A Computer Aided Tolerancing Tool based on Kinematic Analogies

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Abstract

A computer aided tolerancing tool is presented that assists the designer in functional tolerance specification. The theoretical concepts for subsequent tolerance analysis are also provided. The computer aided tolerancing tool is part of a feature based object oriented (re)–design support system, called FROOM. FROOM's assembly modelling capabilities provide basic information for functional tolerance specification. Assembly constraints are satisfied by means of degrees of freedom (DOF) analysis. This method is based on the use of kinematic analogies. The rotations and translations (macro–DOF's) that components are allowed to have, are inferred using this technique. The tolerance representation in FROOM is based on the TTRS method, by Clément et al., which is also based on kinematic analogies. In this method, the small displacements that are allowed in the tolerance zone can be described by a tolerance torsor or transformation matrix. Using the tolerance torsor or transformation matrix, tolerances are described as constraints. The small displacements that are still allowed by means of the torsor are referred to as micro–DOF's. For tolerance analysis, the torsor approach offers a mathematically correct description of tolerance zones, although a lot of equations are generated. These are reduced by applying a kind of degrees of freedom analysis considering both the macro–DOF's and the micro DOF's (tolerances).

Keywords: tolerance representation, tolerance specification, tolerance analysis.

1. Introduction

An introduction into some drawbacks of existing tolerancing standards is provided in section 1.1. The FROOM system, whose tolerancing module is the focus of the remainder of this paper, is introduced in section 1.2. Section 1.3 provides an overview of the remainder of the paper.

1.1 Some drawbacks of existing tolerancing standards

There are some problems related to present tolerancing standards. For instance, the standards do not provide for a theoretic tolerancing model that is consistent with today's 3D geometry models (usually solid models). This is due to the fact that when tolerancing standards emerged, no 3D solid models existed and a lot of freedom was left to the human interpretation of the specified tolerances. Therefore, tolerancing standards are drawing oriented, giving rise to ambiguities when having to be processed by computer. Another drawback of today's tolerancing standards is that they do not provide for a method for tolerance specification. Preferably, tolerance specification should be performed using a functional point of view in the first place: functional tolerancing.

Functional tolerancing will prevent redundant or unnecessary tolerances to be specified and will therefore support the design of manufacturable products at reasonable cost. The reason that the tolerancing standards do not provide for a method for functional tolerancing is mainly due to the standards being part oriented and not assembly oriented. It is the assembly where functioning aspects are (often implicitly) available and not in the individual parts. Therefore, current tolerancing standards can be compared to a language with a large and rich syntax but with not enough semantics and methods, at least to perform computer aided tolerancing. In [Ballu 93a,b], [Koplewicz 93] and [Srinivasan 93] more detailed overviews are provided regarding the insufficiencies in current tolerancing standards as well as current efforts to overcome these.

1.2 FROOM

FROOM is a prototype of a re–design support system, currently under development. The development of FROOM has been motivated by the following facts. First, for re–design tasks no proper computer support tools are available yet. Second, CAD tools that integrate well with automated process planning (CAPP) systems are only scarcely available. If available, they are most often not suitable in practice, especially in situations where design and manufacturing are performed in different companies. The CAPP system that is of particular interest to us, is the PART system [Houten 91], which has been developed in the authors' laboratory. FROOM is an acronym for Feature and Relation based Object Oriented Modelling.

FROOM is a feature based system, allowing the modelling of both components and assemblies. Features in the FROOM context can be design form features, manufacturing form features and even abstract features. For a review of feature–based design, refer to [Salomons 93a]. FROOM employs conceptual graphs for knowledge representation. The conceptual graphs allow re–design support to be provided and facilitate the link with CAPP [Salomons 94a]. The latter subject will not be addressed in this paper, however. Figure 1 shows Froom's system architecture.

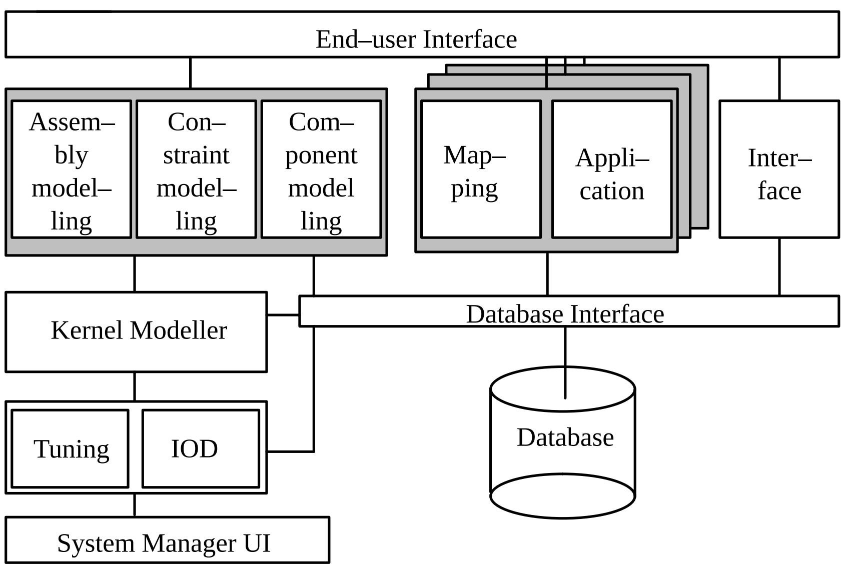


Figure 1 System architecture of FROOM; IOD means interactive object definition. The architecture does not include functionality for supporting collaborative engineering tasks.

Figure 1 shows that two different kinds of users are distinguished: end–users and system manager users. End–users are the designers who design assemblies, components etc.; they perform the actual design tasks. System managers customize the system to a certain application domain, company and user (group). They define the features to be used, the catalogues from which selections can be made etc.. For these two different user groups, separate user interfaces are available. In the modelling module of FROOM, components, assemblies and constraints can be modelled by the end–user. The tolerancing tool, which is the focus of this paper, is part of the constraint modelling module of FROOM. The modelling module has access to the kernel modeller, which offers the basic geometry processing functionality. In FROOM, the commercially available ACIS™ kernel modeller is used for this [Spatial 94]. The application module includes the possible mappings to applications and the applications themselves. An example of an application might be manufacturability evaluation. For more details on FROOM, refer to [Salomons 93b,c, 94a,b, 95].

1.3 Overview of the paper

An overview is provided of previous work in computer aided tolerancing in section 2. Based on this, the overall design of the FROOM tolerancing module is outlined in section 3. In section 4 the FROOM assembly modelling module is briefly examined as an important precondition for automatic tolerance specification (section 6). Tolerance representation in FROOM is addressed in section 5. Tolerance analysis is described in more detail in section 7. This section provides extensions to theories as developed by Rivière [Rivière 94] and Gaunet [Gaunet 94]. Apart from the overall synthesis as provided in this paper, these extensions are the main contributions of this paper. Finally, conclusions and recommendations are provided in section 8.

2. Review of previous research in computer aided tolerancing

Research in computer aided tolerancing is subdivided in tolerance representation (2.1), tolerance specification (2.2), tolerance analysis (2.3) and tolerance synthesis (2.4).

2.1 Tolerance representation

A lot of (possibly theoretically and mathematically correct) approaches for the computer representation of tolerances as proposed in literature do not sufficiently comply with the international standards. For human designers, these approaches will not be sufficient to replace the tolerancing standards as they are not expressive enough; their ”syntax and semantics” are too limited. Moreover, it will be hard to introduce a completely new and different tolerancing standard on a world wide basis even if it has enough expressiveness to humans as well as provide for a mathematically correct model for use in computer systems. Therefore, it seems more appropriate to look for a theoretical model of tolerancing, or tolerance representation scheme, that is in accordance with the international standards.

An early example of previous work in order to come to a mathematically correct model for tolerancing is the solids offset approach by Requicha [Requicha 83], extended in [Requicha 84, 86]. In this approach, nominal surfaces are given a pair of offset surfaces to determine the tolerance zones. This approach differs from the tolerancing standards [Farmer 86]. Because the individual pairs of offset surfaces are combined to obtain a composite tolerance zone of the entire solid, the individual tolerances cease to be independent constraints. Other early work has been described in [Johnson 85] and [Ranyak 88].

Jayaraman and Srinivasan introduced the notion of virtual boundary requirement in an attempt to redefine the notion of tolerances from the viewpoint of functional tolerances [Jayaraman 89], [Srinivasan 89]. Jayaraman and Srinivasan note that the real purpose of tolerances is to characterize functional requirements. Two functional constraints are cited that are believed to account for most tolerances; the maintenance of material bulk in critical locations and spatial relationships for assembly. These requirements can be captured as virtual boundary requirements. Virtual Boundary Requirements can be considered as a collection of virtual half spaces. The interpretation of datums in this approach is not in accordance with the standards.

Turner proposed a feasibility space approach [Turner 88, 93] which however does not seem to be suitable for 3D tolerance representation because of a too high complexity and doubtful usefulness in 3D.

Wirtz used a vectorial approach in which each tolerance is represented as a limit on the components of a vector that relates a given toleranced feature to a given reference feature [Wirtz 93]. The vectorial approach by Wirtz also seems insufficient: it is not close to the standards and is oriented too much towards older dimensioning and tolerancing practices. Form tolerances, for instance, are not accounted for.

Building on their earlier work in the field of computer aided inspection (e.g. [Bourdet 79]) and the work by Requicha and Jayaraman and Srinivasan and Wirtz, Clément et al. arrive at a tolerance representation model which is compatible with the standards and which seems to be theoretically and mathematically sound [Clément 91, 93, 94], [Desrochers 94]. Using the theory of the set of displacements by Hervé [Hervé 78], Clément et al. have proven that there are only seven elementary face types: spherical face, planar face, cylinder face, helical face, rotational face, prismatic face and ”any” face. When these 7 face types are combined, 28 cases of combination can

elementary surface	MGDE element	MGDE symbol
sphere	point	
plane	plane	
cylinder	line	
helical	point and line or line and plane	
rotational	point and line	
prismatic	line and plane	
any	point and line and plane	

Table 1 The MGDE associated with each elementary surface (after [Clément 91]).

be found. These combinations of faces are also called TTRS: Technologically and Topologically Related Surfaces. There is a finite number (≥ 44) of reclassifications of TTRS which denote the theoretical number of different tolerances (cases). On this basis, a computer system can automatically propose tolerance types employing an assembly model (geometric tolerances). Also reference (datum) elements can be determined (semi–)automatically. These elements are called MGDE: Minimum Geometric Datum Element; see table 1 for the MGDEs related to each elementary surface as discerned by Clément et al.. The same set of MGDE is used for determining the datum of composed TTRS. When two elementary surfaces are combined in a TTRS, the resulting TTRS can be classified into one of the seven basic classes depending on the types of surfaces involved and the geometric relations between them. See table 2 for the reclassifications of TTRS. In this approach, tolerances are represented vectorially, in so–called torsors, allowing for tolerance analysis. This has been addressed in [Gaunet 94] and [Rivière 94].

2.2 Tolerance specification

Tolerance specification is the activity of specifying tolerances; defining the tolerance types and related tolerance values (geometric tolerances are assumed). Tolerance specification is preferably carried out in conformance with the tolerancing standards (e.g. ISO 1101 [ISO 83], ANSI Y14.5M [ANSI 82]). However, the standards do not give a method of how tolerances should be specified. Therefore, research into the specification of tolerances seems to be required. Nevertheless, not a lot of research into this field has been performed. The main contributions are described in a number of publications by prof. A. Clément et al..

This approach to tolerance specification as presented in [Charles 89], [Clément 91, 93, 94] and [Dufossé 93] is an exception to most previous approaches towards tolerance specification. Clément et al. propose a method to determine *tolerance types* automatically from the assembly model, resulting in a truly *functional tolerance specification*. Most other approaches in computer

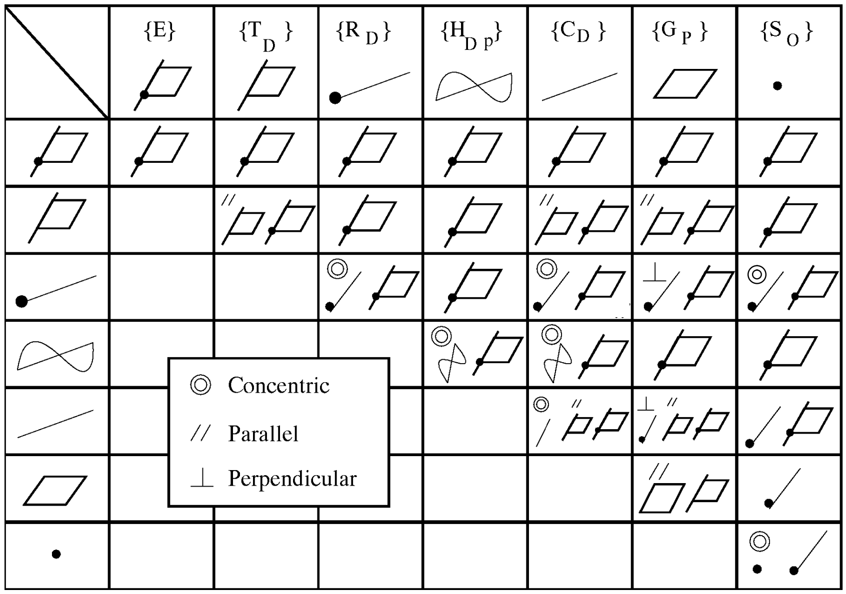


Table 2 Cases of tolerancing between surfaces, including the resulting MGDEs (redrawn after [Clément 91]). Note that the concentric relation in some cases – e.g. in case of the surface of revolution – sphere association – is not literally a concentric relation; it represents the intersection or coincidence of both MGDE.

aided tolerancing are based on manual tolerance specification, usually starting out from single components. The tolerance specification method by Clément et al. is carried out on the basis of face associations between the different components in the assembly, also called the mating function [Briard 89]. By finding kinematic loops (in the graph representing the assembly), faces are found on individual components which can be toleranced relative to one another [Dufossé 93].

2.3 Tolerance analysis

Tolerance analysis is a method to verify the proper functioning of the assembly after tolerances have been specified. Most often, tolerance analysis is performed by verifying two aspects:

- verifying assemblability of the assembly; the feasibility of assembly (fit).
- verifying if specified clearances between parts are still met; the quality of assembly (clearance).

Tolerance analysis can be carried out by determining the tolerance zones belonging to the specified tolerance types. Generally, two types of tolerance analysis are distinguished:

1. statistical tolerance analysis; in this case statistical methods are used together with accompanying probability distributions.
2. worst case analysis; the study of extreme cases (like for example MMC and LMC). Examples of previous research in this case are described in [Hillyard 78] and [Turner 88].

One of the first approaches in the field of tolerance analysis has been carried out by Bjørke in which tolerance chains in 2D mechanisms were used to calculate (maximum) clearance [Bjørke 78]. However, this work was mainly restricted to conventional plus/minus tolerances in the 2D case.

In tolerance analysis and tolerance synthesis, a mathematically correct model of tolerances is required to be able to compare the influence of different types of tolerances, and to verify how they propagate in 3D. In tolerance analysis literature most often the worst case approach is followed accompanied by some additional assumptions: form tolerances are often assumed to be negligible or to be present within position, orientation and size defects. Little is known towards actual 3D tolerance analysis. Gaunet seems to be on the right track, presenting a 3D tolerance analysis framework on the basis of the tolerancing method by Clément et al. [Gaunet 94]. However, only a 2.5 D tolerance analysis example is provided.

2.4 Tolerance synthesis

Tolerance synthesis is regarded as optimizing and completing the (functional) tolerance specification, taking into account manufacturing and inspection concerns. In this field relatively few research has been published. Most approaches that have been published are based on the optimization of a cost function. One of the most interesting methods seems to be the method by Nassef and ElMaraghy, using genetic algorithms [Nassef 93]. The general system of which this synthesis approach is a part is described in [ElMaraghy 93]. Simulated annealing has been employed for tolerance synthesis by Zhang et al. [Zhang 93].

3. Overall design of the tolerancing module in FROOM

A first design iteration of the design of the FROOM tolerancing functionality has been described in [Jonge Poerink 94]. Figure 2 shows the main function of a tolerancing module in a simplified way. Figure 2 also shows that besides the tolerance types and values, a lot of additional data can

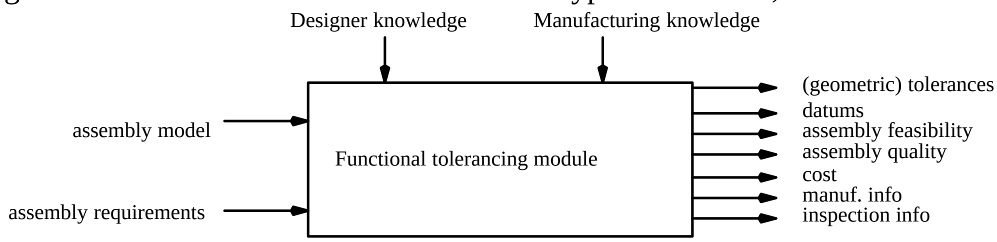


Figure 2 Main function of the tolerancing module.

be generated by a tolerancing module, e.g. on feasibility and quality of assembly, cost etc.. The

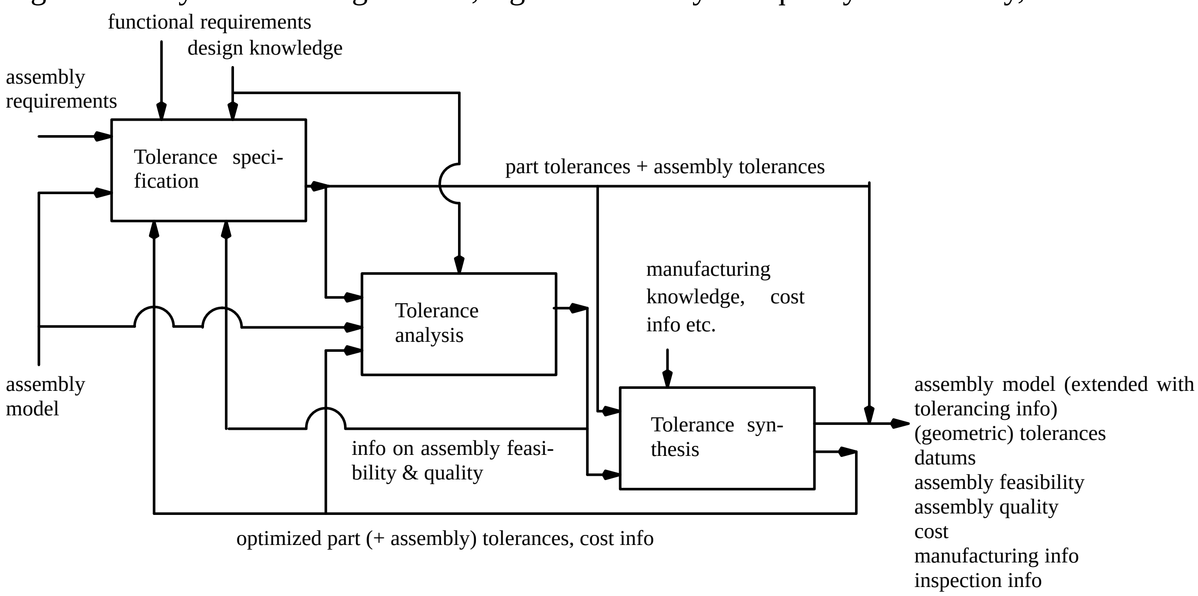


Figure 3 The main functions: tolerance specification, analysis and synthesis and their connections.

areas of tolerance specification, analysis and synthesis can be seen as functions that need to be present in a re–design support system (figure 3). The functional decomposition of figure 3, was decomposed further resulting in the following subfunctions: tolerance representation, tolerance specification, tolerance analysis, tolerance synthesis, tolerance presentation, storage and retrieval [Salomons 95]. For each of these functions a selection was made as to how to fulfil it:

1. For tolerance representation, i.e. the computer internal mathematical model of the tolerances, the vectorial torsor approach combined with the TTRS and MGDE concepts as described in e.g. [Clément 93, 94] and [Rivière 93] has been selected. The main reason for this was that most other methods were not in conformance with the tolerancing standards. Another reason for selecting the torsor approach is that it is based on a strong mathematic background in kinematics, e.g. [Hervé 78]. Using this mathematic background can offer great advantages in tolerance analysis and synthesis; see also section 5.
2. For tolerance specification, essentially the method as proposed by Clément et al. [Clément 91] was adopted, merely because there are no other methods for functional tolerance specification and because of the straightforward and logical nature of this approach. More details are provided in sections 4 and 6.
3. For tolerance analysis, the torsor approach offers a mathematically correct description of tolerance zones and can be used for tolerance analysis applications. Thus, the approach as described in [Gaunet 94] seems favorable. However, in Gaunet's approach to tolerance analysis, a lot of equations are generated that need to be solved. Although a matrix approach as described in [Rivière 94] reduces some of the redundant equations, a great number of equations remain [Jonge–Poerink 94]. Therefore, the sets of equations that are generated in the approach by Rivière or Gaunet may be reduced by applying a kind of degrees of freedom analysis considering both the macro–DOF's as studied by Kramer [Kramer 92] and the micro DOF's (tolerances) as described in [Clément 93, 94]. This is detailed further in section 7.
4. For tolerance synthesis, none of the methods as presented in literature seems to be appropriate. Almost all tolerance synthesis methods employ statistical techniques and a cost function in order to achieve tolerance optimization. In the cost function, for each machining operation, the relation between the cost versus tolerance (values) is expressed. However, during design it is often not known in advance which processes and/or machines are going to be used for manufacturing the individual parts. Therefore, it is difficult to derive a reliable cost function. One could therefore better try to achieve a generic indication of production cost. In this case, it is useful to employ general knowledge of the average cost of the manufacturing processes available in combination with a tolerance factor by which different types of tolerances can be unified (compared). Tolerance factors as proposed by Boerma [Boerma 90] may be used for this. Due to space limitations this is not discussed further; refer to [Salomons 95].
5. Tolerance presentation, i.e. presenting the tolerance information to the user, can be performed in a variety of ways. Tolerance presentation should be supported as much as possible according to the standards to avoid confusion. However, it will not always be possible to show specified tolerances or present results from tolerance analysis and synthesis according to the standards, especially in 3D views. Therefore, graphs and tables could be used in addition to a presentation method which conforms to the standards.
6. Storage of tolerance related information such as the tolerance types and values, results of tolerance analysis and synthesis, can be performed in a number of ways. However, as a relational database was already selected for other data storage functions in FROOM, the most appropriate way to store the tolerancing related information was in a relational database. The data structures needed can be based on the information needed in TTRS en MGDE elements.
7. Assembly/component information retrieval (including tolerancing information) can be performed by queries on the relational database.

4. Assembly modelling in FROOM

For assembly representation in FROOM, conceptual graphs are used as an internal representation format as well as for the external presentation to the end–user [Salomons 94a, 95]. The conceptual graph representing the assembly consists of the objects that make up the assembly and the connection relations between these objects.

Assembly constraints can be specified by means of symbolic constraints like: ”against”, ”align/fit”, ”contact”, ”orient” and ”parallel offset”. The implementation of these relations (assembly constraint satisfaction) is based on a slight modification of the work presented by Liu and Nnaji [Liu 91] and Kramer [Kramer 92]. This method of constraint satisfaction symbolically reasons about the geometry and based on this, a so–called plan fragment is generated. A plan fragment contains the actions that should be performed in terms of rotations and translations in order to satisfy the previously specified assembly constraint(s). This approach allows for automatically determining the kinematic degrees of freedom of each component and for automatically determining the functional surfaces which is important for tolerancing. More details on the assembly modelling module of FROOM can be found in [Salomons 94a, 95].

5. Tolerance representation in FROOM

In the following the concepts of TTRS, MGDE and torsor are elaborated further as these are central to the tolerance representation FROOM. Finally, the FROOM internal tolerance representation format containing TTRS, MGDE and torsors is discussed.

5.1 The applied TTRS concept

In tolerancing, surfaces are often associated two by two. The 7 elementary types of surfaces that were derived by Clément et al. can be associated two by two. The association of surfaces is central to the definition of TTRS (Technologically and Topologically Related Surfaces) as has been proposed by Clément et al. [Briard 89], [Clément 91]:

A TTRS is defined as an assembly formed by two surfaces (or between a surface and a TTRS or between two TTRS) belonging to the same solid (topological aspect) and located in the same kinematic loop in a given mechanism (technological aspect).

A somewhat different, more generic, definition of TTRS is provided by Rivière [Rivière 93]:

A TTRS is a pair of surfaces (or TTRS) belonging to the same solid which are associated because of functional reasons.

Clément et al. have made an extensive classification of all possible associations of the 7 elementary face types, and thus TTRS. One would expect 7 x 7 = 49 different types of association. However, only relative positions of two surfaces are of interest, so that for instance the association of cylindrical surface – prismatic surface is equivalent to the association prismatic surface – cylindrical surface. Therefore, only 28 cases of surface association remain (i.e. 7 + 6 + 5 + 4 + 3 + 2 + 1 cases or one half of a 7 x 7 matrix including the diagonal); see table 2. Although surface associations can be regarded as ”symmetric”, tolerances cannot. This is due to the fact that there is both a reference and a toleranced face. Therefore, some tolerancing cases which are ”below” the diagonal in table 2, and shown blank, should also be regarded [Clément 94].

When two elementary surfaces are combined in a TTRS, the resulting TTRS can be classified into one of the seven basic classes depending on the types of surfaces involved and the geometric relations between them. For instance, if two cylindrical surfaces are associated in a TTRS and if their axes are coaxial, the resulting TTRS is classified as a cylindrical surface. This is the case because both cylindrical surfaces remain invariant to rotations around their common axis and translations along it. Another example could be the association of two parallel cylindrical faces; these are only invariant for translations along the direction of the (parallel) axes and therefore the association results in a prismatic TTRS. A systematic analysis of all possible cases of object reclassification has revealed (at least) 44 cases of tolerancing [Rivière 93], [Gaunet 94]. This is shown in table 2. It must be noted here that the number of 44 was obtained by counting the number of cases in table 2 and could grow if more special cases and cases below the diagonal are considered. Therefore, the actual number of the tolerancing cases is less important than the fact that there is only a finite number of tolerancing cases.

5.2 The applied MGDE concept

The Minimum Geometric Datum Element, or MGDE, of a TTRS is the minimum set of points, lines or planes necessary and sufficient to define the reference frame corresponding to the invariant sub–group of that TTRS. The concept of MGDE also has been proposed by Clément et al. [Clément 91, 93]. The MGDE remains invariant for the displacement it is defining. The MGDE is a set of a reference point, reference line and a reference plane, but not all these elements are always necessary to define the MGDE sufficiently. The MGDE for a cylinder, for example, is the axis of the cylinder (i.e. the MGDE consists of a reference line only). Table 1 shows for each elementary surface its associated MGDE as well as the symbol used for the MGDE.

5.3 The applied torsor concept

The torsor concept has originated in the field of metrology, e.g. [Bourdet 79]. For every point M in Euclidian space, its small displacements can be described by two vectors: $\vec{D}_M$ and $\vec{\theta}$. The $\vec{D}_M$ vector represents 3 translations along perpendicular directions, two by two. The $\vec{\theta}$ vector corresponds to three small rotations around perpendicular directions, two by two. These two vectors define a so–called small displacement screw model or torsor $T_{M,\theta}$ so that [Rivière 93]:

$$\vec{\theta} = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} \quad \vec{D}_M = \begin{bmatrix} u \\ v \\ w \end{bmatrix} \qquad T_{M,\theta} = \begin{bmatrix} \vec{D}_M \\ \vec{\theta} \end{bmatrix} = \begin{bmatrix} u \\ v \\ w \\ \alpha \\ \beta \\ \gamma \end{bmatrix}$$

For each tolerance related to a TTRS, the tolerance zone can be represented as a tolerance torsor, which represents the small displacements that are possible within the tolerance zone. In general the torsor contains three translation parameters and three rotation parameters. However, as the elementary surfaces, and thus the TTRS, usually have some invariances, often some of the torsor parameters reduce to 0. For instance, a cylinder has two invariances; one translation along the axis (x direction) and and a rotation around the axis (x direction). Therefore, a cylindrical tolerance zone (figure 4) obtains the same invariances:

$$T_{general} = \begin{bmatrix} D_M \\ \theta \end{bmatrix} = \begin{bmatrix} u \\ v \\ w \\ \alpha \\ \beta \\ \gamma \end{bmatrix}$$

$$T_{cyl} = \begin{bmatrix} D_{cyl} \\ \theta \end{bmatrix} = \begin{bmatrix} 0 \\ v \\ w \\ 0 \\ \beta \\ \gamma \end{bmatrix}$$

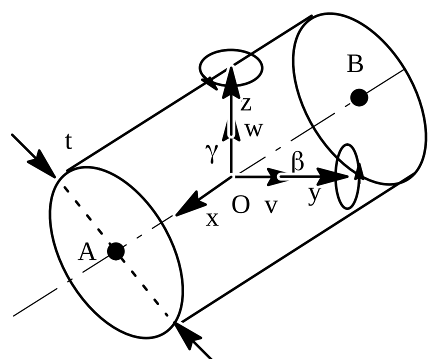


Figure 4 Cylindrical tolerance zone (redrawn after [Gaunet 94]).

Instead of torsors, matrices can be used for representing small displacements [Rivière 94]. Let us again consider a cylindrical tolerance zone of diameter t. The toleranced element is the straight segment AB which could for instance correspond to the axis of a cylinder surface (figure 4). The coordinate system is again defined in the point O in the center of AB and the x direction is the direction from B to A. Now suppose we are considering a position tolerance of the cylinder surface with respect to a complete reference system. The group of displacements that does not leave the segment AB globally invariant can be represented by the transformation matrix D(v,w, β, γ), expressed in the coordinate system (O, x,y,z) [Rivière 94]:

$$D(v, w, \beta, \gamma) = \begin{bmatrix} C\gamma C\beta & -S\gamma & C\gamma S\beta & 0 \\ S\gamma C\beta & C\gamma & S\gamma S\beta & v \\ -S\beta & 0 & C\beta & w \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \text{(Eq. 1)}$$

As the segment AB remains in the cylindrical tolerance zone, the following constraints hold [Rivière 94]:

$$\sqrt{{Y_{dA}}^2 + {Z_{dA}}^2} \le \frac{t}{2} \qquad \text{with} \quad \begin{bmatrix} X_{dA} \\ Y_{dA} \\ Z_{dA} \end{bmatrix} = D * A$$

$$\sqrt{{Y_{dB}}^2 + {Z_{dB}}^2} \le \frac{t}{2} \qquad \text{with} \quad \begin{bmatrix} X_{dB} \\ Y_{dB} \\ Z_{dB} \end{bmatrix} = D * B \qquad \begin{matrix} 0 \le \beta \le \pi \\ 0 \le \gamma \le \pi \end{matrix}$$

Note that the range of the angles β and γ can be restricted further to: $0 \le \beta \le \frac{t}{2.OA}$ and $0 \le \gamma \le \frac{t}{2.OA}$.

5.4 Tolerance representation implementation in FROOM

The data structure in which both TTRS, MGDE and torsor matrix are stored in FROOM is shown in figure 5. This structure is stored in the Oracle™ database. How TTRS, MGDEs and torsors are determined, is elaborated as part of the tolerance specification functionality.

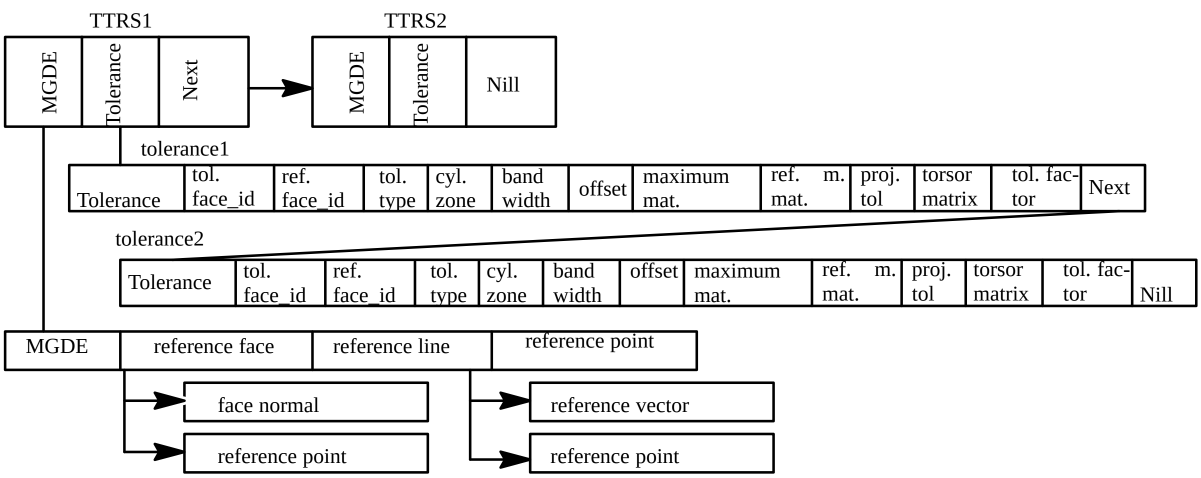


Figure 5 Data structure for TTRS, MGDE and torsors in FROOM.

6. Tolerance specification in FROOM

Tolerance specification includes specifying the tolerance types and values for nominal geometry parts. In current tolerancing practice, designers have to manually specify both tolerance types and values either on a drawing or in a CAD system. The tolerance specification in this case depends largely on the designer’s judgement and experience. Therefore, different designers will possibly arrive at different tolerance specifications for the same nominal geometry. Thus, a true functional tolerance specification is not guaranteed. Functional tolerance specification can be performed semi–automatically when an assembly model is present, assuming that the assembly model contains (implicit) functional information [Clément 91]. Clément et al. have presented a method to detect functional surfaces in an assembly graph. Having determined the functional surfaces in an assembly, the functional surfaces on each component are associated; TTRS are built. Also, the datum elements (MGDE) are automatically created, together with the creation of the TTRS. The type of association indicates the type of tolerance that, from a functional point of view, needs to be specified. The user has to determine the tolerance value. In the following we will elaborate on the theory by which the functional surfaces (TTRS) and datum elements (MGDE) can be determined (section 6.1). Then, implementation in FROOM is addressed (section 6.2).

6.1 Applied tolerance specification theory

Suppose we have an assembly graph showing the components, the assembly relations like ”against” and ”align/fits” as well as the surfaces of the components involved in these mating constraints. These surfaces are considered as functional surfaces. The functional surfaces and the assembly relations give rise to ”kinematic” loops in the assembly graph. The kinematic loops in the assembly graph in a way overconstrain the nominal positions of the components in the loop (when expressing the positions in terms of algebraic equations). Tolerancing the functional surfaces involved in the loop is a way to relax the overconstraints [Dufossé 93]. Figure 6 gives an example; the parts are represented by circles while the mating relations are expressed as arcs and functional surfaces are shown as partial circles that are placed on their component circles. If part I is positioned relative to part II, it can easily be observed that part I’s x–position is overconstrained by the two align/fit relations. If the holes in part II would have had bottom faces, part I’s y–position would also have been overconstrained. Different arc types denote different assem–

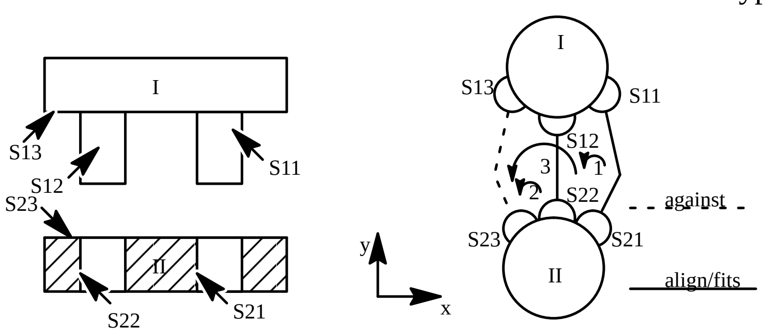


Figure 6 Loops between functional surfaces in an assembly graph.

bly relations and thus different directions in which tolerances work. Note that in figure 6 there is only one loop working in the same direction. Thus, the faces S11 and S12 on part I have to be associated as well as S22 and S21 on part II. Table 2 is used for determining the type of the resulting TTRS. Now on each part, one functional face remains. As the remaining faces are functional, they also should be toleranced. This can be done by relating the remaining surfaces to the previously built TTRS and/or by tolerancing the respective MGDE’s [Desrochers 94].

The shortest kinematic loops one can detect are those between two parts. Two different types of kinematic loops are then possible [Jonge Poerink 94]:

1. loops consisting of two surfaces on each part.
2. loops consisting of one surface on one part and two on the other.

The latter case can provide some problems, however. If a loop consisting of one surface on one part and two on the other is the first loop considered for the part with one functional face in the loop, a TTRS cannot be built. The surface can only be toleranced by a form tolerance. Therefore, these types of loops should be avoided as starting loops.

In the assembly graph loops can easily be detected using standard graph theory like e.g. [Even 79] and [Noltemeier 76]. A kinematic loop in the assembly graph always consists of more than one part and no more than two surfaces on each part in the loop. There can be more than one loop between several parts and one surface can participate in several loops. As there can be more than one loop, the order in which surfaces are to be associated is not trivial. In fact, it depends on the order in which the loops are processed. Also, loops should be independent; they may not be linear combinations of others. Therefore, once the loops have been detected, the following still has to be resolved:

- determining the starting loop.
- determining the sequence in which the loops have to be processed.

Clément et al. have proposed the following criteria, that are mainly related to technological reasons, for the detection of the starting loop [Clément 91], [ISMCM]:

- one dimensional loops passing through contacts having their normal vectors pointing in opposite directions (tolerance chain).
- shortest independent loops; i.e. loops involving the least number of parts.
- choosing loops generating the TTRS that were designated using dimensioning assistance or through technical functions.
- choosing loops passing through surfaces designated as references (MGDE).

The above rules apply to one direction in the assembly graph. For all directions in the graph, all loops in the same direction have to be found and based on the above rules, the loops have to be sorted. By doing this, TTRS trees per direction and per component can be constructed. If these TTRS, possibly in combination with remaining functional faces, are unrelated, they can be associated on their turn. This can be done by considering the previously built TTRS and shortest loops containing multiple directions as well as MGDE. In this way, a sensible dimensioning of every subassembly is allowed, as well as the generation of the shortest tolerancing chain. However, the previous criteria may not always be sufficient, and therefore users should be enabled to select the order in which loops are to be processed [Clément 91]. Jonge Poerink added some additional criteria for loop selection [Jonge Poerink 94]:

- Avoid loops that contain only one surface on a part.
- Choose loops containing the simplest basic geometry.

Automatic tolerance value specification is not examined. Checking for coherence and completeness of specified tolerances is also not addressed, although these can be considered as part of tolerance specification.

6.2 Tolerance specification as implemented in FROOM

Based upon the theory as presented above, FROOM is able to generate tolerance types automatically based on a conceptual graph representation of an assembly model. The rules for loop detection, constructing MGDE, TTRS etc. are implemented using C++. Figure 7 provides an example of tolerance specification of the gearpump which will be addressed later for explaining the concepts regarding tolerance analysis. Apart from an automatic tolerance specification, a manual tolerance specification is desired; not all tolerances should by definition be of functional importance. The manual tolerance specification in FROOM is currently under development.

7. Tolerance analysis in FROOM

This section only summarizes the theory of tolerance analysis as is to be applied in FROOM; it provides an extension to the theories on tolerance analysis by Gaunet [Gaunet 94] and Rivière [Rivière 94]. The implementation is currently under development and has not been completed;

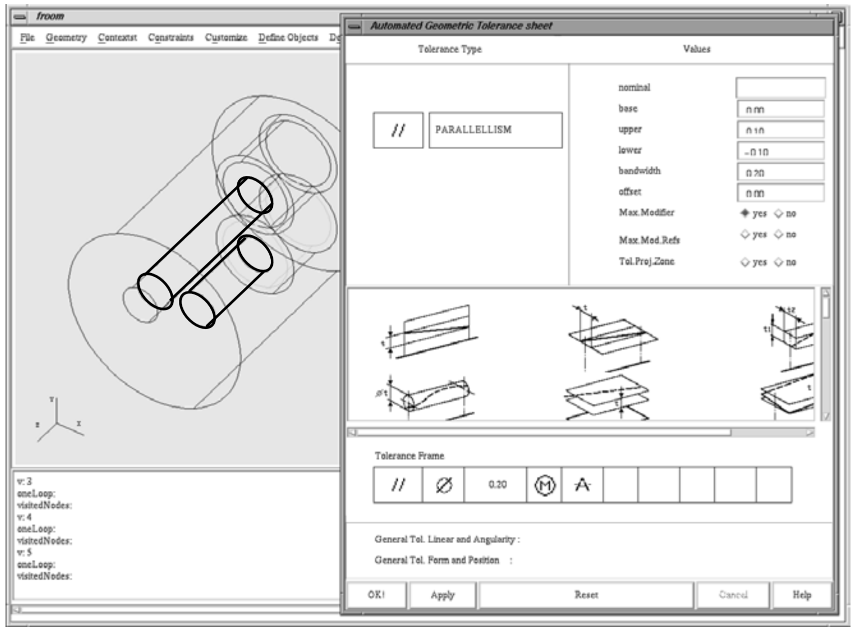


Figure 7 The system proposes a parallelism tolerance between the two gear holes.

therefore it is not discussed. A distinction is made between analyzing the feasibility of assembly (7.1) and the quality of assembly (7.2).

7.1 Analyzing the feasibility of assembly

Feasibility of assembly can be considered by checking face associations two by two, without considering the other faces on a part or in the remainder of the mechanism. Tolerance torsors can be used to calculate softgages for each surface. Softgages are the nominal geometry combined with their virtual tolerance zones. Softgages can be calculated for most material condition and least material condition. Using MMC softgages, a check for feasibility of assembly can be performed.

For each point on a surface, its virtual displacement can be calculated using the torsor related to that surface (association). As there can be infinitely many points on the surface, usually a restricted number of discrete points is chosen which are used to construct the softgage. The more points are chosen, the better the approximation of the softgage. The comparison of the softgages of two different surfaces then results in insight into the feasibility of assembly; interference or not. If there is interference, either the nominal dimension values or the tolerance values can be changed, in order to achieve non–interference.

7.2 Analyzing the quality of assembly

As in both the torsor approach as presented by Gaunet [Gaunet 94] and the matrix approach by Rivière et al. [Rivière 94], a great number of constraints and parameters appear, it seems of great interest to reduce the number of constraints and parameters before trying to solve the constraint set. Also, in the currently presented tolerance analysis approaches it is difficult to find the directions in each ”joint” which influence the most the clearance one is interested in. In fact, this seems to be the main obstacle in applying the torsor approach to 3D tolerance analysis. Including an analogy to degrees of freedom analysis as proposed by Kramer seems to be a possible solution to both problems; by determining the relevant directions for each joint in which calculations should be made this could reduce the constraint/parameter set. To illustrate this idea, an example of a gearpump, similar to [Farmer 86], [Gaunet 94] and [Rivière 94] is presented. The example includes TTRS and MGDE construction.

7.2.1 A simple example

The pump considered is an assembly of a housing and two shafts; the cover and the bolts fitting the cover on the housing are not considered. The shafts are considered to contain the gears. Figure 8 shows the pump with only some size and fit tolerances.

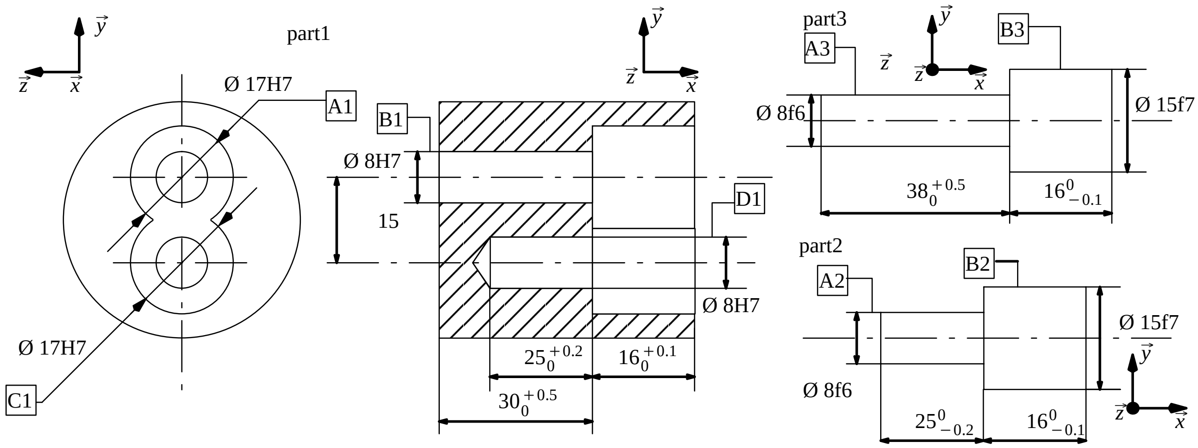


Figure 8 The pump (dimensions and size and fit tolerances after [Rivière 94]).

Tolerance specification for the gearpump example

As has been explained previously, tolerance types can be determined by detecting kinematic loops from the assembly graph. Figure 9 shows the assembly graph and the loops in the case of the pump. The faces A1, B1, C1 and D1 are surfaces of part 1 (the housing), faces A2 and B2

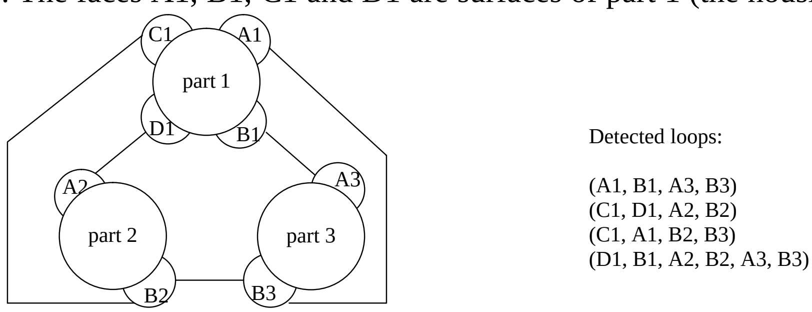


Figure 9 The assembly graph of the pump mechanism and the loops that can be detected.

are surfaces of part 2 (the small shaft) while faces A3 and B3 are surfaces on part 3 (the large shaft).

According to the two most important criteria for loop sequencing, i.e. shortest loops and one dimensional loops, the loops (A1, B1, A3, B3), (C1, D1, A2, B2) and (A1, C1, B2, B3) would be the first to be processed. However, the loop (A1, C1, B2, B3) is dependent on the other two shortest loops. Therefore it is a dependent loop and will not be considered. As the two remaining loops are of equal size and contain cylindrical surfaces, either loop can be used as a starting loop for building the TTRS. Consequently, the loop that is not chosen as the first, has to be used as second. Suppose we choose loop (C1, D1, A2, B2) as the first loop. Then loop (A1, B1, A3, B3) will be the second loop. The remaining loop is (D1, B1, A2, B2, A3, B3). According to the main loop criteria, the sequence would then be:

1. (C1, D1, A2, B2)
2. (A1, B1, A3, B3)
3. (A1, C1, B2, B3) (not considered; dependent loop)
4. (D1, B1, A2, B2, A3, B3)

The first loop contains 4 surfaces: surface C1 and D1 on part 1 and surface A2 and B2 on part 2. Therefore, the first TTRS of part 1 contains surfaces C1 and D1. According to table 2, the resulting class is a cylinder (C1 and D1 are both cylindrical surfaces and are concentric). According to table 2, the MGDE for this TTRS can be the axis (of surface D1 or C1). Surfaces A2 and B2 on part 2 also result in a cylindrical TTRS as they are concentric cylinders as well. The second loop can be processed similar to the first. The third loop contains only faces that are already used in previous TTRS and none of these faces is a reference face or MGDE. Therefore, the third loop is a dependent loop and not relevant to building a TTRS. In the last loop, all faces have been used in previous TTRS, but the axes of D1 and B1 are MGDEs. Also, because of the link B2–B3, it is an independent loop. Therefore, faces D1 and B1 on part 1 are related; the resulting TTRS is prismatic. According to table 2, the resulting MGDE is a combination of a line and a planar face. The resulting TTRS trees that can be generated from the loops of the functional faces of the parts of pump are shown in figure 10.

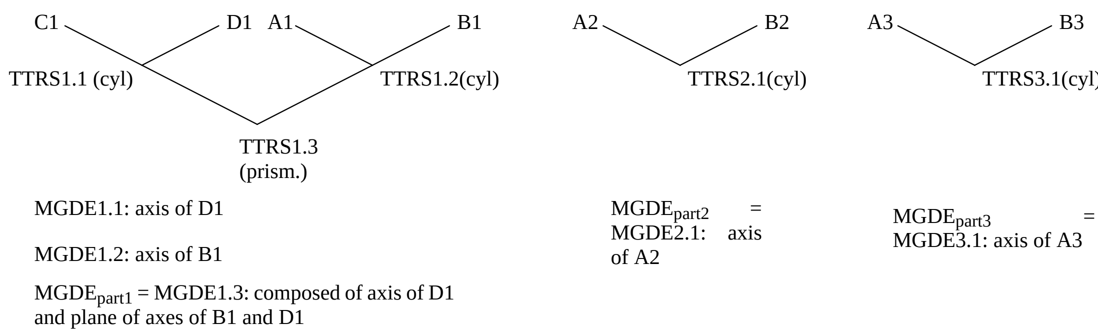


Figure 10 The resulting TTRS trees for each part of the pump.

Based on the TTRS trees that are obtained, geometric tolerance types can be proposed automatically by the system. For TTRS1.1 this results in a concentricity tolerance between face C1 and D1 and a cylindricity tolerance on each individual face. TTRS1.2 is similar to TTRS1.1. TTRS2.1 and TTRS 3.1 are also similar to TTRS1.1. For TTRS 1.3 it is necessary to tolerance D1 and B1. As the faces D1 and B1 are nominally two parallel cylindrical faces, a parallelism and a position tolerance are applied to both surfaces. The proposed geometric tolerances are shown in figure 11. The arrows point from reference to toleranced face. Note that for reasons of

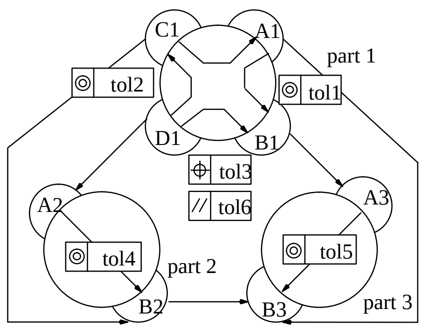


Figure 11 The geometric tolerance types that can automatically be proposed.

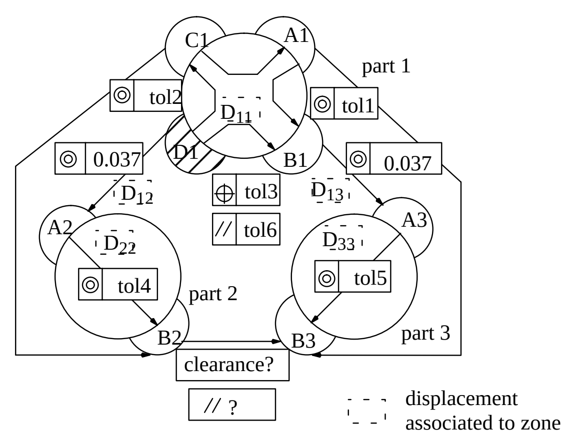


Figure 12 The assembly graph with both the inter–piece contacts and the inner piece TTRS symbolized as tolerances and displacements (slightly adapted from [Rivière 94]).

simplicity, unlike the example of figure 6, all tolerance directions are the same. If, for instance we would have associated the planar face of the bottom of holes A1 and C1 with the corresponding faces of the gears, different directions would have been introduced. Also, if the cover would have been included in the example, this would have been the case.

Tolerance analysis in the gearpump example

Suppose we are interested in the clearance between the two gears: between face B2 and face B3. Figure 12 shows the assembly graph once more; the clearances are now expressed as "tolerances" in the graph. The clearance of interest is part of two of the previously determined loops. Normally, one would have to calculate the clearances resulting from both loops independently. From this, the clearance that is most significant should then be determined. This criterion has not been identified in previous literature. In our example, however, we will only consider the loop (D1, B1, A2, B2, A3, B3), as the other loop has been shown to be dependent. Note that in this case, the tolerance analysis problem is primarily a 2D problem due to the simplified model. In figure 12, D1 has been hatched to indicate that the displacements of surfaces in the loop influencing the clearance are calculated relative to D1. The situation has been outlined in figure 13.

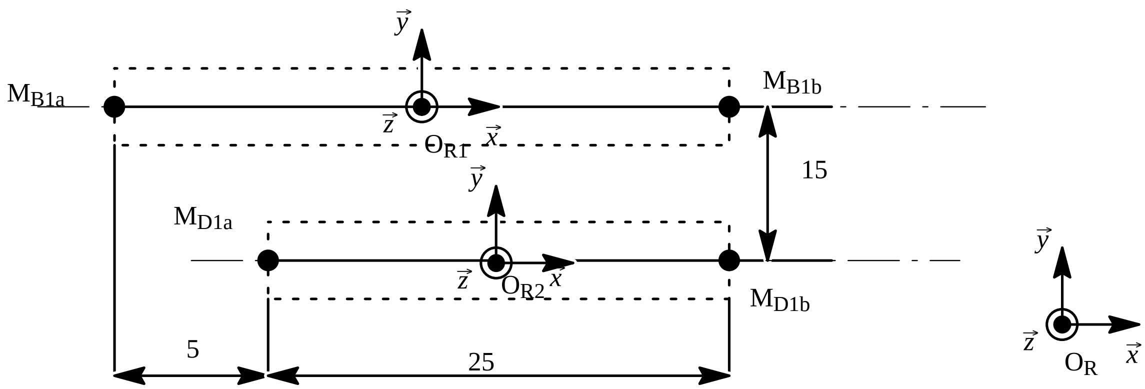


Figure 13 Tolerance zones between surface B1 and D1 (redrawn after [Rivière 94]).

We will now consider the constraints involved in the displacement of B1 relative to O_{R1}. Suppose R is the mechanism coordinate frame and R1 and R2 are those of surfaces B1 and D1. If $P_{R \to R1}$ is the transformation matrix of coordinate frame R to frame R1, then [Rivière 94]:

$$\begin{bmatrix} xM \\ yM \\ zM \\ 1 \end{bmatrix}_{(O_1,\vec{x}_1,\vec{y}_1,\vec{z}_1)} = \left[P_{R \to R_1}\right] * \begin{bmatrix} xM \\ yM \\ zM \\ 1 \end{bmatrix}_{(O,\vec{x},\vec{y},\vec{z})}$$

The matrix D_{11} representing the tolerance zone of surface B1 with respect to O_{R1} can be written as in equation (Eq. 1). The vector of displacement MM' of a point M in the tolerance zone to point M' expressed in R_1 is now [Rivière 94]:

$$\left[MM'\right]_{R_1} = \left[D - I\right] * \left[P_{R \to R_1}\right] * \left[M\right]_R$$

The constraints to be fulfilled at the extremes M_A and M_B of the cylindrical tolerance zone are [Rivière 94]:

$$\left[M_{B1a}\vec{M'}_{B1a}\right]_{R_1} . \vec{y}_1 \le \frac{tol3}{2} \qquad \left[M_{B1b}\vec{M'}_{B1b}\right]_{R_1} . \vec{y}_1 \le \frac{tol3}{2} \qquad 0 \le \gamma \le \pi$$

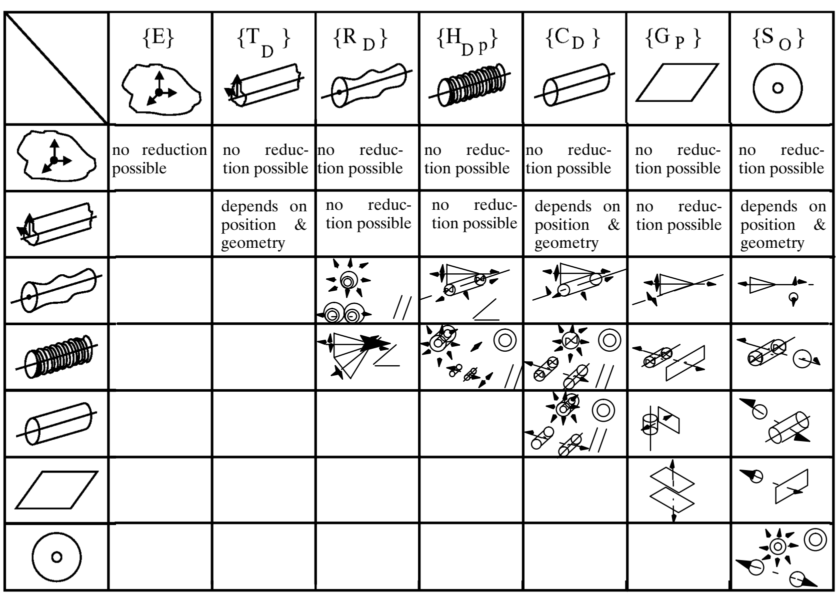
The other constraints for each tolerance are derived similarly. Using these constraints, the minimum/maximum clearances can be calculated. In the example of the pump [Rivière 94], human interpretation easily concludes that only the y–direction is of importance for clearance calculations. However, a computer needs generic methods which also help out in more complex cases.

In the following paragraph, this is detailed further and can be seen as an extension to the theory by Rivière [Rivière 94].

7.2.2 An extension to Rivière's method for 3D tolerance analysis

In the pump example, when calculating the clearances, two halves of the kinematic loop are "walked through" starting from a reference face (face D1). In the case where one aims at reducing the constraint set, it is preferable to start out at the faces that one is interested in (i.e. the faces forming the clearance or orientation). Based on the face types involved, it will then be possible to determine a kind of virtual plan fragment, analogous to the plan fragments by Kramer [Kramer 92]. By virtual is meant that only directions are considered along which translations (in case of clearance) or rotations (in case of orientations) can be carried out. Thus, no values are given to these virtual plan fragments; as a matter of fact these are the values one is finally interested in as they determine the clearance or orientation one would like to know. The two components participating in the clearance or orientation are called primary components from now on. Once a virtual plan fragment has been calculated for each primary component, i.e. a direction in which clearance or orientation are maximum, the two halves of the kinematic loop can be traversed back to the reference face. During this process, constraints can be reduced by reasoning on the macro–DOF's (actual kinematic degrees of freedom; large displacements) in relation to the micro–DOF's (tolerances).

In the case of our pump we are interested in the clearance between the two gears (part 2 and 3). These gears are parallel cylindrical surfaces (simplified geometry), and have each one rotational macro–DOF around axes that are parallel. By means of a kind of plan fragment table we may be able to find the virtual plan fragment (direction in which the clearance is maximum). In fact, the plan fragment table should contain all possible cases of the face associations as mentioned in table 2. However, the faces that are associated must be part of different solids now. Table 3 shows a preliminary version of such a virtual plan fragment table for clearances. Table 4 illustrates some



	{E}	{T$_D$}	{R$_D$}	{H$_{Dp}$}	{C$_D$}	{G$_P$}	{S$_O$}
	no reduction possible	no reduction possible	no reduction possible	no reduction possible	no reduction possible	no reduction possible	no reduction possible
		depends on position & geometry	no reduction possible	no reduction possible	depends on position & geometry	no reduction possible	depends on position & geometry

Table 3 Preliminary virtual plan fragment table for (maximum) clearances.

of the cases of table 3 in some more detail. In the case of two parallel cylinders (that do not share their axes), the virtual pan fragment can be determined by calculating the direction of the line perpendicular to the two axes (or MGDE): see table 4.

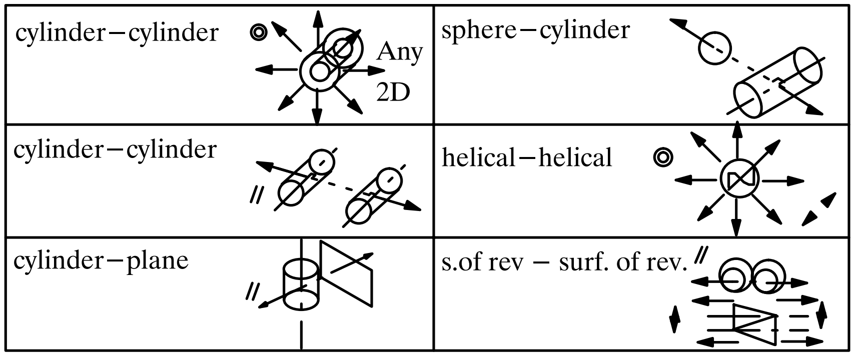


Table 4 Some cases of table 4 in more detail.

Now, for the directions of D_{22} and D_{33} that are internal to parts 2 and 3 respectively and that form part of the two halves of the kinematic loop, only the direction determined by the virtual plan fragment is important. The same holds for the directions of D_{12} and D_{13}, which represent the clearances between parts 2 and 1 and parts 3 and 1. In fact, in these latter clearance zones, the virtual plan fragments can be executed once the values of the translations (or rotations) are known in order to find the maximum clearance. As we now arrived on the base component, i.e. the component with 0 macro–DOF's, no new virtual plan fragments are needed. It is sufficient to observe D_{11} in the required direction. Thus, using an algorithmic approach, we can arrive at the same set of equations as those that were manually derived in [Rivière 94]. In this example, however, no directions had to be taken into account for a certain tolerance zone in the chain for which that tolerance zone is invariant, but for which the clearance one is interested in is not invariant. Especially in these cases reasoning on the macro–DOF's is necessary.

Although only one simple example was shown in which a kind of DOF analysis has been used to reduce the sets of equations and to determine directions in which to calculate, this seems to open the way for true 3–D tolerance analysis.

8. Conclusions and recommendations

A tolerancing tool has been presented which allows for functional tolerance specification. Tolerance specification is based on the method by Clément. This method has been verified by the current research and shown to be a "sound" method although further evaluation in practice seems desirable. Tolerance representation is based on the TTRS concept. For tolerance analysis, an extension has been presented to the methods by Gaunet and Rivière. The extension for tolerance analysis combines the kinematics approach for small displacements (tolerances) with that of large displacements. However, these extensions have not yet been implemented and verified. In the future this should be done. More effort in the field of checking completeness and coherence of tolerance specifications seems required as well. Also, tolerance synthesis should be developed in combination with the presented tolerance specification and analysis methods.

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