SELF-ORGANIZATION IN SURF ZONE MORPHODYNAMICS: ALONGSHORE UNIFORM INSTABILITIES

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Abstract: The simple cross-shore sediment transport model put forward by Plant et al. (2001) is used to predict equilibrium beach profiles. The stability of such profiles with respect to alongshore uniform perturbations is also analyzed. Preliminary results indicate the growth of an unstable mode with a nice resemblance to observed break point bars. Although the mechanism was already suggested by previous empirical and qualitative studies, the present contribution provides a promising quantitative model to explain the formation and migration of such bars.

INTRODUCTION

As any other nonlinear dynamical system, the surf zone environment has the capability of developing regular patterns whose complexity bears little resemblance to the homogeneity of the external forcing. The free instabilities of the alongshore uniform equilibrium provide an explanation for the occurrence of alongshore rhyhtmic patterns like crescentic bars and transverse/oblique bars (Falqués et al, 2000; Ribas et al., 2000; Caballeria et al., 2001; and references herein). It is therefore conceivable that linear (i.e. no rhythmic) shore-parallel bars should stem from this type of stability analyses as instability modes with zero alongshore wavenumber, $\kappa_y = 0$. This is not the case so far because those stability analyses are based on current-driven sediment transport (which is zero in the alongshore uniform situation) whereas the cross-shore transports related to

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gravity, undertow and wave nonlinearities are assumed to be in balance. On the other hand, the dynamics in the alongshore uniform situation depends on the unbalance between those cross-shore transport sources. Surprisingly enough, the 1D instability problem, which is the easiest mathematical one, turns out to be more difficult to deal with than the 2D one. This is a consequence of the difficulty of a reasonably accurate modelling of cross-shore sediment transport which, as shown by Gallagher et al. (1998), leads to a lack of predictive skill of the dynamics of cross-shore beach profiles.

Plant et al. (2001) have recently developed a simple model for the cross-shore transport, based on earlier models of sediment transport (Bagnold, 1963) that, combined with a nearshore wave transformation (Thornton and Guza, 1983), seem to capture the essential facts of the dynamics of cross-shore beach profiles. This opens a way to extend the stability analysis of a long straight coast to allow for alongshore uniform modes and provides a possible explanation for the formation of shore-parallel bars.

Alongshore bars are found in almost all sandy coasts. Some bar systems are well known for their seasonal formation and subsequent disappearance (see for example, Winant et al., 1975). The seasonal cycle consists of formation during large storms, and subsequent disappearance during less energetic conditions. Bars can disappear by migration onshore and welding to the subaerial beach (for instance Wright and Short, 1984). In contrast, other bar systems are known for their persistence over several years to decades (Lippmann et al., 1993; Ruessink and Kroon, 1994; Plant et al., 1999). Figure 1 shows an example of such a persistent bar in Duck's beach, in the U.S. Atlantic Coast. In these cases, interannual cycles of bar formation near the shoreline, net offshore migration, and subsequent bar destruction has been observed.

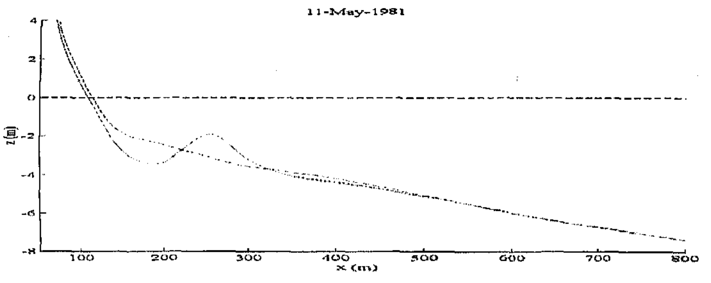

Fig. 1. Beach profile of surveyed bathymetry on May 11 (1981) in Duck's beach, in the U.S. Atlantic Coast. Continue line is the instantaneous interpolated profile and dashed line corresponds to the 16-year mean profile.

The structure of the paper consists of a short review of the formulation for the sediment transport and the waves transformation, the equilibrium or basic state that results from it, the linear instability method applied to the equations and the preliminary results of this stability analysis.

FORMULATION

This work is the natural continuation of the paper by Plant et al. (2001), where a detailed explanation of the theory and the governing equations related to the new cross-shore sediment transport formulation, used in both papers, is enclosed. Nevertheless, a short summary about the most important characteristics of this new formulation is given here in order to understand the results and the physical interpretation.

Given an arbitrary depth profile and offshore wave statistics, a simple wave transformation model can be used to predict the rms wave height over the entire domain. In general, these wave-averaged statistical models include a conservative energy flux term associated with shoaling and a dissipation term. For instance, assuming that wave heights are Rayleigh distributed and using the analogy to dissipation in a periodic hydraulic jump, Thornton and Guza (1983) proposed the following expression

$$\frac{\partial E}{\partial t}+\frac{\partial\left(Ec_g\right)}{\partial x}=-\frac{3}{16}\rho g B^3 f_p\sqrt{\pi}\,\frac{H_{rms}^5}{h^2\gamma^2}\left(1-\left(1+\frac{H_{rms}^2}{h^2\gamma^2}\right)^{-5/2}\right) \tag{1}$$

where E is the local wave energy density and c_g is the wave group velocity, which we will compute using the shallow water approximations to linear wave theory, ρ is the water density, H_{rms} is the root mean square wave height, h is the local still water depth, g is the gravity acceleration, B is a coefficient describing the type of breaking, f_p is the peak frequency of the assumed narrow band wave spectrum and γ is a model coefficient related to the "saturation" value of H_{rms}/h. The cross-shore coordinate, x, increases in the offshore direction. Because the time scale for morphologic response is expected to be much longer than the inherent wave time scale, all the quantities are averaged over the wave period. Therefore, the time-averaged quantities are allowed to vary slowly, in the morphologic time scale.

The second dynamic equation used is a sediment mass conservation equation.

$$\frac{\partial h}{\partial t}=\frac{\partial Q}{\partial x} \tag{2}$$

The sediment flux is the most relevant magnitude of the model. As it was done in other cross-shore profile evolution models (Bowen, 1980), we suppose alongshore uniformity. In addition we assume incident waves driving an onshore-directed mass flux of water above trough level. This input of mass must be balanced by a near-bed offshore-directed mean flow (undertow) that we suppose to be depth uniform below trough level. Both the wave orbital induced velocities and the near-bed mean flow are important to the large-scale cross-shore transport (Gallagher et al., 1998). The magnitude of these currents is obtained from the shallow water approximation to linear wave theory (orbital motion) and second order theories (undertow). What is important to beach profile evolution is the relative strengths and cross-shore structure of the processes driving onshore and offshore transport. We will use a particular transport formulation that captures the role of these processes, in spite of its simplicity. It has been described recently by Plant et al. (2001). In general, it has been shown there that a reasonable parameterization of cross-shore sediment flux consists of a magnitude term due to wave stirring, $q(x,t)$, multiplied by a term that describes the relative importance of the different transport processes, $r(x,t)$.

$$Q(x,t) = q(x,t) \cdot r(x,t) \tag{3}$$

with $q(x,t) = \dfrac{c_2 \rho \sqrt{g}}{16\sqrt{2}\tan\phi} H_{rms}{}^3 h^{-3/2}$ and $r(x,t) = r_0 \dfrac{\partial h}{\partial x} - \left(\dfrac{y}{y_c}\right)^p \left(1 - \dfrac{y}{y_c}\right)$

where c_2 is a reduced friction coeficient relating bottom shear stress and sediment flux, ϕ is the angle of repose of sediment, r_0, p and y_c are some constant parameters of the model and $y=H_{rms}/h$ is the so-called relative wave height. The magnitude term is based on Bagnold's bedload formulation applied to oscillatory currents (Bagnold, 1963), although using suspended sediment formulation wouldn't change qualitatively the results. The first term in $r(x,t)$ accounts for the down slope motion of sediment and the second term is a balance between the onshore transport due cross-correlations between sediment load and fluid velocity (essentially because of wave skewness) and the offshore transport caused by undertow. The assumption made is that this balance is a nonlinear function (p controls the degree of nonlinearity) of the relative wave height. . The most important parameter of this model is the critical relative wave height, y_c. Small values of y/y_c correspond to dominance of onshore transport (waves are neither feeling strongly enough the bottom nor breaking, so they cannot drive undertow), whereas large values of y/y_c mean dominance of offshore transport due to undertow. The value of y_c indicates whether this undertow offshore transport will dominate sooner or later with respect to the relative wave height.

This form of transport model has been suggested by other authors (Horikawa, 1981) and it also fits the sediment transport experimental observations presented in Plant et al. (2001), in case of $p=1$. Setting $p=0$ yields to a more simple formula which deduced theoretically in that paper, if we take specific values for the model parameters. In that simple theoretical model, the critical relative wave height, y_c, is proportional to the nonlinear correlations between sediment load and wave induced currents.

In order to obtain non-dimensional equations, some non-dimensional variables are defined. They are related to the dimensional variables via characteristic depth, h_0, cross-shore length, x_0, time, T_m, volume transport magnitude, Q_0, and relative wave magnitude, y_c, scales, such that $h=h^*h_0$, $x=x^*x_0$, $t_m=t^*T_m$, $Q=Q^*Q_0$ and $y=y^*y_c$. The scales are chosen to make the non-dimensional variables O(1) and depth and cross-shore length scales are appropriate to the nearshore environment. This requires that

$$h_0 = \frac{0.05g}{2\pi f_p} \approx 8m \qquad x_0 = \frac{10h_0}{\tan\phi} \approx 160m \qquad T_m = \frac{16\sqrt{2}x_0 tan\phi}{\mu c_2 \sqrt{g h_0} y_c^3} \approx 10^7 y_c^{-3} s \tag{4}$$

Typical values for some of the scaling parameters are: $c_2 = 0.00015$, $tan\phi = 0.5$, $f_p = 0.1\ s^{-1}$ and $\mu = 2$, where μ accounts for the porosity of the bed. The final non-dimensional sediment flux is the following (dropping tildes from the non-dimensional variables)

$$Q = y^3 h^{3/2}\left(s_0 \frac{\partial h}{\partial x} - y^p(1-y)\right) \quad (5)$$

where s_0 is a parameter that controls the magnitude of the down slope component of the transport. The final non-dimensional wave transformation equation, written in terms of the relative wave height is

$$\frac{\partial y}{\partial x} = -\frac{5y}{4\sqrt{h}}\frac{\partial h}{\partial x} + \frac{3B^3\sqrt{\pi} f_p y_c^3 x_0}{4\gamma_m^2\sqrt{gh_0 h}} y^4 \cdot \left(1 - \left(1 + \left(\frac{yy_c}{\gamma_m}\right)^2\right)^{-5/2}\right) \quad (6)$$

The variations in time of the energy are not taken into account (quasi-steady hypotesis) because morphologic time scale, T_m, is much larger than hydrodinamic time scales.

EQUILIBRIUM BEACH PROFILES

The equilibrium state or basic state is computed by setting the net sediment transport equal to zero along all the domain ($Q=0$). We have solved Eqns. 5 and 6 with the boundary conditions $h_{eq}(0) = 0$, $h_{eq}(\infty) = h_s$ and $y_{eq}(\infty) = y_s$, where h_s and y_s are the conditions at the seaward boundary. The results depend mainly on the ratio between the critical relative wave height and saturation value in the dissipation formula, y_c/γ. When $y_c > \gamma$, nearly convex profiles are obtained, typical of reflective beaches, with a small terrace close to the shoreline. Otherwise, we have profiles with long terraces, characteristic of dissipative beaches (see Figure 2).

In the seaward part of these profiles, when the incident wave height is not saturated at all, the predicted steep slope is prominent because the down slope motion of sediment is the only way to balance the onshore transport due to wave skewness. As y reaches the value of $y_c / 2$, waves start breaking and being saturated so that they start driving undertow offshore transport, that compensates the onshore entrance of sediment. Then, down slope transport is not necessary anymore and profile can become almost plane. When $y_c < \gamma$, undertow starts to dominate the offshore transport much earlier than wave height reaches the saturation value. Consequently waves never reach this saturation and the beach remains nearly plane for a long area, as can be seen in natural dissipative saturated beaches. Otherwise, when $y_c > \gamma$, offshore component of transport due to gravity is dominant in a larger domain of y so that the undertow domain in x is very small (waves are already saturated when it starts acting). In these steep concave-up profiles, the breaker area is very small and the final y is larger than γ (over saturation). Raubenheimer et al. (1996) measured and computed the relative wave height saturation value in both steep and terraced beaches and they concluded that this value is larger in steep profiles.

The two limit profiles we have obtained fit very well with the beach profiles observed in nature, as can be seen in literature (for instance, Wright et al., 1979). Large values of y_c are obtained mainly in case of big correlation between sediment load and fluid velocities. There are two characteristics that can lead to this effect, both large skewness magnitude (obtained in case of long period waves because short ones are more dissipative and linear) and coarse grains in sediment. Fine grains lead to less correlation because they must be stirred before being transported so that there is a delay, whereas coarse grains react quickier to undertow. Summing up, we obtain steep concave-up reflective beaches in case of coarse grains and long period waves and terraced dissipative beaches in case of fine grains and short period waves, in agreement with experimental data.

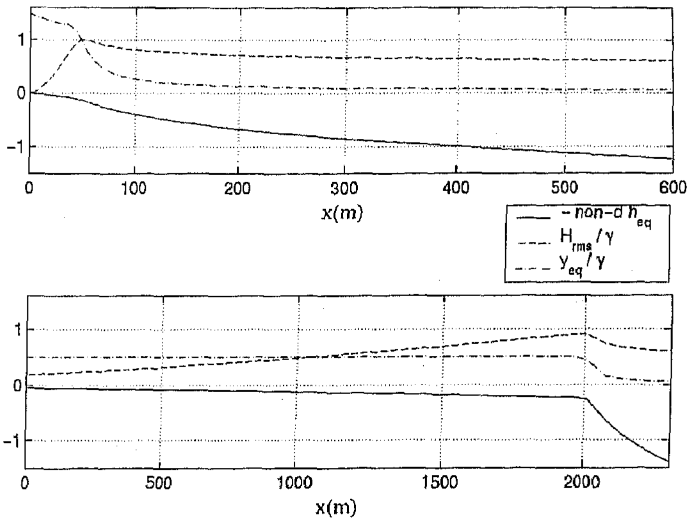

Fig. 2. Solution for the equilibrium or basic state with two different values of y_c/γ.
Top: *Convex profile* ($y_c = 1'5\ \gamma$). **Bottom:** *Terraced profile* ($y_c = 0.5\ \gamma$). Parameter values are $p = 1$, $s_0 = 0.25$, $B = 1$, $f_p = 0.1$ and $\gamma = 0.5$. The horizontal axis corresponds to the dimensional cross-shore coordinate, x. The continue line is the non-dimensional bottom depth, that is equal to minus the water height (h), the dashed line is the dimensional wave height and the dash-pointed line is the dimensional relative wave height. These two last magnitudes are scaled by γ in this graph.

LINEAR STABILITY ANALYSIS

Our next objective is to analyze the stability of the equilibrium profiles with respect to relatively small perturbations in the bathymetry and wave height. The aim here is to see if there are preferred locations, length scales or wave conditions for the growth of initially small perturbations. We are also interested in knowing the shape of these unstable perturbations in order to find the possible growth of longshore bars as an instability of the equilibrium beach profile. The perturbations added to the equilibrium variables are supposed to have an exponential time dependence.

$$\big(h(x,t), y(x,t)\big) = (h_{eq}(x), y_{eq}(x)) + \big(\eta(x), \alpha(x)\big) \cdot e^{\omega \cdot t} \qquad (7)$$

where (h_{eq}, y_{eq}) are the equilibrium water depth and relative wave height, (η, α) gives the cross-shore profile of the perturbation and $\omega = \omega_r + i\omega_i$ is the frecuency that can have an imaginary part. The perturbation will grow if $\omega_r > 0$. Inserting Eq. 7 into Eqns. 5 and 6 and neglecting the quadratic terms of the perturbations and the higher order in the derivatives yields to the non-dimensional linear equations. The equation giving the growth of bottom perturbation is

$$\frac{\partial \eta}{\partial t} = \frac{\partial}{\partial x}\left(F_0 s_0 \frac{\partial \eta}{\partial x}\right) + \frac{\partial (F_0 F_\alpha \alpha)}{\partial x} \tag{8}$$

where $F_0 = y_{eq}^3 h_{eq}$ $\qquad F_\alpha = -y_{eq}^{p-1}\left(p - y_{eq}(1+p)\right)$

The first term in the right hand side is the transport related to down slope motion due to gravity and it only gives diffusion and migration of the bar. The second one is the sediment flux caused by perturbations in relative wave height and can lead to instabilities. In this last term, F_α gives the competition between correlation offshore transport and undertow onshore one. The wave transformation linear equation is given by

$$0 = S_1\eta + S_2 \frac{\partial \eta}{\partial x} + S_3\alpha + S_4 \frac{\partial \alpha}{\partial x} \tag{9}$$

with S_i being the correspondant coefficients which are not given here for the sake of brevity. Some boundary conditions are needed for solving this dynamical system. Far offshore, both perturbations are considered to be zero. Setting the boundary conditions in the coast line is not obvious and will be discitued in the next section.

PRELIMINARY RESULTS OF THE STABILITY ANALYSIS

The linear Eqns. 11 and 12 have been solved with a numerical spectral method using a Chebishev polinomial expansion. Yet the numerical results of this instability analysis are still preliminar, it is already possible to demonstrate that the growth of break point longshore bars can be seen as a self-organization mechanism of the surf zone morphodynamics. For the moment, only one numerically reliable unstable mode ($\omega_r > 0$) has been found. The most important parameter of the model is again the critical relative wave height, y_c, but the boundary conditions used can also be a key point. As a first approach, we have set the perturbations equal to zero in the coastline. The obtained solutions only have real part and also $\omega_i = 0$. An example of the cross-shore shape of the solutions can be seen in Figure 3 (in case of a convex profile). The perturbation in water depth can be seen as a static longshore bar settled in the place where waves start breaking. The maximum of the bar is situated at the non-dimensional point $x = 0.35$ that corresponds to the "breaker point" (where $y = y_c / 2$).

The physical origin of the instability can be understood by a close examination of the competition between the onshore transport due to correlations between waves and sediment load, Q_c, and the offshore transport because of undertow, Q_u. From Eq. 5 the dependence on the relative wave heigth, y, is $Q_c \approx -y^p$ and $Q_u \approx y^{p+1}$. They vanish far offshore ($y = 0$) and they increase in magnitude moving shoreward (increasing y). However, the crucial point is that Q_c increases relatively smoothly everywhere while Q_u is negligible out of the surf zone and increases very sharply shoreward in the surf zone. As a result, $\left|\frac{dQ_c}{dy}\right| > \left|\frac{dQ_u}{dy}\right|$ for small $\frac{y}{y_c}$ and $\left|\frac{dQ_c}{dy}\right| < \left|\frac{dQ_u}{dy}\right|$ if $\frac{y}{y_c} \approx 1$. In between, there is a location where an increase in relative wave height produces the same increase in the magnitude of both transport terms ($y=y_c\ /\ 2$ for $p=1$). Consider now a small amplitude longshore bar with the crest at that position. A bar is expected to produce an increase in y as it is confirmed by the numerical solution (see α in Figure 3 Bottom). Then, seaward of the crest this increase in y will produce an increase in Q_c larger than that in Q_u . Consequently, there will be a net onshore transport. On the other hand, shoreward of the crest the increase in y will produce an increase in Q_c smaller than the increase in Q_u and, therefore, an offshore transport. As a result, convergence of sediment will occur over the bar that will therefore grow.

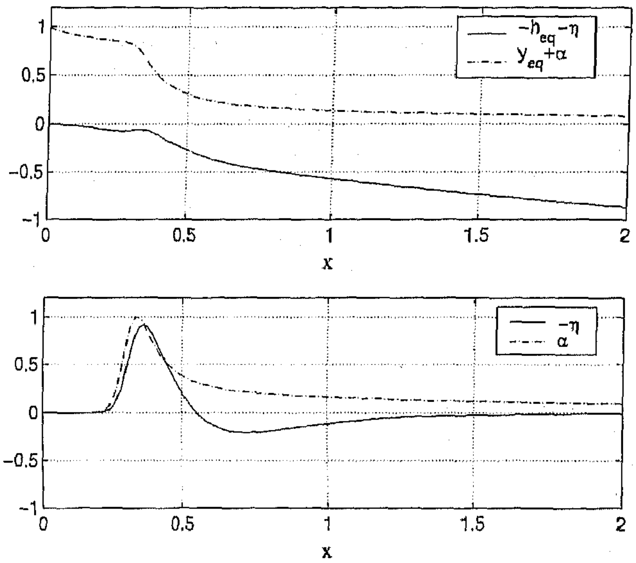

Fig. 3. Result of the linear instability analysis in case of $y_c = 1'5\ \gamma$.
Top: Equilibrium state plus the perturbation. The amplitude of the perturbation is arbitrary. Continue line is the non-dimensional bottom depth (corresponding to minus the water height) and dashed line is the non-dimensional relative wave height. **Bottom:** Perturbations in the bottom depth (continue line) and relative wave height (dashed line). The horizontal axis corresponds to the non-dimensional cross-shore coordinate, x.

DISCUSSION

The results of the stability analysis can be compared to observed sandbar behavior (see for instance, Wright et al., 1984). Because the analysis considered only a single cross-shore profile, we omit comparisons to alongshore variations in bar structure, such as the formation of alongshore-rhythmic topography. Clearly, an immediate comparison can be made between the present model and the early, qualitative break point bar formation model (King and Williams, 1949). The qualitative model was based on empirical reasoning and intuitive understanding of wave driven sediment transport, suggesting that a bar would form at the break point due to convergence of the undertow. However, the early model lacked a quantitative explanation that would apply to a coupled hydrodynamic-sediment transport model. More seriously, it produces ambiguous predictions for the case of arbitrarily-placed bar perturbation: small bars attract wave breaking to their crests and hence sediment convergence. But, a small bar could be placed anywhere on the profile to initiate break point bar formation. Also, the early model did not incorporate statistical variations in the break-point, such as are associated with random wave heights in nature.

In contrast, the present model can explain bar formation even when the initial profile is in equilibrium, and it is based on realistic random wave heights. Furthermore, the potential for growing bars to migrate is allowed in our analysis. Thus, while the qualitatively analysis by King and Williams (1949) showed that bars do in fact form near the break point, the present model contributes a great deal to the quantitative explanation of why break point bars form. Some In nature, as in our results, it appears that an abrupt increase in the relative wave height is required to form a bar (Wright et al., 1985). From Eq. 4, one can compute the time scale of this break point bar. If y_c = 0.75, as in the example given before, the growing time scale is $T_m \approx 9$ months. Smaller time scales can be obtained using suspended sediment transport instead of bedload (see Plant et al., 2001).

CONCLUSIONS

The earlier results of Plant et al. (2001) have been extended with a second type of equilibrium beach profile, previously undiscovered. Both types are consistent with broad classifications of observed dissipative and reflective beaches and their predicted characteristics are in fairly agreement with observations in nature. The stability analysis of such profiles is still under way. But the preliminary found unstable mode has a strong resemblance to a break point bar. The crest of the predicted bar is around the point where most of the waves start breaking. The physical mechanism is related to the unbalance between the transport driven by wave nonlinearities and that driven by undertow that can occur close to that point. Even though the physical mechanism was already suggested on the basis of qualitative and empirical arguments, a quantitative modelling has been now presented.

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