

D/A Resolution Impact on a Poly-phase Multipath Transmitter

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Abstract— In recent publications the Poly-phase multipath technique has been shown to produce a clean output spectrum for a power upconverter (PU) architecture. The technique utilizes frequency independent phase shifts before and after a nonlinear element to cancel out the harmonics and sidebands due to the nonlinearity. A major advantage of this technique is that it circumvents the need to use dedicated RF filters which makes it a potential candidate for cognitive radio transmitters. This paper addresses the requirements on the digital and mixed signal part of such a transmitter. An architecture is proposed based on complex multiplication which can be used to generate the digital multiphase signals required by the multipath technique. Due to equal phase difference of all the paths the same digital hardware could be utilized for carrying out all the phase shifts. When the digital signals pass through a D/A converter which doesn't have a reconstruction filter, the output in this case would be amplitude discrete like that of a zero order hold. The spectrum of this amplitude discrete signal would have distortion components in it. This can be termed as quantization distortion but now in the context of limited D/A resolution. The multipath technique's effect on harmonic cancellation, in the presence of such a quantization distortion is explored in this paper. It is shown through simulation that when using ideal phase shifts the multipath technique is able to cancel most of the harmonics produced by an amplitude discrete representation of pure sinusoids. When (upconversion) mixers are used for the second set of phase shifts then with multipath the highest quantization spurs go down with roughly 8db/bit for a single tone and around 10db/bit for two tone inputs.

Index Terms—cognitive Radio, distortion cancellation, multipath, power upconverter, mixer, polyphase, radio transmitter, Spurious free dynamic range.

I. INTRODUCTION

In conventional multi-standard radio transmitter architectures, dedicated filters are required to suppress unwanted harmonics produced by the power amplifiers and mixers. This would mean that each standard being supported would require its own filters. As new transmitter architectures try to support more frequency bands this topology becomes more and more unpractical. Since filters are discrete components they would also keep on increasing the cost of the design. Figure 1(a) shows such a conventional multi-standard transmitter architecture.

Figure 1(b) shows a much more flexible and wideband architecture which doesn't use any dedicated filters. In this kind of a transmitter most of the control is exercised in the digital domain. In order to relax filter requirements or even

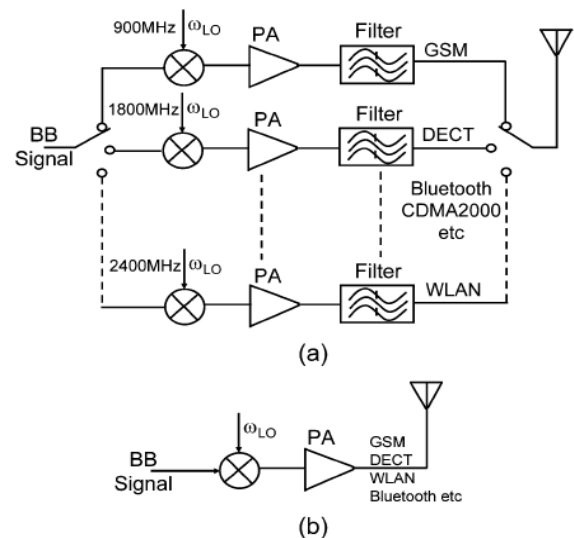


Figure 1. (a) Conventional multi-standard transmitter architecture. (b) Flexible transmitter with no filters

remove the filter altogether, the Power upconverter (PU) designed in [1] using the multipath technique has provided very desirable results. The Weldon transmitter [2] can also eliminate the need for an IF filter by suppressing third and fifth harmonics of IF frequency but the RF filter is still required after the PA as the harmonics of the RF-frequency still need to be suppressed. The multipath technique in combination with duty cycling of the local oscillator (LO) and differential output can cancel most of the harmonics of the PU. Since this can considerably relax the requirements on an RF filter it has the possibility to be used in Software/Cognitive Radio [3] systems. This paper adds on to the previous work by proposing an implementation for the digital part of a transmitter using this PU. The multipath technique's effect on harmonic cancellation with the addition of distortion due to finite D/A resolution is also discussed, both using ideal phase shifts as well as mixers for phase shifts. In this paper the terms amplitude discretization and amplitude quantization refer to the same phenomenon.

This paper is organized as follows, in Section II, the main idea of the multipath technique is summarized, Section III reviews the PU architecture in [1] and shows a possible implementation for the digital part of a transmitter using this technique. Matlab simulation results are shown in Section IV, showing the effect of amplitude discretization of signals in the presence of multipath technique. Conclusions are given in Section V.

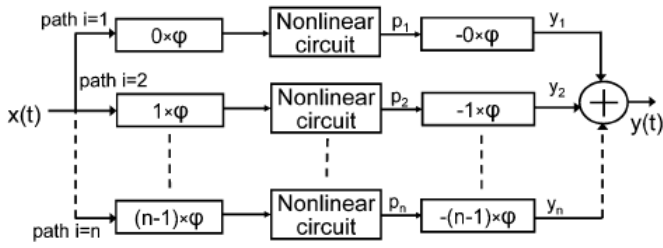


Figure 2: poly-phase n-path circuit

II. POLYPHASE MULTIPATH TECHNIQUE

When a non-linear circuit is excited by a sinusoidal input having a frequency ω , its output spectrum not only contains the frequency component at ω but also multiples of this frequency component at 2ω , 3ω and so on. These higher order terms are the unwanted distortion components. The Polyphase multipath technique is aimed at cancelling these higher order distortion terms [4]. Figure 2 shows an n-path circuit. The idea is to divide the nonlinear circuit into n equal smaller pieces and apply equal but opposite phase shifts before and after each nonlinear circuit.

If the phase shift in path i is $(i-1)\times\Phi$, where Φ is a phase shift constant satisfying $n \times \Phi = 360^\circ$, the output of the multipath circuit would produce the desired harmonic and cancel many of the higher order terms. If the signal $x(t)=\cos(\omega t)$ is applied as input to a weakly non linear system, the output of the i th path can be written as

$$p_i(t) = b_0 + b_1 \cos(\omega t + (i-1)\phi) + b_2 \cos(2\omega t + 2(i-1)\phi) + b_3 \cos(3\omega t + 3(i-1)\phi) + \dots (1)$$

Where $b_0, b_1, b_2, b_3, \dots$ are constants. From (1), it can be seen that the phase of the k th harmonic at the output of the nonlinear circuit rotates by k times the input phase $(i-1)\Phi$. The phase shifters $-(i-1)\Phi$, after the nonlinear blocks are required to align the fundamental components at ω in phase again. The signals at the output of these phase shifters can be written as

$$y_i(t) = b_0 + b_1 \cos(\omega t) + b_2 \cos(2\omega t + (i-1)\phi) + b_3 \cos(3\omega t + 2(i-1)\phi) + \dots (2)$$

In (2), the phase of the fundamental component is identical for all the paths, but the phases of the harmonics are different for each path. If the phase is chosen such that, $\phi = \frac{360^\circ}{n}$, then the higher order terms are cancelled except for the harmonics that satisfy the following equation.

$$k = j \times n + 1, \text{ where } j = 0, 1, 2, 3, \dots (3)$$

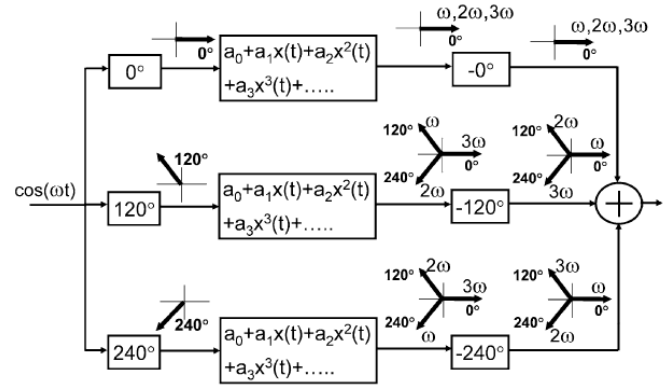


Figure 3: 3-Path circuit cancelling 2nd and 3rd harmonics

The well known differential circuit also exploits such a harmonic cancellation but it only cancels the even order terms. A three path system is shown in Figure 3. In this scheme phase shifts of $0^\circ, 120^\circ$ and 240° are added before and equal but opposite phases after the nonlinear element. As a result the fundamental components add up in phase while the phases of the second and third harmonics cancel each other out. The fourth harmonic would again have the same phase before summation and would not be cancelled. So the first non-cancelled harmonic in an n path system would be the $(n+1)$ th harmonic.

In case of two tones, the phase shift of the $p\omega_1 + q\omega_2$ products at the output of the i th path will be $(p+q-1)(i-1)\Phi$. So the products which satisfy (4) will not be cancelled.

$$p + q = j \times n + 1 \quad \text{where } j = 0, 1, 2, 3, \dots (4)$$

III. Proposed Transmitter

A. Analog Block

The analog part of this transmitter consists of the PU designed in [1]. The first set of phase shifts shown in Figure 2, are discussed in the next sub section. The second set of phase shifts, are implemented using mixers. The phase shifts in analog are often implemented using R-C or L-C networks but they do not provide the phase shift over a wideband as required by flexible multi-standard transmitters. Mixers not only provide wide band phase shifts but at the same time up-

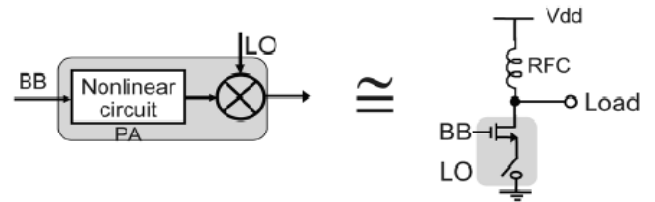


Figure 4. Basic Power Upconverter

convert the input signal. The mixer and power amplifier (PA) are combined into one circuit called a power upconverter (PU) shown in Figure 4. Such a PU when used in a multipath architecture is able to cancel distortion products produced by the PA. A PU using 18 paths shown in [1] uses a 1/3 duty cycle LO to suppress the dominant $3\omega_{LO}+3\omega_{BB}$ up-converted term and a differential circuit to cancel the even order terms produced as a result of using the 1/3 duty cycle. More implementation specific details are discussed in [1]. Measurements carried out on a chip using this circuit showed that the highest non-cancelled harmonic occurs at $17\omega_{LO}-\omega_{bb}$. This harmonic is 31 db below the desired signal. Other harmonic products are effectively suppressed to $\leq -48\text{dbc}$.

B. Proposed Digital Block

Since in a DSP intensive radio transmitter most of the signal processing is done in digital, therefore the first set of phase shifts are proposed to be done in the digital domain. A phase rotator should be added after the baseband processor which generates I and Q data streams. This phase rotator should provide a frequency independent phase shift to all the frequency components within the baseband signal.

When a complex signal such as $I+jQ$ is multiplied with a complex multiplier ($e^{j\theta}$), the resultant output is just the phase shifted version of the input.

$$I' + jQ' = (I + jQ) \cdot (e^{j\theta}) \quad (5)$$

where I' and Q' are phase shifted by the angle θ . The phase rotation is also shown in Figure 5. This phase rotation can be realized using either a complex multiplier or a CORDIC [5] rotator. The CORDIC rotator has the advantage of using only shifts and adds to accomplish complex multiplication. The phases to be generated in the multipath technique are separated by the same number of degrees. If an 18 path circuit is to be used then the phases to be generated are $0^\circ, 20^\circ, 40^\circ, 60^\circ$ etc. up to 340° . This topology has the advantage

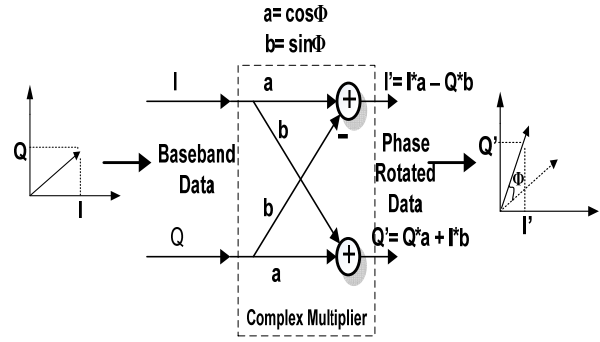


Figure 5.: Phase rotation on the input baseband signal using complex multiplication

that the same hardware used for one phase shift could be utilized to provide phase shifts for all the required phases. As an example, for a 18 path system, once the first I and Q data sample is shifted by 20° , the output can be again fed into the same phase rotator to provide another phase shift of 20° , giving a resultant phase shift of 40° , and this would continue till all 18 phase shifts have been realized. This would require that the intermediate phases for one particular set of samples be stored till all the phases have been generated.

A proposed architecture for implementing an IQ transmitter based on the multipath technique is shown in Figure 6. As suggested the complex multiplier based rotations shown separately in the figure can also be done using a single digital block. The I', I'', I''' etc and Q', Q'', Q''' data streams at the output of the complex multiplier are the phase shifted versions of the original I and Q vectors. After the DA converters the second set of phase shifts can be realized using the PU architecture.

A mathematical analysis for the proposed transmitter (for a three path system) starting from a complex baseband signal $z(t)$ is shown in (6) at the bottom of this page, where the frequency of the baseband is implicitly assumed in $z(t)$. The nonlinearity is assumed to be of the form $a_0x+a_1x^2+a_2x^3+\dots$, where the coefficients of the series (a_0, a_1, a_2) are assumed to be equal. For simplicity the analysis is done in continuous time and complex domain.

$$\begin{aligned}
 & \text{Complex Baseband } z(t) \begin{cases} \xrightarrow{\text{After 1st Phase Shift}} z(t) \cdot e^{j0} \rightarrow \frac{1}{3} [z(t) + z^2(t) + z^3(t) + z^4(t) \dots] \\ \xrightarrow{\text{After non-linearity}} z(t) \cdot e^{j120^\circ} \rightarrow \frac{1}{3} [z(t) \cdot e^{j120^\circ} + z^2(t) \cdot e^{j240^\circ} + z^3(t) \cdot e^{j360^\circ} + z^4(t) \cdot e^{j480^\circ} \dots] \\ \xrightarrow{\text{After non-linearity}} z(t) \cdot e^{j240^\circ} \rightarrow \frac{1}{3} [z(t) \cdot e^{j240^\circ} + z^2(t) \cdot e^{j480^\circ} + z^3(t) \cdot e^{j720^\circ} + z^4(t) \cdot e^{j960^\circ} \dots] \end{cases} \\
 & \xrightarrow{\text{After 2nd phase shifts}} \begin{cases} \frac{1}{3} [z(t) \cdot e^{j(\omega_c t)} + z^2(t) \cdot e^{j(\omega_c t)} + z^3(t) \cdot e^{j(\omega_c t)} + z^4(t) \cdot e^{j(\omega_c t)} \dots] \\ \frac{1}{3} [z(t) \cdot e^{j(\omega_c t)} + z^2(t) \cdot e^{j(\omega_c t + 120^\circ)} + z^3(t) \cdot e^{j(\omega_c t + 240^\circ)} + z^4(t) \cdot e^{j(\omega_c t + 360^\circ)} \dots] \\ \frac{1}{3} [z(t) \cdot e^{j(\omega_c t)} + z^2(t) \cdot e^{j(\omega_c t + 240^\circ)} + z^3(t) \cdot e^{j(\omega_c t + 480^\circ)} + z^4(t) \cdot e^{j(\omega_c t + 720^\circ)} \dots] \end{cases} \\
 & \xrightarrow{\text{After Summation of all paths}} z(t) \cdot e^{j(\omega_c t)} + z^4(t) \cdot e^{j(\omega_c t)} + \dots \quad (6)
 \end{aligned}$$

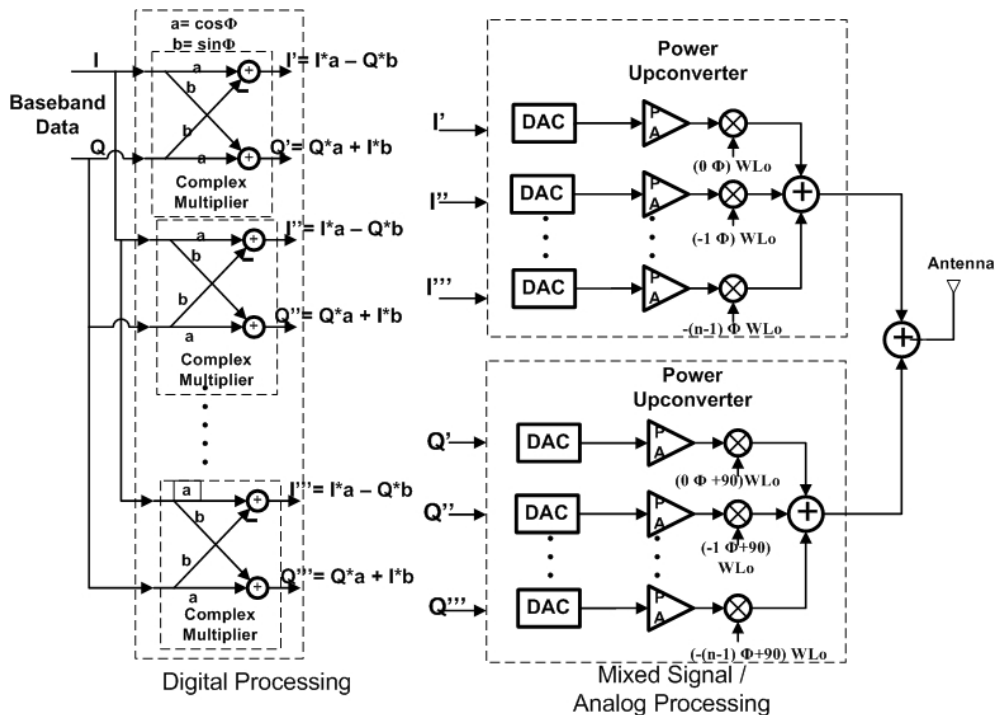


Figure 6: Proposed Architecture for an IQ transmitter using multipath technique

The $1/3$ fraction is due to the equal division of the non-linear elements into 3 equal parts. After summation of all paths, the 2nd and 3rd order terms are cancelled and the first non-cancelled term would be the fourth order term.

C. DAC Implementation issues

The linearity of the DACs and the reconstruction filters after the DACs are vital to obtain reasonable cancellation of the harmonics produced by PU. Using n DACs for n paths could make it difficult to achieve the gain and phase matching requirements derived in [4]. Since each path would add its own mismatch. If a single high speed DAC is used for all the paths then this can mitigate the potential mismatch caused due to multiple parallel DACs and also reduce the total area consumed by the design. This kind of architecture would require that the output of DACs be stored on some memory elements like capacitors and then loaded into the multipath PU at the same time. This loading operation has to be perfectly synchronized for all the paths otherwise it could lead to degradation in phase matching.

Another possible implementation for the DAC could be based on using oversampled single bit semi-digital FIR DAC's [6]. The single bit DAC's are inherently linear and also the semi-digital FIR filter could be more linear than an analog reconstruction filter. The quantization errors in filter coefficients also do not deteriorate the pass-band response as much as the stop-band. The effect on phase linearity could be minimized by using the appropriate number of filter coefficients. The amplitude and phase mismatch requirements are critical for sufficient cancellation of distortion products produced at RF frequency. A discussion on mismatch requirements of the multipath technique is given in [4]. More

work needs to be done to explore the most effective way of implementing the DA converters for the multipath technique.

IV. LIMITED D/A RESOLUTION EFFECTS USING MULTIPATH TECHNIQUE

A. Using Ideal Phase Shifters

In Section III A. it was shown that most of the harmonics produced due to non-linearity in the PU design can be cancelled using the multipath principle. We will now examine the system in which the first set of phase shifts are done in the digital domain.

What would happen if these digital signals are converted to an amplitude discrete representation in the presence of the multipath technique? If the digital inputs applied to a D/A converter are assumed to be single tone or multi-tone sinusoids, what effect does finite D/A resolution have on the output in the presence of multipath? If no reconstruction filtering is done in the D/A converter, what effect will the multipath have in such a case? These questions are investigated first at baseband (upconversion to RF frequency adds further nonlinearity to the transmit path and is analyzed separately) with ideal phase shifters in this section and then with (upconversion) mixers as second set of phase shifts in the next section.

If a pure sinusoid is amplitude quantized in an ADC (resembling a stair-case), then the quantizer noise is purely discrete [7], and the output spectrum shows distortion terms which are multiples of the fundamental frequency.

The output spectrum of D/A conversion with the same resolution (when input is digital sine) in the absence of reconstruction filtering would also show similar distortion as

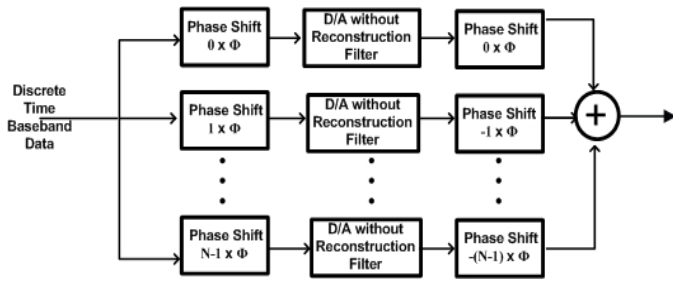


Figure 7: Multipath applied to a uniform DAC without reconstruction filter

in both cases the time domain signal is the same stair-case.

This effect of uniform quantization on an input signal such as $x = A(t) \sin \phi(t)$ is shown in [8] to produce the following output :

$$y = \sum_{p=1}^{\infty} A_p \sin p\phi \quad (7)$$

where A_p is the amplitude of the p^{th} harmonic of the quantized signal. As seen in (7), the phase of the p^{th} harmonic is also p times the input phase. This characteristic of the phase is essential for the multipath technique to work effectively. In this way the uniform quantizer represents the kind of nonlinear system described in (1) in section II. Hence the multipath technique applied to a uniform quantizer, when the input is sinusoid, should be able to cancel out the harmonics produced at the output.

In order to verify the effect of multipath on such a distortion, the model of Figure 7 is used. For simulation purposes the baseband data is assumed to be discrete time representation of a sinusoid. This is because distortion due to amplitude quantization would be easier to predict for a sinusoid as shown in (7).

The D/A conversion block in this case only serves to discretize the amplitude of the input sinusoid. As only the effect of amplitude discretization of input baseband data using the multipath is investigated and not the time discretization effects, therefore the input baseband data in Figure 7 is also assumed to be highly oversampled so in essence it mimics almost a continuous time signal. Phase shifts shown are also assumed to be ideal.

Figure 8 shows Matlab simulation results for the output spectrum of Figure 7 for both single and two tone input data.. First as a test case the input signal was discretized to only two levels, as this corresponds to 1 bit quantization, which is the lowest quantization possible. Figure 8 (a) and (b) show the simulation results of applying one and then four paths to a 1 bit quantized sinusoid. The output of one path shows that the spectrum has only odd harmonic terms ($\omega_1, 3\omega_1, 5\omega_1, \dots$) as the time domain signal is fully symmetric. Four paths were chosen to show that the 3rd harmonic is cancelled and the first non-cancelled harmonic is the fifth (see section II). To show that the multipath also works for different number of paths and quantization levels, Figure 8 (c) shows the spectrum of 2 bit quantized sinusoid after 1 path and (d) shows the spectrum after 6 paths. It can be seen that in this case the first non-cancelled harmonic is the seventh, corresponding to the theory in section II. Simulations for different number of paths and various quantization levels show that the first non-cancelled harmonic of the quantization process is the $(n+1)^{\text{th}}$ where n is the number of paths.

Two tone inputs were also used in order to check whether the outputs are in line with the theory of Section II, Figure 8 (e) shows the spectrum of 1 bit quantized two tone signal after 1 path and Figure 8 (f) shows the spectrum after 4 paths. From the figure it can be seen that for 4 paths, the terms satisfying (4) (i.e. $p\omega_1 + q\omega_2 = j \times n + 1$) which in this case for $n=4$, are

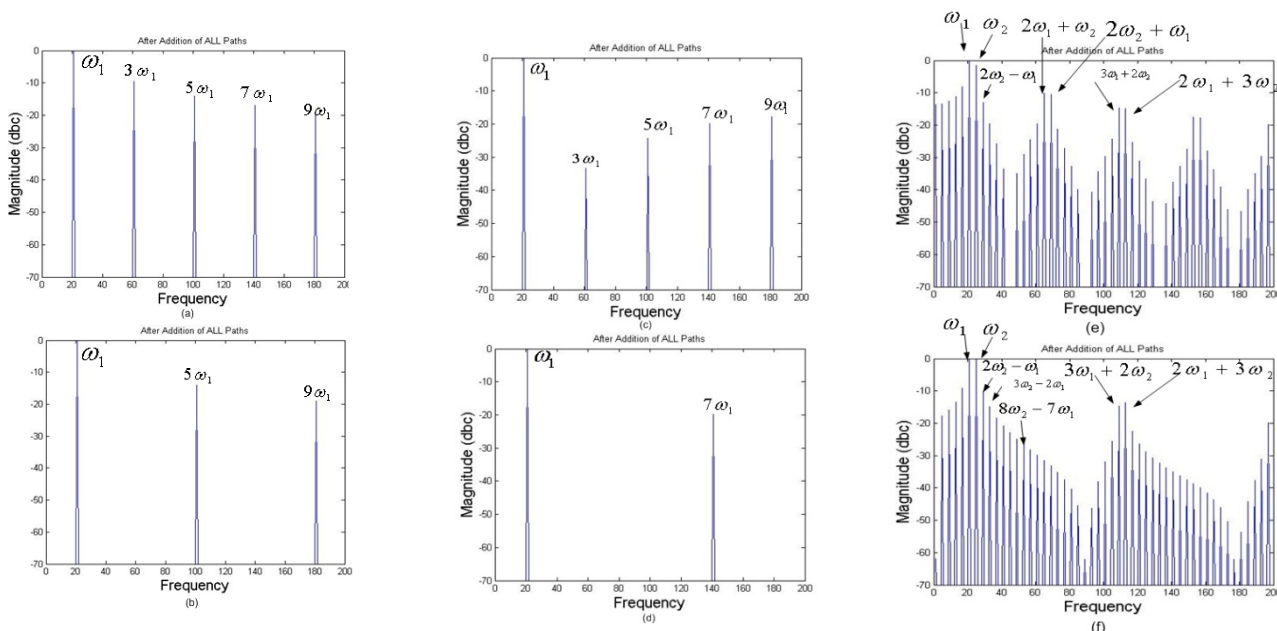


Figure 8: (a) FFT of a 1 bit quantized single tone input after 1 path (b) after 4 paths (c) FFT of 2 bit quantized single tone after 1 path (d) after 6 path (e) FFT of 1 bit quantized 2 tone input after 1 path (f) after 4 paths

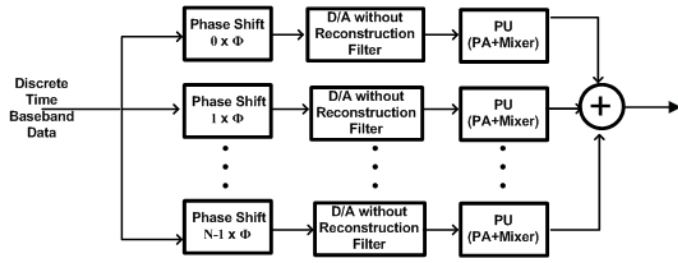


Figure 9: Multipath Transmitter with second set of phase shifts provided by the PU

terms corresponding to $(3+2=1 \times 4+1) 3\omega_1+2\omega_2, 3\omega_2+2\omega_1$ and its adjacent terms remain un-cancelled. Although the terms such as $2\omega_1+\omega_2$ and $2\omega_2+\omega_1$ appear to have been cancelled, in its place there are some other lower magnitude terms in Figure 8 (f) which were not present in Figure 8 (e). These terms seem to be additional products of the intermodulation of ω_1 and ω_2 (which satisfy $p+q=1$), in which case they can also be explained by (4) in section II. Increasing the number of quantization levels or dithering [7] can reduce the highest spur level and make the spectrum of the quantized signal more clean.

From simulation results of single and two tone inputs it can be concluded that the multipath technique using ideal phase

shifts is able to cancel most of the harmonics of a uniform quantizer in line with the theory in section II, however since some significant terms remain, so its usefulness in this context is limited.

B. Using Mixer as Phase Shifter

In previous subsection it was assumed that the phase shifters were ideal. In fact in the multipath PU design [1] the second set of phase shifts are implemented using mixers, as they provide a phase shift over a wideband but they also add nonlinearity. So the question arises what happens after the summation in Figure 9, in the presence of the PU nonlinearity and the distortion caused by amplitude quantization of input data in the D/A block. This model was simulated in Matlab, where the PA nonlinearity was modeled by a power series expansion, the mixer was modeled as an ideal switch, while the first set of phase shifters were again assumed to be ideal. The D/A block is modeled as a uniform quantizer as in the previous subsection without any reconstruction filtering. The input to the switch is a square wave with a 1/3 duty cycle to suppress the 3rd harmonic of the local oscillator (LO) as discussed in [1] and the output is made differential to cancel the even order terms.

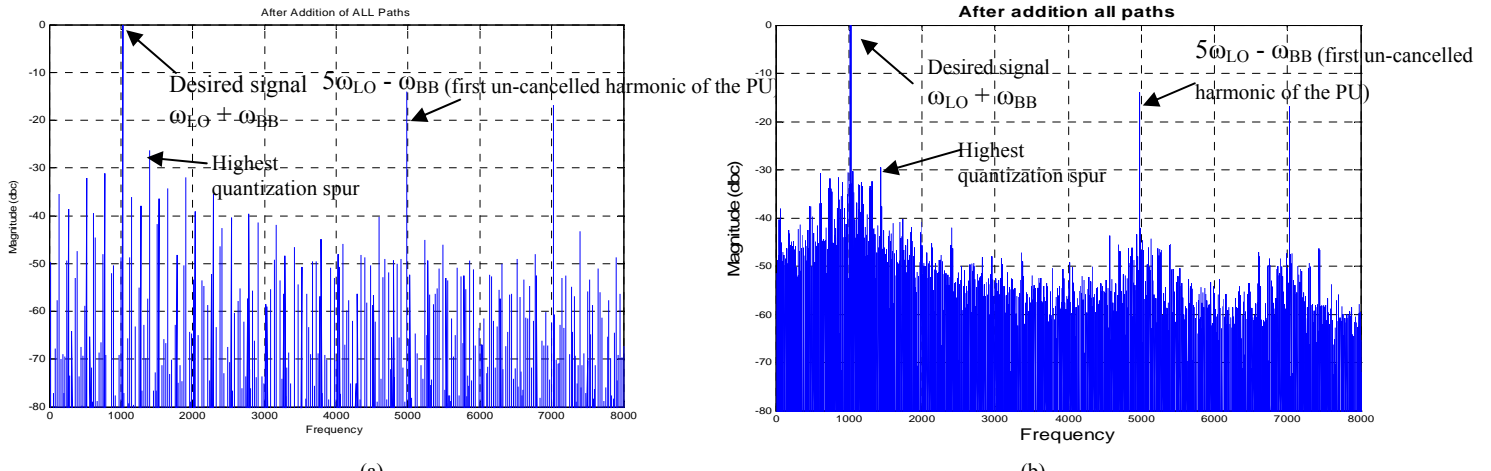


Figure 10: Output spectrum after 6 path and 3 bit quantization (a) for single tone (b) for two tones, both using mixers as second set of phase shifters

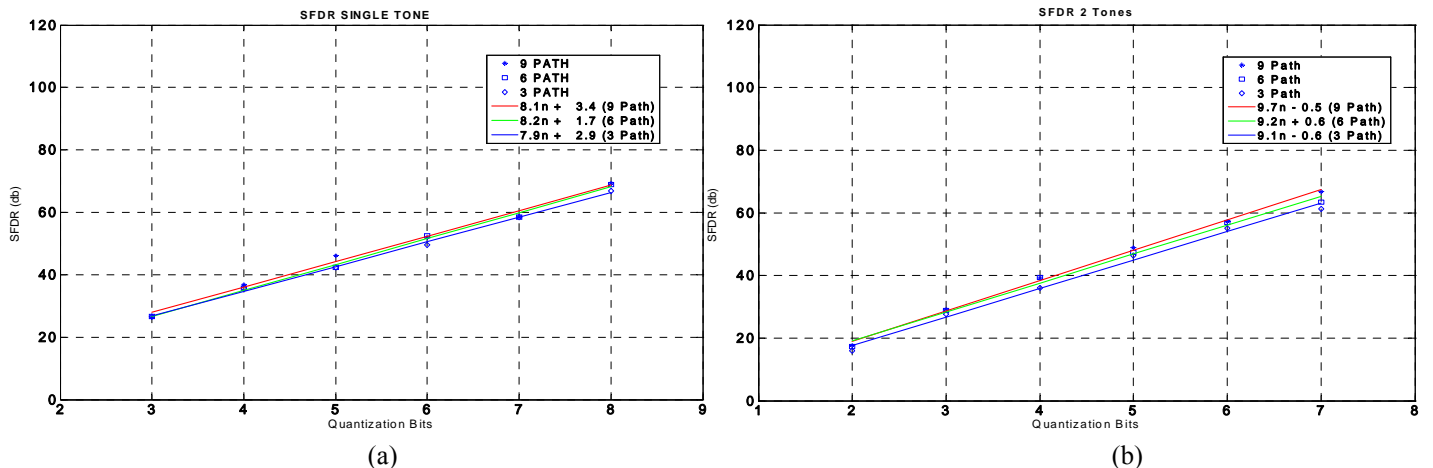


Figure 11 SFDR_Q versus quantization bits for the power up-converter using multipath (a) for single tone (b) two tone input

Since the switching mixer produces both sum and difference frequencies a lot of harmonics are produced, as shown in Figure 10, for a six path system with three bits quantization. The Figure 10 (a) is for single tone input and the Figure 10 (b) for 2 tone inputs which explains the dense nature of the second graph. The figure shows the desired up-converted signal is at $\omega_{LO} + \omega_{bb}$ and the first un-cancelled harmonic of the PU nonlinearity is at $5\omega_{LO} - \omega_{bb}$. According to [1] for a 6 path system the first un-cancelled harmonic of the PU should also occur at $5\omega_{LO} - \omega_{bb}$. Simulations carried out for various quantization levels and paths showed that the non-cancelled harmonics of the PU non-linearity remain the same as predicted by [1]. The analysis to determine which distortion terms produced only due to amplitude quantization are cancelled and which remain un-cancelled after the multipath becomes complex because of the added non-linearity of the PU. Instead of determining which harmonics of the quantization process get cancelled another criteria is used to judge the effect of multipath on the upconverted output spectrum.

The amplitude of the strongest up-converted frequency component produced due to the quantization process relative to the amplitude of the desired signal is determined through simulation. This criterion can also be termed as the spurious free dynamic range (SFDR_Q) of the upconverter where the spurs refer only to the highest upconverted spectral component due to the amplitude quantization. These spurs were plotted for various quantization levels and number of paths. In order to determine the general trend of the SFDR_Q variation with respect to the quantization bits a first order polynomial was fit through these points as shown in Figure 11. The slope of the SFDR_Q curve follows roughly 8db/bit for single tone and around 10db/bit for two tone inputs. This pattern is close to the SFDR_Q figures for ideal quantizers found in literature [9]. The increase in SFDR_Q slope for two tones as compared to the single tone can be attributed to the decrease in correlation between the input signal and the quantization error. The small variation in slope of Figure 11 for different number of paths can be credited to the effect of multipath PU, as it does seem to influence the magnitude of some of the distortion terms of the quantizer, but this effect on the slope of SFDR_Q of roughly ± 0.5 dB is not very significant.

V CONCLUSION

The multipath technique using ideal phase shifts can cancel most of the harmonics produced due to amplitude discrete representation of pure sinusoids. This effect is clearly visible in simulations when using single tone inputs. For two tone inputs the same effect can be seen, but some other lower magnitude spectral terms appear, the cause of which is not proven. Linear fit SFDR_Q plots are utilized to check the effect of multipath on quantization distortion when (upconversion) mixers are employed as second set of phase shifts. The slope of the SFDR_Q curve follows roughly 8db/bit for single tone inputs and close to 10db/bit for two tones. This result can be useful in estimating the DAC resolution for the multipath transmitter.

ACKNOWLEDGEMENT

The authors would like to thank Jaap Haartsen, Andre Kokkler and Niels Moseley from University of Twente for useful discussions.

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