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On the parameterization of all admissable pairs in a class of CCF-ILC algorithms

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***Abstract*— This paper extends some recent results on the parameterization of all admissable pairs in a class of 2-parameter current-cycle-feedback ILC algorithms. In addition, a necessary and sufficient condition is given under which the associated set of equivalent controllers coincides with the set of *all* stabilizing controllers.**

I. Introduction and Problem Statement

Given a plant $P : U \to Y$, $y = Pu$, along with some desired output y_d, the objective in Iterative Learning Control is to construct a sequence of inputs $\{u_0, u_1, \ldots\}$ such that the corresponding sequence of outputs $\{y_0, y_1, \ldots\}$ converges to some limit value $\bar{y} := \lim_{k\to\infty} y_k$ that is close to y_d in some sense. More specifically, the aim is to define an algorithmic procedure prescribing how future inputs may be constructed from recordings of past in- and outputs. The analysis in this section in concerned with one such family of procedures, namely a class of 2-parameter iterations

$$u_{k+1} = Qu_k + Le_k + Ce_{k+1} \qquad (1)$$

Here $e_k := y_d - y_k$ denotes the current tracking error. The parameters Q and L are constrained to be causal, bounded linear operators, that is $Q, L \in \mathcal{RH}_\infty$. The feedback term C is assumed to be stabilizing.

The class of iterations is parameterized by the free parameters Q and L. Of all conceivable combinations (Q, L), those that will generate a *converging* sequence of inputs are of particular interest. Such pairs are called *admissable*. The set of all admissable pairs is denoted by $\mathcal{A}$ [4]. For every $(Q, L) \in \mathcal{A}$, one can define the associated equivalent controller [1], [2], [3], [4], [5], see Figure 1.

$$K = (I - Q)^{-1} (L + C) \qquad (2)$$

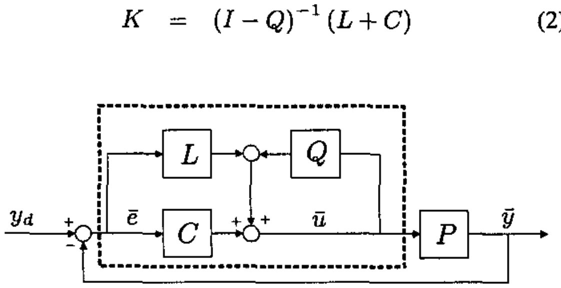

Fig. 1. Equivalent Feedback Controller (dashed).

This equivalent controller has the property that it is always stabilizing. Now let $\mathcal{K}_\mathcal{A}$ denote the set of all equivalent controllers

$$\mathcal{K}_\mathcal{A} := \left\{K : K = (I - Q)^{-1} (L + C)\,;\, (Q, L) \in \mathcal{A}\right\}$$

and let the set of *all* stabilizing controllers be denoted by $\mathcal{K}$. Clearly $\mathcal{K}_\mathcal{A} \subset \mathcal{K}$. Recently it was shown [4] that in case $C = 0$ (in which case P is assumed to be stable) both sets coincide, but that this is not true for all C. In the next section it is shown that the above result is just a special case of a general result which says that $\mathcal{K}_\mathcal{A} = \mathcal{K}$ if and only if $C \in \mathcal{RH}_\infty$.

II. When $\mathcal{K}_\mathcal{A}$ and $\mathcal{K}$ coincide

Theorem 1: Given the class of iterations (1) and the associated sets $\mathcal{K}_\mathcal{A}$ and $\mathcal{K}$. Then, for any stabilizing C, $\mathcal{K}_\mathcal{A} \subseteq \mathcal{K}$ with equality (=) iff C is *strongly* stabilizing, i.e. iff C is stable ($C \in \mathcal{RH}_\infty$) and stabilizing.

Proof: (Sufficiency) Suppose $C \in \mathcal{RH}_\infty$ and let $K \in \mathcal{K}$ be any stabilizing controller. To prove that $K \in \mathcal{K}_\mathcal{A}$, define

$$\begin{aligned} Q &= (K - C)(I + PK)^{-1} P \\ L &= (K - C)(I + PK)^{-1} \end{aligned} \qquad (3)$$

The controller K being stabilizing, by definition all closed-loop transfer matrices are stable. Since C is also stable, so are Q and L. To prove that $(Q, L) \in \mathcal{A}$ note that $(Q - LP)(I + CP)^{-1} = 0$, which is a sufficient condition for admissability [4]. And with $(Q, L) \in \mathcal{A}$, by definition $K \in \mathcal{K}_\mathcal{A}$. ∎

To prove necessity, a few more intermediate results are required.

Lemma 2: Let $K \in \mathcal{K}_\mathcal{A}$ be a controller induced by some admissable pair $(Q, L) \in \mathcal{A}$. Then there exists an "equivalent" [4] pair $(Q_0, L_0) \in \mathcal{A}$ such that $K = (I - Q_0)^{-1} (L_0 + C)$ and $(Q_0 - L_0 P)(I + CP)^{-1} = 0$.

Proof: There exists such $(Q_0, L_0) \in \mathcal{A}$ iff if the following set of equations has a solution $(Q, L) \in \mathcal{A}$.

$$\begin{aligned} (L + C) &= (I - Q) K \\ (Q - LP)(I + CP)^{-1} &= 0 \end{aligned} \qquad (4)$$

The unique solution to the above set of equations is given by

$$\begin{aligned} Q_0 &= (K - C) P (I + KP)^{-1} \\ L_0 &= (K - C)(I + PK)^{-1} \end{aligned} \qquad (5)$$

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From (4) we obtain

$$K - C = L + QK \quad (6)$$

with $Q, L \in \mathcal{RH}_\infty$ by assumption of admissability. After substituting (6) into (5), inspection shows that $Q_0, L_0 \in \mathcal{RH}_\infty$. Hence, by construction (Q_0, L_0) is admissable. This concludes the proof. ∎

Lemma 2 says that among all admissable pairs defining the same equivalent controller, there is one and only one pair (Q_0, L_0) which satisfies the additional constraint. Let $\mathcal{A}_0$ denote the set of all such pairs, i.e.

$$\mathcal{A}_0 = \left\{(Q, L) \in \mathcal{A} : (Q - LP)(I + CP)^{-1} = 0\right\}$$

This set allows for an efficient parameterization

$$\mathcal{A}_0 := \{(Q, L) = (ZN, ZM); Z \in \mathcal{RH}_\infty\} \quad (7)$$

where $P = M^{-1}N$ is a left-coprime factorization over $\mathcal{RH}_\infty$. Through (7) one arrives at a parameterization of the set $\mathcal{K}_\mathcal{A}$.

Lemma 3: Let $C = V^{-1}U$ and $P = M^{-1}N$ be any left-coprime factorization of the plant and the controller respectively. Then the set of all equivalent controllers $\mathcal{K}_\mathcal{A}$ is parameterized by

$$\mathcal{K}_\mathcal{A} = \left\{K = (V - VZM)^{-1}(U + VZN); Z \in \mathcal{RH}_\infty\right\}$$

Proof: The equivalent controller is given by

$$K = (I - Q)^{-1}(L + C)$$

with $(Q, L) \in \mathcal{A}_0$ this evaluates to

$$\begin{aligned} K &= (I - ZM)^{-1}(V^{-1}U + ZN) \\ &= (V - VZM)^{-1}(U + VZN) \quad (8) \end{aligned}$$

This concludes the proof. ∎

Although the above parameterization seems to depend on specific factorizations, in actual fact the choice of coprime factors is immaterial. This is immediate from the fact that left-coprime factors are unique up to a left multiplication with a bistable transfer function.

The next lemma restates an important result on the parameterization of all stabilizing controllers.

Lemma 4 (Youla-Kučera): Given $C = V^{-1}U$ and $P = M^{-1}N$ with U, V and M, N left-coprime. Assume C is stabilizing. Then the set $\mathcal{K}$ of *all* stabilizing controllers is given by [6, Theorem 12.7]

$$\mathcal{K} = \left\{K = \left(V - \tilde{Z}M\right)^{-1}\left(U + \tilde{Z}N\right); \tilde{Z} \in \mathcal{RH}_\infty\right\}$$

Inspection shows that the respective parameterizations of $\mathcal{K}_\mathcal{A}$ (Lemma 3) and $\mathcal{K}$ (Lemma 4) are equivalent if and only if for every $\tilde{Z} \in \mathcal{RH}_\infty$ there exists $Z \in \mathcal{RH}_\infty$ such that $\tilde{Z} = VZ$. The condition for equality is clearly satisfied in case C is strongly stabilizing ($C \in \mathcal{RH}_\infty$) since then V is bistable and Z can be selected as $Z = V^{-1}\tilde{Z}$. The proof of Theorem 1 is now straightforward.

Proof: (of Theorem 1, necessity) Suppose $C \notin \mathcal{RH}_\infty$. Take $\tilde{Z} = I$ and let K be the corresponding stabilizing controller (Lemma 4). By uniqueness of the Youla parameter it is clear that the corresponding controller K belongs to $\mathcal{K}_\mathcal{A}$ iff there exists $Z \in \mathcal{RH}_\infty$ such that $VZ = I$. This however implies that V is bistable and $C = V^{-1}U$ stable, which contradicts the starting assumption. This concludes the proof. ∎

III. Conclusion

The results presented in this paper build on and make use of a framework for analysis that was put forward in a series of recent papers by the same authors. An important conclusion to be drawn from this work is that the choice of the current cycle parameter most definately affects (constrains) the achievable performance.

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