A new, analysis-based, change of measure for tandem queues

Pieter-Tjerk de Boer
Centre for Telematics and Information Technology
& Faculty for Electr. Eng., Math. and Comp. Sci.
University of Twente
The Netherlands
email: ptdeboer@cs.utwente.nl

Werner R.W. Scheinhardt
Centre for Telematics and Information Technology
& Faculty for Electr. Eng., Math. and Comp. Sci.
University of Twente
The Netherlands
email: w.r.w.scheinhardt@utwente.nl

We introduce a simple analytical approximation for the overflow probability of a two-node tandem queue. From this, we derive a change of measure, which turns out to have good performance in almost the entire parameter space. The form of our new change of measure sheds an interesting new light on earlier changes of measure for the same problem, because here the transition zone from one measure to another—that they all have—arises naturally.

Background

Since two decades, the problem of estimating the overflow probability of the total population of two Markovian queues in tandem has been an important benchmark problem in rare-event simulation, and importance sampling in particular. This is because the most straightforward change of measure [PW89] based on large-deviations theory, does not work wel in parts of the parameter space [GK95, dB06]. This is caused by visits of the sample path to the state-space boundary where the second queue is empty: such visits have a very large likelihood-ratio contribution under this change of measure.

Two analytically constructed changes of measure have been proposed in recent years to solve this problem: [DSW07] and [NZ07]. Both of them have a transition zone near the problematic state-space boundary; in this zone, the change of measure gradually changes from one that works well on the boundary to the one from [PW89] that works well in the rest of the state space. They differ mainly in how the transition is made. In [DSW07], the transition is done as a mollification with exponential weights; some insight into the optimal width of this transition zone was recently derived by [BGL09]. In [NZ07], the transition is a straightforward linear interpolation; the width of the transition zone is experimentally optimized case-by-case.

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Our approach

We first derive an analytical approximation for the overflow probability of interest. This is done by "guessing" product-form solutions to the set of equations governing these probabilities, and setting coefficients to satisfy most of them. This (and thus our results) can probably be extended to more complicated networks than the two-node tandem queue.

Next, we plug this approximation into the well-known expression for the zero-variance change of measure, resulting in our new change of measure. This new change of measure exhibits a gradual transition from one measure to another, just like the other changes of measure developed before; the crucial difference however, is that in our case the transition and its width arise *naturally*, as opposed to being inserted artificially to fix the behaviour at the boundary. Interestingly, the *shape* of the transition is different. Also, the limiting change of measure at the state-space boundary is different.

Simulation experiments suggest that the estimator has bounded relative error in *most* of the parameter space. Numerical calculations confirm this. Unfortunately, another numerical calculation shows that in a very narrow region of the parameter space (where the service rates of both queues are almost identical), the estimator has infinite variance. We suspect this can be fixed by a minor adjustment of the change of measure along the state-space boundary where the first queue is empty (as is also done in [DSW07]), but this is still future work, as is the development of analytical (rather than numerical) bounds of the estimator's efficiency.

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