ON STABILITY OF QUEUEING MODELS FOR SLOTTED RING LOCAL AREA NETWORKS

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Abstract.

We study a queueing system which is intended to model local area networks with slotted ring protocols and which generalizes some previously studied models. We define a special type of stability called τ-stability which is related to the slot rotation time τ. We present a condition which is necessary for τ-stability of the queue length process and also show that under certain assumptions this condition is at the same time a sufficient condition.

1. Introduction.

Queueing models are often used for modelling communication networks, as they often lead to reliable estimates for the performance of these networks. Usually it is assumed that queueing models are in some sense 'stable'. However, since there is no unique definition of stability, it is important to make mathematically precise what is meant by 'stability' and to study conditions for 'stability'. In this paper we review various forms of 'stability' and study conditions under which they apply to a queueing system which is intended to model slotted ring local area networks.

The structure of the rest of this paper is as follows. In section 2 we describe the model under study. In section 3 we give several definitions of 'stability' and some preliminary results. In section 4 we formulate and prove a necessary stability condition, while section 5 is devoted to the question of whether this condition is also sufficient. In section 6 we summarize and discuss our findings.

2. Model description.

Let n stations, denoted by $S_1,\dots,S_n$, be connected to a ring-shaped transmission medium. At station S_i packets containing digital information (including an address, which is the index of the addressed station) to be transmitted arrive according to a Poisson process with intensity λ_i. By $\lambda = \lambda_1+\dots+\lambda_n$ we denote the total packet arrival rate. At each station packets awaiting their transmission are stored in a buffer with infinite capacity.

On the ring c identical slots, each capable of containing exactly one packet, rotate with constant rotation time τ. When station S_i is visited by an empty slot and has a packet to transmit, it fills the slot with the packet. The slot subsequently travels along the ring where each station it passes can copy the packet until it reaches the packet's addressed station S_j. At the addressed station the slot is emptied upon which it proceeds empty to the next downstream station. The addresses of packets transmitted from S_i are mutually independent and identically distributed stochastic variables; by p_{ij} we denote the probability that j is the address of a packet transmitted from S_i. It should be noted that the addressed station is not necessarily the or the only station for which the packet is meant, for it can also be copied by stations the slot passes on its way from S_i to S_j. In the present paper we mean by the addressed station always the station that has to empty the slot. Let $\rho_i = \lambda_i\tau/c$ and $\rho = \rho_1+\dots+\rho_n = \lambda\tau/c$. See also figure 1.

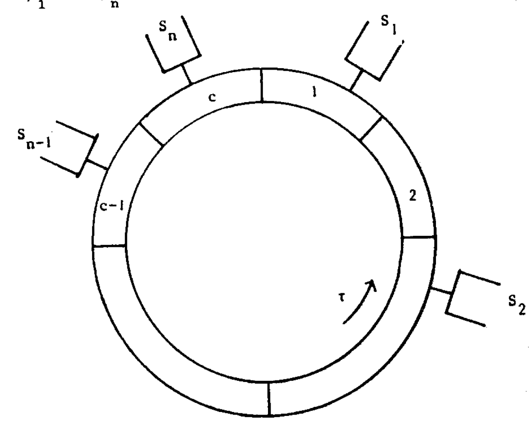

Figure 1. General slotted ring model.

The above model covers some models as special cases. If $p_{ii} = 1$ for all $i = 1,2,\dots,n$, then we obtain a model which was considered in [2], [13] and [16]. The models in [2] and [16] are slightly more general because it is assumed in these references that packets arrive according to batch

Poisson processes, with arbitrarily distributed batch sizes in [2] and geometrically distributed batch sizes in [16].

If $p_{ii} = 0$, for all $i = 1,2,\ldots,n$, then we obtain the so-called *Orwell* protocol, cf. [10]. This model was considered in [15] with batch Poisson arrival processes.

In general the present model is too complex to allow an exact analysis of, e.g., the queue lengths at the stations. In fact, it is already difficult to obtain conditions under which these queueing models admit stationary behaviour. In [14] stability conditions are formulated for some models. However, no formal proofs are given. In this paper we obtain both known and new stability conditions for queueing models of local area networks with slotted ring protocols, by deriving stability conditions for the general model described above.

3. Definitions, general notation and preliminary results.

3.1 Definitions.

For the stochastic processes under study in the present paper different forms of stability may hold, some of which are defined in this subsection. Let $\{ X(t),\ t \ge 0 \}$ and $\{ X_k,\ k = 1,2,\ldots \}$ be m-dimensional stochastic processes with nonnegative components and let x be an m-dimensional vector. By $X(t) \le x$ or $X_k \le x$ we mean componentwise inequality. By $x \to \infty$ we mean that all components of x tend to infinity in an arbitrary way.

Definition 1. $\{ X(t),\ t \ge 0 \}$ is substable if for all initial states $x(0)$

$$\lim_{x\to\infty} \left[\lim_{t\to\infty} \inf P(X(t) \le x \mid X(0) = x(0)) \right] = 1.$$

The notion of substability has also been used in [9] and [12]. In [9] it is shown that $\{ X(t),\ t \ge 0 \}$ is substable if and only if for all initial states $x(0)$ there exists a distribution function $G_{x(0)}$ such that for all x and t

$$G_{x(0)}(x) \le P(X(t) \le x \mid X(0) = x(0)).$$

Definition 2. $\{ X(t),\ t \ge 0 \}$ is stable if there exists a distribution function F such that at all continuity points x of F and for all initial states $x(0)$

$$\lim_{t\to\infty} P(X(t) \le x \mid X(0) = x(0)) = F(x).$$

For the discrete time process $\{ X_k,\ k = 1,2,\ldots \}$ substability and stability are defined similarly. It can easily be seen that a stable process is also substable, while conversely a substable process is only stable if a limiting distribution exists. Thus substability is weaker than stability.

For an important class of stochastic processes, viz. irreducible and aperiodic Markov chains substability and stability are equivalent. Namely, since substability excludes transiency and null-recurrency, an irreducible and aperiodic Markov chain which is substable, is positive recurrent and consequently stable.

The stochastic processes under study in the present paper are aperiodic and irreducible Markov chains only in some special cases. In general it can be seen quite easily that stability does not hold. However, we can still prove a form of stability, which we will call τ-stability and which is stronger than substability but weaker than stability.

Definition 3. $\{ X(t),\ t \ge 0 \}$ is τ-stable if for every $\theta \in [0,\tau[$ the process $\{ X(\theta+k\tau),\ k = 0,1,2,\ldots \}$ is stable.

The importance of τ-stability is that it enables us to define a stationary distribution F of the process at an arbitrary point in time by

$$F(x) = \frac{1}{\tau} \int_0^{\tau} \lim_{k\to\infty} P(X(\theta+k\tau) \le x)\, d\theta, \qquad (1)$$

provided, of course, that this integral exists.

As an aside we note that in [8] the process $\{ X(t),\ t\ge0 \}$ is defined to be stable if $EX_1(t),\ldots,EX_m(t)$ are bounded. Evidently this definition resembles the notion of substability, but is slightly stronger.

3.2 General notation.

In this subsection we define the main random variables needed to describe the system evolution. We denote by $Q_i(t)$ the queue length at S_i at time t, $i = 1,2,\ldots,n$ and let $Q(t) = (Q_1(t),\ldots,Q_n(t))$. Our aim is to derive conditions for τ-stability of $\{ Q(t),\ t \ge 0 \}$. In order to be able to give a complete description of the system at any time we introduce the following quantities. The position of a slot is a real number in $[\ 0\ ,\ \tau\ [$, defined as the time it takes for that slot to reach S_1. We define c reference positions $p_i = (i-1)\tau/c$, $i = 1,2,\ldots,c$. Further, for $j = 1,2,\ldots,c$ we let $A_j(t)$ be a random variable associated with the first slot that reaches p_j at or after time t, assuming the value 0 if the slot is empty and the address of the packet in the slot otherwise. Next, let $A(t) = (A_1(t),\ldots,A_c(t))$ and denote by $N(t)$ the position of the first slot that reaches p_1 after or at time t.

Finally we define $V(t) = (Q(t),A(t),N(t))$, so $\{ V(t),\ t \ge 0 \}$ is a continuous time stochastic process assuming values in $S = \{0,1,2,\ldots\}^n \times \{0,1,\ldots,n\}^c \times (\{0\} \cup\ ]p_c,\tau[)$. Then $V(t)$ completely describes the system at time t.

Throughout the present paper we shall denote a time instant just before t by t^-.

3.3 Preliminary results.

Consider the process $\{ V(\theta+k\tau),\ k = 0,1,2,\ldots \}$ for some fixed $\theta \in [0,\tau[$. It is easily seen that, because of the Poisson arrival processes the process $\{ V(\theta+k\tau),\ k = 0,1,2,\ldots \}$ is a Markov chain. However since $N(\theta+k\tau)$ is constant for every $k = 0,1,2,\ldots$ and because of protocol restrictions $\{ V(\theta+k\tau),\ k = 0,1,2,\ldots \}$ can assume values only in a proper subset of S. In the sequel we will assume that $\{ V(\theta+k\tau),\ k = 0,1,2,\ldots \}$ is defined on a state space S^* consisting of all states that can be reached from the state in which all slots and queues are empty and $N(\theta+k\tau)=N(\theta)$. Because the arrival processes are Poissonian this empty state can also be reached from any state in S^*. Consequently $\{ V(\theta+k\tau),\ k = 0,1,2,\ldots \}$ is irreducible. Because the system can remain empty during one or more rotations, $\{ V(\theta+k\tau),\ k = 0,1,2,\ldots \}$ is also aperiodic. If a stationary distribution exists, then it can of course easily be extended to S by simply assigning stationary probability zero to those states that do not belong to S^*.

Proposition 1. If $\{ Q(t),\ t \geq 0\}$ is substable, then $\{ Q(t),\ t \geq 0\}$ is τ-stable.

Proof. If $\{ Q(t),\ t\geq 0\}$ is substable, then because $N(t)$ and $A(t)$ are bounded, $\{ V(t),\ t \geq 0\}$ is substable. Then, for any $\theta \in [0,\tau[$, the process $\{ V(\theta+k\tau),\ k = 0,1,2,.. \}$ is also substable. Moreover $\{ V(\theta+k\tau),\ k = 0,1,2,.. \}$ is an irreducible and aperiodic Markov chain. Consequently $\{ V(\theta+k\tau),\ k = 0,1,2,.. \}$ is stable and therefore $\{ Q(\theta+k\tau),\ k = 0,1,2,.. \}$ is stable, whence $\{ Q(t),\ t \geq 0\}$ is τ-stable.

□

Remark 1. It may be shown, see [1], that τ-stability of $\{ X(t),\ t \geq 0\}$ in turn implies Cesàro convergence, i.e., there exists a distribution function F such that at all continuity points x of F and for every initial state x(0)

$$\lim_{T\to\infty} \frac{1}{T} \int_0^T P(\ X(t) \leq x \mid X(0) = x(0)\ )dt = F(x).$$

Moreover, F is identical to the distribution function given by (1).

Suppose that d_i, $i = 1,2,\ldots,n$ is the distance between S_i and the next downstream station measured in the time needed for a slot to cover this distance and let the distance vector of the model at hand be $d \equiv (d_1,\ldots,d_n)$. Choose ϵ such that $0 < \epsilon < d_1$ and describe a system with distance vector $d^\epsilon = (d_1-\epsilon, d_2+\epsilon, d_3,\ldots,d_n)$ by corresponding random variables with superscript ϵ. The system with distance vector d^ϵ is obtained from the system with distance vector d by shifting S_2 ϵ time units toward S_1. See also figure 2.

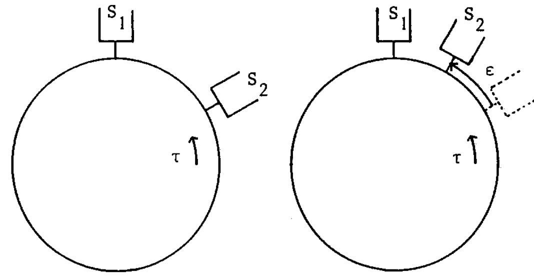

Figure 2. Original and shifted system.

Proposition 2. $\{ Q(t),\ t \geq 0\}$ is τ-stable iff $\{ Q^\epsilon(t),\ t \geq 0\}$ is τ-stable.

Because of its length the proof of proposition 2 is not given here. In [1] the proposition is proved via substability and proposition 1 using a sample path technique. Now, for investigating conditions for τ-stability, we may assume according to proposition 2 that all stations are connected to the ring immediately after each other, as long as their relative positions remain unchanged. Now let $t_i \equiv i\tau/c$, $i = 0,1,2,\ldots$ and assume $N(0) = 0$, then $\{ V(t_k^-),\ k = 0,1,2,\ldots \}$ is a Markov chain. Again it can strictly speaking not be irreducible because $N(t_k^-)$ always assumes the value 0. However we shall tacitly assume that $\{ V(t_k^-),\ k = 0,1,2,\ldots \}$ is defined on a reduced state space on which it is irreducible by definition, as we did for $\{V(\theta+k\tau),\ k = 0,1,\ldots\}$. It is easily seen that $\{ V(t_k^-),\ k = 0,1,2,\ldots \}$ is also aperiodic. Again we observe that if a stationary distribution exists, it can be extended to S.

Remark 2. By proposition 2 we may assume that, for investigating conditions for τ-stability, all stations are connected to the ring at the same position. One wonders whether this insensitivity for mutual distances between the stations might also hold for the stationary simultaneous queue length distribution. This would mean that the above assumption can also be used for studying the stationary simultaneous queue length distribution, so that it would suffice to study the Markov chain $\{ V(t_k^-),\ k = 0,1,2,\ldots \}$. Results in [4] and [5] do not contradict this insensitivity. We think this is so indeed but we were unable to prove this property.

4. A necessary condition for τ-stability.

It will be convenient to define a cyclic summation $\sum_o$ as follows. If $r_1,\ldots,r_n$ are arbitrary numbers, then for $m,k \in \{1,\ldots,n\}$

$$\sum_{i=k}^{m}{}_{o}\, r_i \equiv \begin{cases} \sum_{i=k}^{m} r_i, & \text{if } m \geq k, \\ \sum_{i=k}^{n} r_i + \sum_{i=1}^{m} r_i, & \text{if } m<k. \end{cases}$$

Furthermore we define for $i \in \{1,2,\ldots,n\}$, $b(i) \equiv ((i-2) \bmod n)+1$ and let $S_0 \equiv S_n$.

For $i = 1,\ldots,n$ we define a distance vector $d^{(i)}$ such that all mutual distances are zero except the distance between S_i and the next upstream station, which is equal to τ. In other words, all stations are connected to the ring at the same position and S_i is the first station visited every time a slot arrives at this position. Recall that $\{ V(t_k^-),\ k = 0,1,2,\ldots\}$ is an irreducible and aperiodic Markov chain for any $d^{(i)}$, $i = 1,2,\ldots,n$.

<u>Proposition 3.</u> If $\{ Q(t),\ t \geq 0\}$ is τ-stable, then for $i = 1,2,\ldots,n$

$$\rho_i + \sum_{j=1}^{n} \sum_{k=j}^{b(i)}{}_{o}\, p_{kj}\rho_k < 1. \tag{3}$$

<u>Proof.</u> By virtue of proposition 2 we may assume that all stations are connected to the ring at the same position. Choose an arbitrary first station S_i. Since the Markov chain $\{ V(t_k^-),$ $k = 0,1,2,\ldots\}$ is irreducible, aperiodic and substable it is stable. Thus let $V(t_k^-)$ converge in distribution to a random vector $V=(Q,A,N)$. We will determine the distribution of A_1.

Let $j \in \{1,2,\ldots,n\}$. Because the value of A_1 can only change at the time instants $t_k=k\tau/c$, $k = 0,1,2.,\ldots,$ $P(A_1=j)$ is also the limiting probability that at an arbitrary instant between t_k and t_{k+1} the first slot about to reach S_i contains a packet addressed to S_j. This allows us to use Little's formula in the following way. Let L_j be the expected number of packets in the slot in position 1 addressed to S_j. Of course we now have

$$L_j = P(A_1=j).$$

Let Λ_j be the arrival intensity of packets addressed to S_j at the slot in position 1. The expected sojourn time of a packet in the slot in position 1 is τ/c. From Little's formula we then find

$$P(A_1=j) = \Lambda_j\tau/c. \tag{4}$$

Next we determine Λ_j. The packets addressed to S_j can come from $S_j,S_{j+1},\ldots,S_{i-1}$, but not from $S_i,\ldots,S_{j-1}$ because by the chosen mutual distances, the slots filled by $S_i,\ldots,S_{j-1}$ are emptied the same instant by S_j. It follows that for $j = 1,2,\ldots,n$

$$\Lambda_j = \sum_{k=j}^{b(i)}{}_{o}\, p_{kj}\lambda_k. \tag{5}$$

Using (4) and (5) we find

$$P(A_1=j) = \sum_{k=j}^{b(i)}{}_{o}\, p_{kj}\rho_k,\quad j = 1,2,\ldots,n$$

which immediately yields

$$P(A_1=0) = 1 - \sum_{j=1}^{n} \sum_{k=j}^{b(i)}{}_{o}\, p_{kj}\rho_k. \tag{6}$$

By using Little's formula again we find that the rate of available slots visiting S_i is equal to $P(A_1=0)c/\tau$. Now we create an extra buffer at S_i from which dummy packets are transmitted every time an available slot arrives while the queue at S_i is empty. We assume that a slot with a dummy packet is identified by other stations as available. The stationary expected time between two consecutive transmissions of dummy packets at S_i is $1/\epsilon_i$. Because $P(Q_i=0,A_1=0) > 0$ it follows that $\epsilon_i>0$. Moreover it is easily seen that ϵ_i satisfies the equation

$$\epsilon_i = P(A_1=0)c/\tau - \lambda_i. \tag{7}$$

Combining $\epsilon_i > 0$, (6) and (7) the proposition follows after some rewriting. □

If $p_{ii} = 1$, $i = 1,2,\ldots,n$ we obtain the following result, which can already be found in [14] and [16], however without formal proof.

<u>Corollary 4.</u> If $\{ Q(t),\ t \geq 0\}$ is τ-stable, then

$$\rho + \max_{i=1,\ldots,n} \rho_i < 1. \tag{8}$$

If $p_{ii} = 0$, $i = 1,2,\ldots,n$, then we obtain the following result, which extends known results in [14] and [15].

Corollary 5. If $\{Q(t), t \geq 0\}$ is τ-stable, then for $i = 1,2,\ldots,n$

$$\rho_i + \sum_{j=1}^{n} \sum_{k=j}^{b(i)} {}_{o}\, p_{kj}\rho_k < 1. \qquad (9)$$

If in addition $p_{ij} = (n-1)^{-1}$, $i,j = 1,2,\ldots,n$, $i \neq j$ and all arrival rates are equal, then (9) reduces to

$$\rho < 2-4/(n+2)$$

which was indeed given (without formal proof) in [14] and [15].

5. A sufficient condition for τ-stability.

Before giving a general sufficient condition for substability of $\{Q(t), t \geq 0\}$ we briefly review a well-known result for the $G|G|1$ queue. Therefore we need the following definitions.

Definition 4. A stochastic process $\{X_k, k = 1,2,\ldots\}$ is strictly stationary if for all strictly positive integers m, l, $k_1 < \ldots < k_m$

$$\left\{ X_{k_1},\ldots,X_{k_m} \right\} \text{ and } \left\{ X_{k_1+l},\ldots,X_{k_m+l} \right\}$$

have the same joint distribution.

Definition 5. A stochastic process $\{X_k, k = 1,2,\ldots\}$ defined on a probability space $(\Omega,\mathcal{F},P)$ is asymptotically stationary if there exists a strictly stationary stochastic process $\xi = \{Y_k, k = 1,2,\ldots\}$ defined on a probability space $(\Omega,\mathcal{F},P)$, such that $\xi_N = \{X_{N+k}, k = 1,2,\ldots\}$, which is defined on $(\Omega,\mathcal{F},P_N)$, converges to ξ in the following way. There exists an $\epsilon(N)$ with $\epsilon(N) \to 0$ as $N \to \infty$, such that for any $B \in \mathcal{F}$

$$| P_N(\xi_N \in B) - P(\hat{\xi} \in B) | < \epsilon(N).$$

In [6] it is shown that the waiting times of the customers in the $G|G|1$ queue constitute a substable stochastic process if the sequences of inter-arrival times and service times are asymptotically stationary and the traffic intensity is smaller than one. For the present model we shall make similar assumptions.

Because the arrival processes are Poissonian, the inter-arrival times of packets at each queue are asymptotically stationary with finite limiting expectations. Since in the present model service times are not defined, we introduce the following quantities. A visit of a slot to a station is called successful if the slot is available for transmission of a packet, regardless of whether the station has a packet to transmit or not. The time between two consecutive successful visits at S_j is called a modified service if a packet is transmitted from S_j after the first successful visit or a vacation if no packet is transmitted from S_j after the first successful visit. For $j = 1,2,\ldots,n$ we define $C^*_{j,i}$ as the ith modified service time at S_j, $i=1,2,\ldots$ and $V_{j,i}$ as the ith vacation time at S_j, $i=1,2,\ldots$. Further let $Q^*_{j,i}$ be a random variable associated with the queue length at S_j just before the ith successful visit at S_j, assuming the value 0 or 1 if the queue is empty or non-empty, respectively and $C_{j,i}$ a random variable being the time between the ith and $(i+1)$st successful visit at S_j. We denote by $RC_{j,i}$ the residual time until the next successful visit for the ith packet arriving at an empty queue at S_j. Finally we let $N_j(t)$ and $T_j(t)$ be the number of successful visits and transmissions, respectively, at S_j up to t. The assumption in [6] that the service times are asymptotically stationary is in the present context replaced by the following assumption.

Assumption 1. For all stations are connected to the ring at the same position, we assume that

- for all $j = 1,2,\ldots,n$ $\{C^*_{j,i}, i = 1,2\ldots\}$, $\{V_{j,i}, i = 1,2\ldots\}$, $\{(C_{j,i},Q^*_{j,i}), i = 1,2\ldots\}$ and $\{RC_{j,i}, i = 1,2\ldots\}$ are asymptotically stationary, with finite limiting expectations EC^*_j, EV_j, EC_j, $q_j \equiv EQ^*_j$ and ERC_j,
- the visiting rate of available slots at S_j and the transmission rate of packets from S_j, defined by $\lim_{t\to\infty} EN_j(t)/t$ and $R_j \equiv \lim_{t\to\infty} ET_j(t)/t$, exist and are equal to $1/EC_j$ and q_j/EC_j, respectively.

According to proposition 2 we can restrict ourselves to the situation in which all stations are connected to the ring at the same position. In [12] a general sufficient condition for substability has been formulated, from which it may be derived that, provided assumption 1 is true,

$$\lambda_j EC^*_j < 1, \text{ for every } j = 1,2,\ldots,n \qquad (10)$$

is sufficient for $\{Q(t), t \geq 0\}$ to be substable. Although the proof in [12], which is based on a combination of results of [6] and [7], seems to contain heuristic elements, see also [11], we shall take the validity of the result for granted. In order to check (10), first notice that assumption 1 allows the application of Little's formula on the first queueing position at a station. It is easily verified that EC_j, EC^*_j, q_j and EV_j satisfy the following relation

$$EC_j = q_j EC^*_j + (1-q_j)EV_j, \; j = 1,2,\ldots,n. \qquad (11)$$

Using (11) we will show that

$$\lambda_j EC_j < 1 \text{ for every } j = 1,2,\ldots,n, \qquad (12)$$

implies (10) and is therefore sufficient for substability of $\{ Q(t),\ t \geq 0\}$.

If $q_j=1$ we directly find using (11) that $\lambda_j EC_j^* = \lambda_j EC_j < 1$. So, suppose $q_j < 1$. Consider the stationary probability p_j that the queue at S_j is empty at an arbitrary moment between two successful visits at S_j. By the fact that Poisson arrivals see time averages and subsequent application of Little's formula to the first waiting position of the queue at S_j we get

$$p_j = (1-p_j)\lambda_j ERC_j + p_j\lambda_j EC_j^*,\ j = 1,2,\ldots,n.$$

Now because $0 < p_j \leq q_j < 1$ and $ERC_j \geq \tau/(2c) > 0$ we conclude that $\lambda_j EC_j^* < 1$.

Using proposition 1, it follows that under assumption 1, (12) is sufficient for $\{ Q(t),\ t \geq 0 \}$ to be τ-stable. This allows us to formulate and prove the following proposition.

Proposition 6. Under assumption 1,

$$\rho_i + \sum_{j=1}^{n} \sum_{k=j}^{b(i)}{}_{o}\, p_{kj}\rho_k < 1, \text{ for } i = 1,2,\ldots n,$$

implies that $\{ Q(t),\ t \geq 0\}$ is τ-stable.

Proof. Focus on an arbitrary station S_i. For the transmission rate R_i of packets at S_i one has

$$R_i \leq \lambda_i,\ i = 1,2,\ldots,n. \qquad (13)$$

Now at S_i slots arrive with rate c/τ of which the slots with packets addressed to S_j, $j = 1,2,\ldots,n$ are not available. For any $j = 1,2,..,n$ the rate of slots visiting S_i that are unavailable because they contain a packet addressed to S_j is

$$\sum_{k=j}^{b(i)}{}_{o}\, p_{kj}R_k$$

Therefore the rate of available slots arriving at S_i is:

$$1/EC_i = c/\tau - \sum_{j=1}^{n} \sum_{k=j}^{b(i)}{}_{o}\, p_{kj}R_k,\ i = 1,2,\ldots n.$$

Using (13) we find

$$1/EC_i \geq c/\tau - \sum_{j=1}^{n} \sum_{k=j}^{b(i)}{}_{o}\, p_{kj}\lambda_k,\ i = 1,2,\ldots n,$$

from which it follows that

$$EC_i \leq \left[c/\tau - \sum_{j=1}^{n} \sum_{k=j}^{b(i)}{}_{o}\, p_{kj}\lambda_k \right]^{-1}, i=1,2,\ldots,n.$$

As argued above we must prove that $\lambda_i EC_i < 1$, for which it suffices that

$$\lambda_i\left[c/\tau - \sum_{j=1}^{n} \sum_{k=j}^{b(i)}{}_{o}\, p_{kj}\lambda_k \right]^{-1} < 1\ ,\ i=1,2,\ldots,n$$

which can be rewritten as

$$\rho_i + \sum_{j=1}^{n} \sum_{k=j}^{b(i)}{}_{o}\, p_{kj}\rho_k < 1\ ,\ i=1,2,\ldots,n,$$

which was assumed. □

Using proposition 6 we find that (8) and (9) are also sufficient for τ-stability in the special cases studied in section 4. However, in general we have to be careful since assumption 1 may not be justified, as shown below.

If $p_{ii} = 1$, $i = 1,2,\ldots,n$, then the time between two consecutive successful visits at a queue is bounded, for it can never be larger than $(n+1)\tau$. This implies that all processes of assumption 1 are substable.

If $p_{ii} = 0$, then there are cases for which assumption 1 is not justified, since the involved processes are not substable. By choosing the destination probabilities in a suitable manner it is possible that a station starts monopolising the ring. This may be done by always addressing packets to the first upstream station, so that the station that filled the slot always receives the slot back empty. If we assume $p_{ij} = (n-1)^{-1}$, $i,j = 1,2,\ldots,n$, $i \neq j$, then this case only occurs if $n = 2$. In implementations of the *Orwell* protocol a counter mechanism is provided to avoid this phenomenon, cf. [10] and [14].

Remark 3. Recall that proposition 2 was concerned with small shifts in the system without loosing τ-stability. During the shift a station was not allowed to overtake any other station. For the Orwell protocol we can give an example that the order of the stations indeed matters. Suppose the symmetric destination pattern described above and $n=4$. When we check (9) we find that under assumption 1 $\{ Q(t),\ t \geq 0\}$ is τ-stable if $\rho_1=\rho_3=0.5$ and $\rho_2=\rho_4=0.05$ but not if $\rho_1=\rho_2=0.5$ and $\rho_3=\rho_4=0.05$.

6. Discussion.

In this paper we made mathematically precise what kinds of stability of stochastic processes are relevant to queueing models of local area networks with slotted ring protocols. A new model description was given containing two known models as special cases. We also proved both known and new conditions for τ-stability. For the sufficient conditions we found it inevitable to make assumption 1, of which we suspect that it is justified in most cases.

According to [12] and [9] the arrival process at each station may be generalised to any renewal arrival process, such that $\{V(t),\ t \geq T\}$ does not depend on the arrival process before T. Therefore we may suppose that messages containing a variable number of packets arrive at S_i according to a Poisson process with rate λ_i, where the number of packets in a message is a random variable with a geometric distribution. Denote by B_i the number of packets in a generic batch at S_i and define $\rho_i = \lambda_i EB_i \tau / c$.

If we assume that B_i has an arbitrary distribution, then the necessary conditions in this paper remain valid, although the state spaces of the Markov chains involved may not contain all possible queue length combinations, cf. [3]. The sufficient conditions can not directly be extended because the packet arrival process should be a renewal process, cf. [12].

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