MAMERN11: 4^{th} International Conference on Approximation Methods
and Numerical Modelling in Environment and Natural Resources
Saidia (Morocco), May 23-26, 2011

Physically consistent simulation of transport of inertial particles in porous media

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Keywords: Lagrangian transport model, Particle filtration, Slow flow, Porous media

Abstract. *A new numerical approach is presented for simulating the movement of test particles suspended in an incompressible fluid flowing through a porous matrix. This two-phase particle-laden flow is based on the Navier-Stokes equations for incompressible fluid flow and equations of motion for the individual particles in which Stokes drag is dominant. The Immersed Boundary method is applied to incorporate the geometric complexity of the porous medium. A symmetry-preserving finite volume discretization method in combination with a volume penalization method resolves the flow within the porous material. The new Lagrangian particle tracking is such that for mass-less test particles no (numerical) collision with the coarsely represented porous medium occurs at any spatial resolution.*

1 Introduction

In a wide range of natural processes and industrial applications the motion of small particles suspended in a fluid flowing through a porous medium is one of the major phenomena of interest. Understanding the behavior of particles can help to control and improve processes like pollution percolation into underground areas, oil recovery and various refinement processes.

In this work the emphasis is on capturing the fully detailed motion of particles in a flow through complex porous media. We describe a computational approach for simulating the underlying flow of fluid and the motion of embedded particles in the presence of a porous medium. The employed computational algorithms provide a particle tracking method which can be used both for treating test particles as tracers, that follow precisely the fluid, and as a method for quantifying filtering properties of porous filters. For describing the fluid phase the Eulerian approach is used, while the particle phase is modeled using the Lagrangian transport model. To allow study of complex geometries of various porous materials we employ the Immersed Boundary (IB) method [1].

In the following sections we will shortly describe the mathematical model and the applied numerical methods. Finally, some illustrations based on our simulations for a test geometry will be presented.

2 Mathematical modeling and numerical treatment of two-phase flow in porous media

The system of governing equations is as follows:

$$\nabla \cdot \mathbf{u}_f = 0 \tag{1}$$

$$\frac{\partial \mathbf{u}_f}{\partial t} + \mathbf{u}_f \cdot \nabla \mathbf{u}_f = -\frac{1}{\rho_f}\nabla p + \nu \nabla^2 \mathbf{u}_f + \mathbf{f} \tag{2}$$

$$\frac{\mathrm{d}\mathbf{x}_p}{\mathrm{d}t} = \mathbf{u}_p(t) \tag{3}$$

$$\frac{\mathrm{d}\mathbf{u}_p}{\mathrm{d}t} = \frac{1}{\tau}(\mathbf{u}_f(\mathbf{x}_p, t) - \mathbf{u}_p(t)) \tag{4}$$

with $\nabla \equiv (\partial/\partial x, \partial/\partial y, \partial/\partial z)^T$ representing the vector differential operator and $\nabla^2 \equiv \nabla \cdot \nabla$ the Laplace operator. Here, $\mathbf{u}_f = (u_f, v_f, w_f)^T$ is the fluid velocity, $\mathbf{u}_p = (u_p, v_p, w_p)^T$ is the particle velocity, $\mathbf{x}_p = (x_p, y_p, z_p)^T$ is the particle position, ρ_f is the fluid mass density, p is the pressure and ν is the kinematic viscosity.

The set of equations (1-4) is composed of the Navier-Stokes equations for incompressible fluids describing the fluid phase, Eqs. (1) and (2), and equa-

tions of motion restricted to the dominant drag-force to describe the particle phase [2], Eqs. (3) and (4). One-way coupling of the two phases is considered, implying that the particle phase has no influence on the fluid phase [3]. The term $\mathbf{f}$ is a body force per unit mass, used to approximate no-slip boundary conditions at all solid-fluid interfaces. In this contribution we use a forcing function based on a volume-penalization strategy.

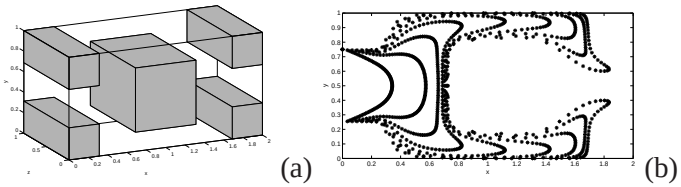
The essential parameter in the equations of motion is the velocity response time or relaxation time τ [4]. Considering the Stokes drag, the particle response time depends on its diameter D_p, the density of the particle ρ_p and the molecular viscosity of the carrier fluid μ: $\tau = (\rho_p D_p^2)/(18\mu)$. The smaller τ, the more sensitive is the particle motion to temporal and spatial variations in the flow field. By making the equation of motion non-dimensional, based on a characteristic length scale and velocity, we obtain a non-dimensional parameter St, denoting the Stokes number, which expresses the inertia of the particle. Another important dimensionless parameter Re (Reynolds number) is introduced in the non-dimensional formulation of Eq.(2), which characterizes the flow regime.

The solution of the governing system of equations has various properties, which represent important physical features of the problem. In order to preserve these properties on the level of the numerical solution, suitable numerical methods need to be developed. For resolving the gas flow we use symmetry-preserving finite-volume discretization on a staggered grid [5]. The IB resolution of the gas flow on the staggered grid results in non-zero velocity fields at the locations at which the solid-liquid interfaces are defined. This plays an important role in the tracking of particles, implying the possibility of a small, unphysical deposition of mass-less test particles on the solid surface. This is not acceptable, since mass-less particles are supposed to follow the streamlines, which are consistent with the no-slip condition on all solid-liquid interfaces. We restrict the velocities to all solid-liquid interfaces in such a way that mass-less particles do not hit the surface of the solid. This is essential when particles are considered as tracers. The new developed approach of interpolating the velocities close to the solid-liquid interface is also important for an accurate estimation of deposition due to impaction for very small particles.

3 Test particles in a structured porous medium

In order to verify and validate the described approach we simulate the motion of an ensemble of particles in a test geometry composed of a staggered arrangement of 3D square rods (Figure 1(a)).

We present the dynamics of particles with $St = 0.05$ for a creeping flow



(a) (b)

Figure 1: Staggered arrangement of square rods in 3D (a). Particle trajectories emanating from a line of initial particle positions (b). Each curve corresponds to a particular moment in time. The particle positions originate from a straight line of initial conditions on the left side of the figure. We simulated the motion of particles with $St = 0.05$ and for a creeping flow of $Re = 1$.

of Reynolds number $Re = 1$. Computed trajectories of the particles smoothly follow the flow streamlines (Figure 1(b)). Our simulations confirm for test particles with no inertia ($St \to 0$, i.e. $\tau \to \infty$) physically consistent behavior: particles move with the flow and never hit the solid walls. For relatively large values of St we observe strong inertial effect on the trajectories of the particles. By following the trajectories for different values of τ and Re, one can quantify the filtering properties of a given geometry.

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