# A Krylov-based exponential time integrator of the incompressible Navier-Stokes equation

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## ABSTRACT

We present an exponential time integrator for the incompressible Navier-Stokes equation based on the exponential block Krylov (EBK) method [2]. The exact solution of the semi-discrete system, given by a discretization of the Navier--Stokes equation in the spatial dimensions, involves the matrix exponential function. So-called *exponential integrators* are usually directly based on the formal expression of the exact solution and rely on Krylov methods, for example, to evaluate the matrix exponential. Recent studies suggest excellent accuracy and stability [4].

The continuity equation is imposed by a projection operator to eliminate the pressure from the Navier-Stokes equation. This operator is included in the definition of the Krylov subspace, and is essentially the discrete counterpart of the Leray projection operator [1], which projects vector fields onto a divergence-free space. The numerical solution then satisfies the discrete continuity equation at all times (with a certain tolerance). We demonstrate our method for a lid-driven cavity flow with time-dependent boundary conditions.

Our exponential integration method also has the potential for parallelization in the time domain, based on the *Paraexp* method [3]. The time-parallel concept has been demonstrated for the viscous Burgers equation [5]. With increasing capabilities of parallel computing, additional parallelism becomes necessary for large-scale numerical simulations of incompressible flows.

## References

[1] H. Amann. On the strong solvability of the Navier–Stokes equations. *Journal of Mathematical Fluid Mechanics*, 2(1):16–98, 2000.
[2] M. Botchev. A block Krylov subspace time-exact solution method for linear ordinary differential equation systems. *Numerical linear algebra with applications*, 20(4):557–574, 2013.
[3] M. J. Gander and S. Güttel. Paraexp: A parallel integrator for linear initial-value problems. *SIAM Journal on Scientific Computing*, 35(2):C123–C142, 2013.
[4] M. Hochbruck and A. Ostermann. Exponential integrators. *Acta Numerica*, 19:209–286, 2010.
[5] G. L. Kooij, M. A. Botchev, and B. J. Geurts. A block Krylov subspace implementation of the time-parallel Paraexp method and its extension for nonlinear partial differential equations. *ArXiv e-prints*, Sept. 2015.