

Modeling and Control of Unmanned Aerial Vehicles: A Port-Hamiltonian Approach

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1 Introduction

Recently, in robotics there is an increasing interest in the field of unmanned aerial vehicles (UAVs) due to the existence of diverse potential applications in the civilian sector.

UAVs are characterized by their under-actuatedness and highly nonlinear and inherently coupled dynamics, which makes the design of an autonomous controller challenging. To address this problem, developing a competent dynamic model of the UAV is an essential step. Such a dynamic model enables the design of robust controller and to evaluate the dynamic response of the UAV in a typical environment.

In this abstract, we present a dynamic model and a tracking controller for UAVs in the port Hamiltonian framework [1].

2 Modeling of the UAV

The dynamic equation of a generic UAV can be derived by considering it as a rigid body in $SE(3)$, on which various wrenches act, such as thrust, gravity, lift, drag, and wind.

The equation of motion of a generic UAV, expressed in coordinates and with respect to a body fixed frame ψ^b , in port-Hamiltonian form [2], is given by

$$\begin{cases} \dot{P}_b^b &= \tilde{P}_b^b \frac{\partial H(P_b^b)}{\partial P_b^b} + W_b^b \\ T_b^{b,0} &= \frac{\partial H(P_b^b)}{\partial P_b^b} \end{cases} \quad (1)$$

where the generalized momentum P_b^b and twist $T_b^{b,0}$ are the state and passive output of the UAV, respectively. Moreover, the energy of the UAV is described by the Hamiltonian function $H(P_b^b) = \frac{1}{2}(P_b^b)^T (I_b^b)^{-1} P_b^b$, and $\tilde{P}_b^b = \begin{pmatrix} \tilde{P}_{b-\omega}^b & \tilde{P}_{b-v}^b \\ \tilde{P}_{b-v}^b & 0 \end{pmatrix}$,

in which $\tilde{P}_{b-\omega}^b$ and \tilde{P}_{b-v}^b are skew symmetric matrices composed of the first and the last three elements of P_b^b . I_b^b is the inertia matrix and W_b^b collects all wrenches acting on the UAV, including those due to the control action.

3 Controller design

The design strategy is recasting the design problem as finding a dynamic system and its proper interconnection so as to achieve the desired task. The role of the controller is to transform the UAV to a passive system with an energy function that attains its minima at the desired configuration.

For design of trajectory tracking controller, define the configuration and momentum errors as $q_e(t) = q(t) - \bar{q}(t)$ and $P_e(t) = P(t) - \bar{P}(t) = I_b^b T_e^{b,0}$, where $\bar{q}(t)$ and $\bar{P}(t)$ are reference state trajectories. The error dynamics of the system, whose Hamiltonian function is $H(q_e, P_e) = \frac{1}{2} q_e^T K_p q_e + \frac{1}{2} P_e^T (I_b^b)^{-1} P_e$, in port-Hamiltonian form is given by

$$\begin{cases} \begin{pmatrix} \dot{q}_e \\ \dot{P}_e \end{pmatrix} &= \begin{pmatrix} 0 & Z \\ -Z & \tilde{P}_b^b \end{pmatrix} \begin{pmatrix} \frac{\partial H(q_e, P_e)}{\partial q_e} \\ \frac{\partial H(q_e, P_e)}{\partial P_e} \end{pmatrix} + \begin{pmatrix} 0 \\ I \end{pmatrix} u_e \\ y_e &= (0 \quad I) \begin{pmatrix} \frac{\partial H(q_e, P_e)}{\partial q_e} \\ \frac{\partial H(q_e, P_e)}{\partial P_e} \end{pmatrix} = T_e^{b,0} \end{cases}$$

where Z is a transformation matrix defined as $\dot{q} = Z T_b^{b,0}$; u_e and y_e are the control input and the passive output of the above system, respectively.

Asymptotic convergence of the above error dynamics to the desired trajectory is ensured by damping injection, i.e., $u_e = -K_d y_e = -K_d T_e^{b,0}$, for any positive definite matrix K_d . As a consequence, the desired behaviour of W_b^b of Eq. 1 for trajectory tracking can be back tracked to

$$W_b^b = \dot{\tilde{P}} - \tilde{P}_b^b \tilde{T}_b^{b,0} - Z K_p q_e - K_d T_e^{b,0} \quad (2)$$

An integral control action can be augmented to Eq. 2 for robustification. Finally, the desired behaviour of W_b^b should be transformed to the actual controller inputs. Stabilization or hovering is a trivial case of the above trajectory tracking problem where the desired reference point is constant.

As a case study, the dynamic model and the robust controller discussed here have been applied to a ducted fan miniature aerial vehicle [3]. The dynamic model and the controller have been validated in simulations [1].

References

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