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SELECTIVE MODE EXCITATION AND DETECTION OF MICROMACHINED RESONATORS

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ABSTRACT

Distributed mechanical systems such as micromachined resonant strain gauges possess an infinite number of modes of vibration. Mostly, one is interested in only one or a few modes. A method is described with which only the desired modes are excited and detected. This is achieved by geometrically shaping the elements used for excitation and detection of the vibration. The method is based on the orthogonality principle, which is valid for a variety of structures and vibrations. In this paper we have restricted ourselves to transversal vibrations of prismatic beams, clamped on both sides (microbridges). The effect of axial stress on the suppression of unwanted modes is discussed and the design rules for obtaining the shapes for most commonly used excitation and detection mechanisms are deduced. Experimental results for some excitation mechanisms are presented.

INTRODUCTION

Micromechanical resonant sensors [1,2,3] are in an increasing field of interest because of their great sensitivity and quasi-digital output. Due to their distributed mechanical character, these systems generally have an infinite number of modes of vibration. By the analytical method of modal analysis [4], it can be made clear that the modes, which are required to describe the dynamical behaviour, are exactly the same modes which are required for the series expansion of the excitation load. Most excitation and detection methods utilize on-chip electrodes. By shaping these electrodes, the excitation load can be controlled, giving us control over which modes are excited and how strong. When no special attention is paid to the design of excitation and detection elements, multiple modes will be excited and detected. However, resonant sensors are usually operated at one eigenfrequency (usually the first). The appearance of other modes can lead to degrading sensor performance. For example, when the resonator is supplied with an electronic circuitry (to provide an oscillator), the oscillator can lock in on the wrong mode. By using selective mode excitation (*SME*) and - detection (*SMD*) it is achieved that the condition for oscillation is met for an unambiguous frequency over a wide frequency range.

THEORY OF SELECTIVE MODE EXCITATION

Small transversal movements of a viscously damped prismatic beam subject to an axial force N and a driving load $P(x,t)$ (see fig. 1) are described by the following linear, inhomogeneous differential equation [2]:

$$\hat{E}Iv''''(x,t) - Nv''(x,t) + \varrho A\ddot{v}(x,t) + c\dot{v}(x,t) = P(x,t) \qquad (1)$$

with $v(x,t)$ the deflection, I the moment of inertia, c the viscous drag parameter (force per unit length per unit velocity) and A the cross-sectional area of the beam ($A=bh_b$ with b the width and h_b the thickness). For beams with $b>>h_b$ holds $\hat{E} = E/(1-\nu^2)$, with E Young's modulus and ν Poisson's ratio of the beam material. ϱ is the density of the beam material. A prime and a dot denote differentiation to x and t, respectively. We assume that separation of space and time coordinates is possible. Further, the excitation term $P(x,t)$ and the displacement $v(x,t)$ are assumed to be harmonic. Note that these assumptions do not mean that the beam moves in phase with the excitation term or that all points of the beam move in phase with each other: the space dependent parts of $P(x,t)$ and $v(x,t)$ are allowed to be complex, and their arguments may depend on x. In this paper, only excitation with distributed forces $D(x,t)$ and distributed moments $m(x,t)$ will be considered. In that case, for $P(x,t)$ holds:

$$P(x,t) = D(x,t) - m'(x,t) \qquad (2)$$

The positive direction of the loads is as shown in fig. 1. It can be shown that, if homogeneous boundary conditions are taken into account, the solution of eq. (1) can be written as the following expansion series [4, pp. 429 ff.]:

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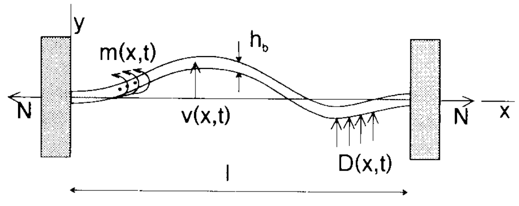

Fig. 1: Cross-section of the structure under consideration.

$$v(x,t) = \sum_{r=1}^{\infty} \eta_r(t) v_r(x) \tag{3}$$

where $v_r(x)$ are solutions of the homogeneous equation of the undamped system (modeshapes):

$$\hat{E}I \; v''''(x) - N\, v''(x) - \varrho A \omega^2 \, v(x) = 0 \tag{4}$$

The modeshapes and corresponding eigenfrequencies ω_r can be calculated using standard mechanics. $\eta(t)$ are the generalized (normal) coordinates. It can be shown that the modes are orthogonal [4, pp. 141 ff.]:

$$\frac{1}{l} \int_0^l v_r(x)\, v_s(x)\, dx = \delta_{r,s} \tag{5}$$

where l is the length of the beam and $\delta_{r,s}$ the Kronecker delta. Note that eq. (5) defines the normalization of the modes. By substituting eq. (3) in eq. (1), using the fact that $v_r(x)$ satisfies eq. (4), multiplying with $v_s(x)$, integrating over the beam-length l and applying eq. (5), we finally find:

$$\ddot{\eta}_r(t) + \dot{\eta}_r(t)\frac{c}{\varrho A} + \omega_r^2 \eta_r(t) = \frac{P_r(t)}{\varrho A l} \qquad r=1,2,... \tag{6}$$

$P_r(t)$ are the expansion coefficients of the excitation term $P(x,t)$, or generalized loads, for which holds:

$$P_r(t) = \int_0^l P(x,t) v_r(x) dx \;\; ; \;\; P(x,t) = \frac{1}{l} \sum_{r=1}^{\infty} P_r(t) v_r(x) \tag{7.a;b}$$

From eq. (6) it is seen that the system can be described as an infinite number of independent lumped elements (modes), every element being driven by its own generalized load. The amplitude of the generalized loads are a good measure for the efficiency with which the modes are excited.

For applications in resonant strain gauges, one is interested in a limited number of modes (mostly the first one) only. For optimum sensor performance, all other modes should be suppressed. It is seen from eq. (6) that only modes for which $P_r(t) \neq 0$ are excited.

Hence, only the modes which are required for the expansion of the excitation load $P(x,t)$ will be excited.

The excitation load is in many cases applied by means of excitation mechanisms utilizing on-chip electrodes. By geometrically shaping these electrodes, we have refined control over the spatial dependence of the excitation load, and thus over the modes being excited.

Some example amplitude bode plots of the very important case where only a single mode is excited are given in fig. 2.

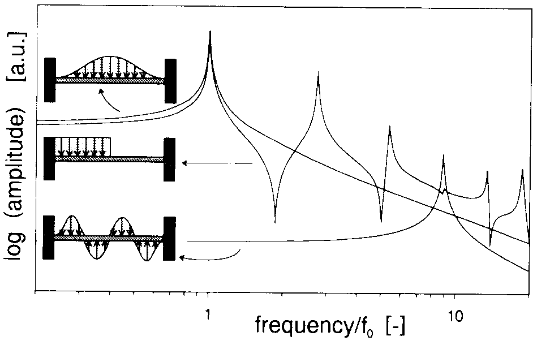

Fig. 2: Computed amplitude bodeplots of a microbridge excited with a rectangular load, and loads for excitation of mode 1 and 4 only. The displacement was computed at 25% of the beam length from the left side.

Axial stress

Resonant sensors operate according to the modulation of the resonance frequency by the measurand. This modulation can be achieved either by changing the mass or the stiffness of the resonator [5]. In both cases, the eigenfrequencies of the resonator are affected. The modeshapes are only sensitive to stiffness modulation, and depend on the axial stress σ ($\sigma = N/A$), thus $v_r = v_r(x,\sigma)$. However, when the stiffness changes, the x-dependence of the load $P(x,t)$ (i.e. the shape of the electrodes) will generally remain constant. Therefore, the modes will be excited with different generalized loads as the stiffness of the structure changes. We will now discuss the effect of the axial stress on the generalized loads.

Suppose that an excitation element has been developed such that at $\sigma = \sigma_0$ the generalized loads are $P_r(\sigma_0, t)$. For the generalized load as a function of the stress σ (i.e. the expansion coefficients of the load, expanded in the modeshapes $v_s(x,\sigma)$) holds (substitution of eq. (7.b) in eq. (7.a)):

$$P_s(\sigma,t) = \sum_{r=1}^{\infty} P_r(\sigma_0, t) H_{r,s}(\sigma,\sigma_0) \tag{8}$$

with

$$H_{r,s}(\sigma,\sigma_0) = \frac{1}{l}\int_0^l v_s(x,\sigma)v_r(x,\sigma_0)dx \tag{9}$$

$H_{r,s}$ is a good measure for the sensitivity of *SME* to axial stress. Note that $H_{r,s}$ equals the Kronecker delta when $\sigma=\sigma_0$. We have evaluated $H_{r,s}(\sigma,\sigma_0)$ for $\sigma_0=0$ for all $r,s \leq 4$. The results are shown in fig. 3. In the special case that only one mode (say mode r) is excited at σ_0, $H_{r,s}$ represents the generalized load of mode s at an axial stress σ relative to the generalized load of mode r at a stress σ_0. In fig. 3, the axial stress is normalized with respect to the absolute value of the buckling stress, for which holds: $\sigma_{buck}= 3.39E\ h_b^2/((1-\nu^2)\ l^2)$. The decrease of $H_{s,s}$ due to an axial stress is of no importance over the entire stress range. When $r \neq s$, and r,s have different parities, mode r remains suppressed for all stresses. However, when r and s have the same parity, $H_{r,s}$ is typically a few percent at the buckling stress.

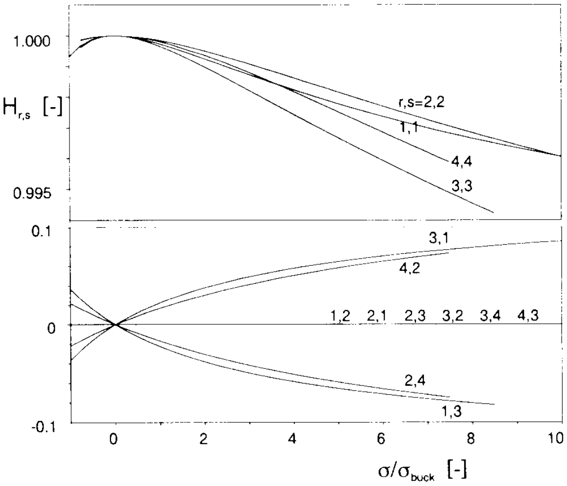

Fig. 3: $H_{r,s}$ *as a function of the normalized axial stress for* $\sigma_0=0$ *and* $r,s \leq 4$.

SME FOR COMMONLY USED EXCITATION MECHANISMS

Micromechanical resonators can be driven by a number of excitation mechanisms. From a mechanical point of view, the mechanisms can be divided into two categories, namely mechanisms which use forces and mechanisms which use moments for excitation of the resonator. Once we have determined $P(x)$ for excitation of the desired mode(s) with the desired efficiency, we can determine the required $D(x)$ and $m(x)$ in 'force -' and 'moment mechanisms' respectively. From eq. (2) it is seen that in force mechanisms the required force $D(x)$ equals $P(x)$, while in moment mechanisms the required moment $m(x)$ must equal a primitive function of $P(x)$. We will now discuss the applicability of *SME* and deduce the required shape of the excitation elements for commonly used excitation mechanisms.

Electrostatic excitation [5,6,7]

Electrostatic excitation is the most commonly applied force mechanism. The Coulomb force between two parallel plates, one of which is placed on the resonator, is used to exert a transversal distributed force $D(x)$ on the resonator. For the component of the load with the frequency of the excitation voltage holds ($v(x,t) << d$):

$$P(x) = D(x) = \varepsilon_0 U_{DC} U_{AC} w(x)/d^2$$

with $w(x)$ the width of the electrodes and ε_0 the permittivity of free space; d, U_{DC} and U_{AC} are the spacing between, and the DC and AC voltages across the electrodes, respectively. The biasing DC voltage is required to get a component with the frequency of the AC voltage. Recalling the outlined statement on the previous page, it will be clear that only modes required for the expansion of $w(x)$ will be excited. To put it differently: when $w(x)$ is chosen proportional to the sum of the desired modes, multiplied with the desired generalized loads, the proper modes will be excited.

Piezoelectric - and dielectric excitation [8,9]

Piezoelectric - and dielectric excitation are typical moment mechanisms. In both mechanisms, the harmonic elongation of a thin layer on top of the beam is used to exert a bending moment on the beam (bimorph excitation). The elongation is obtained by periodically changing the voltage across two electrodes in between which a piezoelectric or dielectric film is sandwiched.

In the piezoelectric case the moment equals [10]: $m(x) = d_{31} U E_f h_b w'(x)/2$, where d_{31} is the piezoelectric constant relating the field strength in the y-direction to the strain in the x-direction, U the voltage across the film, E_f the Youngs modulus of the film-material and $w(x)$ the width of the film.

In the dielectric case, the elongation is a consequence of the shear contraction of the film. The moment can be deduced from [9] and equals: $m(x) = \varepsilon_0 \varepsilon_f U_{DC} U_{AC}\ \nu_f\ h_b\ w'(x)/(2h_f(1-\nu_f))$, with ε_f, ν_f and h_f the dielectric constant, Poisson's ratio and the thickness of the film respectively. Other symbols are defined as above.

In the above equations, the assumption was made that $h_f << h_b$. In Both mechanisms, the moment $m(x)$ is proportional to $w'(x)$, whereas for moment mechanisms $P(x) = -m''(x)$. Therefore, one finds that the right modes are excited when $P(x) \propto w''(x)$.

Thermal excitation [11,12]

The elongation of the top film of the beam can also be accomplished by thermal heating. This can be done by dissipating electrical energy from a resistive film on top of the resonator. It can generally be said that the heat diffusion along the beam axis can be neglected when the changes in $P(x,t)$ are relative small within the length of the reciprocal thermal wavenumber. In that case, the thermally induced moment $m(x)$ is directly proportional to the derivative of the dissipated heat per unit length. For the resistance per unit length of a homogeneous film with constant thickness holds: $R(x) = \varrho_f / h_f w(x)$ [Ω/m], with ϱ_f, h_f and w the resistivity, the thickness and the width of the resistive film, respectively. The dissipated power per unit length equals $i(t)^2 R(x)$, where i is the current through the resistive pattern. Combining the results obtained so far, one finally finds the following (implicit) condition for the film width: $P(x) \propto (1/w(x))''$, with $P(x)$ the load for excitation of the desired modes.

SME of optothermally excited structures can be obtained by irradiating a light absorbing layer with variable width with a homogeneous light-beam. The excitation load will be proportional to the absorbed light power. When the width is chosen $w''(x) \propto P(x)$, with P(x) the desired load, the right modes will be excited.

SELECTIVE MODE DETECTION

In many cases the shapes of the electrodes of the various detection mechanisms can be designed such that only a selective number of generalized (normal) coordinates are sensed. This is advantageous when a non-selective excitation mechanism is applied. However, also when *SME* is applied, the use of selective mode detection has important advantages, as will be pointed out below.

Capacitive detection [5,10]

The configuration used for electrostatic excitation can be applied for detection of the vibrations by keeping the voltage across the capacitor at a constant value. When the dynamic deflection is much smaller than the gap distance d minus the static deflection (due to the required biasing voltage) the following expression holds for the current:

$$i(t) = \frac{U\varepsilon_0}{d^2} \int_0^l w(x) \dot{v}(x,t)\, dx$$

Expanding the electrode width in the modeshapes at σ_0 (i.e. $w(x) = \sum w_r(\sigma_0) v_r(x, \sigma_0)$), and $v(x,t)$ in the modeshapes at σ (see eq. (3)), we obtain:

$$i(\sigma,t) = \frac{U_{DC} l \varepsilon_0}{d^2} \sum_{r=1}^{\infty} \sum_{s=1}^{\infty} w_r(\sigma_0) \dot{\eta}_s(t) H_{r,s}(\sigma,\sigma_0) \qquad (10)$$

where $H_{r,s}$ is defined by eq. (9). When the resonator is operated at σ_0, only modes for which $w_r(\sigma_0) \neq 0$ can be sensed. This means that the electrode width should be chosen proportional to the modeshapes to be detected. The contribution of unwanted modes in the output signal at deviant axial stresses can be treated completely analogue to the contribution of unwanted modes in *SME*. Interesting is the case when both excitation and detection of a single mode is performed. Since $\eta_s \propto P_s$ (eq. (6)), it is immediately seen from eq. (10) and eq. (8) that the contribution of other modes in $i(t)$ at deviant stresses is proportional to $H_{r,s}{}^2$. Thus the presence of unwanted modes is reduced by a factor $H_{r,s}$ as compared to the case when only excitation of a single mode is applied.

Piezoelectric detection

When the voltage in the configuration used for piezoelectric excitation is kept at a constant value, the following expression holds for the current [10]:

$$i(t) = -d_{31} \frac{E}{1-\nu^2} \frac{h_b}{2} \int_0^l w(x) \sum_{r=1}^{\infty} \dot{\eta}_r(t) v_r''(x)\, dx \qquad (11)$$

When no axial load is applied, it can be shown that the functions $v_r''(x)$ represent an orthogonal set. Therefore, if $w(x)$ is expanded in the functions $v_r''(x)$, i.e.:

$$w(x) = \frac{\hat{E}I}{\varrho A l} \sum_{s=1}^{\infty} \frac{1}{\omega_s^2} w_s v_s''(x) \quad ; \quad w_s = \int_0^l w(x) v_s''(x)\, dx \qquad (12.a;b)$$

only modes for which $w_s \neq 0$ can be detected, as can be seen by substituting eq. (12.a) into eq. (11). Eq. (12) is the design rule for the shape of the detection element for a axially unloaded beam. The electrode width should be chosen proportional to $v_r''(x)$ of the desired modes, whereas for selective piezoelectric excitation the width should be chosen such that $w''(x) = v_r(x)$. Since the modeshapes are composed of the trigonometric functions *sin*, *cos*, *sinh* and *cosh*, the conditions for the electrode width of structures with piezoelectric excitation and - detection are the same. Treatment of the effect of axial stress on the suppression of unwanted modes would go beyond the scope of this paper.

Piezoresistive detection

Vibrations can be detected by piezoresistive strain gauges on top of the vibrating structure. For simplicity, it is assumed that the gauge factor of the piezoresistive material $G >> 1$, so the change in the resistance of the resistive pattern is determined by the change in resistivity of the piezoresistive material and not by the change in the dimensions of the strain gauge. This assumption certainly holds for silicon and poly-silicon strain gauges. For the resistance of the pattern holds ($h_b >> h_f$, with h_f the thickness of the film):

$$R(t) = \int_0^l \frac{\varrho_f(t)}{h_f w(x)} dx = R_0 + G\varrho_0 \frac{h_b}{2h_f} \int_0^l \frac{v''(x,t)}{w(x)} dx$$

Here R_0 and ϱ_0 are the resistance and resistivity at zero applied strain, respectively. If $w(x)$ is expanded according to:

$$w(x) = \frac{1}{\sum_{s=1}^{\infty} w_s v_s''(x)} \quad ; \quad w_s = \frac{\hat{E}I}{\varrho A l} \int_0^l \frac{v_s''(x)}{\omega_s^2 w(x)} dx$$

only the modes for which $w_s \neq 0$ can be detected. Thus the shape should be chosen proportional to the reciprocal of the second derivative of the modes to be detected. Just as in the piezoelectric case, the design rule only holds for axially unloaded beams. The effect of an axial load on the suppression of unwanted modes is not discussed here.

Concluding remarks on *SMD*

It was noticed before that for capacitive detection, the application of *SMD*, besides *SME*, leads to a further reduction of the contribution of unwanted modes. Although not worked out in detail, it can be said that this is true for other detection mechanisms as well.

From the above, it appears that the shapes of the electrodes for excitation and detection are the same for the technologically compatible excitation/detection pairs electrostatic excitation/capacitive detection and piezoelectric excitation/detection. This makes the application of one-port resonators, as described in references [10] and [7], very attractive.

EXPERIMENTS

To verify the principle of *SME*, test structures have been realized for both a force - (electrostatic excitation) and a moment mechanism (dielectric excitation). Details of the structures are given in fig. 4. A photo of the dielectrically excited structures is presented in fig. 5. In both types of devices, only one of the electrodes has been shaped. Since both excitation mechanisms are quadratic effects, the applied voltages have been biased with a DC voltage of typically 10 V. The AC driving voltage was 1 V rms. The motion was detected using a Michelson interferometer.

The results of the electrostatically excited structures are shown in fig. 6. The vibrations were measured at approx. 25 % of the beam length from the left side. The AC voltages of the two halfs of the electrode for excitation of the second mode were in antiphase. The results of the dielectrically driven structures are shown in fig. 7. The vibrations of these samples were measured at 10, 50, 25 and 25 % of the beam length from the left side, respectively. The first three resonance frequencies of the test samples are approx. 8, 20 and 40 kHz. Due to mutual thickness differences of the beams, these frequencies are not exactly the same for all samples.

For both excitation mechanisms, the suppression of unwanted modes is clearly recognized. The weak presence of unwanted modes is ascribed to the fact that the shaped electrode did not exactly match the modeshapes (the electrodes were composed of a limited number of rectangles), and to the fact that the beams did not have exactly the same length as the electrodes (due to underetching). Further, the electrode shapes were designed for axially unloaded beams. However, due to the fabrication process, some stress is present in the beams. Also the inhomogeneity of the beam thickness can attribute to the appearance of unwanted modes. For the electrostatic structure, a slight mis-allignment of the shaped electrode with respect to the beam can also contribute.

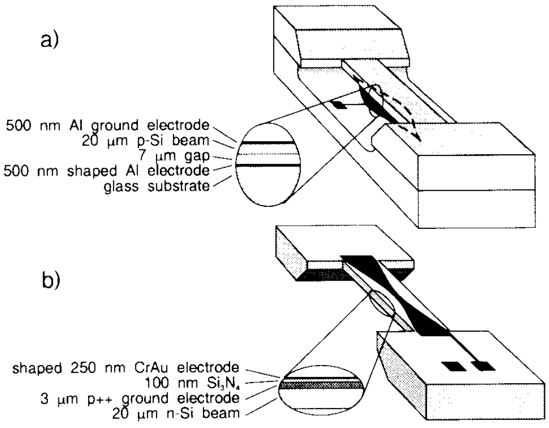

Fig. 4: Test structures with electrostatic excitation (a) and dielectric excitation (b). Dimensions not given in the figure are: l= 5mm and b=1mm.

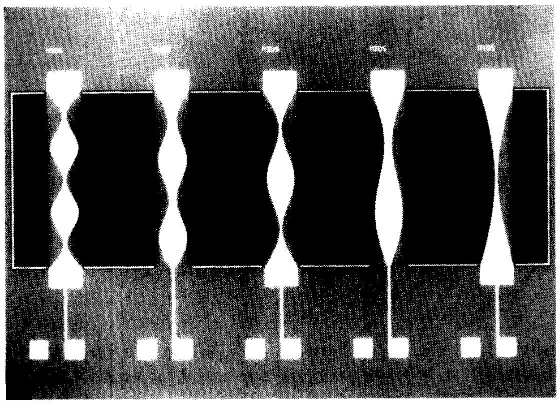

Fig. 5: Photograph of the test structures with dielectric excitation for excitation of modes 1 to 5. The shapes resemble the two times integrated modeshapes.

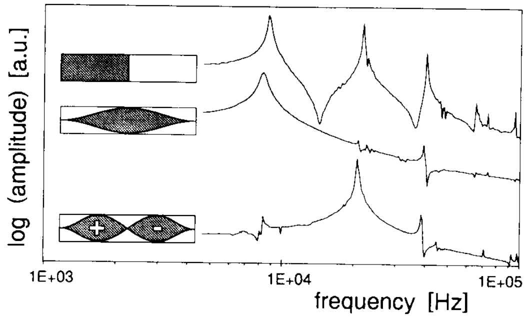

Fig. 6: Amplitude bode plots of the electrostatically excited test samples. From top to bottom: excitation of multiple modes, selective excitation of mode 1 and 2, respectively. The electrode shapes are sketched next to the curves.

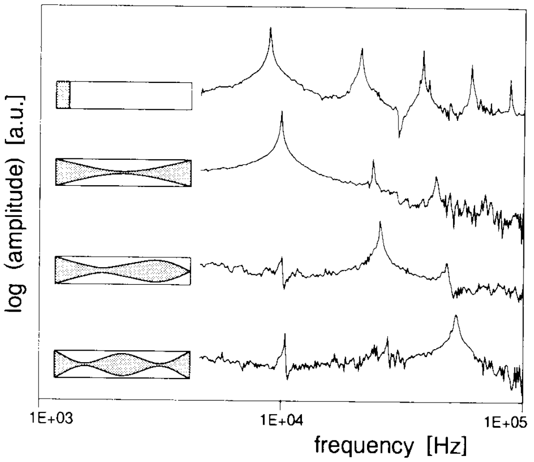

Fig. 7: Amplitude bode plots of the test samples with dielectric excitation. From top to bottom: excitation of multiple modes, selective excitation of modes 1, 2 and 3, respectively.

CONCLUSIONS

To improve the performance of resonant strain gauges, only the modes used for the output signal should be excited and detected. This can be achieved by shaping the electrodes utilized in the excitation and detection mechanisms. A theory and design rules are given with which the shapes of the electrodes of commonly used excitation and detection mechanisms of resonant strain gauges can be designed such that only the desired modes of vibration are excited and detected. The effect of axial stress on the suppression of unwanted modes is discussed. Experiments on single mode excitation of test structures with various excitation mechanisms have been performed. The results show good agreement with the theory.

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