GENERALIZATION AND PERFORMANCE IMPROVEMENT OF A COHERENCE MULTIPLEXING SYSTEM

Arjan Meijerink, Geert H.L.M. Heideman, Wim C. van Etten

University of Twente, Department of Electrical Engineering,
Chair for Telecommunication Engineering
P.O. Box 217, 7500 AE Enschede, The Netherlands
a.meijerink@ieee.org

Abstract

Usually a coherence multiplexing system uses delay-filters at the transmitter and receiver to perform the code. An extension to other filter types is described. Using a continuous source the signal-to-beat noise ratio is proportional to the square of the inverse of the number of simultaneous users. A further extension is made by using a pulsed source and by replacing the filters by banks of filters. Each element of each filter bank also comprizes a unique delay. In that case the *SNR* can be made proportional to the inverse of the number of users, so that more users can be handled simultaneously.

1 Introduction

Coherence multiplexing is a relatively unknown form of optical code division multiple access (OCDMA). It is particularly suitable in access networks for optical communication systems since it imposes less severe constraints on transmitter and receiver components than for instance WDM, which requires very stable lasers and receiver filters in order to avoid crosstalk between adjacent channels. Coherence multiplexing is a technique that utilizes the random phase jitter of a broadband laser or LED, by transmitting two versions of the source signal and correlating these two in the receiver. Coherence multiplexing in its conventional form is extensively discussed in [1] and [2] and is illustrated in Figure 1.

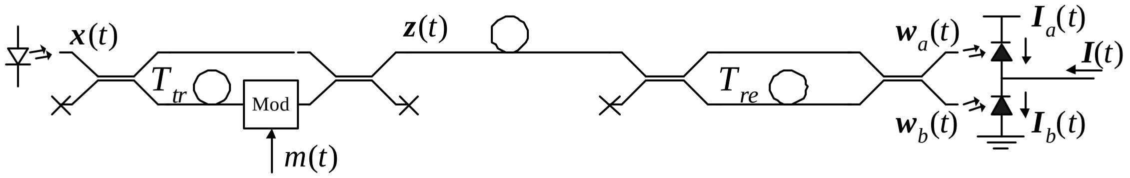

Figure 1: A conventional coherence multiplexing system

The symbols in Figure 1 correspond to the following quantities. $\boldsymbol{x}(t)$, $\boldsymbol{z}(t)$, $\boldsymbol{w}_a(t)$ and $\boldsymbol{w}_b(t)$ are the pre-envelopes of the electrical fields corresponding to the lightwaves at the indicated places. In the receiver, $\boldsymbol{I}_a(t)$ and $\boldsymbol{I}_b(t)$ are the upper and lower photodiode currents, respectively, and $\boldsymbol{I}(t)$ is the difference between these two. $m(t)$ is the information-carrying signal, which is assumed to be a rectangular polar NRZ signal with bit-time T_b. T_{tr} is the difference in delay between the upper and lower transmitter branches, and T_{re} is the difference in delay between the upper and lower receiver branches. The pre-envelopes of the electrical fields and the currents are random processes; all electrical fields are assumed to be circular complex gaussian distributed bandpass signals and the currents are real baseband signals. The left part of Figure 1 represents a transmitter that converts the input signal $\boldsymbol{x}(t)$ into a signal $\boldsymbol{z}(t)$, consisting of two versions of the source signal $\boldsymbol{x}(t)$. One of them is shifted in time with respect to the other by a time T_{tr}, and modulated by $m(t)$. In the receiver, $\boldsymbol{z}(t)$ is splitted up into two versions that are shifted in time with respect to one another by a time T_{re}.

It can be proven that the phase shifts in the right coupler cause the output current to be proportional to the product of the amplitudes of these two versions of $\boldsymbol{z}(t)$ times the cosine of their phase difference, provided that all couplers are lossless and perfectly balanced. The average output current $E[\boldsymbol{I}(t)]$ is thus proportional to the crosscorrelation of these two versions, for zero timeshift. Since the source signal $\boldsymbol{x}(t)$ suffers from phase jitter, two signals can only have a non-zero crosscorrelation function for zero timeshift when they are nearly coherent. Consequently, $\boldsymbol{I}(t)$ will only have a non-zero average if T_{re} and T_{tr} are nearly identical. Choosing the T_{tr}'s sufficiently apart for different transmitters is thus a way to enable the receiver to distinguish between the different transmitted signals. It can be shown (see [2]) that the dominant noise source in the system is interferometric noise, which is caused by the random character of the interfering lightwaves. In the remainder of this paper we will address interferometric noise as beat noise, for convenience. By calculating the autocorrelation function of $\boldsymbol{I}(t)$ and its corresponding power spectral density, it can be shown that the signal-to-beat noise ratio in a coherence multiplexing system with M active transmitters is approximately proportional to $1/M^2$ (see [2]). Particularly for large M, this greatly limits the overall signal-to-noise ratio, and, as a consequence, the number of users for a given quality. It is thus desirable to find a way to reduce the beat noise in the output current. In this paper, an alternative form of a coherence multiplexing system is proposed, in which the signal-to-beat noise ratio is proportional to $1/M$ instead of $1/M^2$.

2 A generalized coherence multiplexing system

A more generalized form of a coherence multiplexing system can be obtained by replacing the delay lines in both the transmitters and receivers by linear filters. This is illustrated in Figure 2.

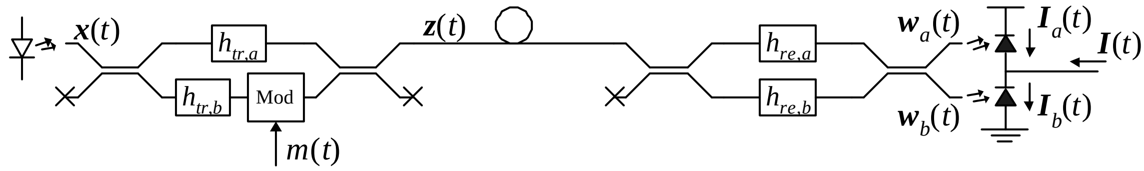

Figure 2: A generalized coherence multiplexing system

The h-symbols in the filter boxes represent the corresponding impulse responses. The generalization serves three purposes:

- Enabling an extension to other filter types;
- Getting a more fundamental understanding of the behaviour of the conventional system;
- Clarifying the analysis of a timeslotted system.

We can express both the average output current and the power spectral density of the beat noise as a function of the impulse responses of the filters in Figure 2, to be able to impose demands on these impulse responses for minimizing bias, crosstalk and noise. Therefore, we have to distinguish M transmitters and M receivers. Each transmitter i has its own transmitter filters $h_{tr,a,i}$ and $h_{tr,b,i}$ and information-carrying signal $m_i(t)$, and each receiver r has its own receiver filters $h_{re,a,r}$ and $h_{re,b,r}$. All transmitters have identically distributed, mutually independent source signals $\boldsymbol{x}_i(t)$, with equal spectrum. It is assumed that losses and dispersion in the fiber can be neglected.

2.1 Average output current

The instantaneous output current $\boldsymbol{I}_r(t)$ of a receiver r can be found by observing one bit period and assuming that $m_i(t)$ is constant during that period. We then write both $\boldsymbol{w}_{a,r}(t)$ and $\boldsymbol{w}_{b,r}(t)$ as a sum of four convolutions, which correspond to the four possible paths that a lightwave can travel from the source to either of the photodiodes, thereby being filtered and/or multiplied by m_i and phase-shifted by 90° when a coupler is 'crossed' (see [3]). $\boldsymbol{I}_r(t)$ is equal to the difference of the powers of $\boldsymbol{w}_{a,r}(t)$ and $\boldsymbol{w}_{b,r}(t)$ times the responsivity R_{pd} of the photodiodes. The average output current $\mathrm{E}[\boldsymbol{I}_r(t)]$ can be found by taking the expected value of the resulting expression. If we define the following inner product:

$$\langle H_1, H_2 \rangle \hat{=} \int_0^{+\infty} H_1(f)\cdot H_2{*}(f)\cdot S_{x*x}(f)\cdot \mathrm{d}f\ , \qquad (1)$$

we can write the average output current as:

$$E[\boldsymbol{I}_r(t)] = \frac{R_{pd}}{16\cdot M^2}\cdot\left(\begin{array}{l} \sum_{i=1}^{M}\left\langle \begin{pmatrix} H_{re,a,r}(f)\cdot H_{re,b,r}{*}(f) \\ +H_{re,a,r}{*}(f)\cdot H_{re,b,r}(f)\end{pmatrix}, \left(\left|H_{tr,a,i}(f)\right|^2 + \left|H_{tr,b,i}(f)\right|^2\right)\right\rangle \\ -\sum_{\substack{i=1\\ i\neq r}}^{M} m_i\cdot\left\langle \begin{pmatrix} H_{re,a,r}(f)\cdot H_{re,b,r}{*}(f) \\ +H_{re,a,r}{*}(f)\cdot H_{re,b,r}(f)\end{pmatrix}, \begin{pmatrix} H_{tr,a,i}(f)\cdot H_{tr,b,i}{*}(f) \\ +H_{tr,a,i}{*}(f)\cdot H_{tr,b,i}(f)\end{pmatrix}\right\rangle \\ -m_r\cdot\left\langle \begin{pmatrix} H_{re,a,r}(f)\cdot H_{re,b,r}{*}(f) \\ +H_{re,a,r}{*}(f)\cdot H_{re,b,r}(f)\end{pmatrix}, \begin{pmatrix} H_{tr,a,r}(f)\cdot H_{tr,b,r}{*}(f) \\ +H_{tr,a,r}{*}(f)\cdot H_{tr,b,r}(f)\end{pmatrix}\right\rangle \end{array}\right) \qquad (2)$$

in which $S_{x*x}(f)$ is defined as the power spectral density function of $\boldsymbol{x}(t)$, and uppercase H's are the transfer functions corresponding to the lowercase h's in Figure 2. The average output current consists of the following terms:

- M bias terms;
- M–1 crosstalk terms;
- One information term.

Ideally, the filters are chosen such that all terms are cancelled except the latter one, which is proportional to the desired information datasignal and which should thus be maximized. We can avoid both crosstalk and bias currents by demanding that:

$$\mathrm{Re}\{H_{re,a,i}\cdot H_{re,b,i}{*}\} \perp |H_{tr,a,j}|^2\ \forall\ i, j \qquad (3)$$

$$\mathrm{Re}\{H_{re,a,i}\cdot H_{re,b,i}{*}\} \perp |H_{tr,b,j}|^2\ \forall\ i, j \qquad (4)$$

$$\mathrm{Re}\{H_{re,a,i}\cdot H_{re,b,i}{*}\} \perp \mathrm{Re}\{H_{tr,a,j}\cdot H_{tr,b,j}{*}\}\ \forall\ i\neq j \qquad (5)$$

Application of the Cauchy-Schwarz inequality proves that the desired information-carrying term is maximized if:

$$\mathrm{Re}\{H_{re,a,i}\cdot H_{re,b,i}{*}\}=\mathrm{Re}\{H_{tr,a,i}\cdot H_{tr,b,i}{*}\}\ \forall\ i \qquad (6)$$

As a result, the filter pairs of a corresponding transmitter–receiver pair should have either equal or complex conjugated transfer functions. The receiver filters should thus either be equal or matched to the corresponding transmitter filters. Both options result in the same average receiver output current.

2.2 Beat noise power spectral density

The power spectral density function of the beat noise can be found by Fourier transforming the covariance function of $\boldsymbol{I}_r(t)$. For computing the performance of the system, only the low-frequency part of this function is interesting, since the information-carrying signal part is confined to this part of the spectrum. One can prove that this can be written as in (7), provided that the receiver filters are either equal or matched to the corresponding transmitter filters, as suggested in section 2.1.

$$S_{\boldsymbol{I}_r\boldsymbol{I}_r}(0)=$$

$$\frac{R_{pd}^2}{256M^4}\cdot\sum_{i=1}^{M}\sum_{j=1}^{M}\int_0^{+\infty}\begin{bmatrix}H_{tr,a,r}*(f)\cdot H_{tr,b,r}(f)\\+H_{tr,a,r}(f)\cdot H_{tr,b,r}*(f)\end{bmatrix}^2\cdot\begin{vmatrix}H_{tr,a,i}(f)\\-m_i\cdot H_{tr,b,i}(f)\end{vmatrix}^2\cdot\begin{vmatrix}H_{tr,a,j}(f)\\-m_j\cdot H_{tr,b,j}(f)\end{vmatrix}^2\cdot S_{\boldsymbol{x}^*\boldsymbol{x}}^2(f)\cdot \mathrm{d}f \tag{7}$$

Expanding this equation results in a sum of $64M^2$ integrals of products of 8 transfer functions and the square of the source spectrum. It is assumed that the filters are chosen such that these integrals are non-zero only when these 8 transfer functions form 4 pairs of complex conjugated transfer functions. In that case only $8M^2+4M+2$ of these integrals remain. Provided that the filters are chosen such that the remaining integrals all account for a same noise contribution, the beat noise spectral density is given by:

$$S_{\boldsymbol{I}_r\boldsymbol{I}_r}(0)\propto\frac{4\cdot M^2+2\cdot M+1}{M^4}\cdot P_{\boldsymbol{x}}\cdot t_c \tag{8}$$

In this expression, $P_{\boldsymbol{x}}$ is the power of the source lightwave and t_c is its coherence time.

2.3 Signal-to-beat noise ratio

The signal-to-beat noise ratio in the output current can be maximized by applying a filter that is matched to the (rectangular) average output current, for example an integrate-and-dump filter. The resulting signal-to-beat noise ratio can then be found by dividing the energy of one output bit by the power spectral density of the beat noise:

$$SNR_b=\frac{\{E[\boldsymbol{I}_r(t)]\}^2\cdot T_b}{S_{\boldsymbol{I}_r\boldsymbol{I}_r}(0)}\propto\frac{2}{4\cdot M^2+2\cdot M+1}\cdot\frac{T_b}{t_c} \tag{9}$$

In this equation, T_b is the bit-time. Obviously, the generalized coherence multiplexing system in Figure 2 does not satisfy our goal as far as the improvement of the signal-to-beat noise ratio is concerned; a more advanced structure is needed for that.

3 A timeslotted coherence multiplexing system

In this section, an extension to the generalized coherence multiplexing system in Figure 2 is proposed and analyzed. It will be shown that, using this extension, we can have a signal-to-noise ratio that is proportional to $\frac{1}{M-1}$.

3.1 System description

This is accomplished by subdividing each bit-time T_b of the transmitted signal into $2N$ timeslots of length $T=T_b/2N$. This can be done by replacing the continuous light source by a source that generates light pulses with pulse length T and pulse repetition period T_b. It is assumed that the starting points of the pulses perfectly coincide with the bit boundaries in $m(t)$. The transmitter filters are replaced by banks of N filters, each filter element comprizing a filter and a delay. Each delay is a specific integer multiple of T, as shown in Figure 3. In the receiver, a similar substitution is made for the receiver filters.

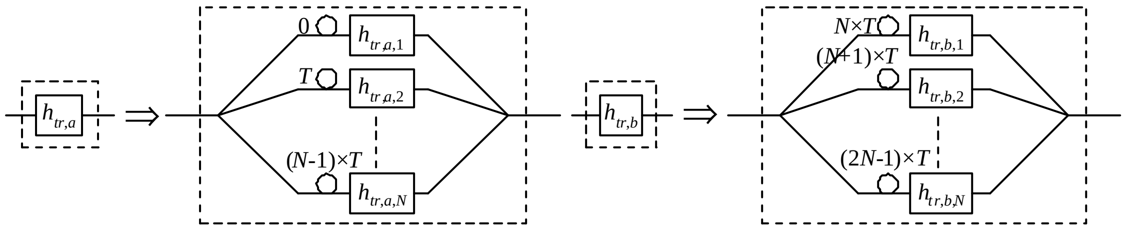

Figure 3: Substitution of transmitter filters

3.2 Analysis of the new system

Since the delays in the transmitter filter banks are multiples of the source pulse width T, a source pulse that is applied to these filter banks is splitted into $2N$ pulses that are perfectly placed behind one another, each pulse being filtered by a different filter. The $2N$ pulses perfectly fit into one bit-time T_b, so that a next source pulse is splitted into another $2N$ pulses, that are perfectly placed behind the first $2N$ pulses. As a result, the signal that is launched into the fiber is slotted in time. In Figure 4, it is illustrated how one source pulse is splitted into $2N$ timeslots.

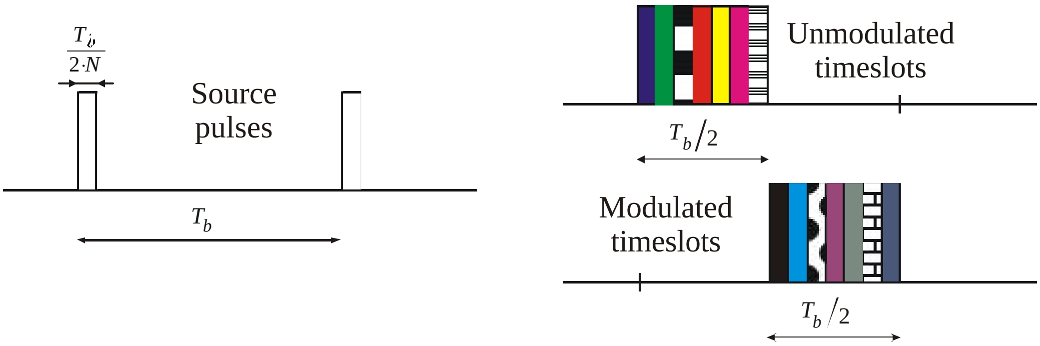

Figure 4: One bit-time of the transmitted signal

The different patterns of the timeslots in Figure 4 represent the different spectral shapes that the lightwaves in the corresponding timeslots have, due to the different filters in the filter banks. Note that the output pulses of the lower filter bank in Figure 3 are modulated by the information-carrying signal $m(t)$ (see Figure 2), so that the last N timeslots in Figure 4 contain a multiplication with m. Since we have M transmitters, each having different filter banks, M of these timeslotted signals are launched into the common fiber. Note that timeslots of different signals may be out of phase, since the transmitters' source modulators will generally not be synchronized in time. In each receiver, a similar procedure is performed on each of the timeslots. As a result, the outputs of the two receiver filter banks are slotted in time as well, each timeslot containing $M \cdot N$ signal components that are each filtered by a different combination of a transmitter bank filter and a receiver bank filter. Each combination of an upper filter bank output signal and a lower filter bank output signal forms a contribution to the receiver output current, resulting in a total of M^2N^2 output current components. Two signals that do not originate from the same transmitter are mutually independent and thus uncorrelated, since they are gaussian with zero mean. This results in an output current contribution with zero average. When two signals do originate from the same transmitter (which applies to $M \cdot N^2$ of these combinations), the average value of the corresponding output current contribution depends on the relation between the four filters that are involved.

The average output current is thus slotted in time as well, having a shape that depends on the relation between the transmitter and receiver filters. Two possible relations between transmitter and receiver filters of a matched transmitter–receiver pair (say transmitter *r* and receiver *r*) are considered:

- Equal filter banks:

$$H_{re,a,r,k} = H_{tr,a,r,k} \text{ and } H_{re,b,r,k} = H_{tr,b,r,k} \ \forall\ k \tag{10}$$

- Matched filter banks:

$$H_{re,a,r,k} = H_{tr,b,r,N+1-k}{}^{*} \text{ and } H_{re,b,r,k} = H_{tr,a,r,N+1-k}{}^{*} \ \forall\ k \tag{11}$$

In the case of equal filter banks, we have N^2 non-zero average output current components, non-uniformly subdivided over $2N–1$ timeslots. In the case of matched filters, we also have N^2 non-zero average output current components, but this time the average output current is concentrated in a single timeslot. This difference is illustrated in Figure 5. Each of the M^2 combinations of two transmitters *i* and *j* contributes to the beat noise in the output current of receiver *r*. By counting the number of beat noise terms for every mutual timing situation of each combination of two transmitters, both for equal and matched filter banks, one can show that the power spectral density of the beat noise in each timeslot of the receiver output current does not depend on the mutual timing of the interfering transmitters.

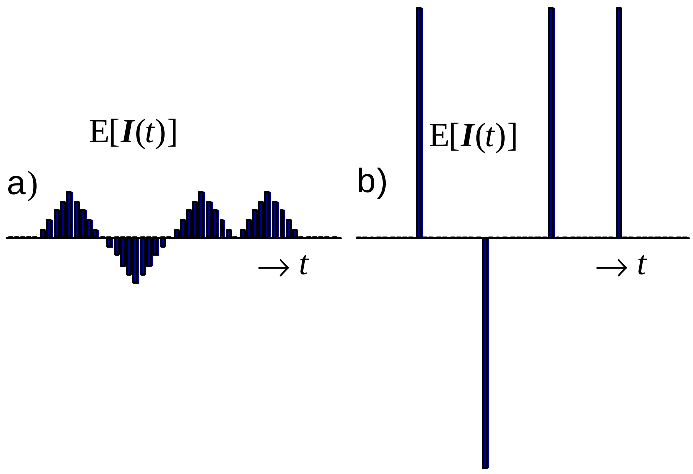

Figure 5: Average receiver output current distribution for *N*=5 after transmitting a {+1,–1,+1,+1}-sequence

a) equal filter banks b) matched filter banks

For equal filter banks, this results in a signal-to-beat noise ratio that is maximized by choosing *N*=1, resulting in a signal-to-beat noise ratio that is given by :

$$SNR_b \propto \frac{1}{M^2+1}\cdot\frac{T_b}{t_c} \approx \frac{1}{M^2}\cdot\frac{T_b}{t_c} \tag{12}$$

Consequently, the timeslotted system with equal filter banks performs only a factor of approximately 2 better than the system with continuous source, for a given number of users *M*. In the matched filter banks case, only one of the timeslots contains information. Therefore, this is the only timeslot that is used for detecting the bits. Consequently, only the power spectral density of the beat noise in this timeslot is interesting. It is given by:

$$S_{\boldsymbol{I}_r\boldsymbol{I}_r}(0) \propto \frac{(M-1)^2\cdot N^2 + 2\cdot(M-1)\cdot N^3 + 2\cdot N^4}{M^4\cdot N^8}\cdot P_{\boldsymbol{x}}\cdot t_c \tag{13}$$

Before we compute the signal-to-beat noise ratio, we have to stress that the bit-time of the output pulse is shortened when N is increased, which decreases the bit energy by an extra factor $1/2N$. The signal-to-beat noise ratio after matched filtering is thus given by:

$$SNR_b \propto \frac{N}{(M-1)^2 + 2\cdot(M-1)\cdot N + 2\cdot N^2}\cdot\frac{T_b}{t_c} \tag{14}$$

This is illustrated in Figure 6.

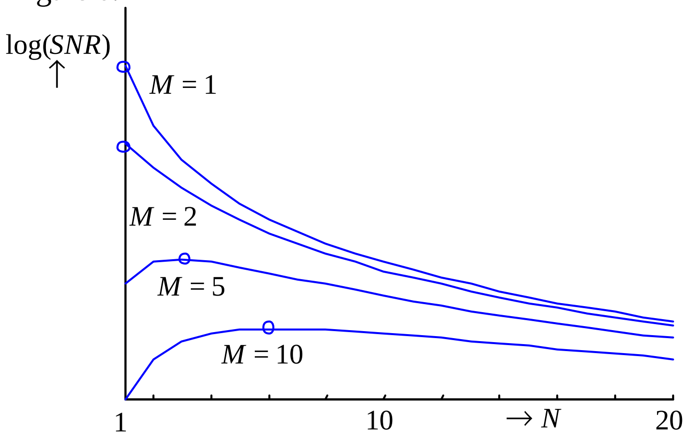

Figure 6: Logarithm of the signal-to-beat noise ratio as a function of the number of filter elements *N* for several values of the number of users *M*

The figure illustrates that the signal-to-beat noise ratio in case of matched filter banks can be maximized by properly choosing N (indicated by the 'o's). The value N_{opt} that optimizes the signal-to-beat noise ratio increases for increasing M. N_{opt} can be found by calculating the zero of the derivative of (13), which results in:

$$N_{opt} = \frac{M-1}{\sqrt{2}} \tag{15}$$

The resulting signal-to-beat noise ratio can be found by substituting (15) in (14), which results in:

$$SNR_{b,max} \propto \frac{1}{2\cdot\left(1+\sqrt{2}\right)}\cdot\frac{1}{M-1}\cdot\frac{T_b}{t_c} \approx \frac{1}{5}\cdot\frac{1}{M}\cdot\frac{T_b}{t_c} \tag{16}$$

This result is illustrated in Figure 7, together with the results for equal filter banks and the continuous source system.

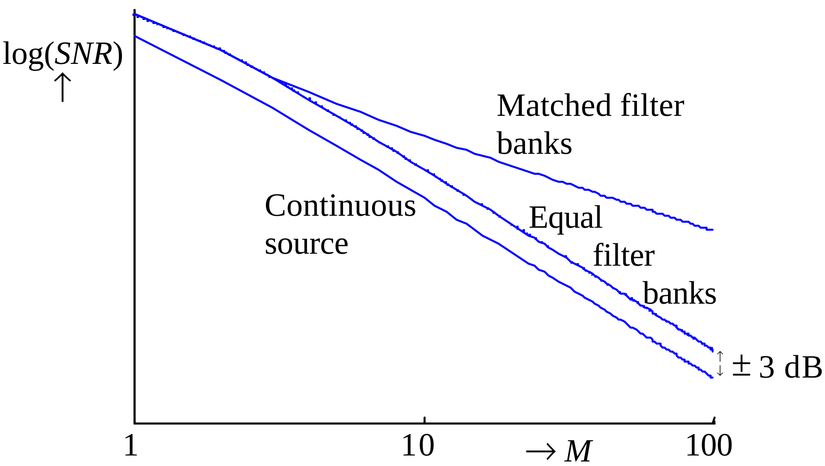

Figure 7: Logarithm of the maximum signal-to-beat noise ratio as a function of the number of users *M*

3.3 Overlapping timeslots

A slight modification is made to the scheme, by subdividing the bit-time into N timeslots instead of $2N$ timeslots, and making the modulated and unmodulated signal parts overlap in time. This can be done by setting the pulse length of the source to $T=T_b/N$, and setting the delays in the lower filter branches equal to the delays in the upper filter branches, as illustrated in Figure 8.

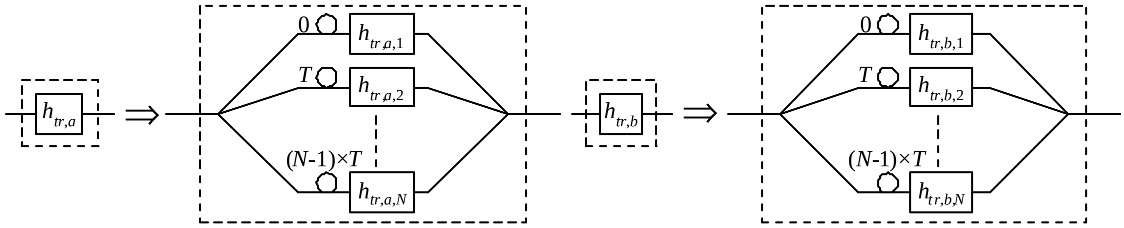

Figure 8: Substitution of transmitter filters

The corresponding average output current is similarly shaped as in the formerly analyzed situation. The difference is that, for both systems having equal N, the output bits in the system with overlapping timeslots are twice as broad with respect to the formerly analyzed system. Therefore, succeeding output bits in the system with overlapping timeslots with equal filter banks overlap in time if $N>1$. This is illustrated in Figure 9.

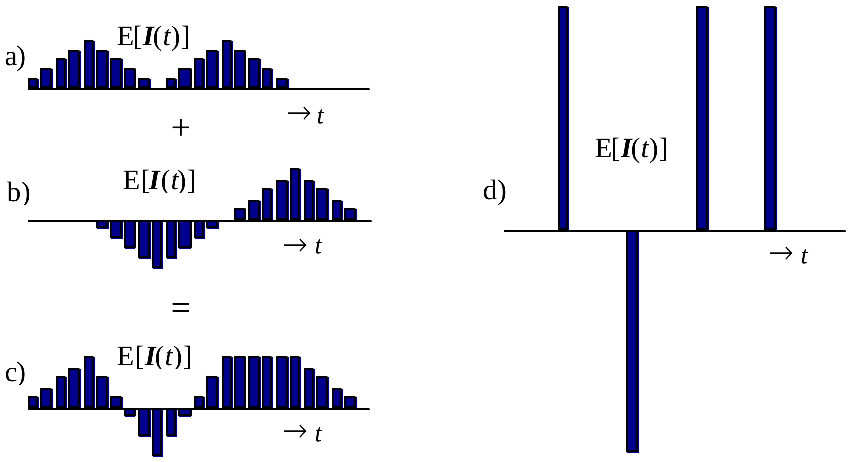

Figure 9: Average receiver output current distribution for N=5 after transmitting a {+1,–1,+1,+1}-sequence

a) odd output bits for equal filter banks
b) even output bits for equal filter banks
c) all output bits for equal filter banks
d) all output bits for matched filter banks

Again, the signal-to-beat noise ratio can be calculated for both the equal and matched filter banks case. For equal filter banks, the signal-to-beat noise ratio is again maximized for N=1, resulting in a continuous source system, with a signal-to-beat noise ratio that is given by (8). For matched filter banks, a similar procedure as described in section 3.2 can be performed to prove that the signal-to-beat noise ratio is maximized by choosing:

$$N_{opt} \approx \sqrt{2} \cdot M \,, \tag{17}$$

resulting in a maximized signal-to-beat noise ratio that is approximately the same as in (16).

3.4 Total system capacity

Using (16), we can calculate the maximum total capacity of the system, for a given upper bound for the probability of bit error p_e. (16) is written as a proportionality, because the proportionality constant depends on the type of filters that are used. If the filters are delay lines, however, we can replace the proportionality sign by an equal-to sign, resulting in (assuming that M>>1):

$$SNR_b \approx \frac{1}{5} \cdot \frac{1}{M} \cdot \frac{T_b}{t_c} \tag{18}$$

If we assume that all transmitters in the system have identical bitrates $1/T_b$, we can write the total capacity C_{tot} of the system as:

$$C_{tot} = M \cdot \frac{1}{T_b} \approx \frac{1}{5} \cdot \frac{1}{SNR_b} \cdot \frac{1}{t_c} \tag{19}$$

Assuming that the beat noise is approximately gaussian, the probability of a bit error p_e is given by

$$p_e = Q\left(\sqrt{SNR}\right), \tag{20}$$

since the output signal has an antipodal signal constellation (see [4]). The signal-to-beat noise ratio should be at least 36 to keep p_e below 10^{-9}, which results in:

$$C_{tot} \approx \frac{1}{180} \cdot \frac{1}{t_c} \tag{21}$$

For a source with a gaussian power spectral density profile, the relation between the coherence time t_c and the linewidth Δl is given by (see [5]):

$$t_c \cong \frac{l^2}{c_0 \cdot \Delta l} \cdot \sqrt{\frac{2 \cdot \ln 2}{p}} \tag{22}$$

in which l and c_0 are the source wavelength and the speed of light, respectively, both in vacuum. Substituting this in (21) gives:

$$C_{tot} \approx 2.5 \cdot 10^6 \cdot \frac{\Delta l}{l^2} \tag{23}$$

For a source wavelength of l=1550 nm, this becomes:

$$C_{tot} \approx 1.0 \cdot 10^{18} \cdot \Delta l \tag{24}$$

As a result, the maximum total system capacity, expressed in Gbit/s, is approximately equal to the the linewidth of the source in nm. A system with light sources with for instance 50 nm linewidth at a center wavelength of 1550 nm thus has a capacity bound of 50 Gbit/s at a bit error rate of 10^{-9}.

4 Conclusions

When the delay lines in a conventional coherence multiplexing system are replaced by filters, we see that a receiver is tuned to a particular transmitter when the receiver filters are either equal or matched to the transmitter filters (where equal delay differences were demanded in the conventional approach). Both choices result in the same received signal power and beat noise power. The received signal-to-beat noise ratio is approximately inversely proportional to the square of the number of users. When a pulsed light source is used and the filters are replaced by banks of filters, each filter element having a delay, the results for equal and matched filter banks are different. In the matched filter banks situation, the signal energy of a received bit is confined to only a small part of the bit-time. Only then the signal-to-beat noise ratio can be made proportional to the inverse of the number of users instead of the inverse of the square of the number of users. A similar performance can be obtained using a system in which the modulated and unmodulated transmission timeslots overlap in time, but this system requires twice as many filter elements for maximizing the performance. Assuming light sources with a gaussian power spectral density profile at a center wavelength of 1550 nm, the maximum system capacity is 1 Gbit/s per nm linewidth, at a bit error rate of 10^{-9}.

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