

Extracting cyclostationary features from single carrier signals

M. Heskamp, C. H. Slump
University of Twente, Faculty of EEMCS
Signals and Systems Group (SaS)
Hogekamp Building,
7522 NB, Enschede, The Netherlands
E-mail: m.heskamp@ewi.utwente.nl

Abstract

This paper contributes to the discussion about the usefulness of cyclostationary feature detection for the purpose of cognitive radio. From a simple but realistic radio signal model and an ideal channel, the power spectral density of the random signal component is derived, and compared with the periodical component that can be retrieved from the signal with a nonlinear operation.

1 Introduction

A cognitive radio (CR) is a radio that opportunistically makes use of licensed frequency bands when the primary user is not using it. In [1] the FCC proposes to use CR to improve spectrum utilization, and asks for comments from industry. A lot of respondents expressed skepticism about the ability of the CR to detect all primary users in a real world scenario. The FCC, however, appears optimistical about the detection problem, as in [1] is claimed that it is feasibly to detect signals as far as 40 dB under the noise floor of the receiver, by making use of the cyclostationarity of radio signals.

In this paper we study the cyclostationary properties of a single carrier phase shift keying (PSK) radio signal. At baseband, a PSK signal can be written as a pulse amplitude modulation (PAM) signal, which can be written as the convolution

$$u(t) = \sum_{n=-\infty}^{\infty} c[n] \cdot p(t - nT) \quad (1)$$

in which $c[n]$ are the complex data symbols, $p(t)$ is the pulse shape and T the symbol duration time.

It is easy to see that this signal is cyclostationary, because the statistics on the sampling moments nT differ from the statistics in between the sampling moments. On the sampling moments the probability density of the amplitude has the same shape as the signal constellation and mainly depends on one single symbol. In between the sampling moments the amplitude is formed by the summation of many neighboring symbols, and tends towards a Gaussian distribution. When measured with a traditional spectrum analyzer, the cyclostationarity of the signal is lost because averaging is done over time frames with a duration that is not an integer multiple of the cyclic frequency. Such spectrum analyzers only measure the power in a certain bandwidth and show no distinction between modulated signals and interference.

If the signal constellation is M-ary phase shift keying (MPSK) we can generate spectral lines by raising the signal to the M-th power. This happens, because the power law multiplies the phase of the signal by M, so on the sampling moment nT , the M complex points from the M-ary PSK constellation are mapped to a constant value.

$$c[n] = \exp(j2\pi\varphi[n]/M) \quad \text{with } \varphi[n] \in \{1 \cdots M\} \quad (2)$$

$$|c[n]|^2 = 1, \quad c[n]^M = \exp(j2\pi\varphi[n]) = 1.$$

In between the sampling moments the phase is not confined to the constellation, so there the signal remains random. So, what we have after raising to the power M is a random signal which exhibits a periodical level crossing, as shown in fig. 1.

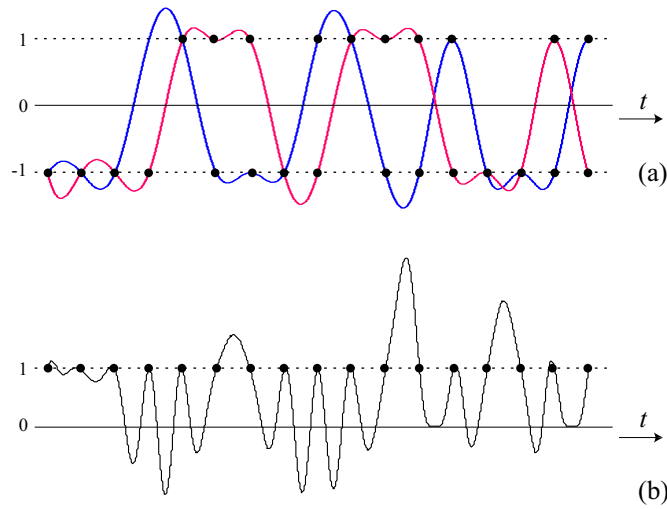


Figure 1: Applying an M-th order nonlinearity to a M-ary PSK signal gives periodical level crossings. (a) Real (blue line) and imaginary (red line) part of a 4-PSK signal $u(t)$. (b) Real part of the signal $v(t) = u(t)^4$.

2 Review of cyclostationarity of a PAM signal

The concept of cyclostationarity was introduced into the telecommunications area mainly by Gardner [2]. In this section we derive the spectral correlation density (SCD) of the signal $u(t)$ given by (1). The expected value of the signal is zero, because the expected value of the complex symbols is zero. Its time dependent autocorrelation function is

$$\begin{aligned}
R_{uu}(t, \tau) &= \mathbb{E}\{u(t + \frac{1}{2}\tau)u^*(t - \frac{1}{2}\tau)\} = \\
&\sum_n \sum_m \mathbb{E}\{c[n]c^*[m]\} p(t + \frac{1}{2}\tau - nT) p^*(t - \frac{1}{2}\tau - mT) = \\
&\sum_n p(t + \frac{1}{2}\tau - nT) p^*(t - \frac{1}{2}\tau - nT) = \\
&p(t + \frac{1}{2}\tau)p^*(t - \frac{1}{2}\tau) * \frac{1}{T} \text{III}(t/T)
\end{aligned} \tag{3}$$

in which $\text{III}(t)$ is the ‘shah’ function which is defined as an infinite train of Dirac delta functions with unit mass and spacing. Taking the two dimensional Fourier transform gives the spectral correlation density [2]

$$S_{uu}(\alpha, f) = \mathcal{F}^2 [R_{uu}(t, \tau)] = P(f + \frac{1}{2}\alpha)P^*(f - \frac{1}{2}\alpha) \text{III}(\alpha T) \tag{4}$$

in which $P(f)$ is the Fourier transform of the pulse shape.

As explained in section 11-4 of [3], a wide sense cyclostationary signal can be converted to a wide sense stationary signal by applying a random time shift. The autocorrelation and spectrum of this time-shifted signal are given by

$$\begin{aligned}
R_{uu}(\tau) &= \frac{1}{T} \int_T R(t, \tau) dt = \frac{1}{T} \int_T \sum_n p(t + \frac{1}{2}\tau - nT) p^*(t - \frac{1}{2}\tau - nT) dt = \\
&\frac{1}{T} \int_{-\infty}^{\infty} p(t + \frac{1}{2}\tau) p^*(t - \frac{1}{2}\tau) dt = \frac{1}{T} p(\tau) * p^*(-\tau)
\end{aligned} \tag{5}$$

$$S_{uu}(f) = \frac{1}{T} |P(f)|^2$$

so to give the signal unity average power, we can normalize the amplitude of the pulse shape according to

$$\int_{-\infty}^{\infty} |p(t)|^2 dt = \int_{-\infty}^{\infty} |P(f)|^2 df = T. \tag{6}$$

If the bandwidth of the pulse shape is reduced to the Nyquist bandwidth, or equivalently a sinc pulse shape is used, we see that (3) and (4) become independent of t and α respectively, so $u(t)$ becomes a wide sense stationary process with autocorrelation and spectrum

$$R_{uu}(\tau) = \text{sinc}(\tau/T) \quad \xLeftrightarrow{\mathcal{F}} \quad S_{uu}(f) = T\Pi(Tf) \tag{7}$$

in which $\Pi()$ denotes a rectangular function with unit height and width. In practice the frequency response of the pulse shape will roll-off to zero slightly beyond the Nyquist frequency, leaving just a small amount of second order cyclostationarity in the signal.

3 Retrieving a periodical component from a PSK signal

Besides the second order cyclostationarity that is present in any PAM signal, we now focus on the higher order cyclostationarity that is present in a PAM signal with a PSK constellation. For this we define the signal

$$v(t) = u(t)^M = a(t) + x(t) \quad (8)$$

in which $a(t)$ is the expected value of $v(t)$ which is a deterministic function, and $x(t)$ is the self-noise which is a zero-mean random (data dependent) process that obscures $a(t)$. First we show that $a(t)$ is a periodical function:

$$\begin{aligned} a(t) &= \mathbb{E}\{u(t)^M\} = \mathbb{E}\left\{\left(\sum_{n=-\infty}^{\infty} c[n] \cdot p(t - nT)\right)^M\right\} = \\ &\sum_{n_1} \sum_{n_2} \dots \sum_{n_M} \mathbb{E}\{c[n_1]c[n_2] \dots c[n_M]\} p(t - n_1T) p(t - n_2T) \dots p(t - n_MT) = \\ &\sum_{n=-\infty}^{\infty} p(t - nT)^M = p(t)^M * \frac{1}{T} \text{III}(t/T) \end{aligned} \quad (9)$$

which has an amplitude density spectrum

$$A(f) = \underbrace{P(f) * P(f) \dots P(f)}_{M \text{ times}} \cdot \text{III}(Tf). \quad (10)$$

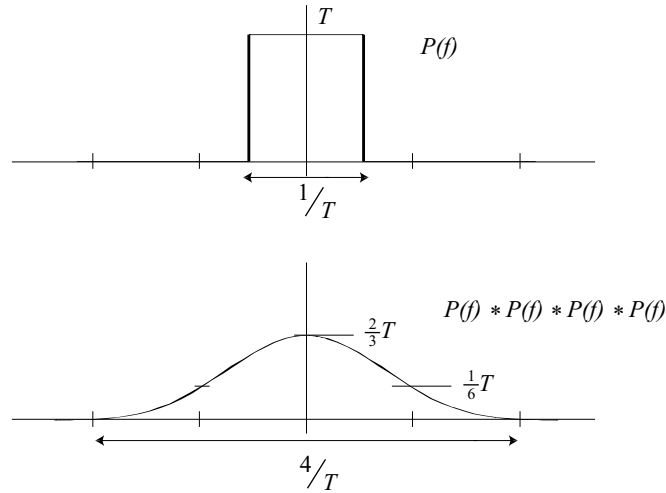


Figure 2: Rectangular function convolved 4 times with itself

Next, we want to know more about the magnitude of the self-noise $x(t)$. For this we first determine the autocorrelation function of $v(t)$ which is given by

$$\begin{aligned}
R_{vv}(t, \tau) &= \mathbb{E}\{(u(t + \frac{1}{2}\tau)u^*(t - \frac{1}{2}\tau))^M\} = \\
&\mathbb{E}\{ (\sum_n \sum_m c[n]c^*[m] p(t + \frac{1}{2}\tau - nT)p^*(t - \frac{1}{2}\tau - mT))^M \} = \\
&\sum_{n_1} \dots \sum_{n_M} \sum_{m_1} \dots \sum_{m_M} \mathbb{E}\{c[n_1] \dots c[n_M] c^*[m_1] \dots c^*[m_M]\} \times \\
&\quad p(t + \frac{1}{2}\tau - n_1T)p^*(t - \frac{1}{2}\tau - m_1T) \dots p(t + \frac{1}{2}\tau - n_MT)p^*(t - \frac{1}{2}\tau - m_MT).
\end{aligned} \tag{11}$$

Because the symbols are independent, the expectation is zero for most combinations of indexes. There are two conditions that can make the expectation non-zero. First, if every n index is paired with an equal m index. Then we have products of the form $c[n] * c[n]^* = |c[n]|^2 = 1$. Second, if all n indexes are the same and all m indexes are the same we have products of the form $c[n]^M = 1$. These two conditions coincide if all indexes are equal, so we have to subtract this term to make it occur only once. If we apply this, (11) reduces to

$$\begin{aligned}
R_{vv}(t, \tau) &= \left(\sum_n p(t + \frac{1}{2}\tau - nT)p^*(t - \frac{1}{2}\tau - nT) \right)^M \\
&+ \sum_n \sum_m p(t + \frac{1}{2}\tau - nT)^M p^*(t - \frac{1}{2}\tau - mT)^M \\
&- \sum_n p(t + \frac{1}{2}\tau - nT)^M p^*(t - \frac{1}{2}\tau - nT)^M \\
&= \\
R_{uu}(t, \tau)^M &+ a(t + \frac{1}{2}\tau)a^*(t - \frac{1}{2}\tau) - p(t + \frac{1}{2}\tau)^M p^*(t - \frac{1}{2}\tau)^M * \frac{1}{T} \text{III}(t/T).
\end{aligned} \tag{12}$$

From this we conclude that the autocorrelation of the self-noise is

$$R_{xx}(t, \tau) = R_{uu}(t, \tau)^M - p(t + \frac{1}{2}\tau)^M p^*(t - \frac{1}{2}\tau)^M * \frac{1}{T} \text{III}(t/T). \tag{13}$$

4 Detecting harmonic components in noise

In the previous section we have shown that a periodical component can be retrieved from a PSK signal. In practice this periodical component will be buried in noise. In this section we focus on the detection of the periodical component. Because for our application it is required to implement the detection algorithms in software and digital hardware, we now switch to sampled signals. Because squaring a signals means roughly doubling of its bandwidth, some factor of over sampling is needed to avoid aliasing.

Suppose we have a signal $a[n]$ that consist of a sum of one or more periodical components. We can write such signal as the sum of complex exponentials

$$a[n] = \sum_i A_i \exp(j2\pi f_i n) \quad (14)$$

in which A_i are the complex amplitudes and f_i the frequencies normalized with respect to the sampling frequency.

The discrete time Fourier transform (DTFT) $A(f)$ of $a[n]$ is the sum of shifted Dirac delta functions

$$A(f) = \sum_i A_i \delta(f - f_i). \quad (15)$$

Next, consider the received signal

$$u[n] = a[n] + x[n] \quad (16)$$

in which $a[n]$ is buried in wide sense stationary zero mean noise $x[n]$ with autocorrelation and spectrum

$$R_{xx}[m] = \mathbb{E}\{x[n] x[n - m]^*\} \xrightarrow{\text{DTFT}} S_{xx}(f). \quad (17)$$

Our goal is to retrieve $A(f)$ from a block of samples from $u[n]$ of length N . For this we apply a window $w_N[n]$ to the signal and do a DTFT

$$u_N[n] = u[n] w_N[n] \xrightarrow{\text{DTFT}} U_N(f) = A_N(f) + X_N(f). \quad (18)$$

The signal component in this estimation is the convolution

$$A_N(f) = A(f) * W_N(f) = \sum_i A_i W_N(f - f_i). \quad (19)$$

In order to resolve all frequency components, it is important that the width of the main lobe of the kernel $W_N(f)$ is smaller than the smallest distance between two frequency components. On the other hand, interference between frequency components can not be completely avoided, since every window of finite duration in the time domain will have non zero side lobes. However, since we are merely interested in detecting the presence of signals, instead of measuring the amplitudes very accurately, we can tolerate some interference between frequency components, and we can set the length of the window inversely proportional to the desired frequency resolution.

Next we look at the noise component

$$X_N(f) = \mathcal{F}\{x[n] w_N[n]\} = \sum_{n=-\infty}^{\infty} x[n] w_N[n] \exp(-j2\pi f n). \quad (20)$$

Because the expectation of $x[n]$ is zero, the expectation of $X_N(f)$ is also zero. The variance of $X_N(f)$ is

$$\begin{aligned}
& \mathbb{E}\{|X_N(f)|^2\} = \\
& \sum_{n_1} \sum_{n_2} \mathbb{E}\{x[n_1]x^*[n_2]\} w[n_1]w^*[n_2] \exp(-j2\pi f(n_1 - n_2)). \tag{21}
\end{aligned}$$

Next we change the summation variables to $n_1 = n$ and $n_2 = n - m$ which gives

$$\begin{aligned}
& \sum_n \sum_m \mathbb{E}\{x[n]x^*[n - m]\} w[n]w^*[n - m] \exp(-j2\pi fm) = \\
& \sum_m R_{xx}[m] w[n] * w^*[-n] \exp(-j2\pi fm) = \tag{22} \\
& S_{xx}(f) * |W_N(f)|^2.
\end{aligned}$$

5 Discussion and Conclusions

In this paper we analyzed what happens to the spectrum of an MPSK signal when it is passed through an M-th order nonlinear operation. The power spectral density of a PSK signal does not have any specific features as can be seen from figure 3a. Equation (10) gives an expression for the periodical components that appears if the signal is raised to the M-th power. Figure 2 shows its graphical representation if the signal has a sinc pulse shape and $M = 4$. From this figure we see that we can expect three delta function in the spectrum of $v(t)$. This is confirmed by the simulation plot in figure 3b. Besides the three delta functions in figure 3b, we also see noise. This noise consist of the AWGN that also is present in figure 3a, and the self-noise that comes from the signal itself. Equation (13) gives an expression for this self-noise. Further research is needed to simplify this expression and to extract possible other signal features from it. If we compare (19) with (22) we see that the signal component in the estimation becomes stronger with respect to the noise component if the resolution bandwidth kernel $|W_N(f)|^2$ tends to a delta function. This can be achieved by making the length N of the window and the FFT size very long.

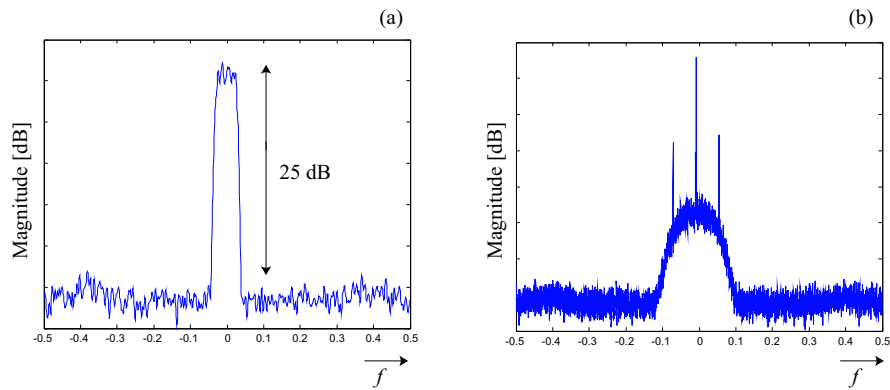


Figure 3: Spectral line generation method applied to simulated 4-PSK signal. (a) Spectrum of the PSK signal. (b) Spectrum after an M-th order nonlinearity.

References

- [1] FCC. Facilitating opportunities for flexible, efficient, and reliable spectrum use employing cognitive radio technologies. ET Docket No. 03-108, December 2003. Notice of Proposed Rulemaking (NPRM).
- [2] W. A. Gardner. *Cyclostationarity in Communications and Signal Processing*. IEEE Press, 1994.
- [3] A. Papoulis. *Probability, Random variables, and Stochastic Processes*. McGraw-Hill, third edition, 1991.