Energy-Delay Trade-off of Wireless Data Collection in the Plane

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Abstract

We analyze the Pareto front of the delay of collecting data from wireless devices located in the plane according to a Poisson process and the energy needed by the devices to transmit their observations. Fundamental bounds on the energy-delay trade-off over the space of all achievable scheduling strategies are provided.

1 Introduction

We consider wireless devices placed in the plane according to a homogeneous Poisson process. The devices have noisy observations of an attribute, e.g. temperature. A collector, positioned at a random location in the plane, is interested in a reliable estimate of this attribute, which is obtained by combining a subset of available observations from the devices. We assume that time is slotted and that the devices are awake independently at random every slot. Our interest is in which wireless devices should be scheduled for transmission and at which time slot such that the collector obtains this reliable estimate.

We focus on two performance measures: i) the delay experienced by the collector until retrieving a reliable estimate of the attribute and ii) the transmission energy used by the wireless devices to transmit their observations, which is assumed to be an increasing function of the distance between the devices and the collector. One could reduce the energy used for transmission by waiting longer until a device located closer to the collector is available for transmission. Alternatively, one could select in every time slot an available device that is closest to the location of the collector, keeping the time needed to retrieve a reliable estimate of the attribute from the devices minimal. In this case, the transmission energy is expected to increase since the devices that are selected can be arbitrarily far from the location of the collector. A trade-off arises between the energy used for data transmission and the time needed for the collector to retrieve a reliable estimate of the attribute.

The problem of scheduling wireless devices located in the plane has been studied in [1–7]. In [1], data from a network of sensors is aggregated at a hub node, which transmits to a base station. Since the base station is located at a considerable distance from the network of sensors, the sensors take turns in serving as a hub node. This minimizes the transmission energy. Simultaneous sensor transmission are possible, provided that the sensors are sufficiently separated in space. In [2], energy minimization within a wireless sensor network is considered such that all the sensors in the network transmit their data within a time threshold. The authors determine the Pareto front of the transmission energy and delay. In our setting, only a subset of the wireless devices is necessary to transmit its observations. Moreover, we consider single-hop transmissions, whereas in [1, 2] multi-hop transmission is considered. Energy-latency trade-offs are analyzed in [3] using a data aggregation tree for wireless sensors.

The problem of collecting observations from a network of wireless devices is closely related to the problem of collecting fragments of a files from a wireless distributed storage network. The energy-delay trade-off for caches in the plane in which data is stored according to partitioning or coding strategies is analyzed in [4,5]. In [6], energy-delay and storage-delay trade-offs are analyzed. In [7], the effect of the topology and density of a network of wireless devices on the trade-off between the energy and the delay associated with the data collection, is analyzed. Closed form expressions for the energy used by a specific set of nodes to transmit to a client are provided in [8].

The contributions of this paper are:

- An inner bound on the Pareto front of the energy-delay tradeoff under a maximum delay constraint.
- Closed form expressions for the performance of a Greedy schedule that minizes the delay. It is shown that performance can expressed in terms of Mahonian numbers.
- Closed form expressions for the performance of a schedule that achieves minimum possible energy consumption under an expected delay constraint.
- We propose a scheduling strategy that achieves expected delay and outperforms any schedule that satisfies a maximum delay constraint.

The remainder of this paper is organizes as follows. In Section 2 we provide an exact problem statement. In Section 3 we present our inner bound. The Greedy schedule is analyzed in Section 4. Finally, we consider expected delay constraints in Section 5.

2 Problem Statement

Consider wireless devices located in the plane according to a homogeneous Poisson process with intensity λ. Let x denote the device that is the $x-$th closest neighbor of the collector in the plane. Let δ_x denote the distance between the collector and the device x.

Each wireless device makes an independent observation on an attribute θ. The observations are subject to independent and identically distributed additive Gaussian noise with variance σ^2, i.e. $X_i \sim \mathcal{N}(\theta, \sigma^2)$. A collector, located at a random location in the plane, is interested in retrieving a reliable estimate $\overline{X}$ with a variance that is below a threshold T. This estimate can be obtained by retrieving an arbitrary set of s observations from the wireless devices such that $\mathrm{Var}(\overline{X}) = \mathrm{Var}(\frac{1}{s}\sum_{i=1}^{s} X_i) = \frac{1}{s^2}\sum_{i=1}^{s} \mathrm{Var}(X_i) = \frac{\sigma^2}{s} < T$.

Every time slot, the wireless devices are awake with probability p, $0 < p < 1$ and asleep with probability $q = 1 - p$. The probability of being awake is independent over time and across the devices. We say that a wireless device is *eligible* to transmit if the device has not transmitted its observation to the collector in previous time slots. Transmission eligibility prevents the collector from receiving the same observation multiple times. A device may be scheduled for transmission only if it is awake and eligible. No restrictions on the maximal transmission range of the wireless devices are imposed, i.e a device at any random location in the plane can transmit its observation to the collector at a corresponding energy.

A centralized scheduler has knowledge about the position of the devices in the plane, their eligibility and whether they are awake at a specific time slot. Our interest is the fundamental performance limits that can be achieved by any possible schedule, The performance measures the we consider are:

1. The energy, denoted by P_s needed by the devices to transmit to the collector. Let the energy used by device x to transmit to the collector be δ_x^a, where $a \geq 1$ is a fixed parameter. The form of the energy function is motivated by, for instance,

the minimum power $P = (e^{2R} - 1)\delta^a$ required to transmit at a fixed rate R over a distance δ given that the capacity of a AWGN channel is $1/2\log(1 + P\delta^{-a})$.

Let $S = \{x_1, ..., x_s\}$ be the set of devices that transmitted their observations to the collector according to a centralized schedule, where the transmission order of the s devices can be any of the $s!$ possible sequences.

The total energy used for transmission by a set $S = \{x_1, ...x_s\}$ of devices such that a reliable estimate is achieved at the collector is:

$$P_s(\delta_{x_1}, .., \delta_{x_s}) = \sum_{i=1}^{s} \delta_{x_i}^a. \tag{1}$$

We are interested in the expected energy $\mathbb{E}[P_s]$ used by the wireless devices to transmit s observations to the collector, where the expectation is taken over the randomness in the Poisson process according to which the devices are located in the plane and the randomness in the awake/asleep status of the wireless devices.

2. The retrieval delay W_s. The retrieval delay W_s is the time until the collector retrieves s observations based on which a reliable estimate is retrieved. Initially we consider a maximum delay within which a collector must have retrieved a reliable estimate. Later, we relax this constraint and consider also the expected delay.

Consider the multi-objective optimization problem which aims at minimizing the energy and the retrieval delay under all possible centralized schedules. In general, a single point that simultaneously minimizes both performance measures does not exist, in which case we do not have a unique optimal solution. Therefore, we characterize the Pareto front [9] of the energy and delay performance measures.

We will make use of the Gamma function, which for $x > 0$ can be represented as $\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}dt$.

3 Inner bound under a maximum delay constraint

In this section we consider a maximum retrieval delay constraint. We provide a lower bound on the expected energy that is required to meet a maximum delay constraint. As a result we provide an inner bound on the energy-delay Pareto front under a maximum delay constraint.

The lower bound is constructed by considering the best possible schedule that could be obtained if the awake/asleep status of the devices would be known ahead of time. Such a schedule corresponds to a minimum matching in a bipartite graph that we describe next.

We consider time slots $1, 2, .., t$, where t is the maximal delay allowed, and the devices that are awake and eligible during those time slots. A matching is an assignment of devices to time slots. In each time slot of the matching, the devices transmit their observations to the collector. Let the weight of a matching be the energy used for transmission according to this matching. We are interested in a matching of size s of minimal weight. This matching provides a transmission schedule for the devices such that the collector retrieves a reliable estimate within a delay of at most t slots.

More precisely, let $G((T, N), E)$ be a bipartite graph with left vertices $T = \{1, \ldots, t\}$ and right vertices $N = \mathbb{N}$, edges $E \subset T \times N$ and weight function $w : E \to \mathbb{R}$. Let T denote the time slots in which the observations are gathered and N the devices in the plane. Devices N are sorted according to their distance to the collector. An edge (i, j) exists if node j is awake in time slot i, i.e. it exists with probability p. Let the weight of an edge w_e, $e = (i, j)$, be the energy needed for device i to transmit to the collector.

In order to compute the actual inner bound, we need to find on $G((T,N),E)$, a minimum matching A of size s. We proceed in two steps. First, we compute a minimum matching A' on graph $G((T,N),E)$ using the Hungarian algorithm [10]. Clearly, the size of A' is t. Next, we determine a matching A of minimum weight of size s starting from the matching A'. More precisely, we consider the matching A' and eliminate the $t-s$ largest weight edges. We are left with a matching A of minimum weight and of size s. We refer to this two step algorithm as the modified Hungarian algorithm. The optimality of the modified Hungarian algorithm follows from the next result, which we state without proof due to space constraints.

Lemma 1. *A minimum weight matching A of size s on $G((T,N),E)$ can be found by eliminating the $t-s$ largest weight edges from the matching given by the Hungarian algorithm applied to $G((T,N),E)$.*

For a given placement of nodes the modified Hungarian algorithm provides the minimum energy at a given delay constraint. The average expected energy follows from taking an average over the node placement. We illustrate the result in Figure 1 in Section 5.

4 Energy at minimum delay

In the previous section we provided a bound on the energy consumption at any given maximum delay constraint $t \geq s$. In this section we provide an exact closed form result for the minimum energy consumption that can be obtained given delay constraint $t = s$.

In particular, we consider a Greedy schedule, which schedules every time slot the closest awake and eligible device. Clearly, the delay in this case is minimal, i.e. $W_s = s$. Also, it is clear that given the delay constraint one cannot achieve lower energy consumption.

Our analysis of the Greedy schedule is based on analyzing the probability that a given set $M = \{x_1, .., x_s\}$, $x_i \in \mathbb{N}, i = \{1, .., s\}$ of devices is scheduled for transmission, where x_i is the x_i-th closest device and $x_1 < x_2 < .. < x_s$.

To illustrate the main idea of our result, consider the case that $s = 3$ and that these devices transmit in order x_1, x_2, x_3. Device x_1 transmits in the first slot if no other device is awake and closer than x_1^{st} device. Also, x_1 has to be awake in the first slot. Thus, x_1 transmits in the first slot with probability $q^{x_1-1}p$. Similarly, in the second slot, x_2 transmits if x_2 is awake and all $x_2 - 1$ closer devices to the collector than x_2 are not awake. Also, recall that x_1 is closer than x_2 with respect to the location of the collector and that x_1 has transmitted in the first slot. Thus, x_2 transmits in the second slot with probability $q^{x_2-1-1}p$. Lastly, x_3 transmits in the third slot if x_3 is awake and all $x_3 - 1$ nodes closer to the collector are not awake. Again, recall that both x_1 and x_2 have transmitted previously. Hence, x_3 transmits in the third slot with probability $q^{x_3-1-2}p$. Therefore, the probability that sequence (x_1, x_2, x_3) transmits under Greedy is:

$$p^3 q^{(x_1+x_2+x_3-1-2-3)} = p^3 q^{(x_1+x_2+x_3-6)}.$$

The general result depends on the order in which the devices transmit. Therefore, we need to notion of the disorder of a permutation.

Definition 1. *The disorder of a permutation is the number of pairs of entries that appear in reversed order. Formally, given a sequence $(x_1, x_2, ..., x_n)$, with $x_1 < x_2 < .. < x_n$, a permutation $(\overline{x_1}, \overline{x_2}, .., \overline{x_n})$ has disorder $\sum_{i=1}^{n} \sum_{j>i}^{n} \mathbb{1}_{(\overline{x_i} > \overline{x_j})}$.*

For example, the disorder of permutation $(1, 2, 4, 3, 5)$ is one since elements 3 and 4 are in disorder. The disorder of $(2, 1, 4, 3, 5)$ is two since elements 1 and 2 are in disorder, as well as elements 3 and 4.

To illustrate the relevance of a disorder of a transmission sequence, we now consider the transmission sequence (x_1, x_3, x_2), which has disorder one. Again, device x_1 transmits in the first slot with probability $q^{x_1-1}p$ and device x_2 transmits in the third slot with probability $q^{x_2-1-1}p$. The only change is in the probability of device x_3 transmitting in the second slot. Since x_3 transmits before x_2 and x_2 is closer to the collector, the probability of x_3 transmiting in the second slot depends on the awake/asleep status of x_2. More precisely, for x_3 to transmit in the second slot, x_2 has to be asleep in this slot. Thus, x_3 transmits in the second slot with probability $q^{x_2-1-2+1}p$. Therefore, the probability that sequence (x_1, x_3, x_2) transmits under Greedy is:

$$p^3 q^{(x_1+x_2+x_3-6+1)}.$$

Observe that in the above expression the term +1 corresponds exactly to the disorder of the transmission sequence. Note also that there are 2 sequences with disorder 1, 2 sequences with disorder 2 and one sequence with disorder 3. By generalizing the above derivations we obtain the following expression for $p_{x_1,x_2,..,x_s}$, the probability that x_1, x_2 and x_3 are transmitting:

$$\begin{aligned} p_{x_1,x_2,x_3} &= p^3 q^{(\sum_{i=1}^3 x_i - 6)} + 2p^3 q^{(\sum_{i=1}^3 x_i - 5)} + 2p^3 q^{(\sum_{i=1}^3 x_i - 4)} + p^3 q^{(\sum_{i=1}^3 x_i - 3)} \\ &= p^3 q^{(\sum_{i=1}^3 x_i - 6)}(1+q)(1+q+q^2). \end{aligned} \tag{2}$$

In general, the probability that a sequence of s devices transmits under Greedy can be expressed in terms of the disorder of that sequence. In addition, the number of sequences that have a given disorder is given by the Mahonian numbers, which are the coefficients in the expansion of $\Pi_{m=0}^{s-1}(1 + x + ... + x^m)$ [11]. This leads to the following theorem, which again we state without proof.

Theorem 1. *The probability that set* $(x_1, x_2, ..., x_s)$ *of devices transmits under Greedy scheduling is:*

$$p_{x_1,x_2,..,x_s} = C_s \cdot q^{(x_1+x_2+..+x_s)},$$

where $C_s = p^s q^{-s(s+1)/2} \Pi_{m=0}^{s-1}(1 + q + ... + q^m)$.

We can now determine the expected energy when Greedy scheduling is used.

Theorem 2. *The expected energy needed to transmit* s *observations to the collector when Greedy scheduling is used, is:*

$$\mathbb{E}[P_s] = \frac{C_s}{(s-1)!} \sum_{x_1=1}^{\infty} \sum_{\substack{x_2=1 \\ x_2 \neq x_1}}^{\infty} .. \sum_{\substack{x_s=1 \\ x_s \neq x_{s-1} \\ \vdots \\ x_s \neq x_1}}^{\infty} \frac{\Gamma(\frac{a}{2} + x_1)}{(\lambda\pi)^{a/2}\Gamma(x_1)} q^{x_1+..+x_s},$$

where $C_s = p^s q^{-s(s+1)/2} \Pi_{m=0}^{s-1}(1 + q + ... + q^m)$.

Proof. Let e_{x_i} denote the expected energy needed for the x_i^{th} closest device to transmit its observations to the collector. From [8] we have

$$e_{x_i} = \frac{\Gamma(a/2 + i)}{(\lambda\pi)^{a/2}\Gamma(i)}. \tag{3}$$

Let β_{x_i} is the probability that device x_i transmits under Greedy schedule. Then, the expected energy used for transmission under Greedy can be written as

$$\mathbb{E}[P_s] = \sum_{x_i=1}^{\infty} e_{x_i}\beta_{x_i} = \frac{1}{(s-1)!}\sum_{x_1=1}^{\infty} .. \sum_{\substack{x_s=1\\ x_s\neq x_{s-1}\\ \vdots\\ x_s\neq x_1}}^{\infty} e_{x_1}p_{x_1,x_2,..,x_s}.$$

The last equality in the above expression is obtained by summing over all sets that include x_i. The result follows from (3) and Theorem 1. □

The expression in Theorem 2 provides a convenient means of computing $\mathbb{E}[P_s]$. As an example, consider $s = 3$ and $a = 2$:

$$\begin{aligned}
\mathbb{E}[P_3] &= \frac{C_3}{2!}\sum_{x_1=1}^{\infty}\sum_{\substack{x_2=1\\ x_2\neq x_1}}^{\infty}\sum_{\substack{x_3=1\\ x_3\neq x_2\\ x_3\neq x_1}}^{\infty}\frac{x_1}{\lambda\pi}q^{x_1+x_2+x_3}\\
&= \frac{C_3}{2\lambda\pi}\sum_{x_1=1}^{\infty}x_1q^{x_1}\sum_{\substack{x_2=1\\ x_2\neq x_1}}^{\infty}q^{x_2}\left(\sum_{x_3=1}^{\infty}q^{x3}-q^{x_2}-q^{x_1}\right)\\
&= \frac{C_3}{2\lambda\pi}q^3\left(\frac{1}{p^4}-\frac{2}{p(1-q^2)^2}-\frac{1}{p^2(1-q^2)}+\frac{2}{(1-q^3)^2}\right).
\end{aligned}$$

5 The benefit of an expected delay constraint

Observe that the minimum expected energy consumption is achieved by scheduling only the s closest devices. Since for any fixed number of time slots there is a positive probability that at least one of these devices is asleep in all these time slots, it is not possible to meet a maximum delay constraint. In this section we relax the delay constraint and analyze the performance of schedules that satisfy an expected delay constraint.

First, we determine the point on the Pareto front given by the minimal expected energy under all achievable schedules and the corresponding expected delay. As indicated above, the minimal expected energy under all achievable schedules is the energy needed for the closest s devices, with respect to the location of the collector in the plane, to transmit their observations.

Theorem 3. *The point on the Pareto front given by the minimal expected energy $\mathbb{E}[P_s^{\min}]$ and its corresponding expected delay $\mathbb{E}[W_s]$ under all achievable schedules:*

$$\left\{(x,y) = \left(\frac{\Gamma(s+\frac{a}{2}+1)}{(\lambda\pi)^{a/2}\cdot(\frac{a}{2}+1)\Gamma(s)},\sum_{i=1}^{s}\frac{1}{1-q^{s-i+1}}\right), x = \mathbb{E}[P_s^{\min}] \text{and } y = \mathbb{E}[W_s]\right\}.$$

Proof. Using the expected energy needed for the x_i^{th} neighbor of the collector in the plane, as in (3), and summing over all s expected energy transmissions [8]:

$$\mathbb{E}[P_s^{\min}] = \sum_{i=1}^{s}\frac{\Gamma(a/2+i)}{(\lambda\pi)^{a/2}\Gamma(i)} = \frac{\Gamma(s+\frac{a}{2}+1)}{(\lambda\pi)^{a/2}(\frac{a}{2}+1)\Gamma(s)}. \tag{4}$$

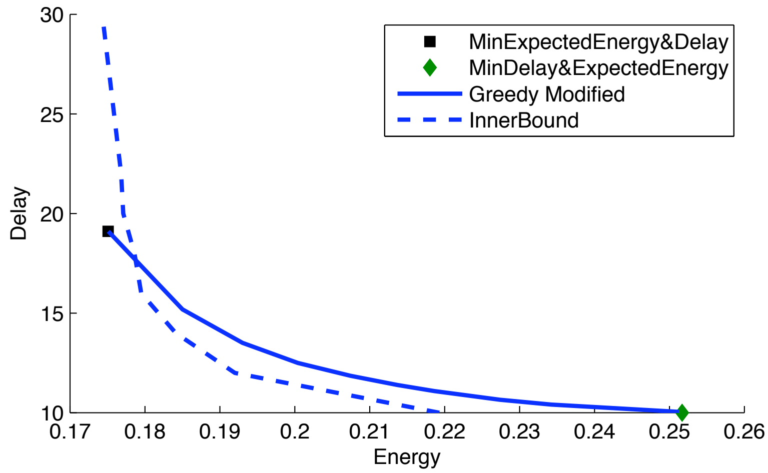

Figure 1: Pareto Front, $\lambda = 100, p = 0.2, s = 10, a = 2$.

The corresponding expected delay is computed as follows. The scheduling scheme is independent and identical over time. Thus, the time until the collector retrieves the i^{th} observation, given that it already received $i-1$ observations, can be viewed as the time until a first success in a Bernoulli trial is achieved, where the success probability T_i is the probability that at least one eligible device is awake. Thus, the probability of success is $T_i = 1 - q^{s-i+1}$. Then, the expected delay is:

$$\mathbb{E}[W_s] = \sum_{i=1}^{s} \frac{1}{T_i} = \sum_{i=1}^{s} \frac{1}{1-q^{s-i+1}}. \tag{5}$$

□

Next, we determine the point on the Pareto front given by the minimal delay and the corresponding minimal expected energy under any achievable schedule. The following result directly follows from Theorem 2.

Corollary 1. *The point on the Pareto front given by the minimal delay and its corresponding minimal expected energy* $\mathbb{E}[P_s]$ *under all achievable schedules:*

$$\left\{ (x,y) = \left(\frac{C_s}{(s-1)!} \sum_{x_1=1}^{\infty} .. \sum_{\substack{x_s=1 \\ \vdots \\ x_s \neq x_1}}^{\infty} \frac{\Gamma(\frac{a}{2}+x_1)}{(\lambda\pi)^{a/2}\Gamma(x_1)} q^{x_1+..+x_s}, s \right), x = \mathbb{E}[P_s], y = W_s^{\min} \right\},$$

where $C_s = p^s q^{-s(s+1)/2} \Pi_{m=0}^{s-1}(1 + x + ... + x^m)$.

Finally, we consider a class of expected delay strategies that interpolate between Theorem 3 and Corollary 1. The strategy considers the $N \geq s$ closest neighbors. In each time slot the closest eligible and awake device among these N devices is scheduled to transmit. We denote this strategy as Modified Greedy. Note that for $N = s$, we have the case of the minimal expected energy and the corresponding expected delay of Theorem 3. For $\lim N \to \infty$ we approach the minimal delay and the corresponding expected energy of Corollary 1.

Figure 1 shows the Pareto front of the expected energy and delay and the inner bound of the Pareto front, as described in Section 3. An important observation from Figure 1 is that for small energy consumption, schedules that satisfy only the relaxed, expected delay constraint, can outperform all schedules that satisfy a maximum delay constraint.

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