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A TRANSFER FUNCTION APPROACH TO STRUCTURAL VIBRATIONS INDUCED BY THERMOACOUSTIC SOURCES

Rob Huls, André de Boer, Jim Kok

University of Twente
P.O. Box 217, 7500 AE Enschede, The Netherlands
r.a.huls@utwente.nl

Abstract

To decrease NOx emissions from a combustion system, lean premixed combustion in combination with an annular combustor is used. One of the disadvantages is an increase in sound pressure levels in the combustion system, resulting in an increased excitation of the surrounding structure, the liner. This causes fatigue, which limits the life time of the combustor. To model the interaction between flame, acoustics and structure, a transfer function approach is used. In this approach, the components are represented by the frequency dependent linear transfer between their inputs and outputs. For the flame a low pass filter with convective time delay is used as transfer function between velocity perturbations at the burner outlet and the flame as acoustic volume source. The acoustic transfer from volume source to velocity perturbation at the burner outlet is obtained from a harmonic finite element analysis, in which a temperature field from CFD calculations is used. The calculated response is subsequently curve-fitted using a pole-zero model to allow for fast calculations. The finite element model includes the two way coupling between structural vibrations and acoustics, which allows extraction of the vibration levels. The different transfers are finally coupled in one model.

Results show frequencies of high acoustic response which are susceptible to thermoacoustic instability. Damping mechanisms and the phase relation between the different components determine stable or unstable behavior and the amplitude of the resulting perturbations. Furthermore there are also frequencies of high structural response. Especially when the two coincide, the risk of structural damage is high, whereas when they move away from each other, the risk decreases.

INTRODUCTION

To decrease NOx emissions from an industrial gas turbine lean premixed combustion (using a surplus of air to burn the fuel) in combination with an annular combustor is

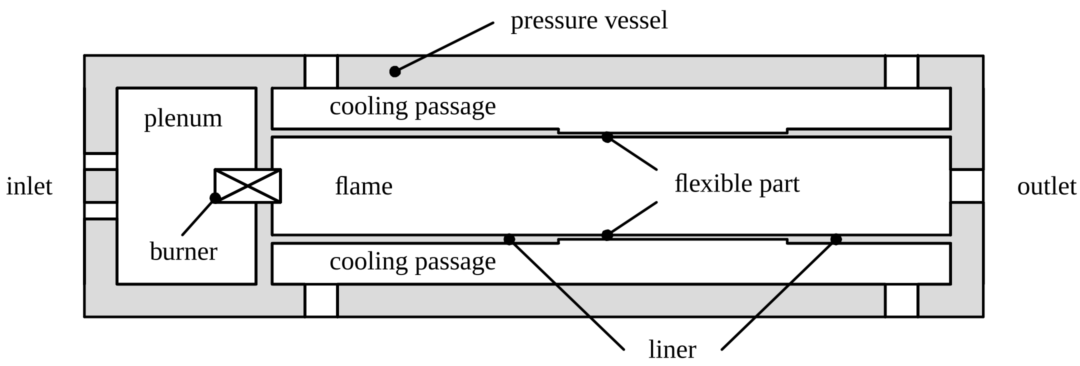

Figure 1: Schematic drawing of the geometry

used.[7] The lean combustion has the side effect of higher sound pressure levels in the combustion system. On the other hand, the annular liner (the surrounding plate-like structure of the combustion chamber) has a relatively low stiffness and can therefore vibrate excessively due to these sound pressures. This limits the life of the combustor and the range of operability, because of failure due to fatigue.

There are two approaches to overcome this problem. The most common approach is to decrease the acoustic pressure levels in the combustor.[6] The other approach is to decrease the vibration level of the liner by, for instance, increasing the damping. This requires investigation of the fluid structure interaction between the liner and the combustion chamber. The European DESIRE project (Design and demonstration of highly reliable low NOx combustion systems for gas turbines) is concerned with this interaction, in which the structural response will be measured in a 500 kW test-rig of square cross section. In this paper a model is made of the coupled problem, based on transfer functions, that describes the interaction between combustion, acoustics and structural vibration. The frequency range of interest is 10-500 Hz. Models are made up to 700 Hz, because this facilitates accurate fitting up to 500 Hz.

The geometry, taken from the test-rig (figure 1), subsequently consists of an inlet, a plenum chamber, the burner, a square combustion section, a square section where structural vibrations can be measured and a circular contraction as outlet. Upstream of the plenum chamber a set of small holes provides an acoustically hard boundary. Downstream of the structural section the boundary is a disk with a hole. This will

parameter	**description**	**value**	**parameter**	**description**	**value**
E	Young's modulus	130 GPa	D_{in}	inside width	150 mm
ν	Poisson's ratio	0.30	μ	damping at outlet	30%
ρ_s	density solid	7800 $\mathrm{kg\,m^{-3}}$	ζ	structural damping	2%
t	liner thickness	1.5 mm	L	chamber length	1.8 m

Table 1: Material properties and dimensions

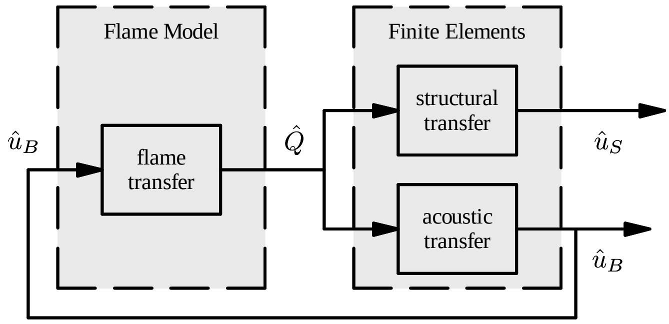

Figure 2: Coupling of flame and vibroacoustic system using transfer functions

probably not be an acoustically hard end, so an absorption coefficient is used. The structural section contains a thinner (1.5 mm) liner section of length 400 mm. The ends of this part are welded to the thicker (4 mm) remaining liner section and therefore a clamped condition is used. The whole liner is embedded in a pressure vessel. The effects of this on the structural vibrations is not taken into account in this paper, but can be significant.[4] The relevant properties are listed in table 1.

MODELING

The problem considered essentially consists of three different parts, being the flame as acoustic source, the acoustic pressure field generated and the structural vibration as a consequence of this. The last two are incorporated in one fully coupled finite element model (figure 2). From this model the transfer from the flame as volume source to the acoustic velocity perturbation at the burner outlet is determined (figure 3). Furthermore transfer from the volume source to structural vibrations can also be deduced. The flame is represented by a flame transfer function, which describes the transfer from the acoustic velocity perturbation at the burner outlet to the flame as acoustic volume source. The different transfer functions are subsequently coupled. For the coupling of acoustics and flame a similar approach was used by Fannin.[3] Both submodels will now be described in more detail.

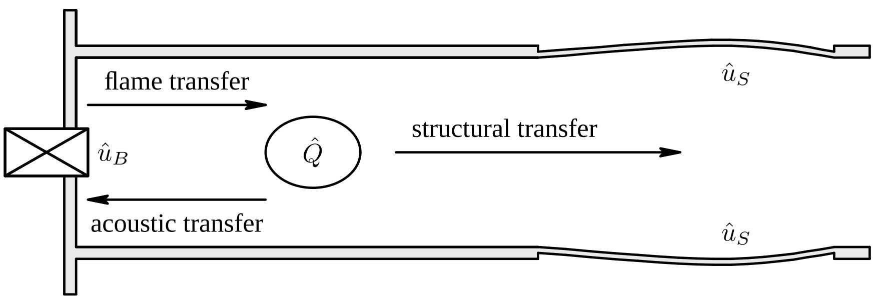

Figure 3: Schematic picture of the transfer functions

Flame model

The heat release from the flame responds to the acoustic particle velocity perpendicular to the burner outlet, because these perturbations cause modulations in the supply of combustible mixture to the flame. The transfer to the heat release rate by the flame is a low pass filter combined with a time delay. The time delay is caused by the time it takes for a disturbance to convect from the burner outlet to the flame front. The low pass behavior is related to the size of the flame. A disturbance burns during its passage of the flame front. Disturbances which convect less than this distance over one period are averaged out, causing low pass behavior. The flame model can be written as[1]

$$\frac{\hat{q}}{q_0} = \left[\frac{1}{1+\mathrm{i}\omega\tau_1}\right]\left[\frac{\hat{u}_B}{u_{0,B}}\right]\mathrm{e}^{\mathrm{i}\omega(t-\tau)} \tag{1}$$

in which a subscript zero denotes a mean value and a hat the amplitude of the perturbation. Furthermore u_B is the velocity at the burner outlet, q denotes heat release rate, τ_1 determines the cutoff frequency of the low pass filter, t is time, τ is the time delay (τ=$x_f/u_{0,B}$ with x_f the flame position relative to the burner outlet, or the time delay is determined directly from CFD results) and ω the angular frequency. To translate the unsteady heat release rate to a volume source as input for the finite element model the following relation is used[2]

$$\hat{Q} = \frac{\gamma - 1}{\rho c^2}\hat{q} \tag{2}$$

in which $\hat{Q}$ denotes the equivalent acoustic volume source. The flame transfer function ($\hat{Q}/\hat{u}_B$) is depicted in figure 4. The properties used are listed in table 2.

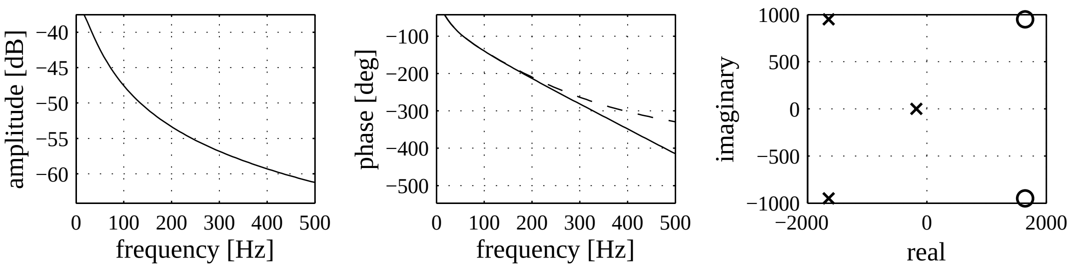

Figure 4: Amplitude (left) and phase (middle) of the flame transfer function, the dashed line is the 2nd order Padé approximation, and the pole zero map of the Padé approximation (right)

The time delay can, in a frequency domain description, be implemented as a decreasing phase with constant slope as function of frequency. In a pole-zero description a Padé approximation can be used, which has a constant gain and approximately the correct decreasing phase. The Padé approximation is also depicted in figure 4.

Vibroacoustic model

parameter	description	value	parameter	description	value
c_0	cold speed of sound	343 $\mathrm{m\,s^{-1}}$	$u_{0,B}$	burner outlet velocity	27 $\mathrm{ms^{-}}$
ρ_0	cold density of air	1.21 $\mathrm{kg\,m^{-3}}$	x_f	position of flame	0.05 m
p_0	mean pressure	1.5 bar	q_0	mean thermal power	150 kW
γ	ratio spec. heats	1.4			

Table 2: Numerical values for the flame model

The coupled vibroacoustic model is made of a combination of linear shell (shell63) and linear acoustic (fluid30) elements using the finite element package Ansys®. An unstructured mesh made of triangular and tetrahedral elements made by Femap® is used (figure 5).

The acoustic and shell elements are coupled using special elements that include fluid structure interaction (figure 5). These elements ensure that the acoustic particle velocity normal to the surface is the same as the structural normal velocity and that the dynamic equilibrium equations are satisfied.

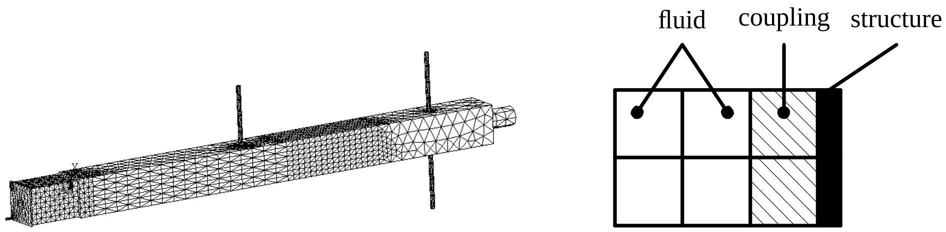

Figure 5: Left shows the mesh that was used and right the method of coupling

The coupling method between structure and acoustics has been experimentally validated on a less complex setup.[5] Furthermore it has been used to investigate the influence of the cooling passage on the vibration behavior of the liner.[4]

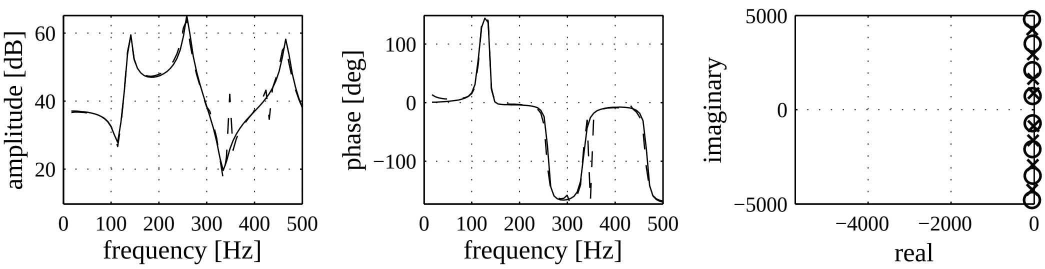

Figure 6: Amplitude (left) and phase response (middle) of the finite element model (solid line) and the pole zero fit (dashed line). Right shows the pole zero map of the fit

For the acoustic properties during combustion, a temperature field taken from a steady CFD RaNS calculation of a quarter section is used. It typically consists of cold zones, a flame front with very steep temperature gradients and hot zones. The temperature field is therefore divided in a hot and a cold section and the properties of the gasses are used according to this spatial distribution, which is interpolated on the finite element mesh. The structural damping is taken as 2% (constant stiffness matrix multiplier), which is an estimation of the damping introduced by the highly turbulent cooling flow between liner and pressure vessel. The properties of the structure are taken at elevated temperature as listed in table 1. A harmonic analysis is performed on the model. The resulting frequency response functions are curve fitted to obtain a pole zero description of the different parts. These can then be used in the coupled description. The acoustic transfer function and the fit are depicted in figure 6.

RESULTS

Unstable frequencies

There are two well-known types of methods from control theory to evaluate system stability, being root-locus methods and frequency domain methods. Whereas the root-locus methods are easier to understand and evaluate, the frequency domain method has some other advantages, two important ones being the ability to include measured data (for instance a measured flame transfer function) and easier inclusion of the time delay in the flame transfer function.

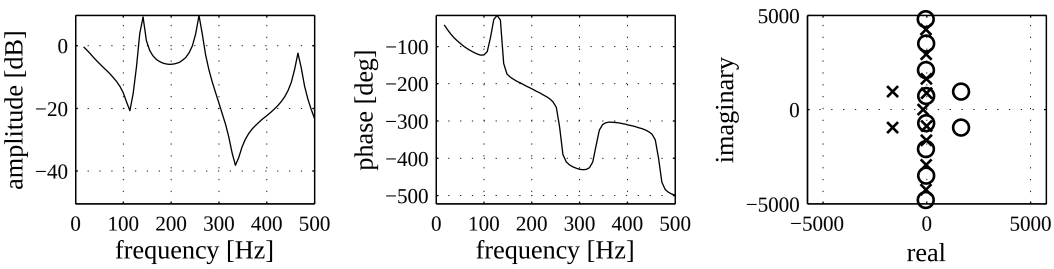

Figure 7: Open loop frequency response (left), phase response (middle) and pole zero map (right)

open loop gain exceeds one and the phase angle is -180 degrees at the same time (for a negative feedback system). Whether this condition is met is largely determined by two phenomena. The gain often exceeds one in regions near acoustic resonances of the combustion chamber. The phase angle is largely determined by the transport delay of the flame. For the parameters used in this paper the most likely frequency for which unstable behavior is expected will therefore be around 150 Hz (figure 7), due to the large acoustic gain provided by first acoustic eigenfrequency. The stability depends on the gain and phase of the flame transfer function. A higher gain will create a wider risk area in the frequency domain. It is therefore very important that the time delay is captured well by the Padé approximation. For the system calculated here, the closed loop is just stable (figure 8), but for a slightly different flame or acoustic response, the system can become unstable. It can also be seen that the second resonance around 260 Hz is stabilized by the flame, because the flame extracts energy from it. For the region of interest (266 Hz) the Padé approximation is fairly well (figure 4). The behavior is also confirmed by time domain simulations using the actual time delay.

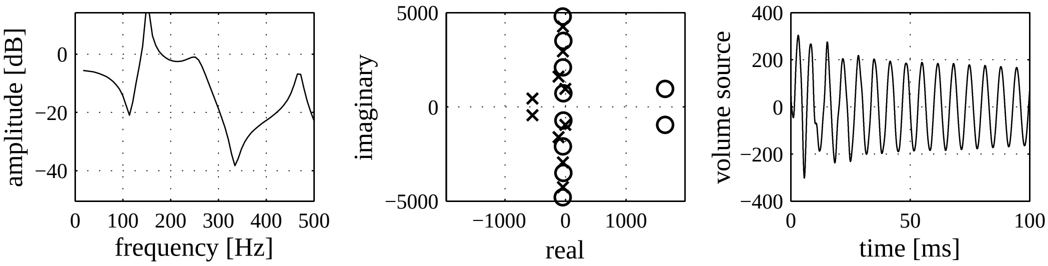

Figure 8: Closed loop frequency response (left), pole zero map (middle) and impulse response (right) showing the just stable behavior of the first acoustic eigenfrequency

Structural vibration

The structural response of the system is also determined with the finite element calculations. The transfer, again from the volume source, is depicted in figure 9 and the structural response for the closed loop system is also shown. There is a high structural response near the two acoustic resonance frequencies seen previously.

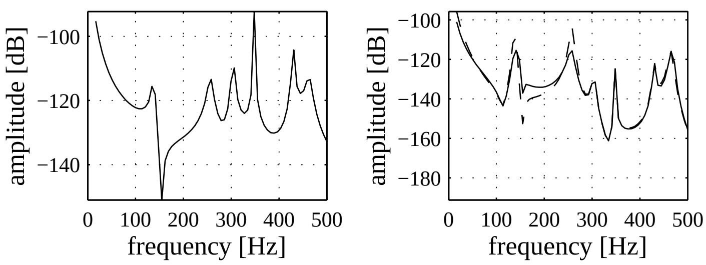

Figure 9: Transfer from volume source to spatial rms structural response and the response for the open (dashed) and closed (line) loop system

The response for the closed loop model is again smaller for the second resonance, because damping is added by the flame. The first acoustic resonance behaves somewhat differently. For this mode the flame adds some energy to the acoustic mode, but not enough to compensate for the damping losses and cause unstable behavior. But the structural vibration level decreases. This is because the acoustic resonance frequency shifts away from the structural resonance and therefore the structural response is less.

Around 450 Hz structural resonances and an acoustic resonance coincide, which leads to high structural response, yet due to the low pass character of the flame there is little input from the flame.

CONCLUSION

A transfer function based method has been developed to predict the vibroacoustic behavior of a combustion system with a flame as acoustic source. Using this method a simple flame model combined with transfer functions from a finite element model can predict the acoustic and structural response of a combustion system. Unstable behavior can be captured and the structural response can be calculated for the closed loop system. Better approximations for the flame transfer function can be easily included, especially if a good curve fit can be made of the transfer function.

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REFERENCES

[1] G.J. Bloxsidge, A.P. Dowling, and P.J. Langhorne. Reheat buzz, and acoustically coupled combustion instability. part 2. theory. *Journal of Fluid Mechanics*, 193:445–473, 1988.

[2] A.P. Dowling. Singing flames - the coupling of acoustics and combustion. In *Proceedings of the International Congress on Sound and Vibration 10*. The International Institute of Acoustics and Vibration (IIAV), 2003.

[3] C.A. Fannin. *Linear Modeling and Analysis of Thermoacoustic Instabilities in a Gas Turbine Combustor*. PhD thesis, Virginina Polytechnic Institute and State University, 2000.

[4] R.A. Huls, J.B.W. Kok, and A. de Boer. Vibration of the liner in an industrial combustion system due to an acoustic field. In *Proceedings of the International Forum on Aeroelasticity and Structural Dynamics*. Nederlandse Vereniging voor Luchtvaarttechniek (NVvL), 2003.

[5] R.A. Huls, J.F. van Kampen, J.B.W. Kok, and A. de Boer. Fluid structure interaction to predict liner vibrations in an industrial combustion system. In *Proceedings of the Tenth International Congress on Sound and Vibration*. KTH Stockholm, 2003.

[6] U. Krüger, J. Hüren, S. Hoffman, W. Krebs, P. Flohr, and D. Bohn. Prediction and measurement of thermoacoustic improvements in gas turbine with annular combustion systems. *Journal of Engineering for Gas Turbines and Power*, 123:557–566, July 2001.

[7] A.H. Lefebvre. *Gas Turbine combustion*. Taylor and Francis, 2nd edition, 1999.