STIFFNESS ANALYSIS OF PARALLEL LEAF-SPRING FLEXURES

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ABSTRACT

Approximate straight displacements are often made using a parallel leaf-spring flexure. This flexure serves as a typical case for studying the influence of shear and the compliance of the reinforced mid sections of the leaf-springs in the support stiffnesses c_z and c_y. The conclusions drawn, however, hold true for the rotational stiffness c_{rx} also, while the stiffness in the ry and rz direction can be approximated based on c_z and c_y. It turns out that shear plays an important role for short relatively wide leaf-springs at small deflections. The compliance of a reinforcement needs to be taken into account for determining the support stiffness at small deflections.

INTRODUCTION

A design principle that plays an important role in precision manipulation is determinism [1]. This rule promotes the use of flexure mechanisms because these mechanisms do not suffer from friction or backlash and therefore result in highly repeatable behavior. Approximate straight displacements are often made with two parallel leaf-springs in a parallel leaf-spring flexure as is shown in Figure. 1.

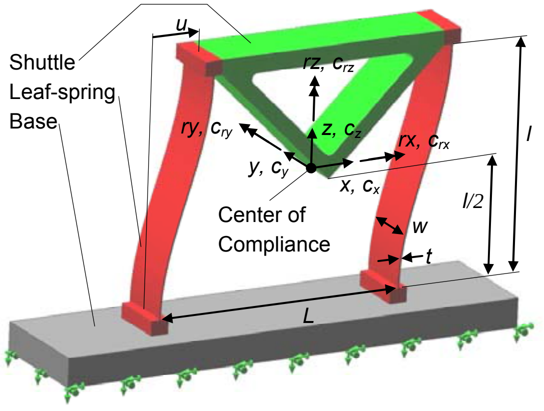

FIGURE 1. Parallel leaf-spring guidance.

Applications for flexure-based parallel or straight guidances can be found in precision motion or alignment of various sensors, optical components and slits. In MEMS for example comb-drive electrostatic actuators make use of flexure-based straight guidances to yield a relatively high axial stiffness to prevent actuator pull-in. Many types of elastic straight guidances exist, such as folded flexures, compound leaf-spring flexures, crab leg flexures and double parallel leaf-spring flexures in its basic form the parallel leaf-spring flexure serves as a typical case which will be evaluated in the paper.

A drawback of a parallel guidance with prismatic leaf-springs is the limited capacity of managing compressive loads. The danger of buckling is very real. By reinforcing the mid-sections (Figure 2) of the leaf-springs the maximum allowable compressive force and the support stiffness (c_y, c_z, c_{rx}, c_{ry} and c_{rz}) increase significantly [1][2] if the displacement u is externally constrained. However, also the bending stresses increase. A trade-off needs to be made.

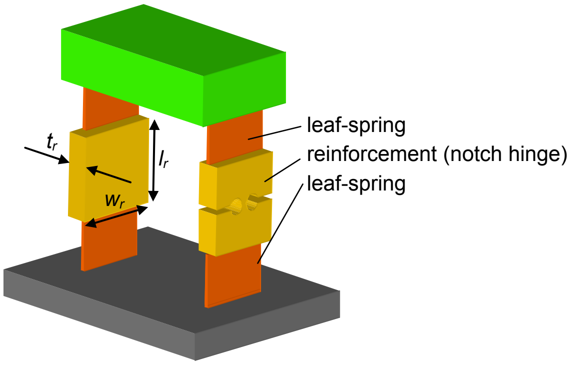

FIGURE 2. Exactly constrained parallel leaf-spring mechanism with reinforcements.

Although extensive analysis of the parallel leaf-spring flexure has been reported in the past [2] and so-called rules of thumb exist [1], they are generally restricted by the assumption that the length over width ratio of the leaf-springs is large so shearing effects do not need to be taken into account, reinforcements are thick enough to be taken rigid, the shuttle has a prescribed position *u* by a rigid external support in *x*- or drive direction, and anticlastic curving effects are small. We present the dominating support stiffnesses c_z and c_y taking into account flexibility of the reinforcement and shearing effects. The support stiffness c_{rx} has been calculated as well but is not presented in this paper. The conclusions drawn, however, hold true also for c_{rx}. The stiffness in the *ry* and *rz* direction can be approximated for $L >> t$, if the distance *L* between the leaf-springs is known:

$$c_{ry} = \tfrac{1}{2} c_z L^2 \quad ; \; c_{rz} = \tfrac{1}{2} c_y L^2 \quad (1)$$

The stiffness change in the driving direction is generally small. However for leaf-springs with aspect ratio $w^2 / l \cdot t > 4$ the transverse stress due to Poisson contraction can be significant for deflections *u/l* larger than 0.1 [2].

Ideally the shuttle has a stiff external support in the drive or *x*-direction. However, in general the external support has a limited stiffness c_d, the 'drive-stiffness'. The strong influence of ratio c_d/c_x on c_z has been shown [4], and is in particularly important for MEMS applications.

SUPPORT STIFFNESS

By reinforcing the mid-sections of flexures, as shown in Figure 2, the maximum allowable compressive force increases significantly [1] if c_d / c_x is large. At the same time the reinforcement is advantageous for the support stiffness. The bending stress however increases unfavorable. As a fair compromise often the reinforcement factor p = 5/7 is applied as a rule of thumb [1], in which $l_r = p \cdot l$ is the reinforcement length. Van Eijk [2] provide rules of thumb for the stiffness in *z*-direction in case the reinforcements are thick enough to be taken rigid, the drive stiffness can be approximated at infinity, the *l/w* ratio is large so shearing effects do not need to be taken into account. We present the stiffnesses c_y and c_z taking into account limited reinforcement thickness and shearing.

The stiffnesses c_y and c_z of the shuttle at the center of compliance (shown in Figure 1) as a function of the relative displacement *u/t* are calculated using the flexible multibody software package SPACAR which uses beam theory including shear. The drive stiffness has been taken infinite. Figures 3a and 3b show a comparison of the relative stiffnesses c_y / c_{y0} and c_z / c_{z0} between a rigidly reinforced, p = 5/7, parallel leaf-spring guidance and a prismatic leaf-spring guidance. The stiffnesses are scaled by the respective stiffnesses at zero deflection c_{y0} and c_{z0}. The stiffnesses of the prismatic leaf-spring parallel guidance at zero deflection c_{y0} can be calculated by:

$$c_{z0} = \frac{2Ewt}{l(1-p)} \quad (2)$$

$$c_{y0} = \left(\frac{12+11\nu}{10(1+\nu)} \frac{b}{Gtw} + \frac{4b^3 + 12ab^2 + 12a^2b}{Etw^3} \right)^{-1} \quad (3)$$

$$a = \frac{pl}{2} \;\; ; \;\; b = \frac{l(1-p)}{2} \quad (4)$$

The c_z/c_{z0} stiffness is independent of the ratio length over width (*l/w*). The c_{z0}-stiffness is increased by a factor of 3.5 due to reinforcing. Loading the shuttle in *z*-direction causes a 1st order bending-mode in the leaf-spring at deflection when loaded in the *z*-direction. Reinforcement stiffens this 1st order bending-mode effectively, as the bending length is shortened.

Leaf-springs with a small *l/w* ratio show a greater decrease in c_y-stiffness during deflection in the *x*-direction than leaf-springs with a large *l/w*. This can be explained as follows: during deflection the c_y-stiffness becomes a series stiffness of bending and torsion stiffness. The torsional component is caused by a combination of *u*-deflection and a force in *y*-direction of a moment in *rx*-direction. A small *l/w* results in a relatively large c_{y0}-stiffness in relation to torsion stiffness of the leaf-springs, as the bending stiffness is proportional to tw^3 and the torsion stiffness is more or less proportional to t^3w for these types of cross-sections. The large c_{y0}-stiffness for small *w/l* due to large *w/t* ratios is compromised more by the torsion stiffness at *u*-deflection than that of leaf-springs with a large *w/l.* The c_{rx}-stiffness shows comparable behavior.

The ratio c_y / c_{y0} becomes worse by reinforcing leaf-springs for *l/w* < 2. A reinforced parallel leaf-spring flexure is much stiffer than a comparable prismatic version at zero deflection. Both are deformed predominately by shear, but the reinforced version is deformed much less due to shorter flexure parts. At deflection however the *y*-stiffness becomes a series stiffness of bending, shear and torsion stiffness. The torque due to deflection and a force in *y*-direction is at its maximum near the shuttle and base in the flexures. Therefore the resulting twist of a reinforced leaf-spring is not that much smaller than the twist in a prismatic leaf-spring. In a series stiffness the most compliant link predominantly determines the overall stiffness, which in this case becomes the torsional compliance at large *u*-deflections.

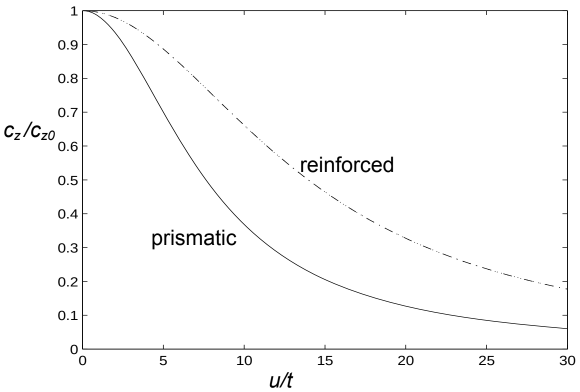

FIGURE 3a. The c_z/c_{z0} stiffness comparison between a reinforced and a prismatic parallel leaf-spring guidance.

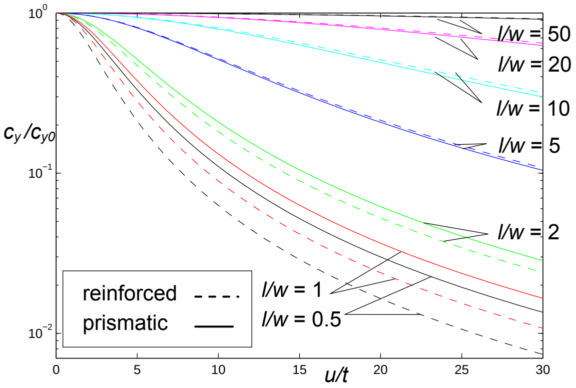

FIGURE 3b. The c_y/c_{y0} stiffness comparison between a reinforced and a prismatic parallel leaf-spring guidance for various l/w ratios.

SHEARING DECREASES THE SUPPORT STIFFNESS

Shearing effects decrease the c_y-stiffness especially for small *l/w* at small deflections. To show the influence of shearing effects on the c_y/c_{y0} stiffness a comparison between a parallel prismatic leaf-spring guidance with and without shearing effects taken into account is given in Figure 4. The c_y/c_{y0} stiffness needs to be calculated taking shearing effects into account for $l/w \leq 2$ and $u/t \leq 5$. The larger the deflection the less the shearing plays a significant role in the c_y-stiffness.

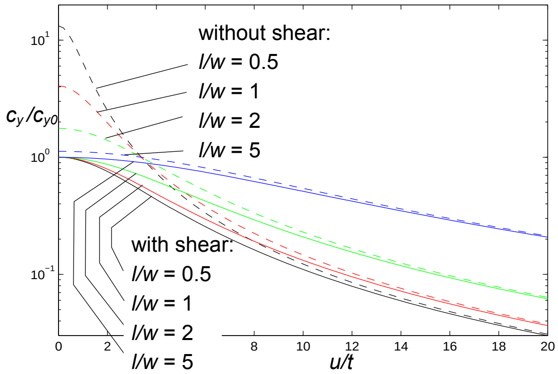

FIGURE 4. The c_y/c_{y0} stiffness comparison between the parallel leaf-spring guidance with and without shearing effects taken into account.

REINFORCEMENT LENGTH

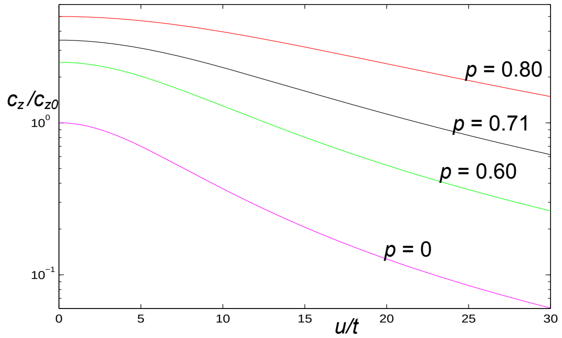

FIGURE 5a. The c_z/c_{z0} stiffness of a reinforced parallel leaf-spring guidance for various reinforcement factors p.

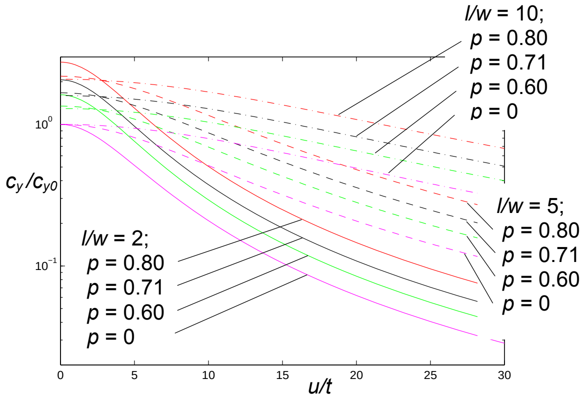

FIGURE 5b. The c_y/c_{y0} stiffness of a reinforced parallel leaf-spring guidance for various reinforcement factors p.

Figures 5a and 5b show the effect of the reinforcement factor *p* on the c_z- and c_y-stiffness for l/w = 0.5, 2 and 5, for a rigidly reinforced parallel leaf-spring guidance. The stiffnesses are scaled by the respective stiffnesses at zero deflection of a prismatic leaf-spring guidance. The larger p the higher the y- and z-stiffness, however also the higher the x-stiffness [1]:

$$c_x = \frac{Ewt^3}{l^3}\frac{1}{1-p^3} \tag{5}$$

and the bending stress [1].

$$\sigma = \frac{3Ehu}{l^2}\frac{1}{1-p^3} \tag{6}$$

As a compromise p = 5/7 is often used, but for specific requirements p can be tuned.

COMPLIANCE OF THE REINFORCEMENT

To find the influence of the reinforcement thickness t_r on the stiffness the compliance of the reinforcements is taken into account. Figures 6a and 6b show the effects for various t_r/t on the c_z- and c_y-stiffness for p = 5/7. The stiffnesses are scaled by the respective stiffnesses at zero deflection of a prismatic leaf-spring guidance.

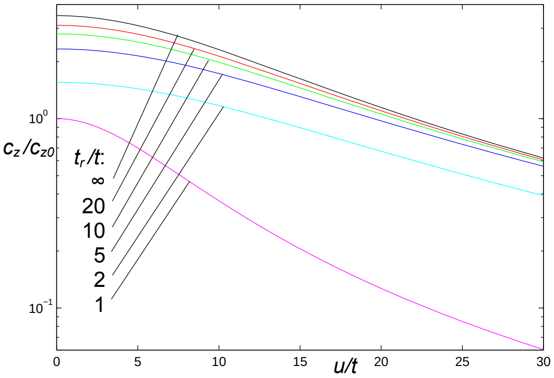

FIGURE 6a. The c_z/c_{z0} stiffness of a reinforced parallel leaf-spring guidance for various relative reinforcement thicknesses t_r/t (p = 5/7).

Even with a reinforcement thickness of t_r = 20t the reinforcement cannot be taken as a rigid body for small deflections. The support stiffness increases only slightly at large deflections when reinforcing more than 5 to 10 times the leaf-spring thickness. For l/w ≥ 5 and at limited deformations u/t < 10 it could make sense to take the reinforcement thickness t_r ≥ 10t. Furthermore, with respect to dynamic properties the effects of increased mass of the reinforcement which cause lowered vibration mode frequencies should be considered. For relatively large deformations u/t > 10, as a compromise between increased support stiffness and dynamic properties the reinforcement thickness can be taken t_r ≤ 5t.

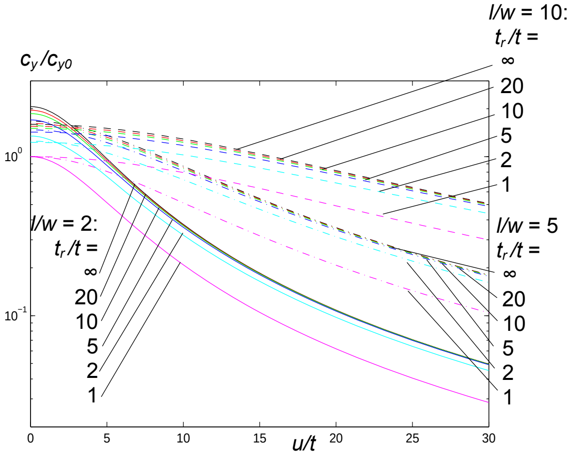

FIGURE 6b. The c_y/c_{y0} stiffness of a reinforced parallel leaf-spring guidance for various relative reinforcement thicknesses t_r/t (p = 5/7).

CONCLUSION

It can be concluded that for the c_y/c_{y0} stiffness of a parallel leaf-spring flexure shear plays an important role for l/w ≤ 2 and u/t ≤ 5. But shear can be neglected for deflections u/t > 12. The compliance of a reinforcement even for t_r = 20t needs to be taken into account for determining the support stiffness at small deformations. For u/l > 10 the optimal z-stiffness is approached for t_r/t > 5, and the optimal y-stiffness is approached for t_r/t > 2. A trade-off is required between the support stiffnesses due to a large reinforcement factor p and the stress in the leaf-springs.

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