GNSS Signal Acquisition in Harsh Urban Environments

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***Abstract*—In urban environment, Global Navigation Satellite System (GNSS) signals are impaired by non-line-of-sight fading conditions and by the presence of potential sources of electromagnetic disturbance. This paper analyzes the impact of the wireless propagation medium and aggregate network interference by measuring the degradation of the acquisition performance expressed in terms of receiver operating characteristics (ROC). The presented framework allows to realistically evaluate the GNSS acquisition performance by jointly considering the effect of radio signal propagation conditions, interfering nodes spatial distribution, and other relevant environment dependent parameters. By means of numerical examples we elucidate the need for alternative positioning techniques in harsh urban environments.**

***Index Terms*—acquisition performance, GNSS, aggregate network interference, urban radio propagation channel**

I. Introduction

Recently, the widespread use of mobile devices has led to a substantial increase of radio transmitters, and consequently, a drastic proliferation of potential sources of interference. Considering that GNSS is a critical infrastructure and bearing in mind the low GNSS signal power, it is relevant to study the impact of multiple interferers to assess the acquisition performance in dense urban environment. Although legal policies are established to protect the GNSS bands, there exist future realistic scenarios such as multi-constellation GNSS [1], the deployment of pseudolites [2] and ultra wideband (UWB) transmitters, where the interference can originate from multiple transmitters, and where literature specifically warns for the severe interference effects and the resulting performance degradation inflicted on GNSS receivers. Another possible threat are cognitive radio (CR) networks, which have been proposed recently to alleviate the problem of inefficiently utilized spectrum by allowing cognitive devices to coexist with licensed users, given that the interference caused to the licensed users can be limited. The frequency bands used for DVB-T transmissions are a possible candidate for opportunistic spectrum access (OSA) [3], yet the DVB-T harmonics are known to coincide with the GPS L1 or Galileo E1 bands. Therefore, cognitive devices which are allowed to transmit in the UHF IV band when the digital television (DTV) broadcasting system is inactive, might create harmful interference to GNSS systems due to amplifiers non-linear behavior. Although literature mentions different types of interference that can affect GNSS receivers [4], [5], a theoretical framework that accounts for the effects of multiple sources of interference and for the channel fading affecting both signal of interest (SoI) and interfering signals is still missing.

In this paper, we propose a framework for the GNSS acquisition performance that jointly accounts for aggregate network interference and different channel conditions for the SoI and the interfering signals. The acquisition performance is characterized by means of mathematical expressions of the probability of detection ($\mathbb{P}_{\rm d}$) and the probability of false alarm ($\mathbb{P}_{\rm fa}$). The framework is of interest for the correct setting of the detection threshold in realistic (future) signal conditions which guarantees a minimum required acquisition performance. The main contribution of this work is the adoption of aggregate network interference in a theoretical framework that evaluates the GNSS acquisition performance. Moreover, our results illustrate the necessity for alternative positioning techniques due to the considerable performance degradation in dense urban environment.

The remainder of the paper is organized as follows. In Section II and III the signal and system model are presented, introducing the assumptions that have been made. In Section IV the acquisition performance in presence of aggregate network interference is discussed. The reduction of the acquisition performance is illustrated by numerical results in Section V and Section VI presents the conclusions.

II. Signal Model

After filtering and downconversion in the receiver front-end, the kth sample of the received signal entering the acquisition block has the following form

$$s[k] = \sum_{l=1}^{N_{\rm sat}} r_l[k] + i[k] + n[k] \tag{1}$$

where $s[k]$ is composed of $N_{\rm sat}$ satellite signals $r_l[k]$, an interference term $i[k]$ and the noise term $n[k]$. We assume the noise samples to be independent and to feature a normal distribution $\mathcal{N}_{\rm c}(0, N_0 f_{\rm s}/2)$, with $f_{\rm s}$ the sampling frequency and N_0 the noise spectral density. The kth sample of the GNSS

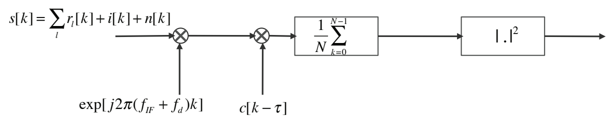

Fig. 1. Schematic of the different processing steps for the calculation of the search space.

signal received from a single satellite can be represented as

$$r_l[k]=\sqrt{2P}h_l c_l[k-\tau_{\mathrm{c},l}]d_l[k-\tau_{\mathrm{c},l}]\cos[2\pi(f_{\mathrm{IF}}+f_{\mathrm{d},l})k+\phi_l] \quad (2)$$

where P is the GNSS received signal power, h_l represents the fading affecting the lth satellite signal, c_l is the code with corresponding code phase $\tau_{\mathrm{c},l}$, f_{IF} and $f_{\mathrm{d},l}$ are the intermediate frequency and the Doppler frequency, and ϕ_l is the carrier phase error. For reasons of simplicity, we suppose the data bit d_l to be 1. Since the GPS L1 and Galileo E1 are operating in protected spectrum, we consider as source of interference the harmonics or intermodulation products of emissions in the UHF IV and UHF V frequency bands. The main objective of the acquisition is to determine the code phases $\tau_{\mathrm{c},1}, \tau_{\mathrm{c},2}, ..., \tau_{\mathrm{c},N_{\mathrm{sat}}}$ and Doppler frequencies $f_{\mathrm{d},1}, f_{\mathrm{d},2}, ..., f_{\mathrm{d},N_{\mathrm{sat}}}$ of the satellites in view.

III. System Model

The acquisition of GNSS signals is a classical detection problem where a signal impaired by noise and interference has to be identified. Prior to the tracking of GNSS signals, the receiver identifies which satellites can be used to determine a position and time solution and provides a rough estimation of the code phase and the Doppler frequency of the present satellite signals. In the receiver acquisition block, the signal as defined in (1) is first downconverted to baseband. Subsequently, the downconverted signal is correlated with a local replica of the code and is integrated over the integration interval which is an integer a times longer than the code period length N. As shown in Fig. 1, the unknown phase of the incoming signal is finally removed by taking the squared absolute value of the complex variable.

The acquisition process is a binary decision problem with two hypothesis. The $\mathcal{H}_1$ hypothesis corresponds with the scenario where the signal is present and correctly aligned with the local replica at the receiver. The null-hypothesis $\mathcal{H}_0$ corresponds to the case where the SoI is not present, or present but incorrectly aligned with the local replica. The acquisition performance is measured in terms of the probability of detection and the probability of false alarm. The probability of detection $\mathbb{P}_{\mathrm{d}}$ is the probability that the decision statistic V surpasses the threshold β in the presence of the SoI and can be expressed as $\mathbb{P}_{\mathrm{d}}(\beta)=\mathbb{P}(V>\beta|\mathcal{H}_1)$. The probability of false alarm $\mathbb{P}_{\mathrm{fa}}$ is the probability that V exceeds β in absence of SoI or when the signal is not correctly aligned with the local replica, and can be expressed as $\mathbb{P}_{\mathrm{fa}}(\beta)=\mathbb{P}(V>\beta|\mathcal{H}_0)$.

Code synchronization has to be accomplished over the code phase and Doppler frequency. These two variables constitute a two- dimensional search space, which is discretized into different cells that correspond with a range of possible values of the code phase and the Doppler frequency. The code phase $\tau_{\mathrm{c},l}$ of the different satellites in view are chosen from a finite set $\{\tau_1,\tau_2,\ldots,\tau_{aN}\}$ with $\tau_p=(p-1)\Delta\tau$, where we choose $\Delta\tau$ equal to the chip time to allow a tractable analysis. As for the Doppler frequency, the value is chosen from the finite set $\{f_1,f_2,\ldots,f_L\}$, with $f_q=f_{\min}+(q-1)\Delta f$, where the frequency resolution Δf and $f_{\min}$ are chosen according to the specifications of the application. In order to define the cell statistics[1], we characterize the different contributions to the cell values and we define the search space as $\bar{X}=\{X[\tau_p,f_q]:1\le p\le aN, 1\le q\le L\}$, and each cell of $\bar{X}$ is given by

$$\begin{aligned}&X[\tau_p,f_q]\\&=\left|\frac{1}{aN}\sum_{k=1}^{aN}\left[\sum_{l=1}^{N_{\mathrm{sat}}}r_l[k]+i[k]+n[k]\right]c[k-\tau_p]e^{j2\pi(f_{\mathrm{IF}}+f_q)k}\right|^2\\&=\Big|X_{\mathrm{r}}[\tau_p,f_q]+X_{\mathrm{i}}[\tau_p,f_q]+X_{\mathrm{n}}[\tau_p,f_q]\Big|^2 \quad (3)\end{aligned}$$

with $X_{\mathrm{r}}[\tau_p,f_q]$, $X_{\mathrm{i}}[\tau_p,f_q]$ and $X_{\mathrm{n}}[\tau_p,f_q]$ the contributions of the satellite signals, the interference and the noise, respectively.[2] The noise term X_{n} results from the downconversion and correlation with the local replica of the noise term in (1). The downconversion yields a complex Gaussian random variable (r.v.) with variance of the real and the imaginary parts equal to $N_0 f_{\mathrm{s}}/4$. The correlation with the local replica yields the mean value of N zero-mean, complex Gaussian r.v.'s, and thus, $X_{\mathrm{n}}\sim\mathcal{N}_c(0,\sigma_{\mathrm{n}}^2)$ with $\sigma_{\mathrm{n}}^2=N_0 f_{\mathrm{s}}/(2N)=N_0/(2T_{\mathrm{per}})$, where $T_{\mathrm{per}}=NT_{\mathrm{c}}$ is the code period and T_{c} is the chip time. Note that, in order to have independent noise samples, the sampling rate is $1/T_{\mathrm{c}}$. Although each interfering signal does not necessarily feature a zero-mean Gaussian distribution, it can be shown that the contribution to the decision variable produced by the despreading of the interfering signal can be often approximated by a Gaussian random variable [6], [7]. When the Gaussian approximation of the contribution to the decision variable produced by the despreading of the interfering signal is not accurate, the proposed framework yields a pessimistic performance analysis [8]. When we consider a network of interferers, we apply a stochastic geometry approach to capture the randomness of the topology and model the spatial distribution of the interferer locations according to a homogeneous Poisson point process [9]. Without loss of generality, we consider the receiver located at the origin of an infinite plane, and we express the aggregate interference measured at the origin as

$$X_{\mathrm{i}}=\sum_{m=1}^{\infty}i_m. \quad (4)$$

[1]We consider a single non-coherent computation method of the search space cells.

[2]Without loss of generality we assume $a=1$.

The mth interfering signal in (4) can be written as

$$i_m = \frac{1}{R_m^{\nu}} g_m (I_{m,1} + jI_{m,2}) \quad (5)$$

where $I_{m,1}$ and $I_{m,2}$ are two i.i.d. Gaussian r.v.'s with zero mean and variance $\sigma_I^2/2$. The term σ_I^2 represents the interferer transmission power at a distance of 1 meter (far-field assumption) in the affected GNSS band. The r.v. g_m represent the fading that affects the mth interferer. As in the far-field, the signal power decays with $1/R_m^{2\nu}$, where R_m is the distance of node m with respect to the victim receiver and ν is the amplitude path loss exponent. It is worth to notice that since $I_{m,1}$ and $I_{m,2}$ are two i.i.d Gaussian r.v.'s with mean equal to zero, I_m is circular symmetric (CS). We suppose that there is no coordination between the different transmitters and thus, they transmit asynchronously and independently. Under such conditions, it can be shown that X_{i} follows a symmetric stable distribution [9]–[11]

$$X_{\mathrm{i}} \sim \mathcal{S}_c(\alpha = 2/\nu, \beta = 0, \gamma = \pi\lambda C_{2/\nu}^{-1} \mathbb{E}\{|g_m I_{m,p}|^{2/\nu}\}) \quad (6)$$

with $C_x = \frac{1-x}{\Gamma(2-x)\cos(\pi x/2)}$.

Although the search space $\bar{X}$ is two-dimensional, we consider in this work the Doppler frequency known, thus leading to a one-dimensional search space that is function of the code phase. We refer to [12] for several acquisition techniques that include also the estimation of the Doppler frequency. For a known Doppler frequency, a cell of the search space $X[\tau] \in \bar{X}$ can be written as[3]

$$X[\tau] = \left| \sum_{l=1}^{N_{\mathrm{sat}}} \sqrt{P} h_l R_l[\tau] e^{-j\phi_l} + X_{\mathrm{i}}[\tau] + X_{\mathrm{n}}[\tau] \right|^2 \quad (7)$$

where $R_l[\tau]$ is the cross-correlation function between the code under search and the code of the lth satellite. We consider the set of $\{h_l\}$ as independent and identically distributed (i.i.d.), with a constant value over the integration time and average fading power $\mathbb{E}\{h_l^2\} = 1$. Without loss of generality, let satellite 1 be the satellite under search. The cell of the search space can now be written as

$$X[\tau] = \left| \sqrt{P} h_1 R_1[\tau] e^{-j\phi_1} + \underbrace{\sum_{l=2}^{N_{\mathrm{sat}}} \sqrt{P} h_l R_l[\tau] e^{-j\phi_l}}_{X_c[\tau]} + X_{\mathrm{i}}[\tau] + X_{\mathrm{n}}[\tau] \right|^2 \quad (8)$$

where $X_{\mathrm{c}}[\tau]$ is the contribution of the cross-correlation noise to the value of a random search space cell[4]. The distribution of $X_{\mathrm{c}}[\tau]$ can be well approximated by a complex, zero-mean Gaussian distributed r.v. [5]. The variance of $X_{\mathrm{c}}[\tau]$ can be written as

$$\sigma_{\mathrm{c}}^2 = \left[\mathbb{E}\{h_l^2\}(N_{\mathrm{sat}} - 1)P\right] \sigma_{\mathrm{cross}}^2/2 \quad (9)$$

where $\sigma_{\mathrm{cross}}^2$ is the variance of the cross-correlation originating from a single satellite.

In this work, we adopt the Generalized Likelihood Ratio Test (GLRT), which has been introduced in [13]. In general, the goal of a decision strategy is to maximize the probability of detection and to minimize the probability of false alarm, which are conflicting objectives. The GLRT leads to select the maximum of the search space defined as

$$V = \max\{\bar{X}\}. \quad (10)$$

The decision is then taken by comparing V with a threshold. In the GLRT strategy, the Neyman-Pearson criterion is applied. For a selected probability of false alarm, a threshold that maximizes the probability of detection is chosen, such that the GLRT strategy is the optimal acquisition strategy when the signal conditions are perfectly known.

IV. Acquisition Performance

In this section, we propose an analytical approach for the evaluation of the acquisition performance that is based on the characteristic function (CF) of the decision variable. The scenario where the interference stems from a network of interferers is of increasing importance, as reported in recent literature [9], [10], [14]. We analyze the impact of this type of interference on the acquisition of the satellite signal for the GLRT acquisition strategy.

A. Probability of Detection

$\mathbb{P}_{\mathrm{d}}$ is determined examining the cell corresponding with the correct code phase. In the GLRT strategy, the maximum value of the entire search space is compared with a threshold. Let X_1 denote the cell value corresponding to the correct code phase τ_1 and the search space with exclusion of the cell X_1 is denoted by $\bar{X}_- = \bar{X} \backslash \{X_1\}$. Considering a relatively strong satellite signal power, we suppose that $X_1 = \max\{\bar{X}\} = X_{(1)}$. In this case, the probability of detection can be found by applying the inversion theorem and is given by [15]

$$\begin{aligned} &\mathbb{P}_{\mathrm{d}}(\beta | \max\{\bar{X}_-\} < X_1) = \mathbb{P}\{X_1 > \beta\} \\ &= \frac{1}{2} - \frac{1}{2\pi} \int_0^{\infty} \mathfrak{Re}\left\{ \frac{\psi_{X_1}(-j\omega)e^{j\omega\beta} - \psi_{X_1}(j\omega)e^{-j\omega\beta}}{j\omega} \right\} d\omega \end{aligned} \quad (11)$$

where $\psi_{X_1}(j\omega)$ is the CF of the decision variable X_1. It can be shown by simulation that the acquisition performance conditioned on $X_1 = X_{(1)}$ is a very good approximation of the unconditional acquisition performance in the region of interest, i.e. for $\mathbb{P}_{\mathrm{fa}} < 0.5$. By using the CF of X_1, we can easily include in the analysis the effect of fading on the SoI and on the interferers. To define the statistics of the decision variable, we analyze the different contributions to the search space cell values. In the presence of aggregate interference, the decision variable can be expressed as

$$X_1 = |\underbrace{\sqrt{P} h_1 e^{-j\phi} + X_{\mathrm{i}}}_{D} + X_{\mathrm{c}} + X_{\mathrm{n}}|^2 \quad (12)$$

[3]To reduce the complexity of notation, the index p is further discarded.

[4]We consider the maximum of the auto-correlation equal to 1.

$$\psi_{X_1}(j\omega) = \frac{1}{1-2j\omega(P/2+\sigma_{\rm nc}^2)}\left\{1+\theta_\nu 2^{1/\nu}\gamma\cos\left(\frac{\pi}{2\nu}\right)\left|\frac{j\omega}{1-2j\omega(P/2+\sigma_{\rm nc}^2)}\right|^{1/\nu}\right. \\ \left.\times\left[1-\mathrm{sign}\left(\frac{j\omega}{1-2j\omega(P/2+\sigma_{\rm nc}^2)}\right)\tan\left(\frac{\pi}{2\nu}\right)\right]\right\}^{-k_\nu}. \quad (21)$$

where D stands for the contribution to the decision variable of the SoI and the aggregate interference. The sum of the noise and cross-correlation noise is a Gaussian r.v. with variance $\sigma_{\rm nc}^2 = \sigma_{\rm n}^2 + \sigma_{\rm c}^2$. Conditioning on D, X_1 follows a non-central chi-square distribution with two degrees of freedom $X_1 \sim \chi^2_{\rm nc}(D^2, \sigma^2_{\rm nc})$, where D^2 represents the non-centrality term. The CF of X_1 conditioned on D can be written as

$$\psi_{X_1|D}(j\omega) = \frac{1}{1-2j\omega\sigma^2_{\rm nc}}\exp\left(\frac{j\omega D^2}{1-2j\omega\sigma^2_{\rm nc}}\right). \quad (13)$$

By taking the expectation over D, the CF of X_1 can be expressed as

$$\psi_{X_1}(j\omega) = \frac{1}{1-2j\omega\sigma^2_{\rm nc}}\psi_{D^2}\left(\frac{j\omega}{1-2j\omega\sigma^2_{\rm nc}}\right). \quad (14)$$

We discuss now two relevant fading distributions for the SoI. The Ricean distribution is frequently used as outdoor channel model [16], while an indoor environment can be modeled using a Rayleigh fading channel [17].

1) Rayleigh fading for the signal of interest: We now consider the case of h_1 distributed according to the Rayleigh distribution. Conditioning on $X_{\rm i}$, D^2 follows a non-central chi-square distribution with two degrees of freedom $D^2_{|X_{\rm i}} \sim \chi^2_{\rm nc}(X_{\rm i}^2, P\sigma^2_{h_1})$. Therefore, the CF of D^2 conditioned on $X_{\rm i}$ can be written as

$$\psi_{D^2|X_{\rm i}}(j\omega) = \frac{1}{1-2j\omega P\sigma^2_{h_1}}\exp\left(\frac{j\omega X_{\rm i}^2}{1-2j\omega P\sigma^2_{h_1}}\right) \quad (15)$$

with $\sigma^2_{h_1} = 1/2$. By inserting (15) in (14), the CF of X_1 conditioned on $X_{\rm i}$ can be expressed as follows

$$\psi_{X_1|X_{\rm i}}(j\omega) = \frac{1}{1-2j\omega(P/2+\sigma^2_{\rm nc})} \exp\left(\frac{j\omega X_{\rm i}^2}{1-2j\omega(P/2+\sigma^2_{\rm nc})}\right). \quad (16)$$

By taking the expectation over $X_{\rm i}$, the CF of the decision variable can be expressed as

$$\psi_{X_1}(j\omega) = \frac{1}{1-2j\omega(P/2+\sigma^2_{\rm nc})} \psi_{X_{\rm i}^2}\left(\frac{j\omega}{1-2j\omega(P/2+\sigma^2_{\rm nc})}\right). \quad (17)$$

Consider a symmetric stable distribution $X \sim \mathcal{S}(\alpha, 0, \gamma)$, then X can be decomposed as $X = \sqrt{U}G$, where $U \sim \mathcal{S}(\alpha/2, 1, \cos(\pi\alpha/4))$ and $G \sim \mathcal{N}_{\rm c}(0, 2\gamma^{2/\alpha})$, with U and G independent r.v.'s [18]. By using the decomposition property of symmetric stable distributions, the aggregate interference term can be written as $X_{\rm i} = \sqrt{U}G$. Therefore, the square of the aggregate interference can be expressed as:

$$X_{\rm i}^2 = 2\gamma^\nu UC \quad (18)$$

where C is a central chi-square random variable with two degrees of freedom. Conditioning on C and using the scaling property of a stable random variable[5], $X_{\rm i}^2$ conditioned on C follows a stable distribution and therefore, the CF of $X_{\rm i}^2$ conditioned on C is given by

$$\psi_{X_{\rm i}^2|C}(j\omega) = \exp\left\{-(2C)^{1/\nu}\gamma\cos\left(\frac{\pi}{2\nu}\right)|j\omega|^{1/\nu} \left[1-\mathrm{sign}(j\omega)\tan\left(\frac{\pi}{2\nu}\right)\right]\right\}. \quad (19)$$

The r.v. $C^{1/\nu}$ can be approximated by a Gamma r.v. Z [11]. By taking the expectation over Z, we can express the CF of $X_{\rm i}^2$ as

$$\psi_{X_{\rm i}^2}(j\omega) = \left(1+\theta_\nu 2^{1/\nu}\gamma\cos\left(\frac{\pi}{2\nu}\right)|j\omega|^{1/\nu} \left[1-\mathrm{sign}(j\omega)\tan\left(\frac{\pi}{2\nu}\right)\right]\right)^{-k_\nu}. \quad (20)$$

Note that the first and second moment of $C^{1/\nu}$ can be expressed as $2^{1/\nu}\Gamma(\frac{N}{2}+\frac{1}{\nu})/\Gamma(\frac{N}{2})$ and $4^{1/\nu}\Gamma(\frac{N}{2}+\frac{2}{\nu})/\Gamma(\frac{N}{2})$. In order to estimate the shape parameter k_ν and the scale parameter θ_ν of the Gamma r.v. Z, we use the method of the moments by imposing the equivalence of the first two moments of the Gamma distribution with the first two moments of $C^{1/\nu}$. By using (20) and (17), the closed form expression of the CF of X_1 can be written as in (21) at the top of this page. Note that, when λ tends to zero (i.e. the dispersion γ tends to zero), (21) reduces to the scenario without interference where the signal of interest is subject to a Rayleigh fading channel. Inserting (21) in (11), $\mathbb{P}_{\rm d}$ can be obtained.

2) Ricean fading for the signal of interest: For h_1 that follows a Rician distribution, we cannot obtain a closed form expression of the CF. However, using the decomposition property for symmetric stable distributions, $X_{\rm i}$ can be expressed as $X_{\rm i} = \sqrt{U}G$ with U and G defined as in Section IV-A1. Therefore, conditioning on U, we find now that [9]

$$(X_{\rm i} + X_{\rm n} + X_{\rm n})_{|U} \sim \mathcal{N}_{\rm c}(0, \sigma^2_{\rm nc} + U2\gamma^{2/\alpha}). \quad (22)$$

Conditioning on h_1, the r.v. X_1 follows a non-central χ^2 distribution with 2 degrees of freedom and non-centrality

[5]If $X \sim \mathcal{S}(\alpha, \beta, \gamma)$, then $kX \sim \mathcal{S}(\alpha, \mathrm{sign}(k)\beta, |k|^\alpha\gamma)$

parameter $\mu_{X_1} = h_1^2 P$. The CF of X_1 conditioned on h_1 can be expressed as

$$\psi_{X_1|h_1}(j\omega) = \mathbb{E}\{e^{j\omega X_1|h_1}\} = \frac{1}{1-2j\omega\sigma_{\text{tot}}^2} \exp\left(\frac{j\omega h_1^2 P}{1-2j\omega\sigma_{\text{tot}}^2}\right) \tag{23}$$

where $\sigma_{\text{tot}}^2 = \sigma_{\text{nc}}^2 + U 2\gamma^{2/\alpha}$. Taking the expectation over h_1, (23) yields

$$\psi_{X_1}(j\omega) = \frac{1}{1-2j\omega\sigma_{\text{tot}}^2}\psi_{h_1^2}\left(\frac{j\omega P}{1-2j\omega\sigma_{\text{tot}}^2}\right) \tag{24}$$

where $\psi_{h_1^2}$ is the CF of the fading power. In case of Ricean fading, the fading power features a non-central chi-square for which the CF is known in closed form. $\mathbb{P}_{\text{d}}$ conditioned on U can be found by (11), and $\mathbb{P}_{\text{d}}$ can be derived by numerically averaging over a large set of realizations of U.

B. False Alarm Probability

A cell of the search space with no signal of interest can be expressed as $X[\tau] = |X_{\text{i}}[\tau] + X_{\text{c}}[\tau] + X_{\text{n}}[\tau]|^2$. The contribution of the aggregate interference to the search space can be represented by a vector $\bar{X}_{\text{i}}$ composed of aN elements. Since $\bar{X}_{\text{i}}$ is a multivariate symmetric stable r.v., the vector can be decomposed as

$$\bar{X}_{\text{i}} = \sqrt{U}\bar{G} \tag{25}$$

where $U \sim \mathcal{S}(\alpha/2, 1, \cos(\pi\alpha/4))$ and $\bar{G}$ is a aN-dimensional Gaussian random vector with $\bar{G} \sim \mathcal{N}_{\text{c}}(0, 2\gamma^{2/\alpha})$. Conditioning on U, for each cell of the search space where no SoI is present we have

$$X_{\text{i}}[\tau] \sim \mathcal{N}_c(0, U 2\gamma^{2/\alpha}). \tag{26}$$

Therefore, the cell values of the search space can be found by merging the Gaussian r.v.'s X_{n}, X_{c}, and $X_{\text{i}|U}$. When no SoI is present, the decision variable is the maximum of a set of exponentially distributed r.v.'s, which follows a generalized exponential distribution [19]. Therefore, $\mathbb{P}_{\text{fa}}$ conditioned on U can be calculated as follows

$$\mathbb{P}_{\text{fa}}(\beta|U) = 1 - F_X(\beta; N, \zeta) = 1 - (1 - e^{-\zeta\beta})^N \tag{27}$$

where $F_X(x; \rho, \zeta)$ is the cumulative distribution function (CDF) of the generalized exponential distribution with ρ and ζ the scale and shape parameters, respectively. The $\mathbb{P}_{\text{fa}}$ can be obtained by averaging over a large set of realizations of the stable distribution U.

Note on independence: The vector $\bar{X}_{\text{i}}$ is given by

$$\bar{X}_{\text{i}} = \sum_{m=1}^{\infty} \frac{g_m}{R_m^\nu} \bar{I}_m \tag{28}$$

where $\bar{I}_m$ is a vector of uncorrelated complex Gaussian r.v.'s. From (28), we can conclude that the components of $\bar{X}_{\text{i}}$ are identically distributed, yet mutually dependent. Bearing in mind that the elements of $\bar{X}_{\text{i}}$ are not independent, the search space cell that contains the SoI is not independent of the rest of the search space. For the scenario of Rayleigh fading affecting the SoI, $\mathbb{P}_{\text{fa}}$ is calculated using a set of realizations of the aggregate interference, while $\mathbb{P}_{\text{d}}$ is calculated based on the closed form expression of the CF of the decision statistic, thus neglecting the dependence of the search space cell containing the SoI and the rest of the search space. It can be shown through simulation that this approximation is accurate.

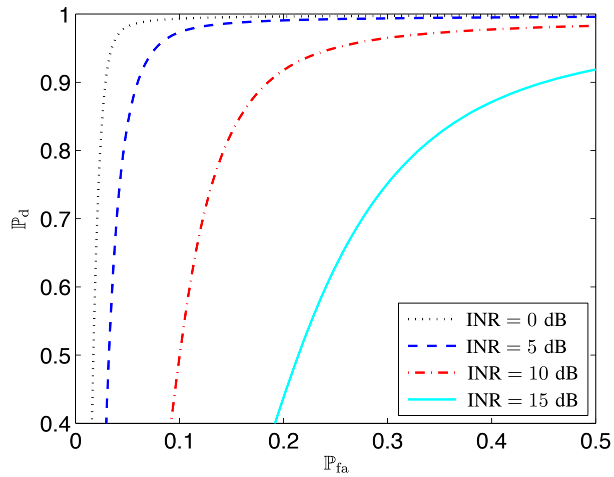

Fig. 2. ROC curves for the GLRT method (SNR = 15 dB; Rice factor $K = \infty$; $\lambda = 0.01/m^2$) for varying values of INR.

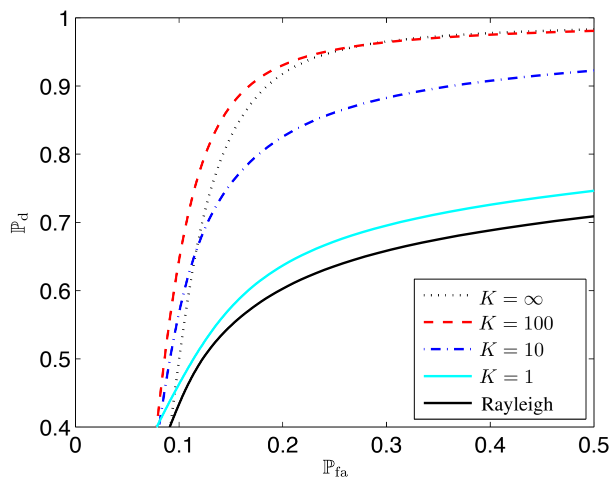

Fig. 3. ROC curves for the GLRT method in the presence of a network of spatially distributed cognitive devices (SNR = 15 dB; INR = 5dB; $\lambda = 0.01/m^2$, $\nu = 1.5$). The impact of the fading distribution (Ricean and Rayleigh) with regard to the SoI is considered.

V. Numerical results

In this section, we evaluate the acquisition performance using the expressions developed in Section IV. In order to reduce the number of scenarios, we only consider Rayleigh fading for the interfering nodes which is realistic in challenging channel conditions, while for the SoI different fading distributions are considered. For simulation of the aggregate interference, 10^6 realizations of the stable r.v. have been generated. Figure 2 illustrates the effect of the transmission power of the cognitive devices. The figure shows the ROC

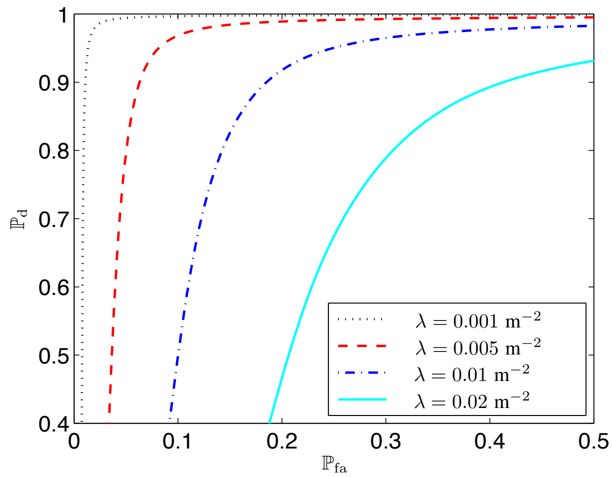

Fig. 4. ROC curves for the GLRT method (SNR = 15 dB; Rice factor $K = 10$; INR = 10 dB), in a Ricean fading channel for varying values of the density λ.

curves as a function of the interference-to-noise ratio (INR) for a constant value of K, which is the parameter for Ricean fading that represent the ratio between the energy of the line-of-sight (LOS) component and the energy of the other multipath components. For INR = 15 dB, the reduction of the acquisition performance is considerable. Figure 3 demonstrates the effect of different types of fading relative to the SoI. As expected, for higher values of K (stronger LOS), the ROC curve approaches the acquisition performance when there is no fading on the SoI. In case of Rayleigh fading (e.g. indoor environment), the acquisition performance is insufficient for practical applications. In Figure 4, we show the effect of the interferer density on the acquisition performance. In the three examples, we notice that the performance deteriorates quickly with increasing interferer transmission power, decreasing K, and increasing node density.

VI. Conclusions

In this paper, we analyse the acquisition performance of GNSS signals in realistic urban scenarios, challenged by the presence of network interference. We derive analytical expressions of the detection and false alarm probability for the GLRT acquisition strategy, that account for the most relevant network parameters such as the interferer node density, the transmission power of the nodes and the fading distribution for the interferers and the signal of interest. The analytical framework proposed in this paper allows to understand the effect of several environment related parameters on the acquisition performance of the GNSS signal. The framework can be used to determine threshold values for the discussed parameters corresponding with a minimum acquisition performance. Moreover, the presented results illustrate that the acquisition performance is severely affected in dense urban environment and suggest the use of alternative positioning techniques.

VII. Acknowledgment

This work was supported, in part, by the A*STAR Research Attachment Programme, the SUTD-MIT International Design Centre under Grant IDSF1200106OH, and the European Commission Marie Curie International Outgoing Fellowship under Grant 2010-272923.

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