Abstract

Micro-electro-mechanical systems (MEMS) are often designed on scales at which electrostatic forces are capable of moving or deforming the parts of the system. In this regime accurate prediction of device behavior may require 3D coupled simulations between the electrostatic and mechanical domains.

We have recently developed CoSolve-EM, a coupled solver for 3D quasi-static electro-mechanics. In this paper, we demonstrate the application of CoSolve-EM to five classes of electro-mechanical problems that are often intractable to other techniques. These classes are: devices with electrostatic pull-in instabilities, devices in which precise deformations are required, devices driven by multiple conductors, capacitive sensors that make use of surface contact, and actuators that make use of surface contact.

Introduction

Previous approaches to this problem are: direct coupling of finite elements (FE) to a parallel plate approximation for the electrostatic force [1]; direct coupling of full FE solvers in both domains [2]; and external, boundary-based coupling between a mechanical FE solver and an electrostatic boundary element (BE) solver, using either relaxation or SNGCR techniques for convergence [3]. CoSolve-EM implements the last of these approaches.

General Coupled Electro-Mechanics

When designing electro-mechanical actuators one typically applies a voltage to the undeformed device, as in Figure 1A. This induces charges on the surface of the conductors in the problem, and those charges induce surface normal pressures over the device. We refer to these pressures as the “electrostatic load” which may be calculated using Equation 1, in which \( p \) is the normal outward pressure on a conductor, \( \sigma \) is the charge density at that point, and \( \varepsilon \) is the dielectric constant of the material outside the conductor.

The electrostatic load causes the device to deform. In general such deformation will lead to reorganization of all surface charges (and thus pressures) on the device, as illustrated in Figure 1B.

\[
\rho = \frac{\sigma^2}{2\varepsilon}
\]

We are principally interested in that subset of MEMS devices in which this reorganization of charge is large enough to cause further deformation. We consider such devices to exhibit “coupled electro-mechanical behavior.” In order to model them we must find self-consistent solutions in which the electrostatic loads are exactly balanced by the stresses of the solid deformation.

CoSolve-EM Software

CoSolve-EM is a coupled solver for Electro-Mechanics. It finds the self-consistent solution to the problem above using an external, boundary-based coupling between a mechanical FE solver and an electrostatic BE solver, implementing both relaxation and SNGCR techniques for convergence [3].

The two external solvers employed by CoSolve-EM are the commercial mechanical FE package ABAQUS [4], and a version of the electrostatic multipole-accelerated BE solver FASTCAP [5]. CoSolve-EM also includes a 3D visualizer, Geomview, from the University of Minnesota [6]. The CoSolve core is written in C++, with its user interface in Tk/tcl [7]. CoSolve-EM makes use of MemBase, our class library for doing general...
manipulations of meshed 3D models. MemBase
gives CoSolve-EM the ability to break models up,
add them together, extract surfaces, analyze
structures for connectivity, and find all external and
internal boundary regions. Using CoSolve-EM we
can take one meshed model of a device, represented
either in PATRAN Neutral File format or in I-DEAS
Universal File format, and do all further electrostatic,
mechanical or electro-mechanical modeling from
the CoSolve-EM interface. CoSolve-EM is also
useful for modeling general capacitance sensors,
even those without significant electro-mechanical
coupling (see the touch-down pressure sensor
example below).

CoSolve-EM is built to use a heterogeneous cluster
of workstations. In our current system the CoSolve-
EM core runs on a Sparc10 while the solvers
ABAQUS and FASTCAP are run on a DEC
3000/800 (alpha CPU with OSF/1). We also have
installed CoSolve-EM with ABAQUS and FASTCAP
on a single Sparc10.

The Examples
We now present five examples of the use of CoSolve-
EM. In each case, only one meshed model is
constructed, and only one run of CoSolve-EM is
required. The five are: a torsion mirror, a
deformable plate, a model of comb levitation, a
touch-down capacitive pressure sensor, and a curved
electrode actuator. We illustrate different aspects of
electro-mechanical problems with each example, and
also demonstrate different types of analysis that can
be performed with CoSolve-EM.

CoSolve-EM implements two different algorithms
for finding coupled electro-mechanical solutions,
relaxation and SNGCR. Each has its merits.
Relaxation is simple and often fast in practical
examples, but can fail to converge for some
problems. SNGCR is more reliable, but is more
computationally expensive, and in our current
implementation may not be used in problems
containing surface contact.

Both the torsion mirror and the deformable plate are
usually solvable with relaxation. The comb
levitation problem generally requires (as do all
levitation problems) the use of SNGCR. The touch-
down pressure sensor, while not strictly a coupled
problem, is easy to treat with CoSolve-EM. The last
element, a curved electrode actuator, is a contact
problem and thus must be handled by relaxation.
However, the problem also contains some levitation,
and would be more accurately solved by SNGCR.

This problem is discussed further in the section on
the curved electrode actuator.

In Table 1 we summarize the sizes and some timing
results for the five examples of this paper. We use
the number of surface panels to indicate the size of
each problem, and the time is the real elapsed time
the user observes for one correct self-consistent
solution. These times are not very well defined,
because, even within one model, the difficulty in
getting a converged solution actually can vary by a
factor of 5 or more. We observe a great variation in
elapsed time vs. voltage on a single problem if it has
a pull-in instability or a touch-down. We are
currently working on better techniques for
benchmarking coupled electro-mechanics.

<table>
<thead>
<tr>
<th>Example</th>
<th>Size</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Torsion Mirror</td>
<td>2830</td>
<td>1750</td>
</tr>
<tr>
<td>Deformable Plate</td>
<td>1580</td>
<td>800</td>
</tr>
<tr>
<td>Comb Levitation</td>
<td>1837</td>
<td>1500</td>
</tr>
<tr>
<td>Touch-Down P Sensor</td>
<td>3728</td>
<td>600*</td>
</tr>
<tr>
<td>Curved Electrode Actuator</td>
<td>4720</td>
<td>2700</td>
</tr>
</tbody>
</table>

*The touch-down sensor is not a coupled problem, so this
time is for a single deformation and capacitance extraction.

Torsion Mirror

Figure 2 Torsion Mirror Plate, at 4.0 V. Mirror
tip displacement is 0.65 μm, corresponding to a tilt
angle of 1.3°. The figure is scaled by 5x in Z for
visualization.

The torsion mirror is shown in Figure 2. The mirror
plate is fixed at the ends of either arm, and able to
twist about them. There are two electrodes below the
mirror, and a ground plane below everything. A voltage is placed on one electrode (the forward one) and all other conductors are held at 0 V.

One question to ask of this model is: What is the pull-in voltage of this device? [See [8] for more extensive discussions of pull-in in MEMS.] CoSolve-EM simulates pull-in by finding self-consistent solutions at a sequence of applied voltages. This is shown in Figure 3. For this model pull-in occurs between 4.25 V and 4.5 V. Figure 2 shows the deformation at 4.0 V.

Displacement vs. Voltage

![Graph of displacement vs. voltage for the tilting mirror plate. CoSolve-EM detects when the solution has gone beyond pull-in and sets the displacement to 0, thus the last two points at 4.5 V and 5.0 V are shown as 0 displacement. Pull-in occurs between 4.25 and 4.5 V.](image)

Controlled Deformation of Plates

In applications such as adaptive optics, one may be interested in producing a controlled deformation in a reflective or refractive plate. To test the modeling of such a device, we built the following model: a square deformable plate 4000 x 4000 x 0.5 μm suspended 5.5 μm above a 5000 x 5000 μm ground plane. Between these plates three square 500 x 500 μm driving electrodes are positioned 3 μm above the ground plane. The driving electrodes are placed to one side of the center of the system; they have centers at \(x = -1000\), and \(y = 1000, 0, \) and 1000.

When we apply voltages to the electrodes we can control the deformation of the plate. Figure 4 shows two X cross-sections of the deformed structure. Each cross-section is formed by taking the x-z node coordinates of nodes within 100 μm of \(y = 1000\) (the figure only displays nodes from the bottom face of the plate, the electrodes and the ground plane.)

The circles are for the solution in which 0 V is applied to electrode s1, and 30 V is applied to s2, while the crosses are for the solution in which 30 V are applied to each of those electrodes. In both cases the plate, s3 and the ground plane are all held at 0 V.

![X cross-section at x = -1000, for two different electrostatic loads on s1 and s2 (s3, the plate and the ground plane are held at 0 V.)](image)

![An exaggerated view of the plate deformation under a load of s1=40 V and s2=30 V. This view looks along the y axis and the electrodes line up on the left.](image)

Comb Finger Levitation

Levitation [9] is an effect often encountered in comb-drive designs. It only appears in problems with more than two conductors, and its modeling
usually requires going beyond a parallel plate approximation for the capacitance. It is also a coupled problem. Figure 6 shows a test model we use to explore levitation effects on structures like comb fingers. The model has three fingers, each 20 \( \times 1 \times 1 \) \( \mu \)m suspended 2 \( \mu \)m over a ground plane. The central finger is fixed at its left end, and bends up under the levitation force. The outer two fingers are fixed.

![Comb fingers example](image)

**Figure 6** Comb fingers example. The example contains 3 fingers over a ground plane.

This example demonstrates two capabilities of CoSolve-EM. First, we can use the MemBase class library to apply translations to arbitrary parts of the model (the ground plane in this case) and second, we have used the SNGCR procedure for solving this model, since at high enough voltages all levitation problems begin to fail to converge under simple relaxation.

**Touch-Down Pressure Sensors**

Touch-down-mode capacitive pressure sensors provide an interesting example of devices in which it is important to estimate the capacitance of a deformed structure.

In this example we build a test model from a 150 \( \mu \)m radius circular plate 10 \( \mu \)m thick, 1 \( \mu \)m above a ground plane. The rim of the circular plate is fixed in CoSolve-EM and a rigid surface 0.5 \( \mu \)m above the ground plane and appropriate interface elements are created in CoSolve-EM. The device is meant to operate in a regime in which the pressure is high enough to cause the plate to touch down on the rigid surface.

![Image in CoSolve](image)

**Figure 8** Image in CoSolve of the model, at 20 MPa. We show the bottom face of the plate and the ground plane. The Z scale is magnified by 100x for this visualization. The plate has radius=150 \( \mu \)m, thickness=10 \( \mu \)m, gap = 1 \( \mu \)m. The interface is 0.25 \( \mu \)m above the ground plane.

Figure 8 shows the bottom surface of the deformed plate. Figure 9 shows the capacitance vs. pressure and contact force vs. pressure for the range 0-20 MPa.

This example provides a demonstration of three features of CoSolve-EM. We can insert interface elements in CoSolve-EM (even if they were not present in the original meshed model) to solve contact problems; we can extract the reaction forces against those interfaces and the capacitance of the deformed structure; and we can vary any applied pressure in the problem to generate figures such as Figure 9.
Curved Electrode Actuators

Curved electrode actuators have been designed and built at Twente University and are discussed in more detail elsewhere [10]. Basically a flat beam is pulled in laterally against a curved electrode containing grounded "bumpers", both positioned above a ground plane. Such a device is shown in Figure 10. These devices have been built with various curvatures and various bumper locations. We model one example of such a device. In the model, x is along the length of the beam, y=-2.0 is the plane of the ground plane, and the principal motion of the beam is along z, towards the fixed electrode. The beam is 515 μm long, 5 μm high and 2 μm wide, made of polysilicon, with E=150 GPa. At x=0 there is a gap of 3 μm between the beam and the fixed electrode. The fixed electrode is 5 μm high, 20 μm wide, and extends 500 μm along x with curve:

$$z = \frac{x^3}{500^3}$$

Such a model is shown in Figure 11. In order to simulate a real device, we insert an interface (using CoSolve) 2 μm away from the curved electrode and attach it to nodes every 50 μm along the movable beam; this models bumpers every 50 μm along the curved electrode.

However the curved electrode actuator is also a levitation problem, and at higher voltages (>70 V) it requires SNGCR to converge. Because of this difficulty we have suppressed levitation in this calculation by fixing the top of the movable beam to move only in x and z. Notwithstanding that simplification, Figure 12 still shows good agreement between model and data out to 80 V, and qualitative agreement everywhere. The model is systematically overestimating the force applied to the beam, as we would expect from the suppression of levitation.
Beta versions of CoSolve-EM are now running at MIT and at UC Berkeley. We intend a general release in 1995.

Acknowledgments
This work is supported by ARPA/ESTO under contract J-FBI-92-196. We would like to acknowledge the contributions of King Yu for work on the Universal File translator and Gregory Pal for work on the MemBase class library. We also acknowledge influential discussions with Jacob White, He Yie, Xuejun Cai and Peter Osterberg.

4 ABAQUS Manual, Hibbitt, Karlsson & Sorenson, Inc. 1080 Main Street, Pawtucket, RI 02860, USA
6 Software Development Group, Geometry Center, University of Minnesota, 1300 South Second Street, Suite 500, Minneapolis, MN 55454, USA, or see http://www.geom.umn.edu/welcome.html
10 R Legtenberg et.al. this volume.