A Robust Cross Coding Scheme for OFDM Systems

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***Abstract*—In wireless OFDM-based systems, coding jointly over all the sub-carriers simultaneously performs better than coding separately per sub-carrier. However, the joint coding is not always optimal because its achievable channel capacity (i.e. the maximum data rate) is inversely proportional to the dynamic range of the channel. In this paper, we propose a novel cross coding scheme to increase the maximum data rate (i.e. the noise floor) achieved in frequency selective channels. The proposed OEC-II is based on fountain codes. The key element in OEC-II is to exchange the modulation order with the discarded sub-carriers. In such case, the system can increase the noise floor by only taking care of the sub-carriers with high energy. By transmitting a fountain-encoded packet over a single sub-carrier, the packets can be discarded if they have encountered a low-energy channel. Fountain codes can recover the source data by only using the surviving packets. With the same effective transmission data rate (i.e. 10.8 Mbits/s), OEC-II with QAM-16 has a SNR gain of around 7 dB over the FEC layer from the IEEE 802.11a standard, around 8 dB over the FEC layer from the IEEE 802.11n standard, around 7 dB over OEC-II with QPSK and around 3 dB over OEC-II with QAM-64.**

I. INTRODUCTION

The wireless channel is a hostile environment and often modeled as a frequency selective fading channel. *Orthogonal Frequency Division Multiplexing* (OFDM) [1] [2] [3] has become a fruitful approach to communicate over such channels, as it eases the equalizer in the receiver [4] [5]. To overcome the effects of noise and interference encountered in the transmission of the signal through the wireless channel, *Error Correcting Codes* are used as a means of utilizing the wireless channel at full capacity [6].

With OFDM, coding is done in the frequency domain. Over a finite block length, coding jointly over all the sub-carriers yields a smaller error probability than can be achieved by coding separately over the sub-carriers at the same rate [4]. The joint coding scheme employs the fact that sub-carriers with high energy can compensate for those in deep fading. OFDM signals in the time domain are distorted by the noise and interference during the transmission, but the signal detection (i.e. error correcting decoding) happens in the frequency domain. The Fourier Transform gives us the constant level of noise floor in the frequency domain for all the sub-carriers. In the frequency selective fading channel, some part of the channel has high energy but some parts suffer deep fading. Because the joint coding scheme treats each sub-carrier equally important, the maximum noise floor depends on the dynamic range of the channel. Higher dynamic range means lower noise floor level. The maximum data rate over the channel is limited by the noise floor.

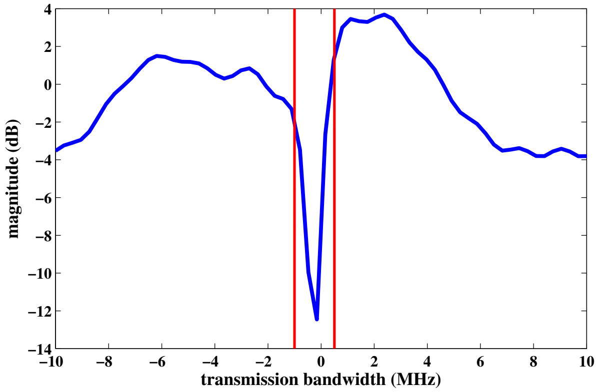

Fig. 1. Example of the baseband transfer function of a frequency selective Channel model A [7].

To relieve the above limit, the question we should answer is: *Can we design a system in such a way that sub-carriers in deep fading can be ignored but that we still achieve reliable communications?* In many frequency selective fading channels, only a small part of the channel has a high dynamic range. Let us take Fig. 1 as an example, the dynamic range of the whole channel is around 16 dB. As we can see, the deep fading part occupies a large part of the dynamic range, but deep fading only happens in the frequency band of $-1 \sim 0.5$ MHz. By discarding this 1.5 MHz sub-band, the dynamic range of the channel is reduced by 50% (i.e. around 8 dB). Obviously, the joint coding scheme does not allow us to discard it. But with fountain codes, we can.

In this paper, we propose opportunistic error correction based on fountain codes that allows us to discard those parts of the channel in deep fading. Fountain codes can reconstruct the original source file by collecting enough fountain-encoded packets [8]. It does not matter which packet is received. We only need to collect enough packets. In other words, fountain-encoded packets are independent with respect to each other [8] [9]. Because fountain codes are designed for erasure channels, error correcting codes are required to transfer the noisy wireless channel into an erasure channel. If a fountain-encoded packet is transmitted over a single sub-carrier, we can reduce the dynamic range of the channel in two possible ways. **Option I** is to exchange the code rate of error correcting codes with the number of sub-carriers in deep fading; **Option II** is to sacrifice the sub-carriers in deep fading by using a higher order modulation. In both cases, the receiver is allowed to discard

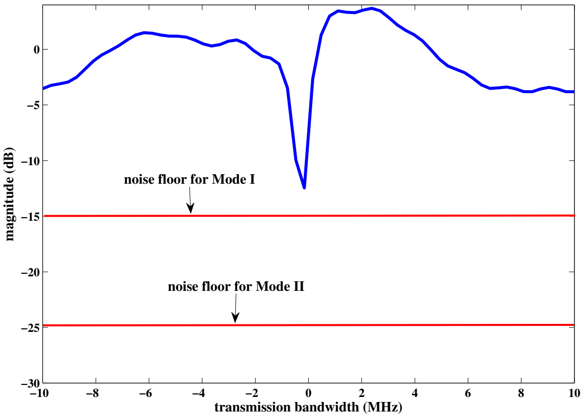

Fig. 2. Example of the difference in the noise floor required by the joint coding scheme and the separate coding scheme.

fountain-encoded packets over the sub-carriers in deep fading. Correspondingly, the level of the noise floor increases.

There are two coding steps in opportunistic error correction. First, source packets are coded jointly over all the sub-carriers by fountain codes; then, each fountain-encoded packet is encoded individually over a single sub-carrier by error correcting codes. This is different from the traditional FEC layer. Here, we call opportunistic error correction based on Option I as OEC-I and the one with Option II as OEC-II. In [10], we have investigated the performance of OEC-I. With the same throughput, simulation results showed that OEC-I has a SNR gain of 7.5 dB comparing to the IEEE 802.11a system [10]. However, the performance of OEC-II is still not clear. In this paper, we study OEC-II with respect to the mitigation of noise and interference.

The paper is organized as follows. Opportunistic error correction is first depicted where we explain why this new method suits for OFDM systems and how it works. In section III, we describe system model by showing how we apply this novel scheme in OFDM systems. After that, we compare its performance with FEC layers from WLAN systems over Channel Model A [7] in the simulation. The paper ends with a discussion of conclusions.

II. Opportunistic Error Correction

In OFDM systems, coding is performed in the frequency domain. Source data can be coded either jointly or separately over all the sub-carriers. That depends on how an encoded packet is transmitted:

- Mode I is to transmit a packet over all the sub-carriers. In such case, the coding is done jointly over all the sub-carriers.
- Mode II is to transmit a packet over a single sub-carrier. With this mode, the coding is performed separately over all the sub-carriers.

Both transmission modes have advantages and disadvantages. Mode I takes the advantage of the fact that sub-carriers with high energy can compensate for those in deep fading. That lets Mode I endure a higher noise floor than Mode II to achieve the same quality of communication. Assuming that some packets are transmitted over a wireless channel as shown in Fig. 1 and that a packet is received correctly when $\mathrm{SNR} \geq 12$ dB. In this example, the maximum noise floor for Mode I is around -15 dB as seen in Fig. 2. But the maximum noise floor for Mode II is around -25 dB, which is determined by the sub-carrier with the lowest energy. However, using Mode II allows the receiver to predict whether the received packet is decodable at a high probability, as each sub-carrier can be modeled as a flat-fading channel. The packets transmitted over the sub-carriers in deep fading can be discarded before going through the whole receive chain. That saves the processing power. Unfortunately, this does not apply to Mode I.

Although Mode I affords a higher noise floor than Mode II, it is still not optimal as there might be a waste of processing power. Also, because each sub-carrier is equally important for Mode I, the noise floor in Mode I is limited to the dynamic range $\mathcal{D}$ of the channel. $\mathcal{D}$ is defined as the dynamic range of the channel. As all the sub-carriers suffer the same level of noise floor, sub-carriers with high energy can not be used at full capacity. Correspondingly, there is a waste of channel capacity in the joint coding scheme. To mitigate this, one way is to reduce $\mathcal{D}$ by neglecting some part of the channel in deep fading. In such case, the noise floor can be increased (i.e. the channel capacity can be increased). To achieve this, we propose a novel cross coding scheme based on fountain codes, as it combines the joint coding scheme and the separate coding scheme together.

Opportunistic error correction is specially designed for OFDM systems. It is based on fountain codes. A fountain code has a similar property to a fountain of water: when you fill a cup from the fountain, you do not care about which drops of water fall in, you only want your cup filled enough to quench your thirst [9]. In other words, fountain-encoded packets are independent to each other. To transmit fountain-encoded packets over the noisy wireless channel, error correcting codes and a certain modulation type have to be applied in each packet. By transmitting a packet over a single sub-carrier, the dynamic range of the channel can be reduced by exchanging the code rate of error correcting codes (i.e. OEC-I) or the modulation order (i.e. OEC-II) with the number of discarded sub-carriers. In such case, we increase the capacity of sub-carriers with high energy by sacrificing those in deep fading. In [10], we have shown that OEC-I increases the total channel capacity (i.e. noise floor) in comparison with the traditional joint coding scheme. In this paper, we investigate whether this also applies to OEC-II.

Fig. 3 shows how OEC-II works. With a fountain code, the transmitter can generate a potential number of fountain-encoded packets. In this paper, the transmitter generates $\mathrm{N_t}$ fountain-encoded packets. Then, each packet is encoded by an error correcting code to make wireless channels behave like an erasure channel. Afterwards, each packet is modulated by

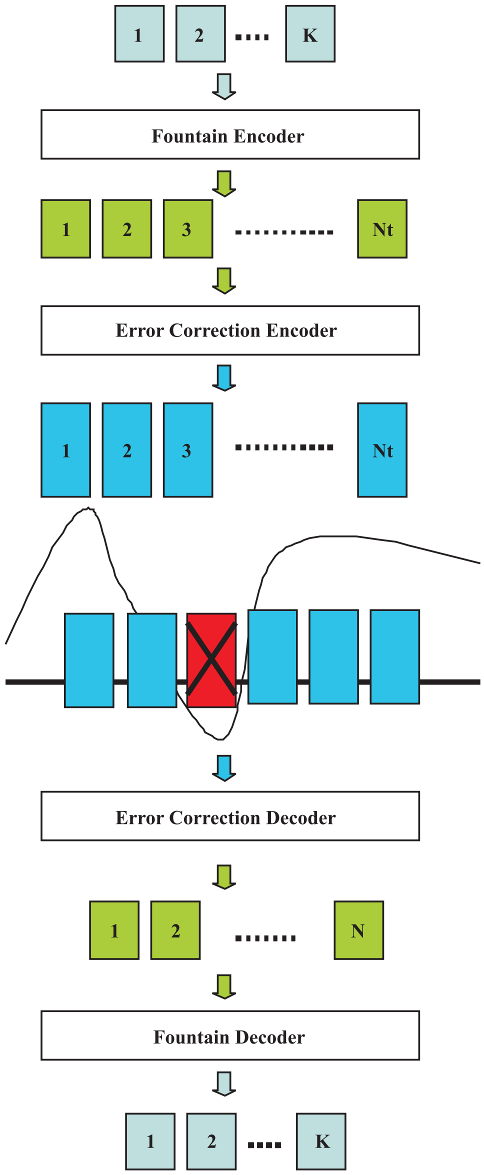

Fig. 3. OEC-II for OFDM systems.

a relatively high order mapping scheme, then transmitted over a single sub-carrier. Thus, multiple packets are transmitted simultaneously, using frequency division multiplexing.

At the receiver side, the channel is first estimated. With the channel knowledge, the receiver makes a decision about which packets can go through the whole receiving chain. That gives the advantage of the separate coding scheme (i.e. save processing power). We assume that $\mathrm{N_r}$ ($\mathrm{N_r} \leq \mathrm{N_t}$) fountain-encoded packets can go through the error correction decoding. Packets only survive if they succeed in the error correction decoder. The fountain decoder can reconstruct the original file by collecting enough packets. The number of fountain-encoded packets N ($\mathrm{K} < \mathrm{N} \leq \mathrm{N_r}$) required at the receiver is slightly larger than the number of source packets K [8]:

$$\mathrm{N} = (1 + \varepsilon)\mathrm{K} \tag{1}$$

where ε is the percentage of extra packets and is called the overhead. For high throughput, ε is expected to be as small as possible. However, fountain codes (e.g. *Luby-Transform* (LT) codes [11]) require a large ε for small block size by only using the *message-passing* algorithm [12] to decode. For example, the practical overhead of LT codes is 14% when $\mathrm{K} = 2000$, which limits its application in the practical system [13]. In [10], we have shown that the overhead is reduced to 3% by combining the messaging-passing algorithm and Gaussian elimination to decode LT codes for $\mathrm{K} \geq 500$ at the expense of a higher complexity.

In this paper, we use LT codes in the proposed error correction layer. This new scheme is generic: any fountain codes (e.g. LT codes, Raptor codes [14] and online codes [15]) can be applied in it. To have a small ε for small block size, we choose to decode LT codes by combining the message passing algorithm and Gaussian elimination in this paper.

III. SYSTEM MODEL

OEC-II can be applied in the OFDM system. In this paper, we take the IEEE 802.11a system as an example of OFDM systems. The FEC layer in the current 802.11a system is based on *Rate Compatible Punctured Codes* (RCPC). These codes only have good performance for random bit errors. Interleaving is employed to reduce burst errors. Each encoded packet is transmitted over all the sub-carriers, as the sub-carriers with high-energy can compensate for those with low-energy. Although this solution works well in practical systems, it is not optimal due to the followings:

- The maximum level of the noise floor endured by the current FEC layer is dependent on the dynamic range of the channel.
- It is not beforehand known whether the received packet is decodable at a high probability. That leads to a waste of processing power.

The above problems can be solved by OEC-II as depicted in Fig. 4. The key idea is to generate additional packets by the fountain encoder. First, source packets are encoded by a LT code which is designed by choosing the parameters $c = 0.03$ and $\sigma = 0.3$ [8]. Then, a 12-bit CRC checksum is added to each fountain-encoded packet before the LDPC encoding is applied. In this paper, we choose the (324,648) LDPC code defined in the IEEE 802.11n standard [16] to encode each fountain-encoded packet. On each sub-carrier, a fountain-encoded packet is transmitted.

At the receiver side, we assume that synchronization and channel estimation are perfect in the simulation. If the SNR of the sub-carrier is equal to or above the threshold, the received fountain-encoded packet will go through the LDPC decoding, otherwise it will be discarded. This means that the receiver is allowed to discard low-energy sub-carriers (i.e. packets) to lower the processing power consumption. After the LDPC decoding, the CRC checksum is used to discard the erroneous packets. As only packets with a high SNR are processed by the receiver, this will not happen often. When the receiver has collected enough fountain-encoded packets, it starts to recover the source data.

IV. PERFORMANCE ANALYSIS

In this section, we analyze the performance of OEC-II by comparing 5 FEC layers. The IEEE 802.11a standard allows 10% packet loss. Because fountain codes promise an error-free communication quality, we compare those FEC layers under the condition of the same effective throughput (i.e. the same effective transmission data rate of 10.8 Mbits/s). **FEC I** is a RCPC with interleaving from the IEEE 802.11a standard with $R_{\mathrm{I}} = 0.5$ and QPSK as the modulation scheme. In **FEC II**, we use the (324,648) LDPC code from the IEEE 802.11n standard to encode source bits. Each encoded packet is mapped to QPSK symbols then transmitted over all the sub-carriers. Correspondingly, FEC II is a joint coding scheme as well. One packet is 54 Bytes[1], so 10% packet loss is equivalent to a BER of 2.3×10^{-4}. **FEC III** is OEC-II based on fountain codes. FEC III employs the same modulation scheme as FEC I and II. In FEC III, the code rate of LT code R_{LT} is around 0.97 and the code rate of LDPC plus CRC $R_{\mathrm{LDPC-CRC}}$ is around 0.48, so its total code rate R_{III} is around 0.47. FEC III is allowed to discard around 4% (i.e. $0.5 \times 0.9/0.47$) of data sub-carriers. **FEC IV** is OEC-II with QAM-16. In this case, around 52% (i.e. $(2 \times 0.5 \times 0.9)/(0.47 \times 4)$) of data sub-carriers can be discarded. **FEC V** is OEC-II with QAM-64 which allows us to discard around 68% (i.e. $(2 \times 0.5 \times 0.9)/(0.47 \times 6)$) of data sub-carriers. In addition, 48 sub-carriers are used to transmit data as defined in [17].

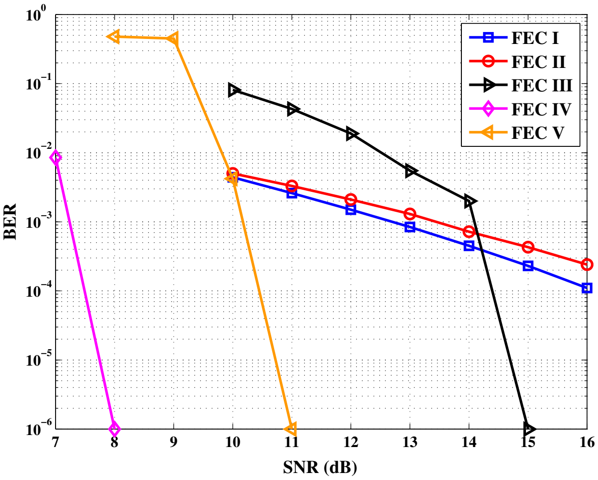

Fig. 5. Performance comparison between FEC I, II, III, IV and V at the data rate of 10.8 Mbits/s over Channel Model A. FEC III, IV and V can achieve error free when the fountain decoder receives enough packets. In the graph, $\mathrm{BER} = 10^{-6}$ corresponds to error-free communications.

Fig. 5 are the simulation results. Each simulation point is based on more than 30 million source bits transmitted over Channel Model A [7]. FEC I has a BER of 2.3×10^{-4} (i.e. 10% packet loss) when SNR = 15 dB; while FEC II reaches this target BER at SNR = 16 dB. The error-free communication quality can be achieved by FEC III at SNR = 15 dB, FEC IV at SNR = 8 dB and FEC V at SNR = 11 dB. Therefore, FEC IV (i.e. OEC-II with QAM-16) performs the best, which is followed by FEC V (i.e. OEC-II with QAM-64) then by FEC III (i.e. OEC-II with QPSK). That shows a tradeoff between the performance of OEC-II (i.e. the total capacity of frequency selective channels) and the modulation order. OEC-II is to exchange the order of modulation with the discarded sub-carriers. Higher order means a lower level of noise floor, but also more discarded sub-carriers which can increase noise floor. As each fountain-encoded packet is transmitted over a single sub-carrier, the tradeoff between the performance of OEC-II and the modulation order can be decided from the channel information.

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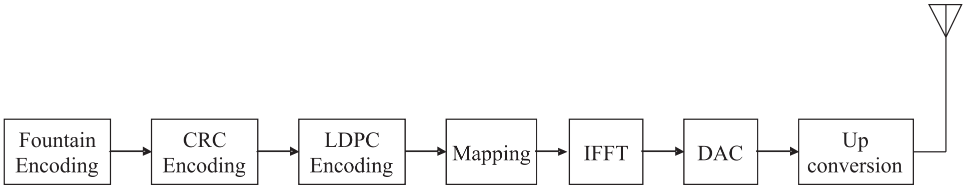
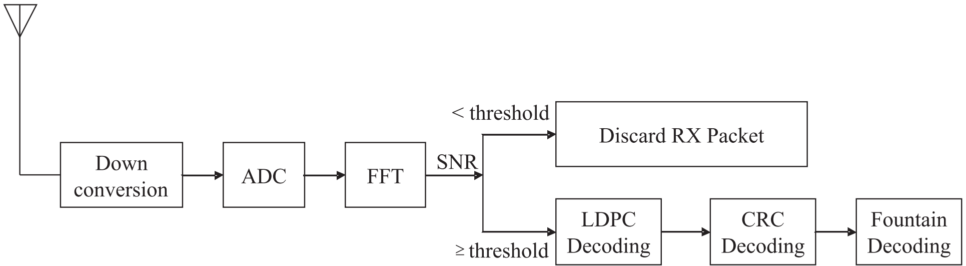
[1] a common value.



(a) Transmitter



(b) Receiver

Fig. 4. The proposed IEEE 802.11a system with OEC-II: transmitter (top) and receiver (bottom). As we assume perfect channel estimation, it is left out in the diagram.

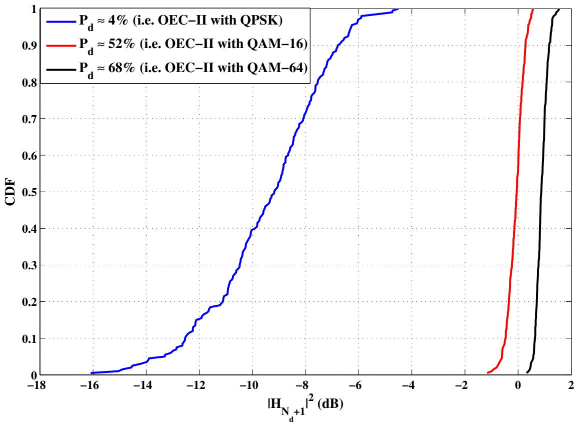


Fig. 6. The cumulative distribution function (CDF) of $|\mathrm{H}_{\mathrm{N_d}+1}|^2$ for different $\mathrm{P_d}$. The curves are based on the 2400 random channels in Fig. 5.

In Fig. 5, one burst data is transmitted over 12 channel conditions. Equivalently, one burst data is transmitted over 576 sub-carriers in total. Here, we define $\mathrm{N_d}$ is the number of discarded sub-carriers per data burst and $\mathrm{P_d}$ is the percentage of discarded sub-carriers which can be expressed as:

$$\mathrm{P_d} = \frac{\mathrm{N_d}}{576} \tag{2}$$

We denote each sub-carrier by $\mathrm{H_k}$ and we sort those 576 sub-carriers by energy, so we have:

$$\left|\mathrm{H_{(k+1)}}\right| > |\mathrm{H_k}| \quad \mathrm{k} = 1, 2, \ldots, 576 \tag{3}$$

Since we give the same noise floor to every data burst, the maximum level of noise floor afforded by OEC-II depends on the minimum value of $\left|\mathrm{H}_{(\mathrm{N_d}+1)}\right|^2$. Fig. 6 shows the cumulative distribution function (CDF) of $\left|\mathrm{H}_{(\mathrm{N_d}+1)}\right|^2$ for different $\mathrm{P_d}$. As we can see, the difference in the minimum $\left|\mathrm{H}_{(\mathrm{N_d}+1)}\right|^2$ between QPSK and QAM-16 is around 15 dB, the difference between QPSK and QAM-64 is around 16 dB and the difference between QAM-16 and QAM-64 is around 1 dB. For the (324,648) LDPC code used in the OEC-II, its performance degrades around 5 dB when the number of bits m per symbol [2] increases by 2 bits (i.e. from QPSK to QAM-16 or from QAM-16 to QAM-64). Therefore, it is profitable to use QAM-16 or QAM-64 instead of QPSK in OEC-II, but it is not worthy to replace QAM-16 by QAM-64 in OEC-II. This phenomenon is observed in Fig. 5 as well. With the same effective transmission data rate (i.e. 10.8 Mbits/s), the simulation result in Fig. 5 shows that OEC-II with QAM-16 has a SNR gain of around 7 dB over the FEC layer from the IEEE 802.11a standard, around 8 dB over the FEC layer from the IEEE 802.11n standard, around 7 dB over OEC-II with QPSK and around 3 dB over OEC-II with QAM-64.

V. CONCLUSIONS

In this paper, we propose a robust cross coding scheme OEC-II based on fountain codes for OFDM systems, which combines the joint coding and the separate coding over all the sub-carriers. The basic idea of OEC-II is to exchange the order of modulation with the discarded sub-carriers. By transmitting a fountain-encoded packet over a single sub-carrier, the receiver is allowed to discard packets over the sub-carriers in deep fading. Therefore, the noise floor (i.e. the data rate of frequency selective channels) can be increased. There is a tradeoff between the performance of OEC-II and the modulation order. OEC-II with QAM-16 performs the best, which is followed by OEC-II with QAM-64 then by OEC-II with QPSK. With the same effective transmission data rate (i.e. 10.8 Mbits/s), the simulation results show that OEC-II with QAM-16 has a SNR gain of around 7 dB over the FEC layer from the IEEE 802.11a standard, around 8 dB over the FEC layer from the IEEE 802.11n standard, around 7 dB over OEC-II with QPSK and around 3 dB over OEC-II with QAM-64.

The performance of OEC-I (i.e. to exchange the code rate of error correcting codes with the discarded sub-carriers) has been studied in [10]. With the same effective throughput, OEC-I has a SNR gain of around 7.5 dB comparing to the conventional 802.11a system. Therefore, further research focuses on the optimization of OEC by combining OEC-I and OEC-II together.

VI. ACKNOWLEDGEMENTS

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REFERENCES

[1] H. Liu and G. Li, *OFDM-Based Broadband Wireless Networks: Design and Optimization*. Wiley-Interscience, 2005.

[2] A. Bahai, B. Saltzberg, and M. Ergen, *Multi-carrier Digital Communications: Theory and Applications of OFDM*. Springer, 2004.

[3] M. Engels, *Wireless OFDM Systems: How to Make Them Work?* Kluwer Academic Publishers, 2002.

[4] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*. New York, NY, USA: Cambridge University Press, 2005.

[5] J. G. Proakis, *Digital Communications*. New York, NY, USA: McGraw Hill, 2001.

[6] A. Carlson, *Communication Systems*. McGraw-Hill New York etc., 1975.

[7] European Telecom Standards Institue, “Channel Models for HIPERLAN/2 in Different Indoor Scenarios,” 1998.

[8] D.J.C. MacKay, “Fountain Codes,” *IEE Communications*, vol. 152, no. 6, pp. 1062–1068, 2005.

[9] M. Mitzenmacher, “Digital Fountains: A Survey and Look Forward,” in *IEEE Information Theory Workshop*, 2004, pp. 271–276.

[10] X. Shao, R. Schiphorst, and C. H. Slump, “An Opportunistic Error Correction Layer for OFDM Systems,” *EURASIP Journal on Wireless Communications and Networking*, 2009.

[11] M. Luby, “LT Codes,” *Proceedings of the 43rd Annual IEEE Symposium on Foundations of Computer Science*, pp. 271–282, 2002.

[12] D.J.C. MacKay, *Information Theory, Inference, and Learning Algorithms*. Cambridge University Press, 2003.

[13] X. Shao, R. Schiphorst, and C. H. Slump, “Opportunistic Error Correction for WLAN Applications,” *Proceedings of the 4th IEEE International Conference on Wireless Communications, Networking and Mobile Computing*, 2008.

[14] A. Shokrollahi, “Raptor Codes,” *Information Theory, IEEE Transactions on*, vol. 52, no. 6, pp. 2551–2567, 2006. [Online]. Available: http://dx.doi.org/10.1109\%2FTIT.2006.874390

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[2] $m = \log_2(M)$, where M is the modulation order.

[15] P. Maymounkov, “Online Codes,” *Research Report TR2002-833, New York University*, 2002.

[16] IEEE, “Draft Standards for Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications, Enhancements for Higher Throughput (IEEE 802.11n Standard, Part 11),” Jan, 2007.

[17] IEEE, “Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications, High-Speed Pysical Layer in the 5 GHz Band (IEEE 802.11a Standard, Part 11),” 1999.