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Tracking Control of Robots Using Only Position Measurements

by

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Abstract - A tracking controller for robots is presented that is based on position measurements only. The controller consists of a computed torque like control part together with an observer part that determines an estimate of the velocity signal. A nice property of this observer is its linearity, which is in fact a consequence of the linearization of the computed torque control part (in the ideal case). Under fairly general conditions on the controller and observer gains, exponential stability of the closed loop system is proven.

1. Introduction

Increasing demands on the performance of robot manipulators has led to the development of various advanced tracking control schemes. One of these schemes is the so-called computed torque controller (Luh et al., 1980). The computed torque method has the property that it feedback linearizes the nonlinear system using exact knowledge of the system dynamics.

The computed torque method requires both position and velocity measurements. Although in general position measurements are rather accurate, this does not hold for velocity measurements. The velocity is determined either by filtering of position measurements, causing difficulties especially at low velocities, or using, for example, tacho signals which in general are noisy. This shows one drawback for the practical implementation of the computed torque method.

In order to overcome this problem we present a computed torque like controller extended with a linear velocity observer. The use of a linear observer has been inspired by the fact that the computed torque method feedback linearizes the robot system in the case that velocity measurements are available. The exponential stability of the closed loop system is shown via Lyapunov's stability theory, and Wen and Bayard (1988).

2. Mathematical Preliminaries

The β-ball lemma (Wen and Bayard, 1988) plays a central role in the later sections. We will present a modified version of this local stability result. For the proof we refer to Wen and Bayard (1988).

Lemma 1. *Given a dynamical system*

$$\dot{x}_i = f_i(x_1, ..., x_m), \quad x_i \in R^n, \quad t \geq 0, \quad i = 1, ..., m \tag{2.1}$$

Let $f_i(.)$ *be locally Lipschitz w.r.t.* $x_1, ..., x_m$. *Suppose a function* $V(.): R^{nxm} \to R^+$ *is given such that*

$$V(x_1, ..., x_m) = \sum_{i,j=1}^{m} x_i^T P_{ij} x_j \tag{2.2}$$

where for each $i = 1, ..., m$ *there exists a* $\xi_i > 0$ *such that*

$$\xi_i \| x_i \|^2 \leq V(x_1, ..., x_m) \tag{2.3}$$

$$\dot{V}(x_1, ..., x_m) = -\sum_{i \in I_1} \left(\alpha_i - \sum_{j \in I_2} \gamma_{ij} \| x_j \| \right) \| x_i \|^2 \tag{2.4}$$

where $\alpha_i, \gamma_{ij} > 0$, $I_2 \subset I_1 \subset \{1, ..., m\}$. *Define* $V_0 \equiv V(x_1(0), ..., x_m(0))$. *If for all* $i \in I_1$

$$\bar{\alpha}_i \equiv \alpha_i - \sum_{j \in I_2} \gamma_{ij} V_0^{\frac{1}{2}} \xi_j^{-\frac{1}{2}} > 0 \tag{2.5}$$

then for all $\kappa_i \in [0, \bar{\alpha}_i]$ *the following inequality holds*

$$\dot{V}(x_1, ..., x_m) \leq -\sum_{i \in I_1} \kappa_i \| x_i \|^2, \quad \text{for all } t \geq 0 \tag{2.6}$$

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3. Dynamics of Rigid Robot Manipulators

In general the dynamics of an n DOF robot manipulator can be described by (Craig, 1986)

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau \tag{3.1}$$

where q is the nx1 vector of generalized coordinates, τ is the nx1 vector of input torques, $M(q)$ represents the positive definite inertia matrix [nxn], $G(q)$ is the nx1 vector of gravitational forces, and $C(q,\dot{q})\dot{q}$ is the nx1 vector of Coriolis and centrifugal forces. In (3.1) the matrix $C(q, \dot{q})$ satisfies

$$C(q, x)y = C(q, y)x \quad \text{for all } x, y \tag{3.2}$$

For revolute robots the matrices $M(q)$ and $C(q, \dot{q})$ are bounded, so

$$\| M^{-1}(q) \| \leq M_M, \quad \| C(q, x) \| \leq C_M \| x \| \quad \text{for all } q, x \tag{3.3}$$

4. Computed Torque Controller with Linear Observer

The computed torque controller (Luh et al., 1980), often proposed in order to achieve tracking control of robot manipulators, is given by

$$\tau = M(q)v + C(q, \dot{q})\dot{q} + G(q) \tag{4.1a}$$

$$v = \ddot{q}_d - K_d \dot{e} - K_p e \tag{4.1b}$$

where $q_d(t)$ represent the desired path, $e = (q - q_d)$ the tracking error, and the controller gains K_d, K_p are symmetric, positive definite matrices. The system (3.1) with controller (4.1) result in the error dynamics

$$\ddot{e} + K_d \dot{e} + K_p e = 0 \tag{4.2}$$

which shows that exponential stability of e can be guaranteed.

The computed torque controller (4.1) needs both position and velocity measurements. The necessity for velocity measurements may be removed by introducing an observer for the velocity signal $\dot{q}$. Dealing with a nonlinear system, a nonlinear observer appears to be most obvious. However, (4.1a) results in the linear system

$$\ddot{q} = v \tag{4.3}$$

for which an observer would be of the form

$$\dot{\hat{q}} = x + L_d(q - \hat{q}) \tag{4.4a}$$

$$\dot{x} = v + L_p(q - \hat{q}) \tag{4.4b}$$

where $\dot{\hat{q}}$ represents the estimate of the velocity $\dot{q}$, and the observer gains L_d, L_p are symmetric, positive definite matrices.

The foregoing suggests the design of a computed torque like controller combined with a linear observer, that is define

$$\tau = M(q)\hat{v} + C(q, \dot{q}_d)\dot{\hat{q}} + G(q) \quad (4.5a)$$

$$\hat{v} = \ddot{q}_d - K_d(\dot{\hat{q}} - \dot{q}_d) - K_p e \quad (4.5b)$$

$$\dot{\hat{q}} = y + L_d(q - \hat{q}) \quad (4.5c)$$

$$\dot{y} = \hat{v} + L_p(q - \hat{q}) \quad (4.5d)$$

5. Proof of Exponential Stability

For the exponential stability proof of the closed loop system (3.1) and (4.5) we define the following constants

$$\overline{\alpha}_1 = L_{d,m} - \lambda - 3\beta - \tfrac{1}{2}\left(\varepsilon^{-2} + \upsilon^{-2}\right)K_{d,M} \quad (5.1a)$$

$$\overline{\alpha}_2 = \lambda\,(L_{p,m} - \lambda\beta) \quad (5.1b)$$

$$\overline{\alpha}_3 = K_{d,m} - \lambda - 3\beta - \tfrac{1}{2}\varepsilon^2 K_{d,M} - \sum_{j=1}^{4} \gamma_{3j} V_0^{\frac{1}{2}} \xi_j^{-\frac{1}{2}} \quad (5.1c)$$

$$\overline{\alpha}_4 = \lambda\,(K_{p,m} - \lambda\beta - \tfrac{1}{2}\lambda\upsilon^2 K_{d,M}) \quad (5.1d)$$

with $\lambda > 0, \varepsilon > 0, \upsilon > 0$ constant, $\lambda\gamma_{31} = \gamma_{32} = \lambda\gamma_{33} = \gamma_{34} = \lambda M_M C_M$, $\beta = \|\dot{q}_d\|\, M_M C_M$, and where $K_{p,m}$, $K_{d,m}$, $L_{p,m}$, $L_{d,m}$ and $K_{d,M}$ denote lower and upper bounds respectively. Furthermore we have

$$\xi_1 = \tfrac{1}{2}(1 - \delta^{-2}), \quad \xi_2 = \tfrac{1}{2}(L_{p,m} + \lambda L_{d,m} - \lambda^2\delta^2)$$

$$\xi_3 = \tfrac{1}{2}(1 - \rho^{-2}), \quad \xi_4 = \tfrac{1}{2}(K_{p,m} + \lambda K_{d,m} - \lambda^2\rho^2) \quad (5.2)$$

where δ, ρ are some constants.

Lemma 2. *Given the robot dynamics (3.1) with the tracking controller (4.5). Assume that $\dot{q}_d$ is bounded. Then the closed loop system is exponentially stable if $\overline{\alpha}_i > 0$, $i = 1, ..., 4$.*

Before proving this result, we notice that indeed the required conditions for exponential stability can be satisfied:

Corollary 3. *There exist controller gains K_d, K_p and observer gains L_d, L_p such that $\overline{\alpha}_i > 0$, $i = 1, ..., 4$.*

Proof of Lemma 2. The error dynamics for the closed-loop system (3.1) and (4.5) are given by

$$M(q)(\ddot{\tilde{q}} + L_d\dot{\tilde{q}} + L_p\tilde{q}) = C(q, \dot{q}_d)\dot{\hat{q}} - C(q, \dot{q})\dot{q} \quad (5.3a)$$

$$M(q)(\ddot{e} + K_d\dot{e} + K_p e - K_d\dot{\tilde{q}}) = C(q, \dot{q}_d)\dot{\hat{q}} - C(q, \dot{q})\dot{q} \quad (5.3b)$$

where $\tilde{q} = (q - \hat{q})$ represents the observer error. Next, consider the following Lyapunov function

$$V(\dot{\tilde{q}}, \tilde{q}, \dot{e}, e) = \tfrac{1}{2}\dot{\tilde{q}}^T\dot{\tilde{q}} + \lambda\dot{\tilde{q}}^T\tilde{q} + \tfrac{1}{2}\tilde{q}^T(L_p + \lambda L_d)\tilde{q} +$$
$$+ \tfrac{1}{2}\dot{e}^T\dot{e} + \lambda\dot{e}^T e + \tfrac{1}{2}e^T(K_p + \lambda K_d)e \quad (5.4)$$

Because by assumption $\overline{\alpha}_i > 0$, $\exists\, \delta, \rho > 1$ such that ξ_i, $i = 1, ..., 4$ as defined in (5.2) satisfy $\xi_i > 0$, $i = 1, ..., 4$. So the function $V(.)$ in (5.4) satisfies condition (2.3) of Lemma 1.

The time-derivative of (5.4) is given by

$$\dot{V}(\dot{\tilde{q}}, \tilde{q}, \dot{e}, e) = -\dot{\tilde{q}}^T(L_d - \lambda I)\dot{\tilde{q}} - \lambda\tilde{q}^T L_p\tilde{q} - \dot{e}^T(K_d - \lambda I)\dot{e} - \lambda e^T K_p e +$$
$$+ (\dot{\tilde{q}} + \lambda\tilde{q} + \dot{e} + \lambda e)^T M^{-1}(q)(C(q, \dot{q}_d)\dot{\hat{q}} - C(q, \dot{q})\dot{q}) +$$
$$+ (\dot{e} + \lambda e)^T K_d\dot{\tilde{q}} \quad (5.5)$$

In the Appendix it is shown that (5.5) can be upper bounded by

$$\dot{V}(\dot{\tilde{q}}, \tilde{q}, \dot{e}, e) \leq -\overline{\alpha}_1\|\dot{\tilde{q}}\|^2 - \overline{\alpha}_2\|\tilde{q}\|^2 - \overline{\alpha}_4\|e\|^2 -$$
$$-(\alpha_3 - \gamma_{31}\|\dot{\tilde{q}}\| - \gamma_{32}\|\tilde{q}\| - \gamma_{33}\|\dot{e}\| - \gamma_{34}\|e\|)\|\dot{e}\|^2 \quad (5.6)$$

This time-derivative satisfies condition (2.4) of Lemma 1. Then using Lemma 1, the sufficient conditions $\overline{\alpha}_i > 0$, $i = 1, ..., 4$, are obtained.

Finally, from (2.6) it follows that

$$\dot{V}(\dot{\tilde{q}}, \tilde{q}, \dot{e}, e) \leq -\kappa_1\|\dot{\tilde{q}}\|^2 - \kappa_2\|\tilde{q}\|^2 - \kappa_3\|\dot{e}\|^2 - \kappa_4\|e\|^2 \quad (5.7)$$

which shows that

$$\dot{V}(\dot{\tilde{q}}, \tilde{q}, \dot{e}, e) \leq -\kappa V(\dot{\tilde{q}}, \tilde{q}, \dot{e}, e) \quad (5.8)$$

for some $\kappa > 0$, and the closed loop system is exponentially stable ■

Proof of Corollary 3. For $L_{p,m}$ large enough, $\overline{\alpha}_2 > 0$. Next observe from (5.4) that V_0 is proportional to $O(K_{d,M})$, so for $K_{d,m}$ large enough and for a suitable choice of ε, $\overline{\alpha}_3 > 0$. The next step is to choose $L_{d,m}$, $K_{p,m}$ such that $\overline{\alpha}_1 > 0$, $\overline{\alpha}_4 > 0$ respectively. ■

It can be shown that the presented control scheme (4.5) also guarantees exponential stability by taking other combinations of the velocity signals $\dot{\hat{q}}$ and $\dot{q}_d$ in the compensation for $C(q, \dot{q})\dot{q}$. So (4.5) belongs to a class of exponentially stable tracking controllers that need only position measurements.

6. Conclusion

An exponentially stabilizing tracking controller for robot systems was presented that needs only position measurements. The controller consists of a computed torque like method combined with an observer. The observer is linear because it effectively exploits the fact that the computed torque controller feedback linearizes the nonlinear robot dynamics in the case that velocity measurements are available.

Appendix

Note that

$$(\dot{e} + \lambda e)^T K_d\dot{\tilde{q}} \leq (\|\dot{e}\| + \lambda\|e\|)\,K_{d,M}\|\dot{\tilde{q}}\| \quad (A.1)$$

Rewriting the cross terms by completion of squares, i.e.

$$\|\dot{e}\|\,\|\dot{\tilde{q}}\| = -\tfrac{1}{2}(\varepsilon\|\dot{e}\| - \varepsilon^{-1}\|\dot{\tilde{q}}\|)^2 + \tfrac{1}{2}\varepsilon^2\|\dot{e}\|^2 + \tfrac{1}{2}\varepsilon^{-2}\|\dot{\tilde{q}}\|^2 \quad (A.2)$$

and similarly for $\|e\|\,\|\dot{\tilde{q}}\|$, an upper bound for (A.1) is given by

$$(\dot{e} + \lambda e)^T K_d\dot{\tilde{q}} \leq \tfrac{1}{2}K_{d,M}((\varepsilon^{-2} + \upsilon^{-2})\|\dot{\tilde{q}}\|^2 + \varepsilon^2\|\dot{e}\|^2 + \lambda^2\upsilon^2\|e\|^2) \quad (A.3)$$

Next, using (3.2) and (3.3) we have

$$(\dot{\tilde{q}} + \lambda\tilde{q} + \dot{e} + \lambda e)^T M^{-1}(q)(C(q, \dot{q}_d)\dot{\hat{q}} - C(q, \dot{q})\dot{q}) \leq$$
$$(\|\dot{\tilde{q}}\| + \lambda\|\tilde{q}\| + \|\dot{e}\| + \lambda\|e\|)\,M_M C_M(\|\dot{q}_d\|\,\|\dot{\tilde{q}}\| + \|\dot{q}_d\|\,\|\dot{e}\| + \|\dot{e}\|^2) \leq$$
$$\beta(3\|\dot{\tilde{q}}\|^2 + \lambda^2\|\tilde{q}\|^2 + 3\|\dot{e}\|^2 + \lambda^2\|e\|^2) + M_M C_M(\|\dot{\tilde{q}}\| + \lambda\|\tilde{q}\| + \|\dot{e}\| + \lambda\|e\|)\|\dot{e}\|^2$$

where we used again the completion of squares as given above. With these upper bounds on the (possibly) nonnegative terms in the Lyapunov derivative (5.5) we obtain (5.6), with the constants $\overline{\alpha}_i$, γ_{3i}, $i = 1, ..., 4$ as defined in (5.1).

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