Digital compensation of nonlinear distortion in loudspeakers

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Abstract

Ideal loudspeakers produce acoustic waves, which are a linear transformation of the electrical input signal. In reality loudspeakers do produce nonlinear distortion as well. This paper presents a method to compensate this distortion in real-time by nonlinear digital signal processing, implemented on a Digital Signal Processor (*i.e.* the TMS320C30 DSP). Based on literature, an electrical equivalent circuit of an electrodynamic loudspeaker is developed first, resulting in a linear lumped parameter model. The parameters in this model are matched with the measurements of a selected test loudspeaker. The linear model is extended to include nonlinear effects by developing the parameters as a function of the voice coil excursion of the loudspeaker in a Taylor series expansion. The resulting nonlinear system is described by a Volterra series. Based on this description an inverse circuit is designed for the second order nonlinear distortion. This circuit is implemented in real-time on the DSP, using a high-level design and code generation system (SPW). Simulations and experiment are presented.

1 Introduction

There is a growing need for small loudspeakers with high performance. In order to have a bass reproduction with enough power, a large voice coil excursion is required. This increases the already inherent nonlinear distortion. With the aid of digital signal processing it is possible to develop nonlinear circuits for manipulating audio signals such that the acoustical nonlinear distortion produced by the loudspeaker is reduced. Such a circuit will be connected between the signal source (preamplifier) and power amplifier that drives the loudspeaker. In this presentation we apply feed forward control to reduce the second order nonlinear distortion, and compare simulations and experiment.

Most methods for distortion reduction in nonlinear systems apply a feedback loop. This approach requires a microphone in the loudspeaker enclosure or an accelerometer attached to the cone. Another method [1] applies two loudspeakers, push-pull mounted with respect to each other, for second order distortion reduction.

2 The loudspeaker model and the distortion reduction structure

In order to apply feed forward control, we need to develop a reliable model of the nonlinear distortion. We assume that the major nonlinearities are time-independent. We use the model of the electrodynamic loudspeaker given in [2], see Fig. 1. Of a selected test loudspeaker mounted in a closed box with dimensions 0.53 x 0.33 x 0.205 m^3, the linear model parameters [3] are determined using an impedance measurement with a HP 3577A network analyzer. The result was verified with a near-field frequency response measurement [4]. The measurements give consistent results.

The self-inductance [1], the stiffness of the system and the coupling factor all depend on the voice coil excursion. For relatively low signal amplitudes, the second order distortion is dominant. This second order distortion is caused by first order nonlinearities. We therefore develop the pertinent parameters L_e, k_t and Bl respectively, as a function of the voice coil ex-

[1] Although in actual measurements often a mixed $j\omega$ and $\sqrt{j\omega}$ dependence of the voice coil impedance is found, for which also a theoretical explanation can be given [5], it will be assumed in this paper that the self-inductance only has $j\omega$ behaviour.

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0-7803-0946-4/93 $3.00 © 1993 IEEE

cursion x in a truncated Taylor series expansion

$$\begin{aligned} L_e &= L_o + l_1 x & (1) \\ k_t &= k_o + k_1 x & (2) \\ Bl &= Bl_0 + b_1 x & (3) \end{aligned}$$

The model now contains three unknown coefficients, *i.e.* l_1, k_1 and b_1 which we determine by driving the loudspeaker with one or two frequencies (intermodulation) and measure the resulting distortion with a microphone (Sennheiser MD 421-2) coupled with a HP 3562A dynamic signal analyzer in the power spectrum mode. With a least-squares fit of several measurements the coefficient values are obtained.

The loudspeaker is described by a nonlinear differential equation in the voice coil excursion:

$$\begin{aligned} E_g &= \alpha x + \beta \dot{x} + \gamma \ddot{x} + \delta \dddot{x} + aE_g x + bx^2 + cx\dot{x} \\ &\quad + dx\ddot{x} + ex\dddot{x} + f\dot{x}^2 + g\dot{x}\ddot{x}, \qquad (4) \end{aligned}$$

in which

$$\begin{aligned} \alpha &= k_0 R_e / Bl_0 \\ \beta &= (R_e R_m + k_0 L_0 + Bl_0^2)/Bl_0 \\ \gamma &= (m_t R_e + L_0 R_m / Bl_0 \\ \delta &= m_t L_0 / Bl_0 \\ a &= -2b_1 / Bl_0 \\ b &= (k_1 R_e Bl_0 + b_1 k_0 R_e)/Bl_0^2 \\ c &= (b_1 R_e R_m + 2 l_1 k_0 Bl_0 + 2 k_1 L_0 Bl_0 \\ &\quad + 3 b_1 Bl_0^2)/Bl_0^2 \\ d &= (b_1 m_t R_e + b_1 L_0 R_m + l_1 R_m Bl_0)/Bl_0^2 \\ e &= (b_1 m_t L_0 + l_1 m_t Bl_0)/Bl_0^2 \\ f &= (l_1 R_m Bl_0 - b_1 L_0 R_m)/Bl_0^2 \\ g &= (l_1 m_t Bl_0 - b_1 m_t L_0)/Bl_0^2, \qquad (5) \end{aligned}$$

where third order distortion terms have been discarded. The coefficients α - δ, a - g are combinations of the linear model parameters of Fig. 1 and the nonlinear parameters. The output $y(t)$ of a nonlinear time-invariant system can be characterized by a Volterra series of convolution integrals [6]:

$$y(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h_1(\tau)\, x(t-\tau)\, d\tau + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h_2(\tau_1, \tau_2)\, x(t-\tau_1)\, x(t-\tau_2)\, d\tau_1\, d\tau_2 + \cdots (6)$$

with $x(t)$ the input signal and where $h_1(\tau)$ and $h_2(\tau_1, \tau_2)$ denote the first and second order Volterra kernel. The Fourier transform of $h_2(\tau_1, \tau_2)$ is:

$$H_2(j\omega_1, j\omega_2) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h_2(\tau_1, \tau_2) e^{-j(\omega_1 \tau_1 + \omega_2 \tau_2)} d\tau_1 d\tau_2 \qquad (7)$$

If the loudspeaker is driven with a voltage of the form $E_g = e^{p_1 t} + e^{p_2 t}$, the voice coil excursion can be written as:

$$\begin{aligned} x(t) &= q_1(p_1)e^{p_1 t} + q_1(p_2)e^{p_2 t} + \\ &\quad q_2(p_1, p_1)e^{2p_1 t} + q_2(p_2, p_2)e^{2p_2 t} + \\ &\quad 2q_2(p_1, p_2)e^{(p_1+p_2)t} \qquad (8) \end{aligned}$$

With (4) we obtain for $q_1(p)$:

$$q_1(p) = \left(\alpha + \beta p + \gamma p^2 + \delta p^3\right)^{-1} \qquad (9)$$

The sound pressure is proportional to the voice coil acceleration. The peak pressure p_N in the near field at the center of a rigid, flat piston is for low frequencies equal to [4]:

$$\begin{aligned} p_N &= \frac{\rho_0 \omega U_0}{\pi a} = \frac{\rho_0 \omega^2 x_0 \pi a^2}{\pi a} = \\ &\quad \rho_0 a \omega^2 x_0 = \rho_0 a |X(j\omega)||H_1(j\omega)| \qquad (10) \end{aligned}$$

with ρ_0 the density of air, U_0 the piston peak-output volume velocity, a the piston radius, x_0 the piston peak excursion, $X(j\omega)$ the Fourier transform of the input signal to the loudspeaker and the transfer function $H_1(j\omega) = (j\omega)^2 q_1(j\omega)$.

For the second order term $q_2(p_1, p_2)$ we obtain:

$$\begin{aligned} q_2(p_1, p_2) &= -\frac{1}{2} q_1(p_1 + p_2) q_1(p_1) q_1(p_2) \times \\ &\quad [e\,(p_1^3 + p_2^3) + g\,(p_1^2 p_2 + p_1 p_2^2] \\ &\quad d\,(p_1^2 + p_2^2) + c\,(p_1 + p_2) + 2f p_1 p_2 + \\ &\quad 2b + a\,(q_1(p_1)^{-1} + q_1(p_2)^{-1}) \end{aligned} \qquad (12)$$

and the pertinent transfer function reads:

$$H_2(j\omega_1, j\omega_2) = (j\omega_1 + j\omega_2)^2 q_2(j\omega_1, j\omega_2). \qquad (13)$$

To determine $H_2(j\omega_1, j\omega_2)$, we drive the loudspeaker with:

$$x(t) = A cos(\omega_1 t) + B cos(\omega_2 t). \qquad (14)$$

The response from $H_2(j\omega_1, j\omega_2)$ is:

$$\begin{aligned} y_2(t) &= \frac{A^2}{2} ||H_2(j\omega_1, -j\omega_1)|| + \\ &\quad \frac{B^2}{2} ||H_2(j\omega_2, -j\omega_2)|| + \\ &\quad \frac{A^2}{2} ||H_2(j\omega_1, j\omega_1)|| cos(2\omega_1 t + \theta_1) + \\ &\quad \frac{B^2}{2} ||H_2(j\omega_2, j\omega_2)|| cos(2\omega_2 t + \theta_2) + \\ &\quad AB ||H_2(j\omega_1, j\omega_2)|| cos((\omega_1 + \omega_2)t + \theta_3) + \\ &\quad AB ||H_2(j\omega_1, -j\omega_2)|| cos((\omega_1 - \omega_2)t + \theta_4) \end{aligned}$$

Driving the loudspeaker with an input signal and measuring the amplitudes of the different frequency components enables us the determine the three unknown parameters: b_1, k_1 and l_1.

With the two transfer functions $H_1(j\omega)$ and $H_2(j\omega_1, j\omega_2)$ in parallel, we arrive at a circuit with the same first and second order response as the model of the loudspeaker. By means of inversion we obtain an inverse circuit. Discretization of this inverse model leads to a structure presented in Fig. 2. The sampling rate is chosen to be 4 kHz, which is sufficiently high as our main frequency range of interest is below 500 Hz. We applied the bilinear transformation to derive the digital filter coefficients from the continuous characteristic. The differentiators were implemented as FIR-filters with linear phase response. The behavior of the designed discrete system is simulated using the high level Signal Processing WorkSystem (SPW) software from Comdisco on the HP - Apollo workstation network in our laboratory. The building blocks of the designed and simulated circuit are converted into C-code by the SPW code generation system. Compilation of this code resulted into the instructions for the TMS320C30 DSP, which is mounted on a PC-board of Loughborough Sound Images, together with AD and DA converters, PC-AT interface, clock generation etc.

The distortion measurements are repeated, now with the DSP being connected in series with the power amplifier and the loudspeaker.

Examples of the obtained results are presented in Fig. 3 for the reduction of the second order harmonic distortion, and in Fig. 4 for the intermodulation with one of the frequencies fixed at 100 Hz.

3 Conclusions and discussion

The theory of Volterra series is well suited for the description of the nonlinear distortion in loudspeakers and for obtaining an inverse compensation circuit. An accurate model of the nonlinear behaviour of the loudspeaker is required. With the three parameters b_1, k_1 and l_1, which can be determined from measurements by the method of least-squares, a good match with the second order nonlinear distortion of a real loudspeaker is achieved. The second order distortion reduction circuit is only usefull in a limited amplitude range. The third order distortion becomes dominant for higher signal amplitude values. The next step in this research is to develop a third order reduction circuit based upon more accurate models. A drawback of the present approach is that the time dependent behaviour of the loudspeaker and the hysteresis [2] cannot be modelled with the Volterra theory.

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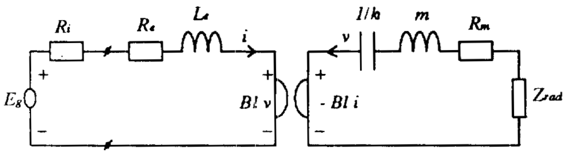

Fig. 1. Lumped parameter equivalent circuit of an electrodynamic loudspeaker

E_g	voltage source [V]
R_i	internal resistance of the source [Ω]
R_e	resistance of the voice coil [Ω]
L_e	inductance of the voice coil [H]
i	current through the voice coil [A]
v	velocity of the voice coil [m/s]
B	air gap flux density [T]
l	effective length of the voice coil wire [m]
$Bl\,v$	induced voltage [V]
$Bl\,i$	Lorentz force on the voice coil [N]
k_t	total stiffness of cone and box [N/m]
m	mass of voice coil and cone [kg]
R_m	mechanical damping [N s/m]
Z_{rad}	radiation resistance [N s/m]

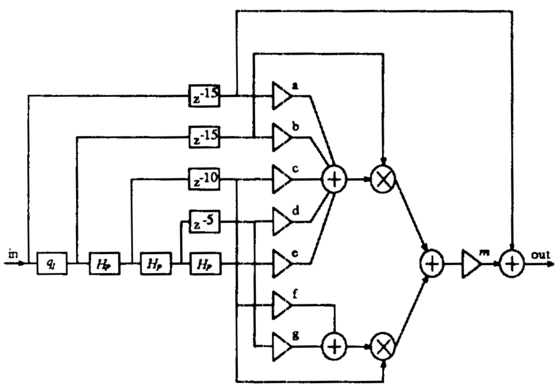

Fig. 2. Second order distortion reduction circuit

H_p	ideal differentiator
m	multiplier
q_1	voice coil excursion as function of the input voltage

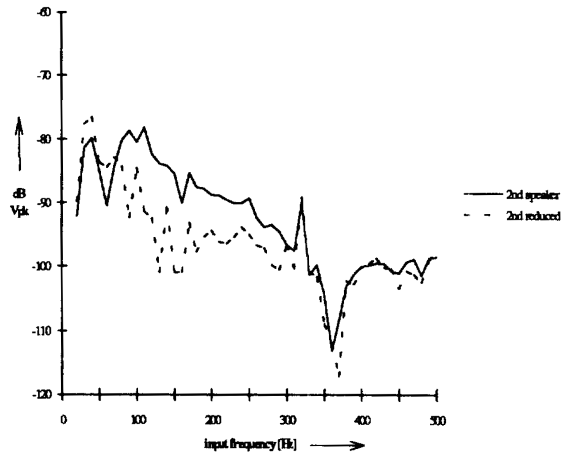

Fig. 3. Second harmonic of the loudspeaker with and without distortion reduction circuit

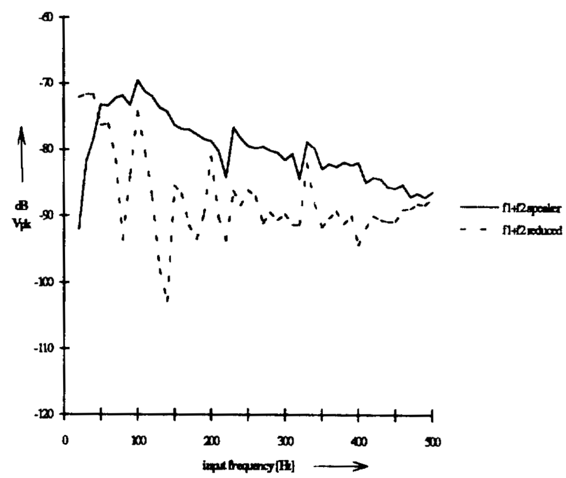

Fig. 4. Frequency f1+f2 from the speaker with and without distortion reduction circuit