

USING EXIT CHARTS TO DEVELOP AN EARLY STOPPING CRITERION FOR TURBO DECODING

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ABSTRACT

Early stopping criteria are used to determine when additional decoder iterations result in little or no improvement in the bit-error-rate (BER) at the output of a Turbo decoder. This paper proposes a stopping criterion based on Extrinsic Information Transfer (EXIT) charts. The generation and properties of EXIT charts are discussed. Convergence and BER predictions obtained using EXIT charts are compared to results obtained using Turbo decoder simulation. Finally this paper describes the use of convergence and BER predictions obtained from EXIT charts to develop a stopping criterion for Turbo decoding.

1. INTRODUCTION

Iterative decoding is a key feature of Turbo decoding. The bit-error-rate (BER) at the output of the decoder decreases with each decoder iteration. However, the improvement in BER decreases with each iteration. The improvement in BER with each iteration also depends on the signal to noise ratio E_b/N_0 , where E_b is the received energy per information bit and N_0 is the noise one sided power spectral density [1]. To determine when additional decoder iterations will provide little or no improvement in BER so called “early stopping criteria” are used. These early stopping criteria attempt to terminate the decoding process without significantly impacting the performance of the Turbo decoder. A recent paper on stopping criteria is [2].

In Extrinsic Information Transfer charts (EXIT charts) [3] transfer characteristics based on mutual information are used to describe the flow of extrinsic information through the soft in/soft out constituent decoders of an iterative decoder. A decoding trajectory visualizes the exchange of extrinsic information between the constituent decoders. The BER can also be visualized in the EXIT charts. This allows the prediction of the BER of the iterative decoder after an arbitrary number of iterations.

This paper describes how EXIT charts can be used to develop an early stopping criterion for Turbo decoding, which can also be used to determine how many decoder iterations are required to achieve a desired BER for a given E_b/N_0 .

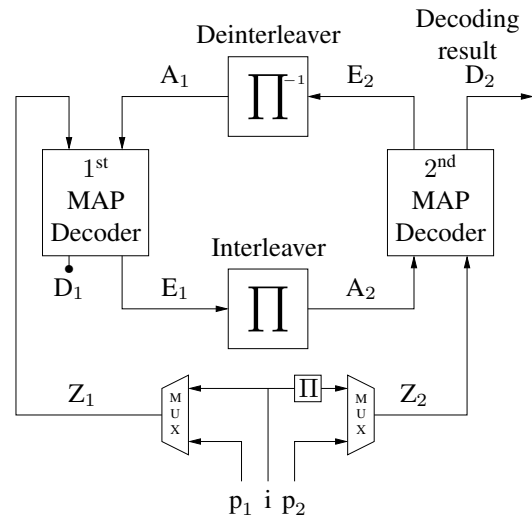


Figure 1: Iterative Decoder

Throughout this paper the UMTS Turbo code [4] is used as an example Turbo code for which an early stopping criterion should be developed. The UMTS Turbo code is a Parallel Concatenated Convolutional (PCC) code with two 8-state constituent encoders. Both constituent encoders are of the recursive systematic type. The polynomial of the feedforward generator is 15 and the polynomial of the feedback generator is 13, both in octal. In simulations of a complete Turbo decoder the UMTS interleaver is used. This paper is organized as follows: In Section 2 EXIT chart generation and the properties of EXIT charts are described. In this section results obtained using EXIT charts are also compared to results from Turbo decoder simulations. Section 3 describes how EXIT charts can be used to develop an early stopping criterion for Turbo decoding. In Section 4 some conclusions are drawn. Finally in Section 5 directions for future research are given.

2. EXIT CHARTS

2.1. EXIT Chart Generation

In Figure 1 an iterative decoder for PCC codes such as the UMTS Turbo code is shown. For each iteration the

first constituent decoder (MAP algorithm) takes intrinsic information (channel observations) Z_1 on the systematic (information) bits i and respective parity bits p_1 and outputs soft values D_1 . The extrinsic information on the systematic bits $E_1 = D_1 - A_1 - Z_1$ is passed through the bit interleaver to become the a priori input A_2 of the second constituent decoder. The second decoder takes the permuted channel observations Z_2 on the systematic bits i and respective parity bits p_2 and feeds back extrinsic information $E_2 = D_2 - A_2 - Z_2$ which becomes a priori knowledge A_1 of the first decoder. The variables $Z_1, A_1, D_1, E_1, Z_2, A_2, D_2$ and E_2 all denote log-likelihood ratios. An EXIT chart for such a decoder is constructed from transfer characteristics of the individual constituent decoders.

Let the binary random variable X denote the transmitted systematic bits with realizations $x \in \{+1, -1\}$. Define I_E as the mutual information between the transmitted systematic bits X and the extrinsic information E of a systematic decoder, i.e. $I_E = I(X; E)$. Define I_A as the mutual information between the transmitted bits X and the a priori information A of a constituent decoder, i.e. $I_A = I(X; A)$. A transfer characteristic T relates I_E to I_A and the signal to noise ratio on the channel

$$I_E = T(I_A, E_b/N_0). \quad (1)$$

The transfer characteristic of a constituent decoder can be determined by Monte Carlo simulation. For this, a priori information A with a certain value of I_A is applied to the a priori input of the constituent decoder. In [3] it is shown that a priori information can be generated with an independent Gaussian random variable with variance σ_A^2 and mean $\mu_A = \sigma_A^2/2$. For this two observations obtained by simulation of Turbo decoders are used. 1) For large interleavers the a priori values A remain fairly uncorrelated from the respective channel observations Z over many iterations. 2) The probability density functions of the extrinsic output values E (a priori values A for the next decoder respectively) approach Gaussian-like distributions with increasing number of iterations. A certain value of I_A can be obtained by choosing σ_A according to [3]

$$\sigma_A = J^{-1}(I_A), \quad (2)$$

with

$$J(\sigma) \triangleq I_A(\sigma_A = \sigma) \quad (3)$$

and

$$I_A(\sigma_A) = 1 - \int_{-\infty}^{\infty} \frac{e^{-((\xi - \sigma_A^2/2)^2/2\sigma_A^2)}}{\sqrt{2\pi}\sigma_A} \cdot \log_2[1 + e^{-\xi}] d\xi. \quad (4)$$

The integral above cannot be solved in closed form and will have to be evaluated numerically.

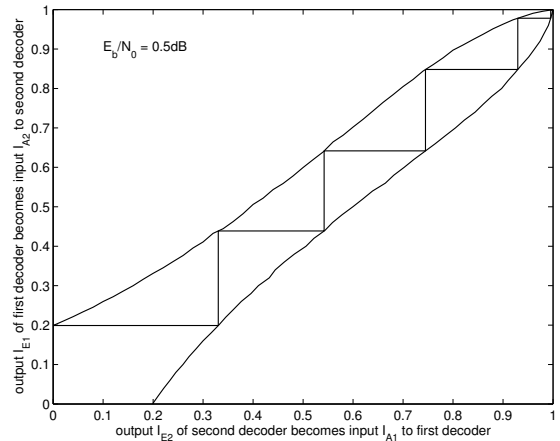


Figure 2: EXIT chart of the UMTS Turbo decoder for $E_b/N_0 = 0.5dB$

The intrinsic information Z can be generated from the transmitted information and parity bits using, for example, an ordinary AWGN channel model.

If the distribution of E is symmetric and consistent the mutual information $I_E = I(X; E)$ for the extrinsic output can be determined from N samples $x_n \cdot E_n$ of the transmitted bits X and the extrinsic output of the constituent decoder E , with [5]:

$$I(X; E) = 1 - E\{\log_2(1 + e^{-E})\} \approx 1 - \frac{1}{N} \sum_{n=1}^N \log_2(1 + e^{-x_n \cdot E_n}). \quad (5)$$

In Figure 2 the transfer characteristics of the two constituent decoders of the UMTS Turbo decoder for an E_b/N_0 of 0.5dB are plotted into a single diagram. The a priori input I_A of the first decoder is on the x -axis, the extrinsic output I_E of the first decoder is on the y -axis. The transfer characteristic of the first decoder is the upper curve in the figure. For the second decoder the axes are swapped, so the a priori input I_A of the second decoder is on the y -axis and its extrinsic output I_E is on the x -axis. The lower curve in the figure is the transfer characteristic of the second decoder. Since the two constituent codes of the UMTS Turbo code have the same generators, the transfer characteristics of the two constituent decoders of the UMTS Turbo decoder are the same. The transfer characteristic of the second decoder can therefore be obtained by mirroring the transfer characteristic of the first decoder in the $x = y$ -axis. This diagram is referred to as EXIT chart since the exchange of extrinsic information can be visualized as an expected decoding trajectory. The expected decoding trajectory is the “stairs”-like curve in Figure 2. The shape of the expected decoding trajectory can be explained as follows: Let n be the iteration index. For $n = 0$

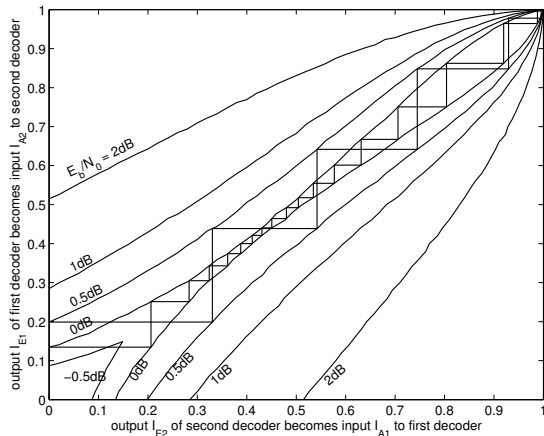


Figure 3: Transfer characteristics of the two constituent decoders of the UMTS Turbo decoder for various E_b/N_0

the decoding process starts at the origin with zero a priori knowledge $I_{A1,0} = 0$. At iteration n the extrinsic output of the first decoder is $I_{E1,n} = T_1(I_{A1,n})$. $I_{E1,n}$ is forwarded to the second decoder to become $I_{A2,n} = I_{E1,n}$. The extrinsic output of the second decoder is $I_{E2,n} = T_2(I_{A2,n})$, which is fed back to the first decoder to become the a priori knowledge $I_{A1,n+1} = I_{E2,n}$. Note that interleaving does not change mutual information.

2.2. Exit Charts for a set of E_b/N_0 values

In Figure 3 the transfer characteristics of the two constituent decoders of the UMTS Turbo decoder are shown for a set of E_b/N_0 -values. The expected decoding trajectories for E_b/N_0 -values of $0.0dB$ and $0.5dB$ are also shown in the figure. From the expected decoding trajectories it can be clearly seen that the Turbo decoder needs a lot more iterations to converge to a mutual information of 1 and thus to a low BER for lower E_b/N_0 values than for higher E_b/N_0 values. For an E_b/N_0 of $-0.5dB$ the Turbo decoder does not converge to a mutual information of 1 at all. When these expected decoding trajectories are representative of the actual decoding trajectories EXIT charts can be used to determine the number of iterations the Turbo decoder requires to converge for a given E_b/N_0 value. If the expected decoding trajectories are representative of the actual decoding trajectories will be verified in Subsection 2.4.

2.3. BER Prediction

In [3] a formula to predict the BERs from an EXIT chart is derived. Under the assumption that the a priori knowledge A and the extrinsic output E are Gaussian distributed this

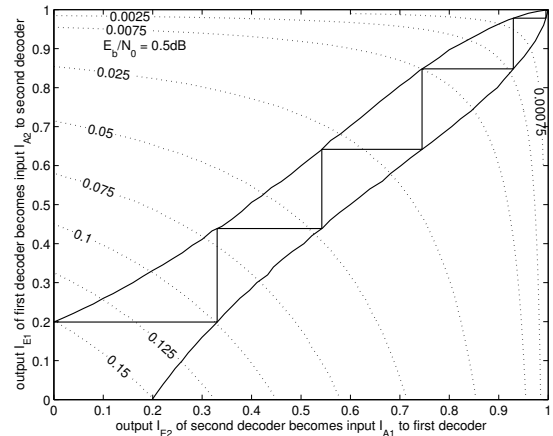


Figure 4: EXIT chart of the UMTS Turbo decoder with predicted BER as contour plot for $E_b/N_0 = 0.5dB$

formula is for an AWGN channel

$$P_b \approx \frac{1}{2} \operatorname{erfc} \left(\frac{\sqrt{8R \cdot \frac{E_b}{N_0} + J^{-1}(I_A)^2 + J^{-1}(I_E)^2}}{2\sqrt{2}} \right), \quad (6)$$

where R is the rate of the PCC code. In Figure 4 the EXIT chart of the UMTS Turbo decoder for an E_b/N_0 of $0.5dB$ is shown. In this figure the BERs predicted from the EXIT chart using the equation above are plotted as dotted contours. When the BER predictions obtained from the EXIT chart are accurate, EXIT charts can also be used to predict the BER that is achieved at the Turbo decoder output, for a given E_b/N_0 value, after a certain number of Turbo decoder iterations. The accuracy of the BER predictions will be verified in the next subsection.

2.4. Comparison with Turbo Decoder Simulations

In Figure 5 the EXIT chart of the UMTS Turbo decoder for an E_b/N_0 of $0.5dB$ is shown again. In the figure the BERs obtained from the EXIT chart are plotted as dotted contours. In addition to the expected decoding trajectory (dashed), the actual decoding trajectory (solid) is plotted. The actual BERs after each decoder iteration are plotted as solid contours. The actual decoding trajectory and the actual BER values are obtained from a simulation of the UMTS Turbo decoder with an interleaver size of 5114 bits.

The decoding trajectory expected from the EXIT chart in Figure 5 matches the actual decoding trajectory exactly for the first two decoder iterations. After two iterations the expected decoding trajectory still matches the actual decoding trajectory pretty well. Both decoding trajectories show that the Turbo decoder has converged after about 5 decoder iterations. This corresponds with simulation re-

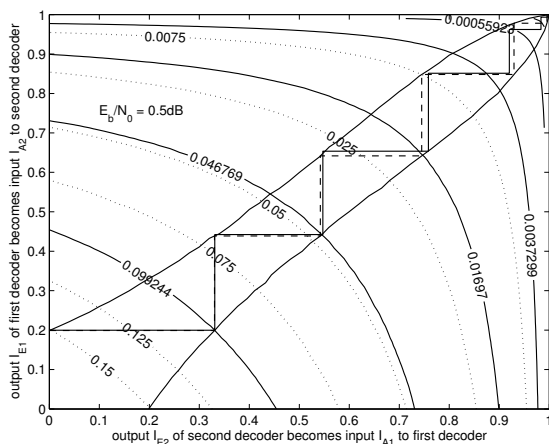


Figure 5: EXIT chart of the UMTS Turbo decoder with actual decoding trajectory and predicted and actual BERs as contour plot for $E_b/N_0 = 0.5dB$

sults of the UMTS Turbo decoder found in literature [6]. The BERs obtained from the EXIT chart match the actual BERs after each decoder iteration pretty well for the first three iterations. After three iterations the BER predictions from the EXIT chart become to optimistic. More simulations are being performed to verify the accuracy of the predictions obtained from EXIT charts in other conditions.

3. USING EXIT CHARTS TO DEVELOP AN EARLY STOPPING CRITERION

A table with the number of iterations a Turbo coder should perform to reach convergence for a given E_b/N_0 value can be constructed from the EXIT charts for a set of E_b/N_0 values. This table is generated and stored in the decoder when the decoder is designed. The decoder can estimate the E_b/N_0 value at run-time and determine the number of iterations it should perform for these conditions from the table. So the table that is generated using EXIT charts can be used as an early stopping criterion.

Using EXIT charts a table can be generated that stores the BER that is achieved after each decoder iteration for a set of E_b/N_0 values. When this table is generated at design-time and stored in the decoder, the decoder can determine the number of iterations it should perform to achieve a certain BER. This can be useful if the decoder knows the quality of service requirements on the BER.

4. CONCLUSION

This paper proposes an early stopping criterion for Turbo decoders based on EXIT charts. This early stopping criterion can be used to determine the number of iterations the

Turbo decoder should perform to converge to low BERs as well as to determine the number of iterations that are required to achieve a certain BER.

5. FUTURE RESEARCH

To implement the proposed EXIT chart based early stopping criterion we will have to study E_b/N_0 estimation algorithms. We can then compare the proposed early stopping criterion to other early stopping criteria, such as [2]. Finally we would like to use the proposed early stopping criterion in our Adaptive Wireless Networking project [7].

6. ACKNOWLEDGEMENT

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7. REFERENCES

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