

# On the Effect of Spectral Location of Interferers on Linearity Requirements for Wideband Cognitive Radio Receivers

Dlovan H. Mahrof, Eric A.M. Klumperink, Senior Member IEEE, Jaap C. Haartsen, Member IEEE, and Bram Nauta, Fellow IEEE

**Abstract**—Since 2008, The Federal Communications Commission (FCC) allows the operation of Cognitive Radio (CR) in unused parts (i.e. white spots) of the DTV spectrum. Due to the nonlinearity of the radio receiver and the existence of strong DTV signals, different types of distortion products will be generated in the CR-receiver. This paper analyzes the spectral location of distortion products across the white spots depending on the location of the DTV signals in the RF spectrum, focusing on 3<sup>rd</sup> order distortion products. Based on this analysis, we show that a receiver is always limited by cross-modulation (XM3) and self-interference products. Thus true distortion free white spots do not exist if DTV signals are present after the RF-band filter. However, XM3 and self-interference distortion products are typically much weaker than 3<sup>rd</sup> order intermodulation (IM3) products. Thus it makes sense to monitor the level and spectral location of interferes and classify the “white spots” into two types, namely IM3-spots and IM3-free spots. This paper derives equations to quantify how much the 3<sup>rd</sup> order linearity requirements are relaxed when the CR operates at an IM3-free spot. The analysis not only takes into account narrowband interferers but also wideband interferers. The analysis is verified by measurements.

**Index Terms**— Cognitive radio (CR), radio frequency (RF), dynamic spectrum, orthogonal frequency division multiplexing (OFDM), radio receiver, linearity requirement, input third intercept point (IIP3), third order intermodulation product (IM3), cross-modulation product (XM3), Self-interference.

## I. INTRODUCTION

COGNITIVE RADIO is a new emerging radio communication paradigm [1] aiming at improving the utilization efficiency of the scarce spectral resources. It senses unused “white spots” in the radio spectrum that is licensed to a primary user and adapts its communication strategy to use these parts while minimizing interference to the primary service.

This work studies requirements on the 3<sup>rd</sup> order linearity of a CR receiver for wireless applications operating in the DTV bands [2].

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Cognitive Radio requires high programmability for radio transmission and reception because the position of the white spots changes dynamically with time (i.e. dynamic spectrum). Traditional radio receivers are narrowband and are typically highly dedicated to a specific RF-band, especially due to the fixed RF-filter (see Fig.1).

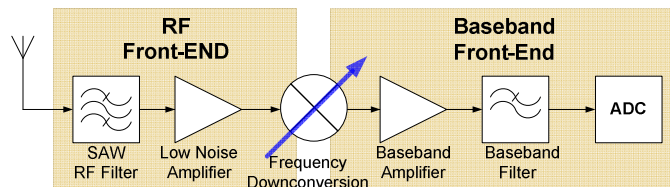


Fig. 1: Traditional narrow-band RF receiver with one RF “Front-End”

The high out-of-band rejection of the RF-filter, usually a high-linearity highly selective Surface Acoustic Wave (SAW) filter, suppresses the interference of out-of-band interferers. Traditional radio standards define in detail how the radio-band is used, e.g. by specifying a multiple access method (TDMA, FDMA, CDMA) and blocker profiles. Consequently in-band signals are “under control”, while RF-filtering reduces the out-of-band interference. In such narrowband systems, typically third order intermodulation products (IM3) produced by nonlinearities in the receiver dominate as is shown in Fig. 2. Both the signal and the IM3 product will be converted to the baseband and their ratio must be higher than the minimum Signal to Distortion Ratio that is required to demodulate the transmitted information with acceptable bit error rate.

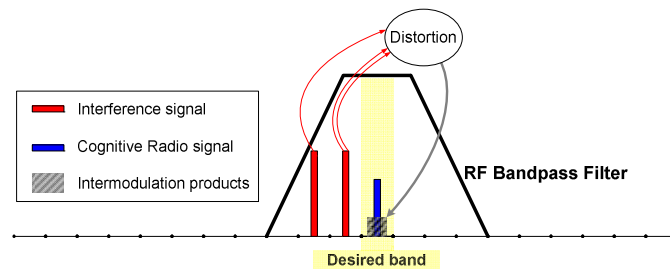


Fig. 2: Corruption of the desired signal due to intermodulation products caused by two interference signals (Conventional narrow-band Radio)

For CR, the distinction between “out-of-band” and “in-band” signals more or less vanishes, as bands are no longer reserved for one or a few radio standards. Also, the power of radio signals is not restricted to a low level, as in ISM bands, in which more freedom is allowed provided low power levels are used (typically 10-100mW). If we want to maximize spectral efficiency in the TV-bands, we like to use white spots between DTV-signals and preferably even in the proximity of DTV signals. In such cases, very steep high-order RF-filters would be needed, which are difficult to implement even with high-Q SAW filters. If the frequency-distance between DTV signals is small, narrowband RF-filters would be needed. To cover a significant part of the TV bands, many filters would be needed. From these observations we conclude that a broadband RF receiver with relaxed RF-filter requirements is highly desired. In this paper we look for ways to benefit from spectral sensing which is available in a CR anyway. We will look at a broadband receiver, but still limit the bandwidth to below an octave so that second order distortion products are largely suppressed by the RF-filter and the 3<sup>rd</sup> order distortion is the main remaining problem.

A broadband RF receiver implies the reception of undesired interference as shown in Fig. 3. The example of a DTV spectrum in Fig. 3 consists of 20 numbered frequency spots: 5 spots contain interference (i.e. DTV signals at spots 1, 4, 11, 16 and 19) and one spot contains the desired signal (i.e. CR signal at spot 8).

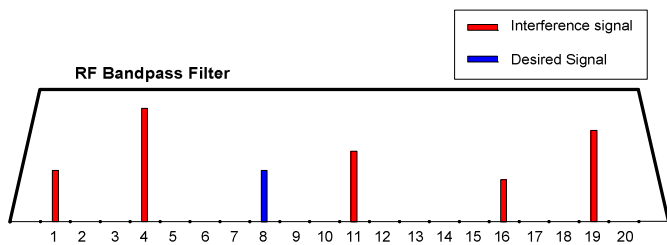


Fig. 3: An instantaneous scenario of the DTV spectrum at the output of a wideband RF bandpass filter with 20 frequency spots

In [3], the use of spectral sensing to relax linearity requirements of a receiver is addressed. The paper assumes a dense spectrum with a large number of interferers. According to the law of large numbers, the distribution of the intermodulation products due to the many interferers becomes even across the spectrum and can be seen as a distortion induced (noise) floor. However, a different picture arises when the spectrum is dominated by a few high power interference signals in the band of the RF bandpass filter. According to our analysis and measurements, different spectral distributions of the interference produce quite different distributions of the distortion products across the spots of the DTV spectrum. Consequently, the linearity requirements of the CR receiver will be different for each white spot. A prediction of the spectral location of the distortions provides the CR with the ability to choose an appropriate white spot. The required information about the level and the spectral location of the interference signals can be obtained from spectrum sensing, which is normally available anyhow in a CR.

The analysis in this paper is based on a CR receiver with direct conversion architecture. The theory can be expanded to other (non-zero) IF architectures, where the image problem must be included as an extra contribution of distortion to the desired signal, but this is outside of the scope of this paper.

The content of this paper is structured as follows. The next section describes the receiver model and nonlinearity model. The spectral location analysis of the 3<sup>rd</sup> order distortion products is done in Section III. The effect of this spectral location of the distortion on the linearity requirement is analyzed in Section IV. Section V verifies our theoretical analysis and presents results from practical measurements. Finally, conclusions will be drawn in Section VI.

## II. BEHAVIOR MODEL FOR NONLINEARITY

It is common to model the nonlinearity of an RF receiver assuming a memory-less weakly nonlinear model [4] with a truncated 3<sup>rd</sup> order Taylor series around the DC operating point:

$$s_{out} = k_1 s_{in} + k_2 s_{in}^2 + k_3 s_{in}^3 \quad (1)$$

where  $k_1$  specifies the gain, while  $k_2$  and  $k_3$  characterize the second and third order nonlinearity of the circuit with input signal  $s_{in}$  and output  $s_{out}$  ( $s$  can be both a voltage or current).  $s_{in}$  contains all the input signals at the output of the RF bandpass filter.

Equation (1) contains three terms. The first term with  $k_1$  is the amplified version of  $s_{in}$ , the linear term. The second and the third term will be called the 2<sup>nd</sup> order and 3<sup>rd</sup> order nonlinear terms, respectively. Those nonlinear terms generate different distortion products. Some of those distortion products fall into the band of interest. By using an octave RF bandpass filter [3] and RF components with differential structure, the effect of 2<sup>nd</sup> order nonlinear term can often be handled. However, the effect of the 3<sup>rd</sup> order nonlinear terms cannot be ignored because their distortion products always fall inside the desired DTV spectrum.

## III. ANALYSIS OF THE SPECTRAL LOCATION OF DISTORTION PRODUCTS

A CR scans the spectrum for white spots, which have a bandwidth in the order of 6-8 MHz in the DTV spectrum. Although the DTV signals and the CR signal are wideband (i.e. the CR occupies the whole spot) signals, we will start the analysis representing them as tones with a power equal to the integral of the power of the wideband signal. Afterwards, the analysis will be extended to deal with the spectral shape of the wideband signals.

Referring to Fig. 4,  $s_{in}$  in equation (1) contains three DTW signals around  $\omega_1$ ,  $\omega_6$  and  $\omega_{19}$ . To not lose the general view, CR signal will be introduced later after specifying the spectral location of the distortion products of the DTW signals.

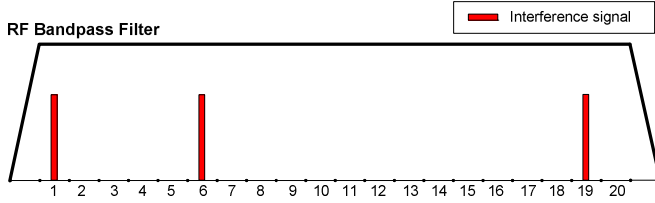


Fig. 4: An instantaneous scenario of the DTW spectrum dominated by three interference signals in the RF passband

$s_{in}$  can be described by the following equation:

$$s_{in} = \text{Re} \left\{ \sqrt{2} \sum_{n=1,6,19} A_{TV_n} e^{j\omega_n t} \right\} \quad (2)$$

where  $A_{TV}$  is the corresponding RMS value of the DTW (single tone) signals. Substituting  $s_{in}$  in the 3<sup>rd</sup> order nonlinear term of equation (1) gives the following distortion products:

$$\begin{aligned} k_3 s_{in}^3 = & \text{Re} \left\{ \sqrt{2} \left( \frac{3}{2} k_3 A_{TV_1}^3 + 3k_3 A_{TV_1} A_{TV_6}^2 + 3k_3 A_{TV_1} A_{TV_{19}}^2 \right) \right\} e^{j\omega_1 t} \\ & + \text{Re} \left\{ \sqrt{2} \left( \frac{3}{2} k_3 A_{TV_6}^3 + 3k_3 A_{TV_6} A_{TV_1}^2 + 3k_3 A_{TV_6} A_{TV_{19}}^2 \right) \right\} e^{j\omega_6 t} \\ & + \text{Re} \left\{ \sqrt{2} \left( \frac{3}{2} k_3 A_{TV_{19}}^3 + 3k_3 A_{TV_{19}} A_{TV_1}^2 + 3k_3 A_{TV_{19}} A_{TV_6}^2 \right) \right\} e^{j\omega_{19} t} \\ & + \text{Re} \left\{ \sqrt{2} \left( 3k_3 A_{TV_1} A_{TV_6} A_{TV_{19}} \right) \right\} e^{j\omega_4 t} \\ & + \text{Re} \left\{ \sqrt{2} \left( \frac{3}{2} k_3 A_{TV_{19}}^3 + 3k_3 A_{TV_{19}} A_{TV_1}^2 + 3k_3 A_{TV_{19}} A_{TV_6}^2 \right) \right\} e^{j\omega_9 t} \end{aligned} \quad (3)$$

Fig. 5 provides a visual way to understand the spectral location of the distortion products that are double underlined in equation (3). These products are important because they fill the white spots, where a CR may operate. They represent two types of intermodulation products:

1. The intermodulation products of two interferers (observe spot 11), which is the traditional IM3 that is already mentioned in literature [5].
2. The intermodulation products of three interferences (spot 14), which is 6 dB higher than the traditional 2-tone IM3 for interferes with equal power. This type of distortion will also be called IM3 in this paper.

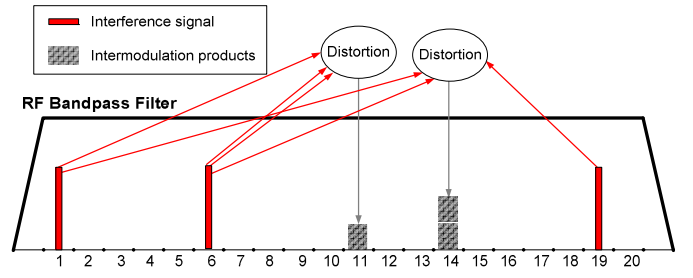


Fig. 5: Spectral location of the intermodulation products for the case of Fig. 4

For simplicity, we will keep the power of the DTW signals equal in this paper. Of course, using equation (3) case with different power can be analyzed easily.

Repeating the same analysis for another DTW spectrum scenario with three large interference signals at different locations, gives the distortion distribution that is depicted in Fig. 6.

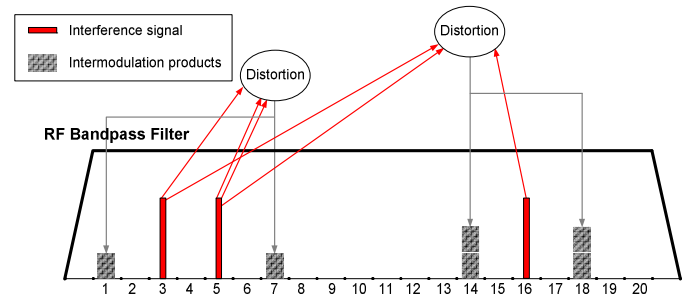


Fig. 6: Spectral location of the intermodulation products for another case

Comparing Fig. 5 with Fig. 6, we recognize that the first figure presents 15 white spots without intermodulation distortion (i.e. IM3-free spots) and 2 white spots with IM3 (i.e. IM3-spots), while the second figure presents 13 IM3-free spots and 4 IM3-spots. For all the possible spectral locations of the three interference signals, the number of the IM3-free spots can be counted and a histogram of it can be plotted as shown in Fig. 7. The expected value is 13 white spots here, with a spread from 8 to 17.

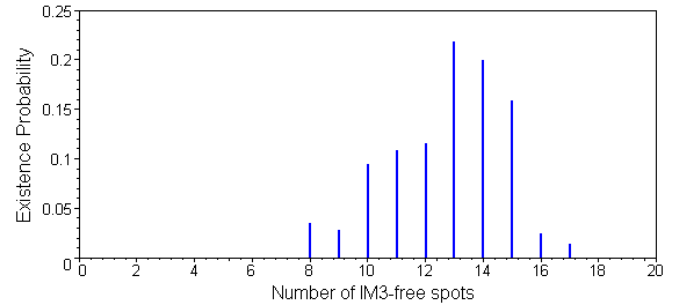


Fig. 7: Probability density of IM3-free spots for all possible scenarios with 3 interferers (Fig. 6 depict one example scenario)

Note that the fact that a spot is IM3-free does not mean that there is no 3<sup>rd</sup> order nonlinearity requirement anymore for the receiver. Actually, the interferers will cause cross-modulation on top of the desired signal as shown in Fig. 8.

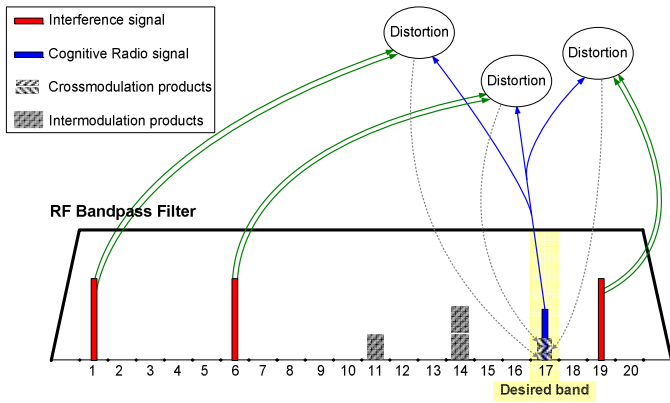


Fig. 8: Cross-modulation products (XM3) due to interferers which cross-modulate the CR signal

To analyze the effect of cross-modulation, the mathematical expression for  $s_{in}$  including a CR at spot 17 is written as follows:

$$s_{in} = \text{Re} \left\{ \sqrt{2} A_{CR} e^{j\omega_{17} t} + \sqrt{2} \sum_{n=1,6,19} A_{TV_n} e^{j\omega_n t} \right\} \quad (4)$$

where  $A_{CR}$  is the corresponding RMS value of the CR signal. Substituting  $s_{in}$  in equation (1) and retaining just the terms that exist in the desired band gives:

$$s_{out}|_{\text{Desired Band}} = \text{Re} \left\{ \underbrace{\sqrt{2} k_1 A_{CR}}_{\text{Desired amplified CR signal}} + \sqrt{2} k_3 \left\{ \underbrace{\frac{3}{2} A_{CR}^3}_{\text{Self interference}} \right. \right. \\ \left. \left. + 3 A_{CR} \left( A_{TV_1}^2 + A_{TV_6}^2 + A_{TV_{19}}^2 \right) \right\} \right\} e^{j\omega_{17} t} \quad (5)$$

Cross-modulation product

where the first term on the right hand is the amplified version of CR signal, the second term is the self-interference product [6] and the last three terms are the mentioned 3<sup>rd</sup> order cross-modulation [7] products (XM3) of the DTV signals. The self-interference product can usually be neglected in comparison to the XM3 products because the DTV signals are usually much stronger than the desired signal. The same analysis is done to show the spectral location of the distortion products of 4 interference signals with high power and the results can be seen in Fig. 9 and Fig. 10 (Expected value = 7).

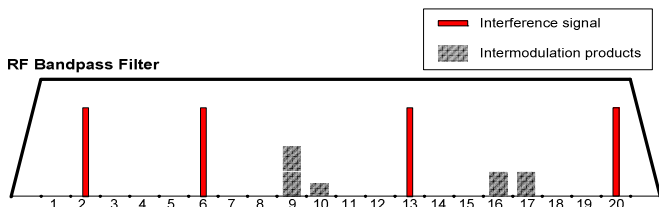


Fig. 9: Spectral location of the intermodulation products for a scenario with 4 interferers

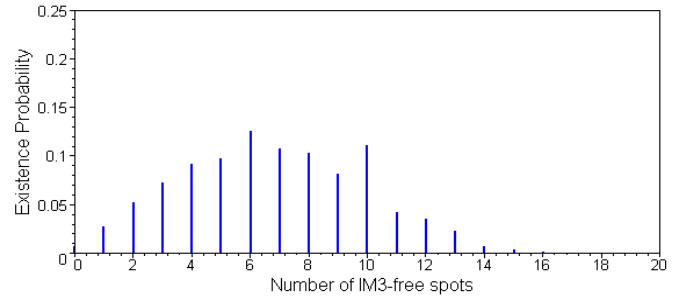


Fig. 10: Probability density of the existence of given number of free spots without intermodulation distortions for scenarios with 4 interferers

As mention previously, both the DTV signals and the CR signal are wideband signals with about the same bandwidth. Therefore their shape can be approximated by square blocks in the frequency domain. The mathematical representation in the time domain of those signals and their distortion products is written in the same way as mentioned previously with the exception that  $A_{TV}$  and  $A_{CR}$  now represent the complex envelope of the DTV signal(s) and CR signal, respectively. In the time domain, the intermodulation and the cross-modulation products are nothing else than a multiplication of three signals, where the involved signals are indicated with arrows. E.g. three arrows constitute the inputs to the distortion circles in Fig. 5, Fig. 6 and Fig. 8, in some cases from two frequencies, in other cases from three frequencies. In the frequency domain, this corresponds to two times convolution of the signals. The result of this convolution is shown in Fig.11 for the interference scenario of Fig. 5, but now with more realistic block-shaped spectra instead of single tones.

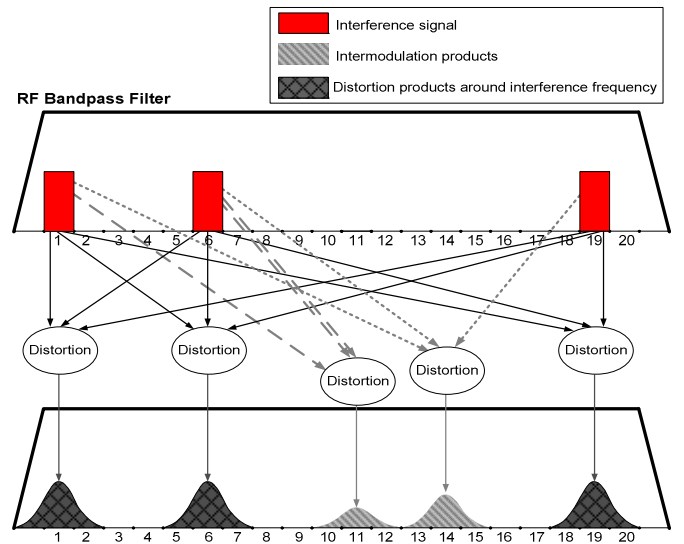


Fig. 11: Bell shapes for the distortion products of three wideband interferers

A Bell shape results, occupying three spots as shown in Fig. 11. Analysis shows that more than 66% of the distortion power is concentrated in the frequency spot in the center of the Bell, while the power spilled over to the adjacent frequency spots is less than 17% of the distortion power each. Referring to equation (3), the bell shapes around spots 1, 6 and 19, each contain a self-interference product and two cross-modulation

products (e.g. the Bell shape around spot 6 contains its self-interference product and the cross-modulation products of the other two interferers of spots 1 and 19 on top of the interferer of spot 6). As a consequence of spreading, the distortion to the adjacent spots of 1, 6 and 19, the spots 2, 5, 7, 18 and 20 will be called adjacent-distortion spots (i.e. the distortion that is adjacent to the interferers). The linearity requirements in those adjacent-distortion spots of Fig. 11 will be higher than that of Fig. 5. Additionally, the bell shape of the IM3 products (i.e. around spot 11 and 14) will reduce the number of IM3-free spots by 4 spots in comparison to Fig. 5, as spots 10, 12, 13 and 15 will contain 17% of the total IM3 power of the Bell shape, hence those spots will be also called IM3 spots. Paper [8] presents an analysis about the interference effects in MB-OFDM systems. It proves that the power of the IM3 resulting from two OFDM signals (QPSK modulation) each with a large number of sub-carriers is higher than that of two tones that have the same power as the OFDM signals:

$$P_{IM3}^+ = P_{IM3} + \Delta_{Wideband} \quad [\text{dBm}] \quad (6)$$

where  $P_{IM3}^+$  and  $P_{IM3}$  represent the intermodulation distortion of two wideband signals and two tones, respectively.  $P_{IM3}^+$  is distributed over three spots (e.g. see spots 10, 11, 12 of Fig. 11).  $\Delta_{Wideband}$  represents a correction term. Instead of dealing with wideband complex signals, equation (6) provides the possibility to analyze the nonlinearity with the traditional two-tone test method and simply add a correction term.

As an example that can be applied to our case, a MatLab simulation setup has been built to simulate the nonlinearity with two real OFDM signals at RF. QPSK modulation was used and the frequency difference between the sub-channels is 4.46 kHz satisfying the DVB-T standard. The resulting IM3 was simulated. The simulation result is shown in Fig. 12, where the number of the sub-carriers per OFDM signal has gradually been increased from 1 to 900. Thus we can compare the IM3 generated for two (narrowband) single tones with that for a truly wideband OFDM signal with many sub-carriers, where the total signal power in all cases is kept constant.

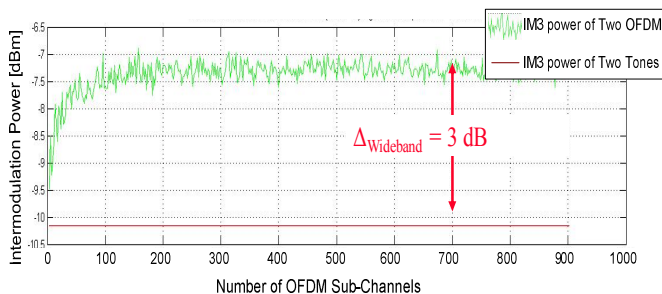


Fig. 12: Comparison of the IM3 power caused by two (single) tones and two multi-tone OFDM signals with a number of sub-carriers from 1 to 900

Fig.12 shows a  $\Delta_{Wideband}$  of about 3 dB. Changing the modulation of the two OFDM signals to 16QAM, we find 3.5 dB. By further increasing the number of the QAM modulation to higher orders, the signal characteristic of the two OFDM

signals resembles the case of having two band limited noise signals. In this case the factor is increased to 4.5 dB. In Table 1, the simulation results have been summarized:

TABLE I  
CORRECTION FACTOR  $\Delta_{Wideband}$  FOR TWO OFDM SIGNALS WITH DIFFERENT MODULATION CHARACTERISTICS

Two Signals	Max. $\Delta_{Wideband}$ [dB]
Two OFDM: QPSK	3
Two OFDM: QAM16	3.5
Two Band limited Noise	4.5

As the purpose of this paper is to compute the linearity requirements for CR, the highest value of the maximum  $\Delta_{Wideband}$  will be taken, which is 4.5 dB.

#### IV. ESTIMATION OF LINEARITY REQUIREMENTS

The linearity of a receiver must be adequate to preserve a minimum Signal to Distortion Ratio needed at the demodulator. As previously depicted in Fig. 5, we distinguish two types of white spots: IM3-free spots and IM3-spots. We will show that the CR requires higher linearity if it operates at IM3-spots than for IM3-free spots with only self-interference and the XM3. This section begins with introducing an estimation of the linearity requirement for the case that the CR is operating at an IM3-free spot and the interferers are modeled by tones. Afterwards the linearity equation will be extended to cover the case of wideband interference signals. Finally, this linearity requirement will be compared to that of a CR operating at an IM3-spot and at a adjacent-distortion spot.

Referring to Fig. 8 and equation (5), the Signal to Distortion Ratio can be written as follows:

$$\frac{S}{D} = \frac{(k_1 A_{CR})^2}{\left(3k_3 A_{CR} \left(\sum_{n=1,6,19} A_{TV_n}^2\right)\right)^2} \quad (7)$$

where the self-interference has been neglected. The linearity requirement can be presented as the ratio between  $k_1$  and  $k_3$ :

$$\left.\frac{k_1}{k_3}\right|_{\text{spot 17}} = 3 \times \sqrt{\frac{S}{D}} \left(\sum_{n=1,6,19} A_{TV_n}^2\right) \quad (8)$$

where  $S/D$  is the required Signal to Distortion Ratio for the CR receiver output. It is proportional to the sum of the power of the DTV signals, which will cross-modulate the receiver CR-signal.

Equation (6) presents the correction factor  $\Delta_{Wideband}$  for the intermodulation power. The same  $\Delta_{Wideband}$  is valid for the XM3 power (the  $\Delta_{Wideband}$  is just the relative difference between the modulation products of the two tones in relation to that of the two wideband signals). Therefore equation (6) provides a useful tool to extend the derivation of (8) to deal

with wideband interference signals as explained in the Appendix:

$$\left| \frac{k_1}{k_3} \right|_{\text{spot 17}} = 3 \times \sqrt{10^{\frac{\Delta_{\text{Wideband}}}{10}}} \times 0.66 \times \sqrt{\frac{S}{D}} \sum_{n=1,6,19} A_{TV_n}^2 \quad (9)$$

Using the mentioned maximum  $\Delta_{\text{Wideband}}$  of 4.5dB gives:

$$\left| \frac{k_1}{k_3} \right|_{\text{spot 17}} = 3 \times 1.4 \times \sqrt{\frac{S}{D}} \sum_{n=1,6,19} A_{TV_n}^2 \quad (10)$$

It is instructive to relate equation (10) to the traditional IIP3 specification. According to the definition of IIP3, the RMS value of IIP3 is related to  $k_1/k_3$  as follows [5]:

$$A_{IIP3} = \frac{\sqrt{\frac{4}{3} \left| \frac{k_1}{k_3} \right|}}{\sqrt{2}} = \sqrt{\frac{4}{6} \left| \frac{k_1}{k_3} \right|} \quad [\text{V}] \quad (11)$$

Substituting equation (10) in (11) gives the following equation:

$$P_{IIP3}|_{\text{spot 17}} = 2 \times 1.4 \times \sqrt{\frac{S}{D}} \sum_{n=1,6,19} P_{TV_n} \quad [\text{W}] \quad (12)$$

We recognize that dealing with wideband signals instead of tones, the IIP3 requirement will be increased by around 1.5 dB (i.e.  $10 \times \log(1.4)$ ). To get a sense of what equation (12) presents, let's assume that the interference signals of Fig. 11 have a power of -10 dBm and the required Signal to Distortion Ratio is 10 dB. Then the required linearity to keep XM3 low enough expressed in terms of IIP3 would be 4 dBm. Increasing the power of the interferences by 10 dB increases the requirement also by 10 dB. Equation (12) can be generalized as follows:

$$P_{IIP3} = 2 \times 1.4 \times \sqrt{\frac{S}{D}} \sum_n P_{TV_n} \quad [\text{W}] \quad (13)$$

where  $n$  refers to all the interference signals that exist in the RF bandpass filter of the CR receiver.

As a final step, let us compare IIP3 of equation (12) (i.e.  $IIP3_{\text{XM3}}$ ) to that IIP3 requirement when the CR operates in IM3-spot ( $IIP3_{\text{IM3}}$ ) like for example spot 11 in Fig. 11. In this case the distortion products will contain the IM3 plus the XM3 products:

$$s_{out}|_{\text{DesiredBand}} = \text{Re} \left\{ \begin{aligned} & \underbrace{\sqrt{2} k_1 A_{CR}}_{\text{Desired amplified CR signal}} \\ & + \sqrt{2} \left( \underbrace{3 k_3 A_{CR} \sum_{n=1,6,19} A_{TV_n}^2}_{\text{Crossmodulation product}} + \underbrace{\frac{3}{2} k_3 A_{TV_1} A_{TV_6}^2}_{\text{Intermodulation product}} \right) \end{aligned} \right\} e^{j\omega_1 t}$$

Writing down the Signal to Distortion Ratio gives:

$$\frac{S}{D} = \frac{(k_1 A_{CR})^2}{\left( \underbrace{k_3 3 A_{CR} \left( \sum_{n=1,6,19} A_{TV_n}^2 \right)}_{\text{Crossmodulation product}} + \underbrace{\left( \frac{3}{2} k_3 A_{TV_1} A_{TV_6}^2 \right)}_{\text{Intermodulation product}} \right)^2}$$

The resulting linearity requirement in terms of  $k_1/k_3$  becomes:

$$\left| \frac{k_1}{k_3} \right|_{\text{spot 11}} = 3 \times \sqrt{\frac{S}{D}} \left( \sum_{n=1,6,19} A_{TV_n}^2 + \frac{1}{2} \frac{A_{TV_1} A_{TV_6}^2}{A_{CR}} \right) \quad (14)$$

Expressing the linearity requirements in IIP3 and extending the derivation to wideband signals gives the following two equations for spot 11 and 10, 12 (see Fig.11):

$$P_{IIP3}|_{\text{spot 11}} = 2 \times \sqrt{10^{\frac{\Delta_{\text{Wideband}}}{10}} \times 0.66}^{1.4} \times \sqrt{\frac{S}{D}} \left[ \sum_{n=1,6,19} P_{TV_n} + \frac{1}{2} \sqrt{\frac{P_{TV_1}}{P_{CR}} P_{TV_6}} \right] \quad [\text{W}] \quad (15)$$

$$P_{IIP3}|_{\text{spot 10,12}} = 2 \times \sqrt{10^{\frac{\Delta_{\text{Wideband}}}{10}} \times 0.17}^{0.7} \times \sqrt{\frac{S}{D}} \left[ \sum_{n=1,6,19} P_{TV_n} + \frac{1}{2} \sqrt{\frac{P_{TV_1}}{P_{CR}} P_{TV_6}} \right] \quad [\text{W}] \quad (16)$$

By comparing equation (15) and (16), the IIP3 requirements of spots 10 or 12 (i.e. the spots adjacent to the Bell center spot) are 3 dB lower than the IIP3 requirement of spot 11 (i.e. the center spot of the Bell shape).

As mentioned previously, the CR requires also linearity requirement in the adjacent-distortion spots (see Fig.11). Repeating the same analysis as previously done and using equation (3), (5) and (11) results:

$$\begin{aligned}
 P_{IIP3}|_{\text{spot } 2} &= 2 \times \sqrt[0.7]{10^{\frac{\Delta_{\text{Wideband}}}{10}} \times 0.17} \\
 &\times \sqrt{\frac{S}{D}} \left[ \sum_{n=1,6,19} P_{TV_n} \right. \\
 &\left. + \left( \frac{1}{2} P_{TV_1} + P_{TV_6} + P_{TV_{19}} \right) \sqrt{\frac{P_{TV_1}}{P_{CR}}} \right] \quad [\text{W}] \quad (17)
 \end{aligned}$$

To get a sense about the linearity requirement of equations (12), (15), (16) and (17), consider the same numbers as for the previous example, while assuming that the CR signal is equal to -60 dBm. Then the different IIP3 requirements are summarized in Table 2.

TABLE 2  
LINEARITY REQUIREMENT OF EQUATIONS (12), (15), (16) AND (17) FOR THE SCENARIO OF FIG. 11

Equation	Spot no.	Spot Description	IIP3 [dBm]
(12)	17	IM3-free spot	4
(15)	11	IM3-spot	36
(16)	10, 12	IM3-spot	33
(17)	2	Adjacent-distortion spot	40

Clearly, there is a very significant benefit in looking for IM3-free spots. Such spots like spot 17 can well make a key difference in the feasibility [9, 10] of CR.

### V. MEASUREMENT RESULTS

The spectral location of the distortion products is the consequence of modeling a memory-less weakly nonlinear device with a Taylor series as presented in equation (1). The model has been verified by building a setup where a Low Noise Amplifier (LNA) from the Mini-Circuits (i.e. ZFL-1000LN) is tested with three interference signals. The power and the spectral location of the distortion products are measured. The LNA has gain of 18 dB, IIP3 of -6 dBm, Output 1 dB compression point of 3 dBm and its operating region is from 0.1 to 1000 Hz (i.e. VHF/UHF band). The interference signals are three carriers with QPSK modulation, which are generated by vector signal generators from Agilent technology. In the first experiment, the power of each interferer is around -26 dBm and their frequency location is around the following values: 548, 563 and 600 MHz. This case looks like the case shown in Fig. 5 and the measurement result is shown in Fig. 13.

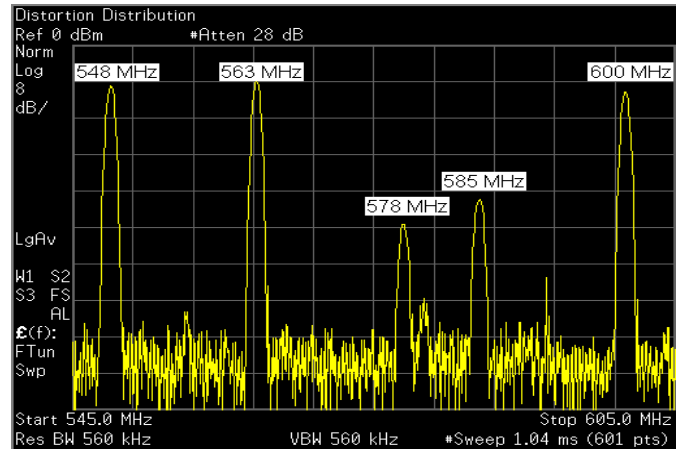


Fig. 13: Spectral location of the distortion products in case of three interferers at 548, 563 and 600 MHz

Comparing to Fig. 5, Fig. 13 indeed contains two distortion products:

1. The first interferer at 548 MHz and the second interferer at 563 MHz generates the traditional IM3 at 578 MHz (i.e. 2\*563-548)
2. The combination of all the three interferers generates the previously mentioned new IM3 at frequency 585 MHz (i.e. 600-(563-548) and (600-563)+548). Shown in Fig. 13, this type of IM3 is higher than the first IM3 by 6 dB as our theory has predicted.

We measured the power of those IM3 products and compared it with the results of our equations as shown in Table 3.

TABLE 3  
COMPARISON BETWEEN THE MEASUREMENT AND THE THEORY RESULTS OF THE IM3 DISTORTION PRODUCTS

IM3 frequency [MHz]	Theory [dBm]	Measurement [dBm]
578	-39.4	-38.7
585	-33.4	-32.5

Another experiment has been done with a scenario that looks like the case shown in Fig. 6. The measurement result is shown in Fig. 14.

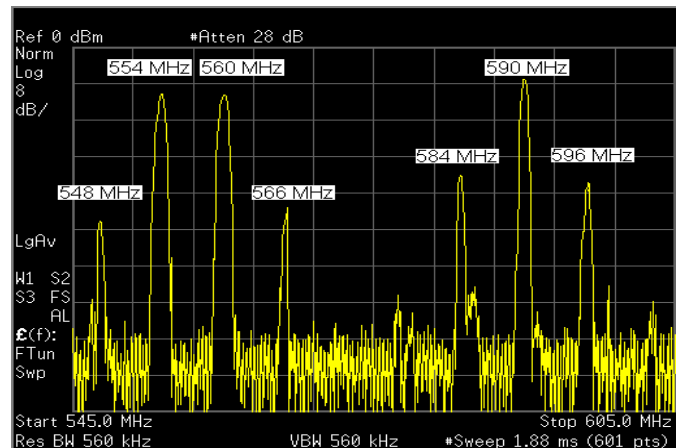


Fig. 14: Spectral location of the distortion products in case of three interferers at 554, 560 and 590 MHz

Consequently, the measurement results verify our theory about the effect of interference locations on the spectral location of the distortions. The IM3 spots and the IM3-free spots are clearly shown in Fig. 13 and Fig. 14.

## VI. CONCLUSION AND OUTLOOK

This paper studies the 3<sup>rd</sup> order nonlinearity requirements for a CR receiver with a wideband RF bandpass filter that operates in the DTV spectrum, where several high powers DTV signals fall in the RF filter-band. Due to the nonlinearity of the RF receiver and the existence of strong interference signals, different types of 3<sup>rd</sup> order distortion products will be produced. The spectral location of these distortion products depends on the locations of the interference signals. Analysis and measurement show that white spots can be classified into two types:

1. IM3-spots where the distortion is dominated by IM3 products. The IM3 products are much stronger than XM3 and self-interference if the interferers are much stronger than the desired signal.
2. IM3-free spots where the CR signal will only be distorted by XM3 and self-interference.

If an IM3-free spot is selected for CR operation, its third order linearity is relaxed. In a scenario with 3 interferers at -10dBm and a CR-signal at -60dBm, the resulting IIP3<sub>XM3</sub>=4dBm, while the IIP3<sub>IM3</sub> is 36 dBm. Since the linearity of any radio design is limited, typically to not much more than 0dBm in CMOS, spectral sensing information about the level and the spectral location of the strong interferers can be used to predict the location of the IM3-free spots. Such a prediction mechanism seems to be almost indispensable for a CR that can work in a wide RF-band. Further work pursuing this direction seems worthwhile.

## APPENDIX

Equation (6) can be rewritten for the cross-modulation products as follows:

$$P_{XM3}^+ = P_{XM3} + \Delta_{Wideband} \quad [\text{dBm}]$$

Converting it to the linear range gives:

$$p_{XM3}^+ = p_{XM3} \times 10^{\frac{\Delta_{Wideband}}{10}} \quad [\text{W}]$$

The shape of this distortion looks like a Bell and covers three spots. However we are just interested in the amount of it in the middle spot, which contains 66% of the total distortion power:

$$p_{XM3}^+ \Big|_{\text{spot17}} = p_{XM3} \times 10^{\frac{\Delta_{Wideband}}{10}} \times 0.66 \quad [\text{W}]$$

Rewriting it according to its RMS values and substituting the crossmodulation product of equation (5) in it gives:

$$\begin{aligned} A_{XM3}^+ &= A_{XM3} \times \sqrt{10^{\frac{\Delta_{Wideband}}{10}} \times 0.66} \\ &= 3k_3 A_{CR} \left( \sum_{n=1,6,19} A_{TV_n}^2 \right) \times \sqrt{10^{\frac{\Delta_{Wideband}}{10}} \times 0.66} \end{aligned}$$

where  $A_{XM3}^+$  and  $A_{XM3}$  represent the RMS value of the crossmodulation products of two wideband signals and two tones, respectively. The next step is substituting it in the Signal to Distortion Ratio, which gives:

$$\frac{S}{D} = \frac{(k_1 A_{CR})^2}{\left( 3k_3 A_{CR} \left( \sum_{n=1,6,19} A_{TV_n}^2 \right) \times \sqrt{10^{\frac{\Delta_{Wideband}}{10}} \times 0.66} \right)^2}$$

This can be rewritten as follows:

$$\left. \frac{k_1}{k_3} \right|_{\text{spot17}} = 3 \times \sqrt{10^{\frac{\Delta_{Wideband}}{10}} \times 0.66} \times \sqrt{\frac{S}{D}} \sum_{n=1,6,19} A_{TV_n}^2$$

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