

# Applying Column Generation to the Discrete Fleet Planning Problem

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**Abstract**—The paper discusses an Integer Linear Programming (ILP) formulation that describes the problem of planning the use of domestic distributed generators, under individual as well as fleet constraints. The planning problem comprises the assignment of time intervals during which the local generator must produce or not. In [1] this ILP is shown to be NP-complete in the strong sense. Heuristic methods have been developed to find solutions in reasonable time.

In this work a different technique is used to overcome the complexity problems. We use column generation to search the possible decision vectors in a faster way. The ILP is slightly adjusted to facilitate the column generation technique to search in a clever way through the set of possible solutions.

To measure the results, the column generation technique is compared to an earlier developed heuristic method. Both the quality of the objective function and the speed of the methods are compared.

**Keywords:** column generation, scheduling

## I. INTRODUCTION

During the last years the electricity supply chain (the production, consumption, distribution, storage and load management with respect to electricity), has changed. For example, the availability of renewable energy sources, the development of distributed electricity generators and the demand for more energy efficient appliances are increasing [4]. A specific type of upcoming distributed electricity generators is microCHP (Combined Heat and Power on a domestic scale). A microCHP produces both heat and electricity for household usage; the electricity can also be delivered back to the electricity grid. This electricity production, while being connected to the grid, gives possibilities to increase stability in the grid, to replace a conventional power plant, and more. To effect these possibilities the individual microCHPs need to be controlled, as standalone devices, but also in cooperation with other microCHPs or generators.

An important part in this control can be contained by online and offline planning problems. In [1] we presented different kinds of scheduling models that are derived from the microCHP problem description. In this paper we focus on the planning problem that combines the planning of production runs of the microCHPs of single households equipped with a heat buffer into the larger concept of a so-called fleet. This Discrete Fleet Planning Problem (DFPP) is shown to be NP-complete in the strong sense. In practice this means that it is

very costly to find an optimal solution for instances containing 10 houses and more. To overcome these complexity problems, in [2] a heuristic is proposed, that iteratively tries to find a better planning, by solving independent dynamic programming formulations for single house problems, and steering these individual DPs by changing artificial steering parameters in the objective function.

### A. Column generation

In this paper we propose an alternative method that tries to find a good solution in reasonable computation time. This method makes use of a technique called column generation. The idea of column generation is to divide the given problem in two parts: a main problem and a sub problem. In the main problem the original problem is optimized for a much smaller part of the solution space, indicated by a set of solution vectors, the so-called ‘columns’. Based on the solution for this main problem, a new column is added to this solution space, as long as it improves the objective value in the main problem. In the sub problem this new column is found, by optimizing the added value of the new column, based on the solution of the main problem.

Column generation is widely applied in practice. An example of the use of this technique is given in [3], where it is applied to solve the Cutting Stock Problem. It is also used to solve the Vehicle Routing Problem, as in [5].

The paper is organised as follows. In Section II the ILP formulation of the DFPP and an earlier developed heuristic are given and commented. Also the idea of adopting column generation to this ILP is explained. Section III shows a comparison between the column generation technique and the heuristic. The paper ends with conclusions and recommendations for future work in Section IV.

## II. PROBLEM AND COLUMN GENERATION APPROACH

In this section our column generation approach is presented. Section II-A gives the complete formulation of the original problem and introduces a heuristic to solve the problem. In Section II-B a simplified formulation is used to solve the problem via column generation.

### A. ILP model and heuristic approach

The DFPP is given by Equations (1)-(19). The formulation uses time intervals  $j = 1, \dots, N_T$  and houses  $i = 1, \dots, N$ . Note that the constraints are applied to all houses, next to the given values for  $j$ .

$$\max \sum_{j=1}^{N_T} \sum_{i=1}^N \hat{\pi}_j e_j^i \quad (1)$$

$$\text{s.t.} \quad \text{start}_j^i \geq x_j^i - x_{j-1}^i \quad j = 2 - MR, \dots, N_T \quad (2)$$

$$\text{start}_j^i \leq x_j^i \quad j = 2 - MR, \dots, N_T \quad (3)$$

$$\text{start}_j^i \leq 1 - x_{j-1}^i \quad j = 2 - MR, \dots, N_T \quad (4)$$

$$\text{stop}_j^i \geq x_{j-1}^i - x_j^i \quad j = 2 - MR, \dots, N_T \quad (5)$$

$$\text{stop}_j^i \leq x_{j-1}^i \quad j = 2 - MR, \dots, N_T \quad (6)$$

$$\text{stop}_j^i \leq 1 - x_j^i \quad j = 2 - MR, \dots, N_T \quad (7)$$

$$g_j^i = G_{\max}^i x_j^i - \sum_{k=0}^{N_{up}^i - 1} \hat{G}_{k+1}^i \text{start}_{j-k}^i + \sum_{k=0}^{N_{down}^i - 1} \check{G}_{k+1}^i \text{stop}_{j-k}^i \quad j = 1, \dots, N_T, \quad (8)$$

$$e_j^i = \alpha^i g_j^i \quad j = 1, \dots, N_T \quad (9)$$

$$x_j^i \geq \sum_{k=j-MR^i+1}^{j-1} \text{start}_k^i \quad j = 1, \dots, N_T \quad (10)$$

$$x_j^i \leq 1 - \sum_{k=j-MO^i+1}^{j-1} \text{stop}_k^i \quad j = 1, \dots, N_T \quad (11)$$

$$hl_1^i = BL^i \quad (12)$$

$$hl_j^i = hl_{j-1}^i + g_{j-1}^i - H_{j-1}^i - K^i \quad j = 2, \dots, N_T + 1 \quad (13)$$

$$0 \leq hl_j^i \leq BC^i \quad j = 1, \dots, N_T + 1 \quad (14)$$

$$\sum_{i=1}^N e_j^i \leq P_j^{upper} \quad j = 1, \dots, N_T \quad (15)$$

$$\sum_{i=1}^N e_j^i \geq P_j^{lower} \quad j = 1, \dots, N_T \quad (16)$$

$$x_j^i \in \{0, 1\} \quad j = 1, \dots, N_T \quad (17)$$

$$\text{start}_j^i \in \{0, 1\} \quad j = 1, \dots, N_T \quad (18)$$

$$\text{stop}_j^i \in \{0, 1\} \quad j = 1, \dots, N_T \quad (19)$$

The formulation uses binary decision variables  $x_j^i$  (17) to represent the on/off state of the microCHP in interval  $j$  in house  $i$ .  $\text{start}_j^i$  (18) and  $\text{stop}_j^i$  (19) are used to describe the intervals in which the microCHPs are started and stopped in Equations (2)-(7). Now, Equation (8) gives the heat generation of the microCHP, which can be derived from the variables  $x_j^i$ ,  $\text{start}_j^i$  and  $\text{stop}_j^i$ . The electricity generation is given in (9). Equations (10) and (11) represent technical constraints of the microCHP; heat buffer constraints are given in (12)-(14). The total electricity output is limited by (15) and (16). Finally, the

objective is to maximize the profit that can be made on an electricity market with prices  $\hat{\pi}_j$  (1).

If we let  $P_j^{upper} \equiv P_j^{lower} \equiv P_j$  for all intervals  $j$ , the DFPP becomes a feasibility problem: can we find an assignment of  $x_j^i$ 's, such that the desired pattern  $P_j$  is acquired. The profit in this case is fixed, if a solution can be found. If we now allow some deviations from the pattern  $P_j$ , Equations (15) and (16) change in:

$$\sum_{i=1}^N e_j^i - ex_j \leq P_j \quad j = 1, \dots, N_T \quad (20)$$

$$\sum_{i=1}^N e_j^i + sl_j \geq P_j, \quad j = 1, \dots, N_T \quad (21)$$

where  $ex_j$  and  $sl_j$  are excess and slack variables to measure the deviation from the desired pattern  $P_j$ . As objective we can minimize the deviations from the pattern:

$$\min \sum_{j=1}^{N_T} (sl_j + ex_j). \quad (22)$$

In the following, we focus on this variant of the DFPP: the problem is to minimize the deviation from a fixed production pattern  $P$ .

1) *Heuristic*: A heuristic to solve this variant of the DFPP is described in [2]. It uses a dynamic programming (DP) formulation to solve the planning problem for single houses. The phases in this DP are represented by the intervals and the states are formed by using similar constraints as in Equations (10)-(14) in the ILP formulation. The objective of the DP is to maximize the profit of the single house, when electricity prices  $\hat{\pi}_j$  are used. Now  $N$  independent DPs are solved and give altogether a certain deviation from the total electricity pattern  $P$ . The idea of the heuristic is to adjust the prices  $\hat{\pi}_j$  locally per house  $i$  to find a better global deviation in total electricity production. A more detailed description of the adjustments of these prices can be found in [2].

### B. Column generation approach

The limitations of the DP formulation are mainly determined by the sizes of the state spaces of allowed state tuples for individual generators. All possible production patterns can be deducted from these state spaces, including the ones that are not likely to be chosen. It would be an improvement to decrease the size of the state space by discarding the production patterns that are not likely to be chosen. However, it is not an easy task to discard these unlikely production patterns, since it is difficult to determine on beforehand whether a pattern will not be chosen. Therefore, the approach to discard patterns probably leads to a way of almost randomly deleting patterns, which can impact the optimality of the solution. More elaborate approaches can be used, but they need information on the original state space, which we want to avoid to improve the speed of the solution technique.

On the other hand, once we start with a strongly reduced state space we can use the information from this small set of patterns to add new patterns to improve the solution. If

the eventual state space that we create is still much smaller than the original state space, we might have a faster solution approach. This is the idea of the column generation technique: starting from a small set of production patterns, we add new patterns until we are satisfied with the global pattern assignment.

1) *Solving the DFPP*: The column generation technique consists of two phases. In the first phase the DFPP is solved; in the second phase a new pattern is added to the current set of feasible patterns.

The column generation technique starts with a relatively small set of feasible patterns  $PS$ . This set grows as the heuristic evolves; for now, it suffices to know that the size of the set does not change while the heuristic is in the phase of solving the DFPP (i.e. assigning patterns to generators to minimize the deviation from the global predefined production plan  $P$ ). Let the size of this set for a certain visit to this phase be given by  $N_P$ , such that  $p^1, \dots, p^{N_P}$  are the available patterns to choose from. For now we assume that these patterns are feasible for all generators (houses). The problem then is to assign exactly one pattern to each generator, such that the global plan  $P$  is approximated as close as possible. To calculate the deviation to this plan we minimize the mismatch rather than the squared mismatch, since we want to avoid quadratic objective functions. We then get the following formulation:

$$\min \sum_{j=1}^{N_T} (sl_j + ex_j) \quad (23)$$

$$s.t. \sum_{i=1}^N \sum_{p=1}^{N_P} p_j^p y_{i,p} + sl_j \geq P_j \quad j = 1, \dots, N_T \quad (24)$$

$$\sum_{i=1}^N \sum_{p=1}^{N_P} p_j^p y_{i,p} - ex_j \leq P_j \quad j = 1, \dots, N_T \quad (25)$$

$$\sum_{p=1}^{N_P} y_{i,p} = 1 \quad i = 1, \dots, N \quad (26)$$

$$sl_j, ex_j \geq 0 \quad j = 1, \dots, N_T \quad (27)$$

$$y_{i,p} \in \{0, 1\}, \quad (28)$$

where  $y_{i,p}$  is a binary decision variable indicating whether pattern  $p^p$  is chosen for generator  $i$  (in this case  $y_{i,p} = 1$ ) or not ( $y_{i,p} = 0$ ). In Equations (24) and (25) slack and excess variables  $sl_j$  and  $ex_j$  are introduced to allow an exact match to the predefined production pattern  $P$ . The sum of slack and excess variables is minimized in Equation (23). Finally, Equation (26) requires that exactly one pattern is chosen for each generator.

2) *Creating new patterns*: The second phase of the column generation technique consists of creating new patterns that can be added to the current pattern set. These new patterns should contribute to the existing set in the sense that they must give possibilities to decrease the objective value in the first phase (i.e. the sum of slack and excess). A new pattern  $p^g$  is only added if it improves the existing solution of the DFPP. To

fulfill this requirement, the new pattern  $p^g$  is only added if

$$\sum_{j=1}^{N_T} \lambda_j p_j^g > 1, \quad (29)$$

where  $\lambda_j$  are the shadow prices associated with Equations (24) and (25).

A new pattern should be a feasible pattern. This can be guaranteed by calculating the heat level of the corresponding generators following the created pattern and checking whether the buffer limits are violated. In the formulation of the second phase, this is simplified by using two parameter sets, specifying in each interval  $j$  the minimum number of intervals where the generator should have run ( $MinOn_j$ ) and the maximum number of intervals where the generator could have run ( $MaxOn_j$ ) up to and including the current interval. Note that these parameters  $MinOn_j$  and  $MaxOn_j$  are also assumed to fulfill the technical constraints in Equations (10) and (11).

Combined with the objective of maximizing  $\sum_{j=1}^{N_T} \lambda_j p_j^g$ , the second phase can be formulated as:

$$\max \sum_{j=1}^{N_T} \lambda_j p_j^g \quad (30)$$

$$s.t. \sum_{k=1}^j p_k^g \leq MaxOn_j \quad j = 1, \dots, N_T \quad (31)$$

$$\sum_{k=1}^j p_k^g \geq MinOn_j \quad j = 1, \dots, N_T \quad (32)$$

$$p_j^g \in \{0, 1\} \quad j = 1, \dots, N_T, \quad (33)$$

where  $p_j^g$  is the binary decision variable to produce in interval  $j$  in pattern  $g$ . The objective is maximized, meaning that, if requirement (29) can be met, an optimal solution to the formulation gives an improvement in the first phase.

3) *The column generation algorithm*: Two ILPs are presented for the two phases. ILP1 is the formulation of the first phase and ILP2 the formulation of the second phase. The complete solution method is given in algorithm 1.

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#### Algorithm 1 Column generation

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init  $PS := \{p^1, \dots, p^{N_P}\}$

solve ILP1

solve ILP2

**while**  $\sum_{j=1}^{N_T} \lambda_j p_j^g > 1$  **do**

$PS \leftarrow PS \cup p^g$

solve ILP1

solve ILP2

**end while**

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If the starting patterns in  $PS$  are chosen feasible, all possible solutions in the eventual set are feasible (in this version we assume all houses to have equal characteristics). Due to the feasibility constraints (31) and (32) for the newly created pattern, the final solution in the first phase needs not

to be optimal. This is due to the choice for the initial set (and added columns), which could imply that not all necessary columns are created in later iterations.

Note that we simplified the problem formulation by looking at the number of generators that are on instead of the total electricity production.

### III. RESULTS

To compare the column generation technique to the iterative heuristic using DPs we created two sets of problem instances. The column generation approach is modeled using AIMMS modeling software and solved by CPLEX 11.1. The heuristic approach is implemented in C++.

Both sets of instances make a planning for 48 intervals, meaning that each interval represents half an hour in a one-day-planning. In these instances, we assume that all houses have similar characteristics, such as heat demand. Due to the use of a heat buffer we have scheduling freedom in the assignment of runs of the microCHP. In all instances, this comes down to the fact that the microCHP in each house should run for minimally 25 intervals and maximally 28 intervals. The sets of instances differ in the total amount of houses that is planned. In the first set, we consider a fleet of 50 houses, while in the second set we look at 100 houses. An instance is completed by defining the production pattern  $P$ . This pattern is a constant pattern ( $P_1 = \dots = P_{N_T}$ ) where the values of  $P_j$  are set to a constant number of houses (from 0 to the number of houses in the fleet). So we have 51 instances in the first set and 101 instances in the second set.

The initial set  $PS$  in the column generation approach is chosen to consist of the two vectors  $MinOn$  and  $MaxOn$ .

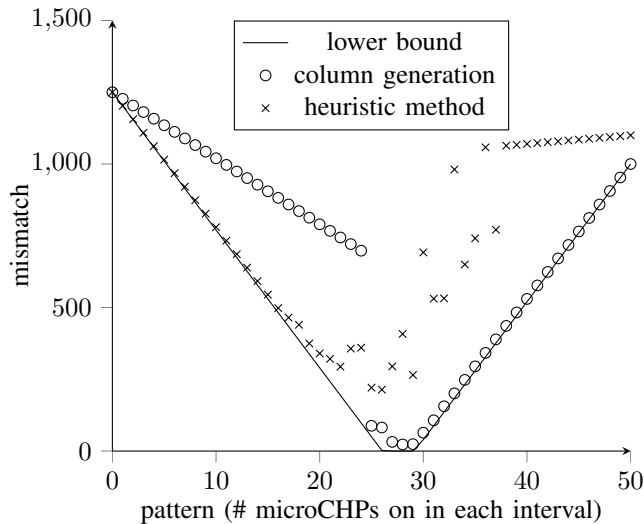


Fig. 1. Mismatch from flat patterns (maximal AIMMS runtime 1 minute per iteration)

The solutions for both approaches are given in Figures 1 and 2, as well as a theoretical lower bound on the deviation from the production pattern. In the figures the deviation (mismatch) is plotted against the production pattern. In Figure 3 the computation times are given of both techniques in both sets

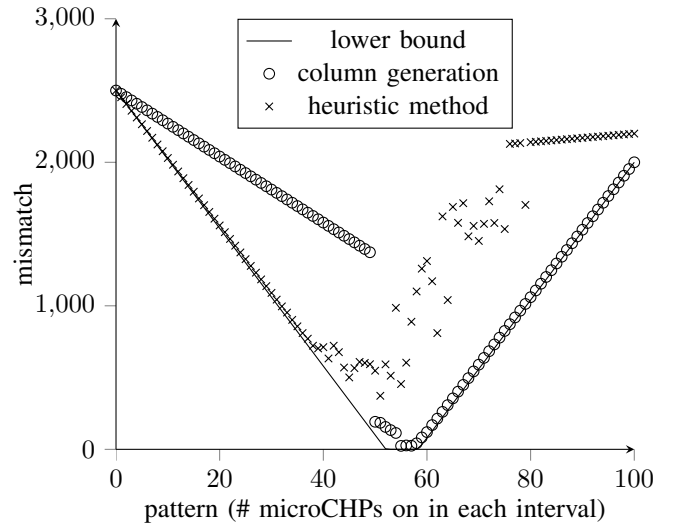


Fig. 2. Mismatch from flat patterns (maximal AIMMS runtime 1 minute per iteration)

of instances. The maximum runtime is set to 60 seconds per problem that is solved in the column generation approach.

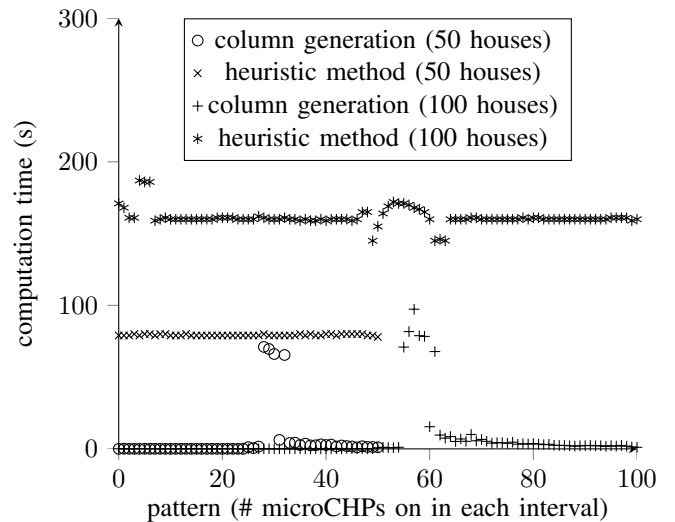


Fig. 3. Computation time

Since both techniques are applied to achieve good approximations of the optimal solution, they both have to perform well regarding objective value and speed. In the first two figures the column generation outperforms the heuristic approach in the instances where the lower bound is (close to) 0. Note that these are the instances that we want to solve in practice. In the instances with a low preferred production pattern  $P$  the column generation approach does not find a feasible new pattern in the first iteration. Therefore it performs bad compared to the heuristic. In these instances the heuristic comes close to the lower bound. The opposite can be said about higher values of  $P$ , where the column generation technique comes close to the lower bound and the heuristic cannot find large improvements.

Regarding computational time the column generation tech-

nique is considerably faster than the heuristic. However, the instances for which the lower bound is close to 0 are more difficult to solve (but still faster than the heuristic).

#### IV. CONCLUDING REMARKS AND RECOMMENDATIONS

In this paper we have shown a first approach in applying the column generation technique to a variant of the Discrete Fleet Planning Problem (DFPP), in which the production of microCHPs of a fleet of houses should be matched to a predefined total production pattern  $P$ . First results indicate that the solutions improve the results that are obtained by applying an earlier developed heuristic, as well in objective value as in computational time.

In future work, this concept of column generation can be applied to the more general DFPP, in which electricity prices play a role as well. It is also important to allow more general house configurations, to which not every production pattern might be applicable.

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