Noise-based frequency offset modulation in wideband frequency-selective fading channels
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A frequency offset modulation scheme using wideband noise carriers is considered. The main advantage of such a scheme is that it enables fast receiver synchronization without channel adaptation, while providing robustness to multipath fading and in-band interference. This is important for low-power wireless systems with bursty traffic, such as sensor networks. In this paper a semi-analytical framework for evaluating its bit error rate performance in wideband frequency-selective fading channels is introduced. Some numerical results are presented, based on channel models developed in the IEEE 802.15.4a channel modeling subgroup. These illustrate that the considered system can be designed with a lower fading margin than a narrowband system.

1 Introduction

Ultra-wideband (UWB) communications [1] has received considerable attention in the last two decades, because of its robustness to multipath fading, immunity to interference, and ability to coexist with other wireless devices sharing the same spectrum space. A common design approach taken in many UWB systems is to employ a Rake receiver configuration to collect the energy of all multipath components (MPCs), where the required number of Rake fingers equals the number of resolvable MPCs. Furthermore, accurate estimation of the channel impulse response is essential for determining the required weight and delay of each Rake finger. This may be prohibitive in wireless systems with bursty traffic (such as sensor networks), where fast receiver synchronization is essential for low-power operation.

These issues can be overcome by using transmitted-reference (TR) modulation [2, 3]. So far TR modulation has mainly been developed within the framework of impulse radio (IR) techniques, where sparse sequences of ultra-short pulses are used as information bearers. The basic idea of TR modulation is that each modulated pulse is accompanied by an unmodulated reference pulse, separated by a delay that is known in the receiver. Hence, the receiver can demodulate the received signal by correlating it with a delayed version of itself. Since the modulated pulse and reference pulse travel through the same channel, each MPC will contain two identically distorted pulses with consistent mutual delay. Therefore, the demodulation does not require a Rake receiver and/or channel estimation, receiver synchronization is limited to the actual symbol timing, and sampling and further signal processing are simply performed at the (relatively low) data rate (rather than at the pulse rate).

Taking inspiration from a similar scheme applied in optical communications [4, 5], it was realized that (random) noise can be used as information bearer instead of (pseudo-random) UWB pulse sequences [6]. This is not only easier to generate, but also obviates
the need of pulse position-dithering schemes for flattening the spectrum of the transmitted signal. Nevertheless, although the scheme looks straightforward (and, in fact, has potential for very simple implementation on an optical chip), both the IR-based scheme and the noise-based scheme pose significant challenges when implemented in the RF domain, because of the wideband delay that is required. Therefore, an alternative scheme was proposed in which the time offset was replaced by a small frequency offset [7], as illustrated in Fig. 1. (A similar scheme was developed independently within the framework of IR by Goeckel and Zhang [8].) In [7] the performance of the noise-based scheme in additive white Gaussian noise (AWGN) was studied, including the effect of multi-user interference, as different users can transmit in the same frequency band by using different frequency offsets. An experimental demonstration using off-the-shelf components was described as well.

The effect of frequency-selective fading on the noise-based TR schemes was considered in [9], using a simple two-tap tapped delay line model with deterministic tap weights. Unfortunately, given the physical nature of this model, it would be inappropriate to consider it as representative of a wideband frequency-selective fading channel. Therefore, we extend the analysis of [9] in this paper by assuming a more general model that can be applied to various channel scenarios. This analysis is briefly summarized in Section 2. Some results are described in Section 3, followed by conclusions in Section 4.

2 System Performance Modeling

The performance analysis of the system in wideband fading channels is based on the widely accepted tapped-delay line model for the equivalent baseband representation of the channel [10]. Its impulse response can be written as

$$ h(t) = \sum_k h_k \delta(t - k\Delta\tau) . \tag{1} $$

The model has tap delays that are chosen as multiples of the so-called delay bin width $\Delta\tau$ (which should be equal to or smaller than the inverse of the system bandwidth in order to satisfy Nyquist’s criterion), and random (complex) tap weights $h_k$ that form the channel vector $\mathbf{h}$.

For simplicity we consider the single-user case (one transmitter and one receiver), and neglect interference from other systems. Furthermore it is assumed that the noise carrier in the transmitter is a Gaussian bandpass signal, which has a rectangular power spectral
density function with width $B$. The modulating data signal is assumed to be an antipodal signal with a data rate $R_b$ that equals the frequency shift $\Delta f_{\text{TX}}$, so that the modulated and unmodulated reference are orthogonal over one symbol period. Also, it is assumed that the modulated and unmodulated reference in the transmitted signal both carry the same amount of power, together resulting in a total transmitted energy per bit $E_b$. $R_b$ and $\Delta f_{\text{TX}}$ are assumed to be much smaller than the coherence bandwidth of the channel, so that inter-symbol interference can be neglected, and the modulated and unmodulated reference essentially experience the same channel distortion. The bandpass filter in the receiver is assumed to have a transfer function with the same rectangular shape as the power spectral density function of the noise carrier. When its width $B$ is assumed much larger than $\Delta f_{\text{TX}}$, this implies that the received signal is passed with only negligible distortion. Assuming AWGN with power spectral density $N_0/2$ at the input of this filter, and a low-frequency oscillator that is locked both in frequency and phase to the oscillator in the transmitter, it follows that the input signal of the (low-pass) detection filter consists of several despreaded terms —of which only the desired (modulated) term appears at baseband— and several (undesired) broadband noise terms. When the detection filter is matched to the despreaded data signal, it follows from the assumptions above that the bandwidths of these noise terms are much larger than the bandwidth of the detection filter. Hence, the Central Limit Theorem applies, so that the decision samples can be assumed Gaussian distributed. Therefore, for calculating the bit error rate (BER), we merely have to calculate the (conditional) mean and variance of the decision samples $D_n$ for given data symbols and given realization of the channel vector $\mathbf{h}$. The bit error rate then follows as

$$P_e = \mathbb{E}_{\mathbf{h}} \left[ Q\left( \sqrt{\text{SNR}(\mathbf{h})} \right) \right],$$

where $\mathbb{E}_{\mathbf{h}}[.]$ denotes averaging over all channel vector realizations, $Q(.)$ is the Gaussian tail probability

$$Q(z) \triangleq \frac{1}{\sqrt{2\pi}} \int_{z}^{\infty} \exp(-x^2/2) \, dx,$$

and $\text{SNR}(\mathbf{h})$ is the signal-to-noise ratio at the output of the decision filter for a given channel vector realization $\mathbf{h}$. The latter can be expressed in the transmitted SNR per bit $E_b/N_0$, the time bandwidth product (or spreading factor) $S \triangleq B/R_b$, and given realizations of the channel vector $\mathbf{h}$ as (the proof is omitted for brevity)

$$\text{SNR}(\mathbf{h}) = \frac{16 \alpha^2(\mathbf{h})(E_b/N_0)^2}{50 \beta(\mathbf{h})(E_b/N_0)^2/S + 40 \alpha(\mathbf{h})E_b/N_0 + 16 S}$$

where $\alpha(\mathbf{h})$ and $\beta(\mathbf{h})$ depend on the channel tap weights as

$$\alpha(\mathbf{h}) \triangleq \sum_k |h_k|^2,$$

$$\beta(\mathbf{h}) \triangleq \sum_k \left| \sum_l h_l h_{l-k}^* \right|^2.$$

The term in the numerator of (4) is proportional to the square of the mean value of the decision sample $D_n$ for given data symbols and channel realization. This mean value is proportional to the energy $\alpha(\mathbf{h})$ of the channel impulse response, which illustrates the inherent Rake-like behavior of TR systems.
The denominator of (4) is proportional to the total variance of the decision samples $D_n$ (which does not depend on the data symbols) for a given channel realization. The three terms represent the noise terms that are generated at the input of the detection filter by mixing the received signal with itself, the received signal with the additive input noise, and the additive input noise with itself, respectively.

The derived expressions can be applied to arbitrary channel models by properly incorporating the statistics of the channel tap weights $h_k$. A common (semi-analytical) approach in UWB research is to generate a large number of channel vector realizations using a channel simulator (or actual channel measurements), and to estimate the average BER by averaging the conditional BER over these realizations [3, 8].

### 3 Results

To illustrate the developed theory, 1,000 channel realizations were generated using the models and corresponding Matlab scripts developed in the IEEE 802.15.4a channel modeling subgroup [11]. These models incorporate the frequency-dependence of the path gain, use a modified Saleh-Valenzuela model for the (clustered) MPC arrivals, and assume Nakagami-distributed small-scale fading of the MPCs. Antenna effects, shadowing, and the time-varying nature of the channel are not incorporated in the models. Specifically, the channel model “CM 4” (indoor office, non-line of sight) was used, which covers the frequency range 3–6 GHz, i.e., corresponding to a system bandwidth $B = 3$ GHz. A slight modification was made to the original Matlab scripts: instead of normalizing the energy $\alpha(h)$ of each individual channel realization to one, the mean energy of all 1,000 realizations was normalized to one. (Otherwise the normalization would in fact remove the actual fading effect.) Note that, due to this normalization, $E_b/N_0$ from now on represents the average received SNR per bit. (We write average received SNR per bit, because the actual received SNR per bit is random. This is both because the transmitted SNR per bit is random —due to the randomness of the noise carrier— and because the channel response is random.)

The resulting average BER has been plotted as a function of $E_b/N_0$ in Fig. 2, for spreading factors $S = 100$ and $S = 200$, respectively. This was done both for our noise-based frequency-offset modulation (N-FOM) system and for the impulse radio frequency-offset modulation (IR-FOM) system presented in [8]. For the latter, the same equations can be used, except that the first term in the denominator of (4) (containing $\beta(h)$) should be omitted [8]. This was done for CM 4, but —for comparison— also for a simple AWGN channel (simply by substituting $\alpha(h) = \beta(h) = 1$ instead of calculating these from channel simulations). Also, the classical results for the BER of binary phase shift-keying (BPSK) in AWGN and Rayleigh fading channels [10] have been plotted.

Several observations can be made from these results:

- Both for the AWGN channel and the wideband fading channel, the results for N-FOM are very similar to the results for IR-FOM at low values of $E_b/N_0$, whereas a BER floor arises for N-FOM at high values of $E_b/N_0$. This floor is caused by the noise that is generated at the detector input by mixing the received noise carrier with itself. The higher the spreading factor $S$, the lower this BER floor [7];
• The BER of the N-FOM system in a wideband fading channel shows an elevated floor with respect to the same system in AWGN. This is caused by the fact that the number of noise terms at the detector input that are caused by mixing the received noise carrier with itself, rapidly increases with an increasing number of delayed versions of the transmitted signal at the receiver input. This signifies the need for choosing a high value for the spreading factor $S$

• As expected, both the IR-FOM and the N-FOM system require a higher average received SNR per bit for the wideband fading channel than for the AWGN channel. However, for low BERs the corresponding fading margins are smaller than for a simple BPSK system in a Rayleigh fading channel, provided that a sufficiently high spreading factor $S$ is chosen;

• Nevertheless, it is obvious from the AWGN curves that the N-FOM and IR-FOM schemes suffer significant inherent power penalties with respect to simple BPSK. From the formulas these are easily shown to be at least 10.8 dB and 7.0 dB, respectively. These penalties are hardly compensated for by the reduction in fading margin, i.e., in a fading channel, N-FOM and IR-FOM are only superior to BPSK in the very low-BER regime. However, it should be noted that N-FOM and IR-FOM are robust to narrowband in-band interference, especially for high spreading factors $S$. (Note that the difference in performance of N-FOM and IR-FOM then becomes negligible.) The effect of in-band interference is not considered in further detail here, but will be considered in a later paper.
4 Conclusions

A semi-analytical framework for evaluating the BER performance of a frequency offset modulation scheme using wideband noise carriers in wideband frequency-selective fading channels has been introduced. Some numerical results were presented to illustrate that such a system can be designed with a lower fading margin than a narrowband BPSK system, provided that the bandwidth of the noise carrier is sufficiently larger than the bit rate of the system and the coherence bandwidth of the channel. Nevertheless, this does not imply that it has a superior performance, as the performance of the system in AWGN is inferior to BPSK. Hence, the justification of the system heavily relies on the performance that is achieved when narrowband interferers are present. This will be discussed in a later paper. For large spreading factors, the performance of the N-FOM system is nearly identical to the performance of an equivalent IR-FOM system. However, the N-FOM system has the advantage that wideband noise carriers are easier to generate than ultra-short-pulse sequences, and do not require additional pulse position-dithering techniques for spectral flattening. These and other implementation aspects are also part of our on-going research, ultimately aimed at full transceiver integration in low-power CMOS chips.

References