

# **A time allocation model analysed and discussed.**

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## CONTENTS

Samenvatting . . . . .	2
1 Introduction . . . . .	3
2 Towards a model specification . . . . .	4
3 Functional Form . . . . .	7
4 Simplified Model . . . . .	12
5 Theoretical Analysis . . . . .	15
6 Conclusions . . . . .	16
References . . . . .	17
Appendix . . . . .	18

## Samenvatting

### *Analyse en discussie van een model voor de verdeling van tijd.*

Deze paper ontwikkelt en bediscussieert een model voor de **allocatie** van tijd **aan** binnenshuis en buitenshuis activiteiten, inclusief het gegenereerde verkeer. Het model gebruikt nuts maximalisatie technieken, waarbij het nut afhangt van het type, de **duur, locatie** en frequentie van de activiteiten. De complexiteit van het ontwikkelde model en problemen rond de parameterschatting hebben geleid tot een simplificatie van het model. Dit versimpelde model beschouwd slechts **één** activiteiten-categorie en de afhankelijkheid van de afstand is afgezwakt. We beschouwen twee manieren om de nutsfunctie te **definiëren** en deze **worden** met elkaar vergeleken. Voor beide model technieken onderzoeken we de gevolgen van verandering van exogene variabelen. Bijvoorbeeld door het **totale** tijdbudget te verhogen **als** gevolg van kortere werkweken. Ook de invloed van een grotere gemiddelde snelheid wordt onderzocht. Tot slot wordt de bruikbaarheid van beide versimpelde modeltechnieken besproken.

## Summary

### *A time allocation model analysed and discussed.*

This paper develops and analyses a model for time allocation on in-home and **out-of-home** activities, including the generated travel. The model uses utility maximisation approach, where the utility depends on activity type, duration, location and frequency of the activity. The complexity of the model and problems with parameter estimation lead to the use of a simplified version of the model. This simplification incorporates one class of out-of-home activity types and the dependency on the location is somewhat weakened. We consider two approaches of defining the utility function, which will be compared. For both model approaches the impact of changes in exogenous variables is analysed. The comparison is made how both models show what happens with the amount of generated travel if the total time budget increases, for example by working less hours. But also the consequences of an increase in average travel speed is **analysed**. A discussion about the usefulness of this model completes the paper.

## 1 Introduction

The past decades mobility has grown considerably. Apart from increasing travel distances also the number of trips and travel time has increased (see e.g. Gommers and **Krikken**, 1992). The question then arises whether this **growth** will continue or whether there will be some limits to it. And if mobility growth is limited, the question is at what level the limits occur and when they occur. To search for these limits a model is developed which allocates time to various activities, out-of-home as well as in-home and to the generated travel.

This paper develops a theoretical framework and gives a model structure (see also earlier papers of author: Kraan, 1994 or 1995). The theory presented here is based on the theory of allocation of time (Becker, 1965; Winston, 1987) combined with aspects of the theory of flexible budgets (**Golob** et al, 1981; **Downes** and **Emmerson**, 1985). Time as well as money will be allocated to various activities and goods, including travel. The allocation is based on utility **maximisation**, where the utility depends on characteristics of outdoor activities, like priority, duration, location and frequency, the effort of travel, the amount of time spent at home and money spent on consumption goods.

Due to the complexity of this model a simplified version is developed, with only one activity type and with fixed unit travel distance. This simplification will help to understand the relationships within the theory. Two modelling approaches, namely with an additive utility function and with a multiplicative utility function, are analysed and compared. For both models we analyse **the** impact of changes in exogenous variables, such as total time budget, travel speed and unit distance.

Finally we draw some conclusions, based on these analyses and discuss the usefulness of **both** model approaches. Furthermore, we give some recommendations for further research, mainly concerned with extensions of the model and parameter estimation.

The contents of this paper is as follows: section 2 describes the assumptions made for developing the model, section 3 gives the complex version of the **model**, section 4 develops a simplified version, section **5** analyses and compares both simplified model approaches and section 6 gives conclusions and recommendations for further research.

## 2 Towards a model specification

In this section the assumptions are discussed for the time allocation model. We can make some assumptions on people's behaviour towards activities and travel, necessary for the model specification. First of all, people make decisions on different time horizons; long, medium and short term decisions. In the long term people choose for instance their dwelling location, workplace and employment. Decisions for a medium term can be choices for car ownership, buying a public transport season ticket, club's membership and thus paying a contribution fee. Choices on a short term are made on a daily or weekly basis, like whether or not to go shopping, visiting friends or relatives, going out (to a pub, the cinema, theatre etc.). Since the scope of the model is to aim at long term decisions of how people would divide their time between the various activities, all these decisions are **modelled** at once, without the distinction between the different time horizons.

People have limited amounts of time and money available. These time and money budgets are given by the employment status (working full time, part time or unemployed) and income. Also the amount of sleep (which differs strongly between children, adults, and elderly people for example) affects the time budget. Within these time and money budgets the total activity pattern has to be scheduled. The key question is how these available time and money is divided over the activities. Activity patterns depend on people's life style. This in turn is dependent on various characteristics, such as gender, age, education level, employment and life cycle stage. Different life styles lead in general to different activity patterns. So the allocation of time and money will also be different for various population groups. The population can be divided into "homogeneous" groups, based on their time and money budgets. On the one extreme we distinguish people with a lot of time but few money, like the unemployed, and on the other extreme people with a lot of money but very few time, like the double income households, where both adults have a full time job. And of course a lot of population groups fall in between these two extremes. We assume that people with comparable budgets and background variables show comparable activity patterns, at least regarding the total time allocated on activities.

Behaviour varies more between individuals than between households. The behaviour of the household members is very diverse, because within a household the members have different time budgets. So for this reason the model is concerned with individual behaviour. Of course the type of household an individual belongs to is important for his or her behaviour. When examining individuals, for instance housewives, there is a big difference in behaviour dependent on the household characteristics, for example whether there are little children in the household or not. So the household type, or life cycle stage is a very important characteristic in defining population groups.

Activities can be undertaken at home or out of home, such that a trip needs to be made. Travel is a demand, derived from activities. The need to perform an activity on a specified location generates travel. For activities with a high priority people do not mind to travel that much, because they have an urgency to get at the desired location. For low priority activities people might reconsider their trip, whether they do want to go **that** far or whether they want **to** go at all. So travel choices do depend on the preferences of activities. This can be interpreted as travel creating some kind of satisfaction, because it gets people to the desired location, to perform an activity.

In economic theory this satisfaction is called utility. So the scheduling of activities can **be** described by micro-economic theories of utility maximisation. This assumes rational behaviour of people. Of course in reality people do not always behave rational, but the outcome of their behaviour can be described by this rationality. The model presented here is based on **micro-**economic theory of consumer behaviour.

All activities performed yield a certain amount of utility. The utility obtained by **performing** an activity depends not only on the type of activity, but **also** on the duration, the location and the frequency with which the activity is undertaken. Total utility is a function of the utility of each activity, the disutility of travel and the utility of goods purchased. Individuals maximise the utility of their total activity pattern under time and money budget constraints.

Activities can be compulsory or discretionary. In our theory activities are placed into three categories: the obligatory activities (**labour**, study or school attendance), maintenance activities (household tasks, like shopping, or private business) and leisure activities (social and recreation, e.g. sports, visiting friends or relatives, or going out). The reason for this **categorisation** is given by the characteristics of **these** activities. For compulsory activities there is no choice in whether or not to perform the activity in the short run and sometimes the duration and frequency are fixed (e.g. **labour**: if one has a job for eight hours a day, one has no choice in whether to go to work and for how long). Only in the long run one might consider it a choice. One can decide to change the working condition, like working less hours. The same yields for the distance. Long term choices concerning the distance for obligatory activities can be interpreted as moving house or changing job, and thus the distance to the workplace. In case of optional activities it is more obvious that one has a choice in frequency, duration and location, even in the short term. The category of maintenance activities consists of activities that are more or less obligatory, in the sense that they have to be done, but there is a large choice in frequency, location and duration. But the frequency must be larger than zero. For example, one has to do their shopping, but one can choose to go every day, to the **little** shops nearby, or to go only once a week to the big shopping mall, further away.

The utility obtained by the activities depends on their characteristics; type, duration, location and frequency. The longer the duration the more satisfaction one obtains, with decreasing marginal utility. Because people want to spend as much time as they can on various activities, but after a while the extra utility obtained from spending an extra time unit to **the** activity decreases.

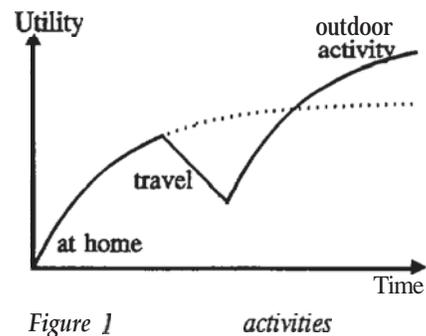
The location of an activity is rendered by the distance travelled. The distance has both a positive and a negative valuation (see also Kitamura, 1995). On the one hand, the longer the distance one is willing to travel the more locations one can reach and the higher the chance that one can reach an attractive location satisfying the needs (for a longer distance larger and more exclusive shops are assumed to be reachable). On the **other** hand the longer the distance the more effort is needed to travel to the location, given by travel time and travel costs. So the satisfaction obtaining from an activity should compensate the effort needed for travel. The interpretation is that people need to spend some effort to obtain a higher utility in the end, than when they would have continued with the previous activity. (See Figure 1.)

People will undertake a second activity if the satisfaction of this second activity at least compensates the extra effort for travel to the second location. But after a while the need to spend some time at home will be strong enough to stop outdoor activities and return home.

The utility of an activity also depends on the frequency. An activity with high priority will be undertaken more frequently than those with low priority. Some activities performed with a higher frequency obtain more utility, even if the total duration stays the same. For example for leisure activities one yields more satisfaction from performing twice a week one hour than from once a week for two hours. This assumption was also stated by Linder (1971) who expected people to split up their time more and more and thus performing more, but shorter activities. But the higher **the** frequency, the higher the extra disutility for travel. Obligatory activities (such as **labour**) have fixed frequencies or at least a minimum value for the frequency, on the short term. Again, for long term decisions the frequency can be taken variable.

The contribution of the frequency to the utility differs for different population groups. For example retired people with much leisure time often go out to do some shopping, while people with full time jobs, thus with few leisure time, like to do their shopping only once a week.

In-home activities yield a similar utility function, except for the distance travelled; because no trips have to be made for activities at home. All activities at home are considered alike; there is



no distinction assumed between the various in-home activities. The model concentrates on out-of-home activities, as those will generate travel. The time spent at home will be considered as a flexible budget constraint. We assume that everyone wants to spend some time at home, irrespective of the activities undertaken.

Also the money available is to be spent on activities, travel and other goods. The amount of goods purchased generate a positive utility. The money available for purchasing goods can be seen as the amount of money not spent on activities and travel, or the money saved. It can be interpreted as a measure of welfare. This measure for goods is considered as the flexible money budget (see Golob *et al*, 1981; or Downes and Emmerson, 1985).

All these assumptions lead to a model, based on micro-economic theories of consumer behaviour to allocate the total time and money available to various activities, at home as well as out-of-home, including the travel generated by the out-of-home activities. In this model individuals maximise the total utility of their activity pattern, under time and money constraints. The next section develops this model.

### 3 Functional Form

The previous section gave some assumptions the model developed here should comply to. The aim of the model is to allocate the total amount of time and money available to various activities and goods. Activities can be performed in-home, with duration  $T_H$ , as well as out-of-home, with duration  $T_i$ . Activities performed out-of-home generate travel over some distance  $d_i$  to reach the desired location. The performance of activities lead to some amount of utility, which is then **maximised** by the individual. The utility of an activity depends on the duration  $T_i$ , location  $d_i$  and **frequency**  $f_i$  of that activity. This relationship differs of course for different types of activities.

Consumer behaviour is considered to be bounded on time and money expenditures. The total time available will be allocated to various activities, both obligatory and discretionary, at home as well as out of home, and to travel. The total amount of money available will be allocated to the activities, travel and other consumption goods. Some activities yield an amount of money, most activities cost an amount of money. These costs can depend on the frequency and duration of the activity and the distance travelled determines the travel costs.

The general model structure can now be formulated as:

$$\begin{aligned}
 & \underset{T_i, d_i, f_i}{\text{Max.}} \quad U_{\text{TOT}}(T_i, d_i, f_i \forall i; T_H, G) \\
 & \text{subject to:} \\
 & \left\{ \begin{array}{l}
 \sum_i \left[ T_i + \frac{d_i}{v_i} \right] + T_H = T_{\text{TOT}} \\
 \sum_i c_i(T_i, d_i, f_i) + G \leq Y \\
 T_i, d_i, f_i \geq 0 \quad \forall i \\
 T_H, G \geq 0
 \end{array} \right. \quad (1)
 \end{aligned}$$

Here  $T_i$ ,  $d_i$  and  $f_i$  are the total time spent on, the distance travelled for, and the frequency of activity  $i$ , respectively.  $T_H$  is the total time spent at home and  $G$  the amount of money spent on various goods and services other than travel or out-of-home activities. The average speed is denoted by  $v_i$ , such that  $(d_i/v_i)$  denotes travel time. The costs going with activity  $i$ ,  $c_i(T_i, d_i, f_i)$  are divided into the costs of the activity itself and travel costs. The costs of the activity consists of fixed costs  $c_i^d$  (e.g. contribution for a sports club, **payed** once, independently of the frequency or duration), costs per time of performance  $c_i^f \cdot f_i$  (e.g. the price of a ticket for the theatre) and variable costs  $c_i^T \cdot T_i$  (for some recreational activities, e.g. going to a fun-fair, recreational shopping, going to a pub: the longer one stays the more money one spends). Most activities cost an amount of money, so  $c_i^T$  is positive. But the activity "labour" yields an amount of money; in this case  $c_i^T$  is negative; so  $c_w = -w \cdot T_w$  + travel costs; with  $w$  the wage rate. Travel costs are given by the variable costs per distance travelled  $c_i^d \cdot d_i$ . The cost function is then defined by  $c_i = c_i^d + c_i^T \cdot T_i + c_i^d \cdot d_i + c_i^f \cdot f_i$ .

The budget constraints are given by  $T_{\text{TOT}}$ , the time budget, and  $Y$ , the money budget. There exists a trade-off between the time and money expenditures through the amount of time spent working,  $T_w$  ( $i = \text{work}$ ). If the time spent working increases, then the variable income  $w \cdot T_w$  increases, thus total income increases and more money can be spent on other activities. On the other hand, by working more time, individuals have less time to spend on other activities. Things are easier to see by rearranging the budget constraints and rewriting them as:

So an increase in  $T_w$  leads to an increase in disposable income, but a decrease in available time.

According to Becker (1965) and Winston (1987) the activity itself does not produce a utility, but through performing an activity one obtains a commodity  $Z_i$  that produces a utility, comparable with production functions (e.g. cooking itself not always provides someone with utility, but the

$$\left\{ \begin{array}{l} \sum_{i \neq w} \left[ T_i + \frac{d_i}{v_i} \right] + \frac{d_w}{v_w} + T_H = T_{TOT} - T_w \\ \sum_{i \neq w} c_i(T_i, d_i, f_i) + c_w^d \cdot d_w + G \leq Y + wT_w \end{array} \right. \quad (2)$$

dinner made as a result of cooking yields satisfaction). This production function  $Z_i$  (or commodity as Becker calls it, or the intensity of the activity, according to Winston) depends on the characteristics of the activity (duration, frequency and distance to the location). Also the time spent at home and the goods purchased generate a similar commodity  $Z_H$  and  $Z_G$ , respectively. For travel we obtain a disutility or resistance  $R_i$ , for all out-of-home activities  $i$ .

Total utility,  $U_{TOT}$ , is now a function of all these commodities and resistances. So  $U_{TOT}$  is then the obtained utility for the total activity pattern:

$$U_{tot} = U(Z_i, R_i, \forall i; Z_H, Z_G) \quad (3)$$

Consider a discretionary outdoor activity  $i$  with duration  $T_i$ , frequency  $f_i$  and distance  $d_i$  between home and the location of the activity. Now the production function  $Z_i$  is a function of  $T_i$ ,  $d_i$  and  $f_i$ . The form of this production function (the utility of a single activity) is described in 2. A mathematical function that meets the requirements of that form is the Cobb-Douglas function and so the commodity of activity  $i$  is given by:

$$Z_i(T_i, d_i, f_i) = T_i^{\beta_i} \cdot d_i^{\gamma_i} \cdot f_i^{\rho_i} \quad (4)$$

where  $\beta_i$ ,  $\gamma_i$  and  $\rho_i$  are parameters with values between 0 and 1. In case there is hardly any choice, *i.e.* for obligatory activities, the same functional form yields, but with very small parameter values for  $\beta_i$ ,  $\gamma_i$  and  $\rho_i$  (approaching zero), such that  $Z_i$  is almost constant. But for modelling long term behaviour even obligatory activities can be **modelled** as being more or less free to choose. The parameters define the relative weight of characteristics  $T_i$ ,  $d_i$  and  $f_i$ .

To reach the location where the activity takes place, the distance  $d_i$  needs to be travelled. The effort to do so (denoted as resistance or disutility) is defined by total travel time and total travel costs. Travel costs are split into variable costs  $c_i^d \cdot d_i$  and fixed costs  $p_o \cdot f_i$ . The last term is interpreted as starting costs: every time one starts to travel it costs a bit extra. Ten short trips of **five** kilometres, for example, costs more than one long trip of fifty kilometres. If time is considered more valuable than monetary costs, then the effort caused by travel time ( $\zeta \cdot d_i / v_i$ ) will be valued more negative than the effort caused by monetary costs ( $\xi \cdot (c_i^d \cdot d_i + p_o \cdot f_i)$ ). In that case  $\zeta > \xi$ . If on the other hand an individual has plenty of time, but money is scarce  $\zeta < \xi$ . The resistance of travel is given by:

$$R_i(d_i, f_i) = -\zeta \cdot \frac{d_i}{v_i} - \xi \cdot (c_i^d \cdot d_i + p_o \cdot f_i) \quad (5)$$

The quotient  $\zeta/\xi$  denotes the value of travel time.

The total time spent on activities at home also contributes to a positive utility, through  $Z_H$ , as well as the total amount of consumption goods purchased, through  $Z_G$ . The total time spent at home acts like a constraint on the total time spent out of home. And the amount of goods can be interpreted as a constraint on the costs of outdoor activities and travel. These can be comparable with the flexible budget approach described in **Golob et al** (1981) or **Downes & Emmerson (1985)**, where the time not spent on travel (substituted for leisure time) returns in the utility function as a variable, as well as the **amount** of money not spent on travel (see equation (6)). This approach considers utility as a function of travel (the number of kilometres travelled), leisure time and goods purchased. From the viewpoint of travel, leisure time can be seen as the time not spent on travel and the amount of goods purchased is interpreted as the amount of money not spent on travel. Substituting this into the utility function yields:

$$\text{Max. } \sum_i \varphi(x_i) + \psi\left(\hat{Y} - \sum_i p_i x_i\right) + \chi\left(\hat{T} - \sum_i t_i x_i\right) \quad (6)$$

Here  $x_i$  denotes in the case of the application by **Golob et al** and **Downes and Emmerson** the amount of kilometres travelled by mode  $i$ .  $p_i$  denotes unit travel costs,  $t_i$  unit travel time,  $\hat{T}$  and  $\hat{Y}$  are the time and money travel budgets, respectively.  $\varphi$ ,  $\psi$  and  $\chi$  are concave functions. The aim of this model was to allocate time and money to travel; irrespective of the activities. The model developed in this thesis is an extension of the model (6) where the total activity pattern is considered and the distances are only part of the total activity pattern.

The time spent on activities at home  $T_H$  can be then regarded as the total time not spent on activities out of home including travel time and can be compared with the argument of function  $\chi$  in (6). The goods purchased is regarded as the amount of money not spent on outdoor activities including travel cost and can be compared with the argument of function  $\psi$  in (6). Since we assume a similar production function for the total time spent at home  $Z_H$ , compared to **out-of-home** activities, and a comparable production function for the purchased goods  $Z_G$ , we define them by:

$$\begin{cases} Z_H(T_H, f_H) = T_H^\delta \cdot f_H^\alpha \\ Z_G(G) = G^\alpha \end{cases} \quad (7)$$

The utility of the total activity pattern over a period  $T_{TOT}$  contains the production function of the separate outdoor activities, the resistance of travel, the production function of activities at home and of the consumption goods purchased. There are many ways to define this total utility function. Referring to Winston (1987, pp 570) total utility  $U_{tot}$  ought to be concave in its arguments.

$$\frac{\partial U_{tot}}{\partial Z} > 0 ; \quad \frac{\partial^2 U_{tot}}{\partial Z^2} < 0 \quad (8)$$

This could lead to the Cobb-Douglas utility function:

$$U_{tot} = \left( \prod_i Z_i^{\alpha_i}(T_i, d_i, f_i) \cdot R_i^{\tau_i}(d_i, f_i) \right) \cdot Z_H^{\alpha_H}(T_H) \cdot Z_G^{\alpha_G}(G) \quad (9)$$

where all powers ( $\alpha$ 's and  $\tau$ 's) lie in the interval ]0,1[. The parameter  $\alpha_i$  defines the relative weight of the priority of the activity, so in case of obligatory activities, such as **labour**, the value of this parameter should be large.

But maximising  $U_{tot}$  gives the same solution as maximising  $\log(U_{tot})$  and this converts to:

$$\log(U_{tot}) = \sum_i \left[ \alpha_i \log(Z_i(T_i, d_i, f_i)) + \tau_i \log(R_i(d_i, f_i)) \right] + \alpha_H \log(Z_H(T_H)) + \alpha_G \log(Z_G(G)) \quad (10)$$

There is some trouble with incorporating the resistance of travel  $R_i$ . This resistance function is linear in the arguments, instead of concave. Furthermore, using this utility function it is not possible to have a zero solution for one or more of the activities. This means that **all** activities have to be performed,  $T_i, d_i$  and  $f_i$  are all  $> 0$ . Since the possibility of not performing an activity should be **captured** we use another functional form. For this we assume strong separability between the components of the total activity pattern, *i.e.* there is no substitution between two different, activities from independent classes  $i$  and  $j$  ( $j \neq i$ ), but the complete categories act as substitutes for one another. Total utility can then be given by:

$$U_{tot} = \sum_i \left( Z_i(T_i, d_i, f_i) + R_i(d_i, f_i) \right) + Z_H(T_H) + Z_G(G) \quad (11)$$

This model is also comparable with the model of daily time allocation to discretionary **out-of-home** activities of Kitamura (1984). The main difference is that Kitamura obtains exogenous

variables as independent variables of the utility function, whereas in this thesis for each population group the model is estimated separately, so the parameters express the exogenous characteristics. The model of Kitamura also includes all kinds of characteristics of obligatory activities, such as distance from home to work and duration of work, whereas the model in this thesis treat them as endogenous variables.

### Simplified Model

Now let us consider some more assumptions on the model to obtain a simplified version of the model given in the previous section. A simplified model gives the opportunity to **analyse** the model **structure** and the assumptions underlying the model. This can lead to understanding the behaviour of the basic relations in the model.

Instead of leaving a choice in the total distance travelled, let us consider the distance constant per activity. So every time an out-of-home activity is undertaken, the location, and thus the distance, is fixed. The total distance is then given by the average distance per activity times the frequency with which the activity is performed.  $d_i = d_i^0 \cdot f_i$ . The average distance  $d_i^0$  is also called unit travel distance.

Furthermore let us consider only one category of all out-of-home activities. So time will be allocated to “staying at home”, “going out”, and travel. The model also determines the frequency with which one is “going out”.

The emphasis is on the time allocation, less on money expenditure. Now suppose there is no money constraint and the income is fixed, *i.e.* not depending on  $T_w$ , then the costs do not have to be considered.

All these assumptions lead to the simplified model:

$$\text{Max.}_{T,f} U_{tot} = T^0 \cdot f^0 + (T_{tot} \cdot T - f \cdot \frac{d^0}{v}) \quad (12)$$

The time constraint  $T + f \cdot \frac{d^0}{v} + T_H = T_{tot}$  has been substituted (for  $T_{tot}$ ) in the utility function. Optimising this utility gives us the necessary conditions

$$\frac{\partial U_{tot}}{\partial T} = \frac{\partial U_{tot}}{\partial f} = 0 \quad (13)$$

This is also a sufficient condition because the utility function considered is concave, *i.e.* So the solution of (13) will be the maximum of the utility function (12). Solving (13) gives us:

$$\begin{cases} \frac{\partial^2 U}{\partial T^2} < 0 \\ \frac{\partial^2 U}{\partial f^2} < 0 \end{cases} \quad (14)$$

$$\begin{cases} \beta T^{\beta-1} f^e = \vartheta \left[ T_{\text{tot}} - T - f \frac{d^0}{v} \right]^{\vartheta-1} \\ e T^\beta f^{e-1} = \vartheta \frac{d^0}{v} \left[ T_{\text{tot}} - T - f \frac{d^0}{v} \right]^{\vartheta-1} \end{cases} \quad (15)$$

This gives us the relation

$$f = \frac{e v}{\beta d^0} T \quad (16)$$

And the equality:

$$\beta^{1-e} e^e T^{\beta+e-1} \left[ \frac{v}{d^0} \right]^e = \vartheta \left[ T_{\text{tot}} - T \left( 1 + \frac{e}{\beta} \right) \right]^{\vartheta-1} \quad (17)$$

This equation is not solvable analytically, but we can still **analyse** some relationships. Equation (17) can be written as  $F(\beta, \rho, \vartheta, T(\beta, \rho, \vartheta)) = 0$ , and then

$$\begin{cases} \frac{\partial T}{\partial \beta} = - \frac{\partial F}{\partial \beta} \div \frac{\partial F}{\partial T} \\ \frac{\partial T}{\partial \rho} = - \frac{\partial F}{\partial \rho} \div \frac{\partial F}{\partial T} \\ \frac{\partial T}{\partial \vartheta} = - \frac{\partial F}{\partial \vartheta} \div \frac{\partial F}{\partial T} \end{cases} \quad (18)$$

We will not give the full analysis here, but only the results. We find that for all parameter values

$$\frac{\partial T}{\partial \beta} > 0; \quad \frac{\partial T}{\partial \vartheta} < 0 \quad (19)$$

and **this** automatically gives

$$\frac{\partial f}{\partial \vartheta} < 0; \quad \frac{\partial T_H}{\partial \vartheta} > 0 \quad (20)$$

The sign of the derivative of  $T$  to  $\rho$  depends on the parameter values. If we require then the derivatives of  $T$  should meet the requirements:

$$\frac{\partial f}{\partial \varrho} > 0; \quad \frac{\partial f}{\partial \beta} < 0; \quad \frac{\partial T_H}{\partial \beta} < 0; \quad \frac{\partial T_H}{\partial \varrho} < 0 \quad (21)$$

$$\frac{\partial T}{\partial \varrho} > -\frac{T}{\varrho}; \quad \frac{\varrho T}{\beta(\beta + \varrho)} < \frac{\partial T}{\partial \beta} < \frac{T}{\beta} \quad (22)$$

In other words  $T$  should be inelastic in both  $\beta$  and  $\rho$ .

$$E_{T,\varrho} = \frac{\partial T}{\partial \varrho} * \frac{\varrho}{T} > -1; \quad 0 < E_{T,\beta} = \frac{\partial T}{\partial \beta} * \frac{\beta}{T} < 1 \quad (23)$$

Now let us see what happens if we change one assumption. The formulation according to (12) is based on separability between out-of-home and in-home activities. This means that there is no substitution between single activities from one category to the other. Only the whole group of activities acts as substitutes or complements for one another (See Deaton and Muellbauer, chapter 5). This can be interpreted as: **first** one chooses to go out or stay at home, the second choice is then what kind of activity to perform. In this simplified model the comparison is made between the total time spent out-of-home and the total time spent at home, irrespective of the type of activity performed. But if we assume substitutability between single activities, for example between watching TV at home and going to the cinema, we might define the utility function as:

$$U_{tot} = T^\beta \cdot f^\varrho \cdot (T_{tot} - T - f \cdot \frac{d^0}{v})^\vartheta \quad (24)$$

Solving again (13) lead to the following equations:

$$\begin{cases} \beta T^{\beta-1} f^\varrho T_H^\vartheta = \vartheta T^\beta f^\varrho T_H^{\vartheta-1} \\ \varrho T^\beta f^{\varrho-1} T_H^\vartheta = \vartheta \frac{d^0}{v} T^\beta f^\varrho T_H^{\vartheta-1} \end{cases} \quad (25)$$

And this gives us the demand functions for  $T$ ,  $f$  and  $T_H$ :

$$\begin{aligned} T &= \frac{\beta}{\beta + \varrho + \vartheta} \cdot T_{tot} \\ f &= \frac{\varrho}{\beta + \varrho + \vartheta} \cdot \frac{v}{d^0} \cdot T_{tot} \\ T_H &= \frac{\vartheta}{\beta + \varrho + \vartheta} \cdot T_{tot} \end{aligned} \quad (26)$$

In this case the parameters  $\beta$ ,  $\rho$  and  $\vartheta$  are given by the ratio between duration out-of-home  $T$ , travel time  $f \cdot d^0/v$  and duration at home  $T_H$ . In this case  $\beta + \rho + \vartheta = 1$ . For these demand functions we obtain

$$\begin{aligned}
\frac{\partial T}{\partial \beta} > 0; & \quad \frac{\partial T}{\partial \rho} < 0; & \quad \frac{\partial T}{\partial \vartheta} < 0 \\
\frac{\partial f}{\partial \beta} < 0; & \quad \frac{\partial f}{\partial \rho} > 0; & \quad \frac{\partial f}{\partial \vartheta} < 0 \\
\frac{\partial T_H}{\partial \beta} < 0; & \quad \frac{\partial T_H}{\partial \rho} < 0; & \quad \frac{\partial T_H}{\partial \vartheta} > 0
\end{aligned} \tag{27}$$

We already mentioned the main difference between both models (given by (12) and (24)). For explaining the differences it helps to draw the indifference curves of the utility (see appendix).

In the case of the additive utilities (model (12)) the curves intersect both axes, which means that both  $T$  or  $T_H$ , can become zero. In the case of the multiplicative utility function (model (24)) both axes are asymptotes, which means that both  $T$  and  $T_H$  can not become zero, they are both strictly positive.

## 5 Theoretical Analysis

In this section we analyse what will happen if some exogenous characteristics, such as the time budget  $T_{tot}$  or average travel speed  $v$  change. If, for instance, the time budget increases, due to shorter work weeks, the extra time will be divided over the time spent at home, the time spent at activities out-of-home and travel time. In the case of the multiplicative utility function (model (24)) this extra time is divided according to the ratios  $\beta:\rho:\vartheta$ . This means that according to this model travel time increases if the total time budget increases in such a way that the ratio between out-of-home duration, in-home duration and travel time stays the same. In the case of additive utility (model (12)) the ratio between the extra time spent on out-of-home activities and on travel is also  $\beta:\rho$ , but the ratio between the extra time spent at home and out-of-home varies and depends on the values of  $T$  and  $T_H$  for given parameters.

If the average travel speed increases the extra time saved can be spent on travel. On the one hand travel distances can increase: people can travel further during the same travel time, on the other hand the number of trips can increase, but this of course is deduced from performing more activities, so the frequency increases. But this extra time can also be spent at the performed activity (out-of-home or in-home), holding the frequency and distances constant.

Some early model exercises show that in case the total time budget increases for both models the time expenditures and the frequency linearly increase. The ratio  $T$ :travel time: $T_H$  remains constant. In the case of the multiplicative utility function this ratio equals  $\beta:\rho:\vartheta$ . In the case of the additive utility function the ratio  $T$ :travel time also equals  $\beta:\rho$ , but the ratio  $T:T_H$  differs and is mostly less than  $\beta:\vartheta$ . In the appendix of this paper the additional charts are given.

The impact of increasing the average travel speed on the model variables shows a typical difference between both model approaches. Again, in the case of the multiplicative utility function the ratios between the three time expenditures is constant and due to the fact that the total time budget stays the same, these time expenditures also remain constant. The only change is seen in the frequency, which increases drastically. In the case of the additive utility function the frequency increases comparably, but we also see changes in the time expenditures. An increase in travel speed makes people to stay less time at home and spend more time at out-of-home activities. Total travel time also increases, although it is not easy to see in the chart.

The impact of changing the unit travel distance shows a comparable difference. Again, the multiplicative utility function keeps the time expenditures constant, so the only change can occur in the frequency. Increasing the unit travel distance leads to an exponential decrease in the frequency. This decrease also occurs at the additive utility function, but there the time expenditures do change. An increase in unit distance causes people to spend more time at home and less out-of-home. The travel time also decreases.

For our aim, to search for limits to mobility growth, by modelling time allocation, we conclude that the multiplicative utility function does not **fulfil** our requirements. At least not this simplified version. Although the model with additive utility function is not analytically solvable, the first results we showed here satisfy our criteria.

Parameter estimations on time expenditure data in the near future can determine the developments of the past twenty years and more model calculations using various future scenarios can explore the developments for the future.

## 6 Conclusions

In this paper a model is developed for the allocation of time to various activities. The aim of this model is to search for limits to mobility growth, based on the limited amount of time and money people have available. Due to the complexity and estimation problems a simplified version was developed, incorporating only one activity type (for out-of-home activities). This simplification uses a fixed unit travel distance for each activity. The utility function used in this model can be derived in two ways, an additive function and a multiplicative (or Cobb-Douglas function). Some early calculations with the model and comparisons between both approaches gave some idea about the model behaviour and we can now discuss the usefulness of the model.

Given the analyses of the previous section, some preliminary conclusions can be drawn. First of all, the multiplicative model framework is a simple one, easy to analyse and to calibrate. But it models constant time expenditures (at least for constant parameter values). If we assume that changes in travel speed will lead to changes in travel time, we better use the additive utility

function. For this model changes in travel speed lead to large changes in frequencies, comparable to the multiplicative utility function, but time expenditures also change (though slightly).

The next step for this research will be to make some extensions to the simplified versions. First we might take the average distance travelled for each activity (the unit distance) variable, instead of constant, and dependent on the frequency. Secondly we will **analyse** the extensions to more activity types. Thirdly, we will incorporate the cost function and the money budget again.

Another step in the research is parameter estimation. In the case of the Cobb-Douglas utility function, the parameters can be taken equal to the proportions of the time expenditures, relative to the total time budget. In that case the parameters count up to 1. For the additive utility function we will need some parameter estimation techniques. In the case of the simplified version we expect no enormous difficulties, but with the extensions of the model fit could be troublesome, given the experiences of the past.

During the presentation of this paper at

*Speurwerk* we hope

to show the estimation results.

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