bored tunnels. After an intensive and careful elicitation process with experts, the model, containing an exhaustive collection of risk factors, their interactions, and the associated probabilistic information, was developed. Further, the ability of the model to provide information to support risk management decisions was verified.

This paper reported on a novel way to represent and analyse risks in tunnel projects. The model shows how tunnel risks can be modelled by Bayesian Networks and used to provide more reliable information to aid decision-making. It is concluded that, despite the complex and uncertain nature of tunnelling risks, the developed model can produce useful results which could guide the allocation of resources to specific risk remedial measures on a cost-efficient basis.

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