

VIBRATION OF THE LINER IN AN INDUSTRIAL COMBUSTION SYSTEM DUE TO AN ACOUSTIC FIELD

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Abstract. The subject of this paper is a numerical study of the properties of the liner of a test rig to be built in the future. The test-rig consists of a flexible tube of square cross-section surrounded by a pressure vessel, also with a square cross-section. At first instance, a two dimensional structural analytical model of the cross-section is made. The influence of the air between liner and pressure vessel and that within the liner on the vibration of the liner is studied using a coupled 2D finite element model. Furthermore the influence of the vibration of the liner on the acoustics of the setup is studied. After this the problem is extended to three dimensions and again the influence of the cavity surrounding the liner is analyzed. Both 2D and 3D results are compared. The cavities are found to substantially influence the structural behavior and therefore they cannot be neglected in predicting the behavior of the liner.

1 INTRODUCTION

To decrease NO_x emissions from an industrial gas turbine lean premixed combustion in combination with an annular combustor is used.⁶ The leaner combustion has the side effect of higher sound pressure levels in the combustion system. On the other hand the annular liner (the plate-like structure surrounding the combustion chamber) does not have a high stiffness and therefore the sound pressures excite the liner. This liner can vibrate excessively which limits the life of the combustor and the range of operability, because it will fail due to fatigue. There are two approaches to decrease this problem. The approach most often taken is to aim for decreasing the pressure levels in the combustor.⁵ The other approach is to directly decrease the vibration level of the liner by, for instance, increasing its damping. This requires investigation of the fluid-structure interaction problem of the liner and the air in the combustion chamber.

The fluid-structure interaction problem is studied within the European project DESIRE (Design and demonstration of highly reliable low NO_x combustion systems for gas turbines). In this framework the University of Twente will build a small test rig (500 kW thermal power) to measure the flame transfer function, the dynamic pressures in the combustion chamber and the vibrational response of the liner. To make the setup as modular as possible, a separate liner section is used to measure structural vibrations. The part on which these measurements will be performed is depicted in figure 1. It consists of a thin part (the vibrating part) clamped between two thicker parts. The latter parts are used for pressure measurements and to exclude end effects as much as possible. This allows for well defined boundary conditions on the thin part of the structure. The length of the thin, flexible section is taken such that the eigenfrequencies will no longer decrease substantially when making the flexible section longer. The first eigenfrequency

has to be in the range of the frequency spectrum of the flame, which turns out to be as low as possible. The width is confined by the fact that a recirculation zone has to emerge behind the swirling burner. The liner section is contained in a pressure vessel which has to withstand the overpressure in the test rig. This paper describes the way in which this square tubular structure is modelled.

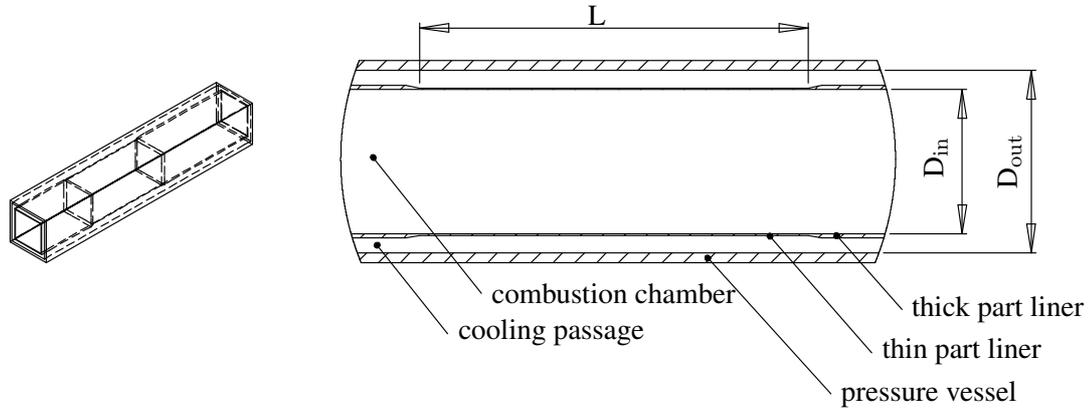


Figure 1: Flexible section of the liner (L) with pressure vessel, left the 3d representation, right a cross-sectional view

2 2D MODEL

2.1 Introduction

A 2D model is constructed first, because its results are easier to interpret than those of a 3D model. An analytical model is made, describing the behavior of only the structure. To include the influence of the air in the combustion chamber and the cooling passage, a finite element model is made. The analytical model serves as a check on the results of the finite element model.

The layout of the 2D model is depicted in figure 2. The model is a cross-section taken from the flexible part of the liner. It consists of an acoustic cavity on the inside which is surrounded by the flexible liner. Outside the liner is the cooling passage which is bounded by a pressure vessel. It is expected that the corners of the liner will not move much in a three dimensional tube when it is not very long (bending stiffness along the length of the liner will be much higher than the local stiffness of the side plates of the tube). The corners are therefore modelled as pinned boundary conditions which only allows for rotation. Furthermore the corners are assumed to remain perpendicular. The pressure vessel is assumed to behave as an acoustically hard wall. The mechanical properties and dimensions used are listed in table 1. The dimensions originate from an experimental setup that will be built at a later stage. The mechanical properties are taken at atmospheric conditions.

2.2 Analytical model

The behavior of the square section can be described by beam modes of each of the sides. All four beams have to vibrate in the same mode shape, because they all vibrate at the same frequency. Because the boundary conditions applied to the model are pinned connections, there can be no

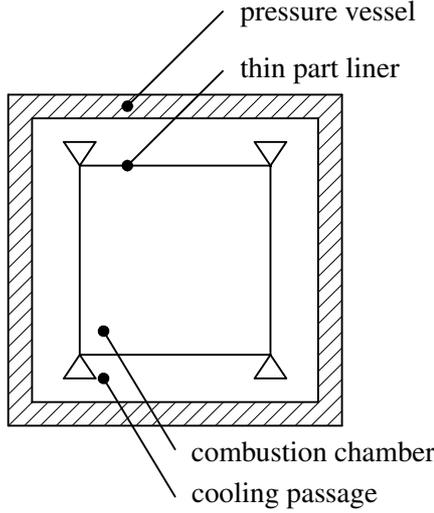


Figure 2: 2D model

symbol	meaning	value
c_0	speed of sound	343 m s^{-1}
ρ_0	density of air	1.29 kg m^{-3}
E	Young's modulus of steel	210 GPa
ν	Poisson's ratio of steel	0.3
ρ_s	density of steel	7800 kg m^{-3}
D_{in}	inside diameter	150 mm
D_{out}	outside diameter	190 mm
t	thickness	1 mm

Table 1: Material properties and dimensions

free end conditions. The three mode shapes remaining are clamped-clamped, clamped-pinned and pinned-pinned. Because the corners have to remain perpendicular, these beam mode shapes give the eigenmodes of the square depicted in figure 3.

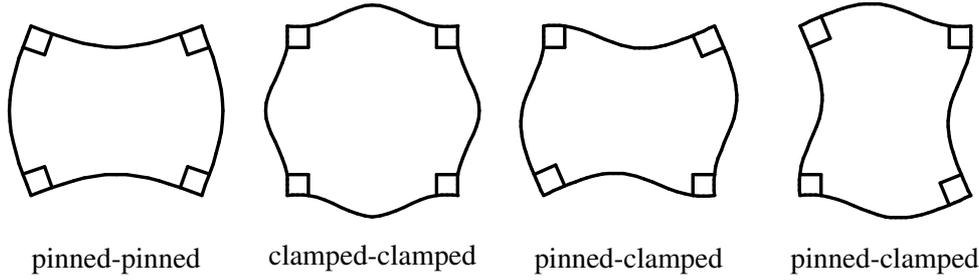


Figure 3: Schematic analytic structural mode shapes, the two figures on the right are 90 degrees rotated versions of the same mode

The eigenfrequencies for the beam modes are given in Blevins² as

$$f_i = \frac{\lambda_i^2}{2\pi L^2} \sqrt{\frac{EI}{m}} \quad (1)$$

mode	pinned-pinned	pinned-clamped	clamped-clamped
1	1π	3.92660231	4.73004074
2	2π	7.06858275	7.85320462
3	3π	10.21017612	10.9956079

Table 2: Value of parameter λ_i in equation 1

In which L is the length of the beam, E is Young's modulus, I is the area moment of inertia about the neutral axis (which would include a term $(1 - \nu^2)$ to account for the fact that the sides

are plates and not beams) and m is the mass per unit length. The parameter λ_i is given in tabular form in Blevins² (see table 2). The resulting eigenfrequencies are listed in table 3.

The modes consisting of clamped-pinned beams are listed twice in table 3. This is because the shape of this beam mode is not symmetric. There are therefore two possible geometric compositions, one being a 90 degrees rotated version of the other, which is also depicted in figure 3.

mode	structural analytic	structural FEM	combustion FEM	cooling FEM	both FEM	mode type
1	109.62	109.62	108.95	106.81	106.20	pinned-pinned
2/3	171.25	171.24	169.54	161.03	159.62	pinned-clamped
4	248.50	248.49	287.09	309.85	340.73	clamped-clamped
5	438.48	438.44	436.87	435.15	433.61	pinned-pinned
6/7		-	-	507.22	505.82	acoustic
8/9	554.95	554.90	549.95	593.72	589.30	pinned-clamped
10	684.99	684.93	678.35	669.47	663.53	clamped-clamped

Table 3: Eigenfrequencies for the 2D model, combustion means with the air in the combustion chamber included, cooling with the air in the cooling passage and both with both cavities included in the model

2.3 2D finite element model

To study the behavior of the system of liner with acoustics on the in- and outside, a finite element model has been made. It consist of linear 2D fluid elements and 2D linear beam elements (figure 4). The elements are coupled using fluid-structure elements that ensure the acoustic velocity and the structural velocity are equal in the normal direction of the plate (the hatched elements in figure 4). Because the fluid and the structure share nodes on the fluid structure interface, two sets of beams are used (the pressure degrees of freedom of the cooling passage and the combustion chamber would otherwise be directly coupled). The displacement degrees of freedom of these beams are coupled. Because Young's modulus is taken half the real value, these coupled beam elements have the same stiffness as the original beam would have. The acoustic cavity elements are then coupled to one beam set and the cooling passage elements to the other. The results show to be mesh independent.

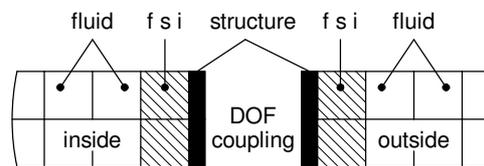


Figure 4: Layout of the fluid-structure interface for finite element modelling

2.4 Influence of cavities on structural behavior

The coupled finite element model can be used to study the influence of the cavities on the

structural behavior of the liner. The eigenfrequencies and mode shapes are therefore evaluated. Four cases are considered:

1. the structure only
2. the structure and the air in the combustion chamber
3. the structure and the air in the cooling passage
4. the structure with both cooling passage and combustion chamber

The eigenfrequencies of the different models are given in table 3. Eigenfrequencies 6 and 7 are only present when the cooling passage is included in the model. This is because this eigenmode is the first circumferential acoustic mode of this gap (figure 5). This circumferential eigenfrequency is given by

$$f = \frac{c_0}{2(D_{out} + D_{in})} \quad (2)$$

in which it is assumed that the acoustic wave travels along the center of the cooling passage. The first circumferential eigenfrequency is then calculated as 504.40 Hz. This is very close to the corresponding finite element result.

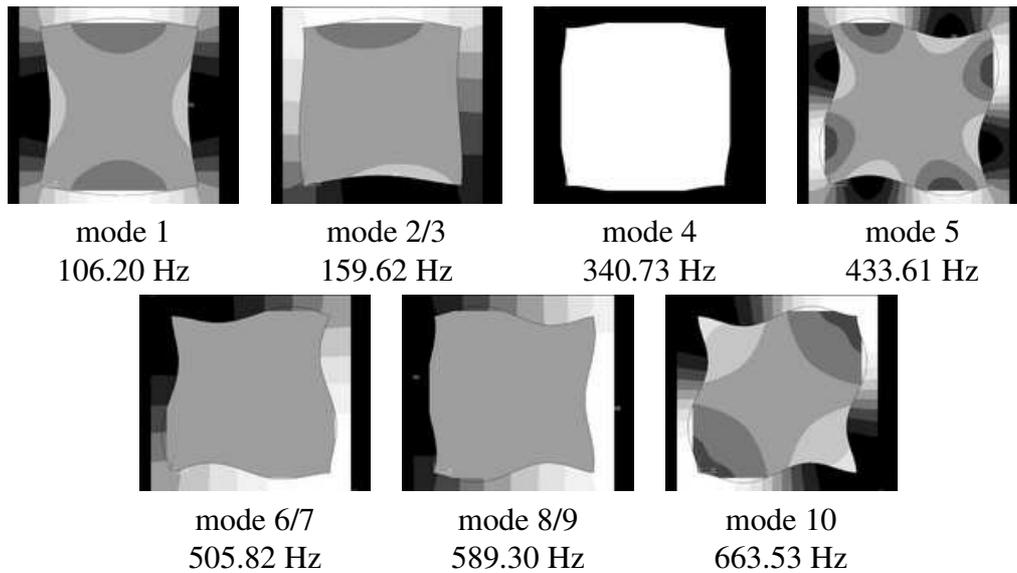


Figure 5: Eigenmodes of the 2D system, dark means high acoustic pressure, light low acoustic pressure, the contour represents the vibration shape of the liner

The fourth eigenmode has a strongly changing eigenfrequency due to the cavities. From the mode shape in figure 5 it can be seen that this is a volumetric mode. When the structure vibrates the volume of the cavity 'breathes'. The cavities act as springs and therefore increase the eigenfrequency. This is seen for both cavity and cooling passage.

When examining the first three and the fifth eigenfrequency, it can be observed that they slightly decrease. This is due to the added mass effect of the air surrounding it. The air is pumped between the bulbs of the vibration shape, which increases the effective mass of the system.

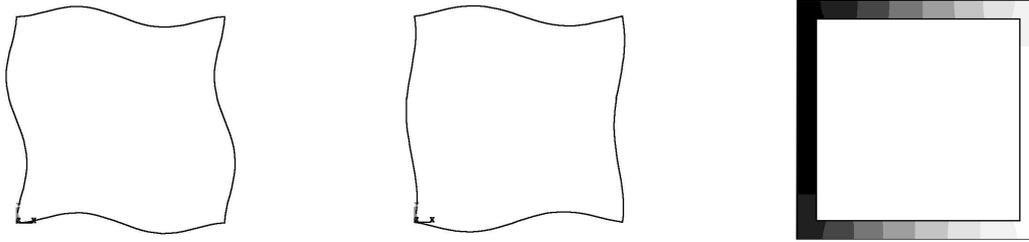


Figure 6: Mode shape 8/9, left without cooling passage, middle with cooling passage, right the pressure in the cooling passage (black is high, white is low)

Beyond these five structural modes, the first acoustic eigenfrequency arises. The interaction between the vibrating structure and the acoustic cavities now becomes more complex, because structurally dominated and acoustically dominated modes become more tightly coupled.¹

Figure 6 shows the structural vibration shape for mode 8 with and without the cooling passage. The pressure distribution in the cooling passage is also shown. The pressure pattern strongly resembles the first acoustic mode of the cooling passage. The structural vibration for the mode with the cooling passage matches spatially to this vibration shape. The vibration shape without cooling passage matches far less well. The influence of air in the cooling passage is very strong for this mode, because the eigenfrequency of the acoustically dominated mode and the structurally dominated mode are close to each other, which causes the high amount of coupling. This is also seen in the eigenfrequency which is strongly influenced.

2.5 Influence of cooling passage vibration amplitude

To evaluate whether the cooling passage substantially influences the vibration amplitudes, the spatial rms value of the displacement of all the nodes of the structure due to a pressure source inside the combustion chamber is depicted in figure 7. This position of the source is also depicted. The rms value is an indication of the vibration level of the liner. The figure shows both this level with and without the cooling passage.

It can be seen that the resonance around 500 Hz is only present for the case with the cooling passage, because this is an eigenfrequency of the cooling passage. It is clear that this acoustic eigenfrequency of the cooling passage leads to a sharp increase in the vibration level of the liner around this frequency and therefore the cooling passage should be included in the calculation.

The resonance around 550 Hz increases in frequency due to the cooling passage, which was already predicted in the modal analysis. The other resonances also behave as predicted with the modal analysis. The first resonance is not visible and the second is only slightly visible. Because the frequency is far below the first acoustic eigenfrequency of the combustion chamber, the pressure source induces an almost constant pressure field in the cavity. This field does not couple with the vibration shapes of the first two eigenfrequencies and therefore these do not show up in the transfer.

What remains are the peaks around 250 Hz. This is the fourth eigenfrequency. As predicted this value shifts towards a higher frequency when the cooling passage is taken into account. The modal analysis however shows that the peak with the cooling passage should be at 340 Hz, and not 265 Hz where it is located here. Without the cooling passage it should be at 287 Hz and not at 240 Hz where it is in figure 7. It seems like the air in the combustion chamber no longer

influences the eigenfrequencies. This is probably because the excitation is a source inside this combustion chamber. This source induces a pressure field on the liner wall which makes the liner vibrate. Because the source is independent of this vibration, the combustion chamber can no longer influence the vibration. The reason that this effect is only observed for the fourth eigenfrequency is that this is the only mode that is substantially influenced by this cavity.



Figure 7: Source position (left) and vibration response of liner (right)

2.6 Influence of structure on acoustic behavior

The structure also influences the acoustic behavior in the cavity. This is clearly observed in the fourth mode, the volumetric mode. When the structure comes into resonance, the pressure in the cavity rises and falls in the same frequency. The non-volumetric modes only influence the local acoustic pressure perturbation, and therefore have little global influence. At higher frequency this influence will grow, because the acoustic eigenfrequencies of the cavity come into range. This effect shows up for the tenth mode.

The acoustic behavior of the cooling passage is strongly influenced by the vibration of the liner. The local perturbations seen in the inside cavity are large enough to fill the entire outer cavity. This may be important, because in a real gas turbine setup, acoustic pressure fluctuations in the cooling air supply can travel downstream to the burner mouth and lead to a thermoacoustic source. This source excites the structure, which closes a potentially unstable loop.

3 3D MODEL

3.1 Introduction

To further examine the behavior of the liner, a 3D model is constructed. This model is more similar to the real setup depicted in figure 1. Only the thin section is modelled, because this section will dominate the dynamic behavior of the liner. The section is clamped at the in- and outlet sides (figure 8), which is different from the 2D boundary condition in the previous section (this model was pinned at the corners, figure 2). The dimensions are taken the same as for the 2D model, with the length being 0.375 meters.

3.2 3D analytical model

The 2D model showed that the eigenmodes of the square section consist of three types of beam modes, being pinned-pinned, pinned-clamped and clamped-clamped. The in- and outlet side are now clamped. It could therefore be expected that the eigenmodes now consist of plate modes

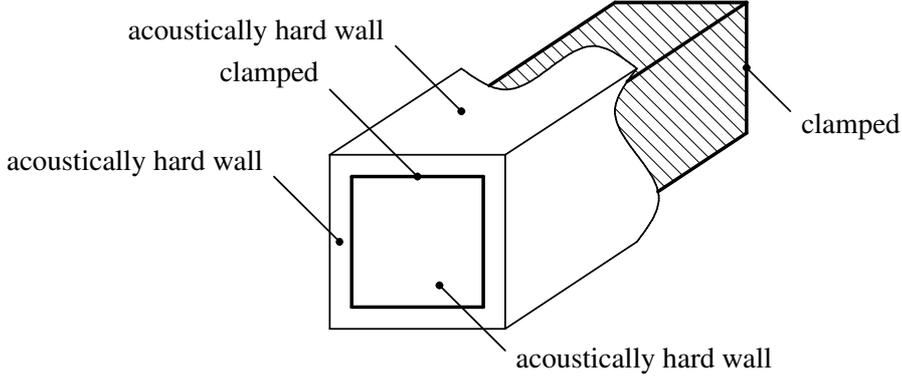


Figure 8: Boundary conditions of the 3D model

with two opposite sides clamped and the other two sides either simply supported (cscs), clamped (cccc) or one clamped and one simply supported (cccs). The eigenfrequencies for plates with different boundary conditions are given in Blevins as

$$f_{ij} = \frac{\lambda_{ij}^2}{2\pi a^2} \sqrt{\frac{Eh^3}{12\gamma(1-\nu^2)}} \quad (3)$$

In which γ is the mass per unit area of the plate and parameter λ_{ij} is tabulated, which is reproduced in table 4 for $a/b=0.4$ (in this case b is the length of the tube and a the width).

mode	cccc	cccs	cscs
1	23.65	23.44	12.13
2	27.82	27.02	18.36
3	35.45	33.80	27.97
4	46.70	44.13	40.75
5	61.55	58.03	41.38

Table 4: Value of λ_{ij} for different plate boundary conditions, $a/b=0.4$

The maximum aspect ratio for which the values of λ_{ij} are listed is 0.4. For a liner of 150x150 mm this yields a length of 375 mm. The resulting eigenfrequencies for this geometry are listed in table 5.

3.3 3D finite element model

The elements applied in the 3D finite element model are linear shell (SHELL63) and linear fluid (FLUID30) elements. The coupling works similar to the 2D model. The model consists of 4800 shell elements and 14801 fluid elements.

3.4 3D structural behavior

The most striking difference with the 2D model is that the modal density has sharply increased. This is because new modes arise in the third direction. It can be seen in table 5 that the eigen-

frequencies of the cccc and cscs modes match very well. The cccc modes do not show up accurately, though. The modes that cannot be identified as either cccc or cscs appear in pairs of two because of their asymmetric nature, but the eigenfrequency does not match accurately with the expected plate mode. The mode shapes from the finite element model show for instance that the second/third mode does not have the same vibration shape on all four sides and therefore the eigenfrequency cannot be predicted using the analytical model.

mode	structural analytic	structural FEM	coupled FEM	mode shape
1	134.73	134.68	125.60	cscs (1,1)
2/3		189.91	179.60	
4	203.92	203.68	177.85	cscs (1,2)
5/6		247.61	224.13	
7	262.68	262.25	288.05	cccc (1,1)
8	308.99	308.12	274.61	cccc (1,2)
9	310.66	309.99	284.90	cscs (1,3)

Table 5: Eigenfrequencies for the 3D model, the coupled model has both combustion chamber and cooling passage included

3.5 3D influence of acoustic cavities

Analogous to the 2D model the acoustic domain on the in- and outside decrease the eigenfrequencies (table 5), except for the seventh mode, as this is the first volumetric mode. Furthermore it can be seen that for modes that have the same vibration pattern on all sides, the eigenfrequencies decrease stronger than for modes for which this is not the case. This is because more air is moved.

When the mode shapes are observed (figure 9) it can be seen that by adding the acoustic cavities the sides influence each other stronger. This can be seen clearly for the second and third mode. The high vibration level on one side makes the low vibration level on the other side become higher. This is caused by a more or less matching pressure field which amplifies the vibration on that side. For the fifth and sixth mode the vibration level is decreases though, because the pressure opposes the vibration.

4 CONSEQUENCES FOR MODELLING THE INTERACTION

In the previous sections it was shown that the acoustic cavities inside and outside the liner substantially influence the dynamic behavior of the liner. They should therefore be included in the dynamic structural analysis of the liner. When using a CFD code to predict the acoustic pressures in the combustion chamber the dynamics of the structure and the fluid would have to be solved simultaneously, which is complicated.³

Another option could be to extract a thermoacoustic source and temperature and density field (which influence the acoustic properties) from CFD and use these data to construct a coupled finite element analysis.⁴ In literature one-dimensional acoustic models are often used similarly because they allow insight in the physical processes driving acoustic instabilities.⁷⁻⁹

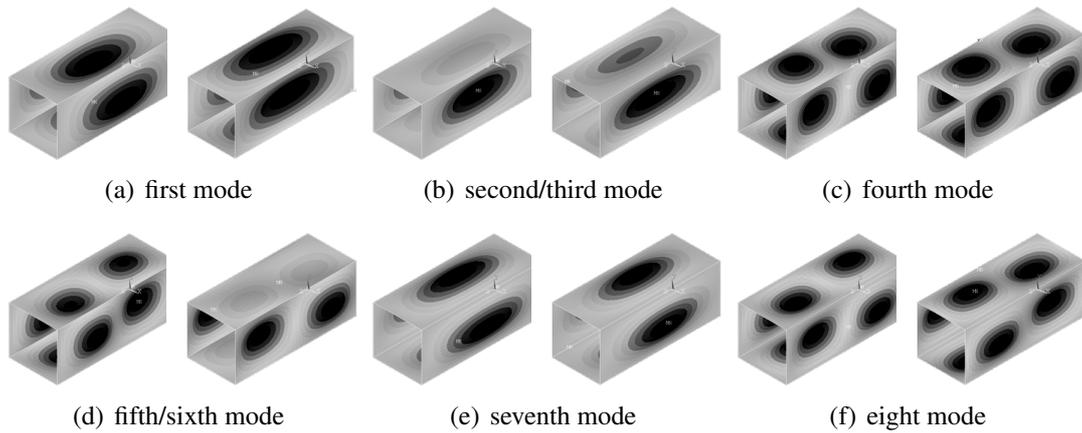


Figure 9: Structural displacement for the 3D-model, left is the structural model, right is the coupled model

5 CONCLUSIONS

This paper discusses the acousto-elastic interaction of a future test rig, consisting of a flexible liner contained in a rigid pressure vessel. The structural behavior of the two-dimensional model can be approximated by beam mode shapes. The structural behavior of the 3D model can partially, but not entirely, be described by plate mode shapes. Finite element calculations of the coupling between the flexible structure and the acoustics inside the liner and between liner and pressure vessel show that these cavities influence the structural behavior strongly. The eigenfrequency of volumetric modes increases, whereas the other eigenfrequencies decrease. Finite element calculations on the 2D model show that the vibration level does not substantially decrease due to the air entrapped between the flexible liner and the pressure vessel. 2D results showed that the strongest influence is by the cooling passage. To obtain accurate predictions of the vibration level of the liner this cooling passage therefore has to be taken into account.

Acknowledgements

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