The influence of viscothermal effects on calibration measurements in a tube

J.W.van Honschoten, H-E.de Bree, F.J.M.van Eerden*, G.J.M. Krijnen

MicroMechanical Transducers group, MESA Research Institute, *Dep. Mechanical Eng.

University of Twente P.O. Box 217, 7500 AE Enschede, The Netherlands.

Email: j.w.vanhonschoten@el.utwente.nl

Abstract

The Microflown, a novel acousic flow sensor that is capable of measuring particle velocity in a fluid, can easily and accurately be calibrated in a standing wave tube. This calibration method in the standing wave tube, is generally most favourable. Even very low frequencies can be measured with this simple set-up. It is therefore of importance to analyse the limits of validity of the calibration method. Here the effects of the viscous and thermal properties of the fluid on the measurements of the transfer function, when calibrating sensors, are considered. It is shown for a broad frequency spectrum, both theoretically as experimentally, that these viscothermal effects are relatively small.

1. Introduction

For the calibration of the Microflown, one of the most useful and simple ways is performing measurements in a standing wave tube.

Since reference particle velocity microphones do not exist, the main concern calibrating the Microflown is involved with the realisation of a known particle velocity with respect to the probe. Measuring the sound pressure in a configuration in which the specific acoustic impedance is known solves this problem. The particle velocity is calculated by dividing the sound pressure by the specific acoustic impedance.

Three possible methods to measure the transfer function of the Microflown are the anechoic room, the long calibration tube and the (relatively short) standing wave tube.

The anechoic room consists of an environment with a simple sound source and non-reflecting walls, approaching free field conditions. The method is easy to use and reliable but it is a very expensive set-up. It can be used in a $50 - 20kHz$ bandwidth.

A low cost solution to approach free-field measuring conditions is to measure in a long tube by means of time frame measurements. The practical effective bandwidth is limited by diameter and length of the tube and is found to be several kilohertz.

In a standing wave tube the relationship between the reference pressure (microphone) and the particle velocity is a bit more complex, but this set-up is small and low cost and therefore most favourable. It is possible as well to calibrate pressure microphones using these standing wave tubes. By the use of two standing wave tubes of different lengths and diameters, a bandwidth of $10Hz$ up to $10kHz$ can be achieved, so calibration at even very low frequencies can be established. Because of the importance of this calibration method, for both very low as high frequencies, the possible influence of viscothermal effects on the calibration should be analysed.

2.1 The transfer function between measured signal and reference pressure

For the calibration of the Microflown and the determination of its frequency dependent sensitivity, generally a standing wave tube is used [1,3].

A Microflown is placed in a rigidly terminated tube with at one end a reference microphone, and at the other end a load speaker generating a broad frequency spectrum. See Fig.1.

The measured transfer function $\frac{u}{p_{ref}}$, where u is the measured particle velocity, the signal of the Microflown, and p_{ref} the pressure measured by the reference microphone, gives quantitative and qualitative information about the sensitivity of the Microflown. This transfer function is frequency dependent, and is slightly influenced by viscothermal effects of the air in the tube. This is seen by considering the coefficients related to the viscous and thermal effects in the equations for the wave propagation.

For harmonic excitation, one solution that will satisfy the wave equation is given by the superposition of the complex pressures associated with a plane wave travelling in positive direction and one in negative direction. The plane wave approximation is allowed for frequencies lower than the cut-off frequency.

The wave propagation in the tube becomes one-dimensional and the acoustic pressure p can be expressed as

$$p(x,t) = (\overline{p}_A e^{\Gamma kx} + \overline{p}_B e^{-\Gamma kx})e^{i\omega t} \tag{1}$$

where x is the distance along the tube, k the wave number, ω the frequency of the sound wave and Γ the propagation coefficient. The complex amplitudes of the forward and backward propagating waves are $\overline{p}_B$ and $\overline{p}_A$. The transfer function of two fixed microphones in a standing wave tube was already analysed by Chung and Blaser [2] in 1980. A variety of literature on the viscothermal wave propagation has been published. Tijdeman [4] and Beltman [5] give a complete overview of the different analytical solutions. Here it is shown that the approach of Zwikker and Kosten (1949) leads to an efficient and accurate solution. It was assumed that the pressure is constant across the tube cross-section, and effects of inertia, compressibility, viscosity and thermal conductivity of the fluid were included as well. For cylindrical tubes, the propagation coefficient Γ reads, according to Zwikker and Kosten,

$$\Gamma = \sqrt{\frac{\gamma}{n(s\sigma)} \frac{J_0(i\sqrt{i}s)}{J_2(i\sqrt{i}s)}} \tag{2}$$

with n a polytropic coefficient characterising the thermodynamic processes in the tube:

$$n = \left(1 + \frac{\gamma - 1}{\gamma} \frac{J_0(i\sqrt{i}s\sigma)}{J_2(i\sqrt{i}s\sigma)}\right)^{-1} \tag{3}$$

Here J_0 and J_2 are respectively the zeroth and second Bessel function of the first kind, $\gamma = \frac{C_p}{C_v}$ the ratio of specific heats, $s = R_e\sqrt{\frac{\omega\rho_0}{\mu}}$ the shear wave number and $\sigma = \sqrt{\frac{\mu C_p}{\lambda}}$ the square root of the Prandtl number.

(R_e represents the equivalent radius of the tube: $2 \times \frac{area}{perimeter}$, ρ_0 the density of the gas, μ the dynamic viscosity and λ is the thermal conductivity coefficient.)

The mentioned viscous and thermal effects can be of significant influence on the wave propagation. For arbitrary cross-sectional shapes the propagation coefficients have already been derived. An important parameter in these models is the shear wave number, $s = R_e\sqrt{\frac{\omega\rho_0}{\mu}}$, which is a measure for the ratio of inertial and viscous effects. A small shear wave number indicates that viscous effects are dominant, in which case the velocity distribution over the cross-section of the tube approaches a Poiseuille flow, whereas a large shear wave number corresponds to a plane wave-profile. Here, for $s << 1$, a tube is called 'narrow', for $s >> 1$ it is called 'wide'.

For 'wide' tubes, eq. 2 can be approximated by the solution of Kirchhoff,

$$\Gamma = i + \frac{1+i}{\sqrt{2}} \left(\frac{\gamma - 1 + \sigma}{s\sigma}\right) \tag{4}$$

If viscothermal effects are neglected, $\Gamma = i$ (Γ is purely imaginary), in general the propagation coefficient is complex, $\Gamma_{re} + i\Gamma_{im}$; the attenuation of the wave is determined by Γ_{re}, and $\frac{c_0}{\Gamma_{im}}$ becomes its phase velocity.

For determining the transfer function $\frac{u}{p_{ref}}$, an expression for the velocity u is needed. With eq. 1 , and applying the linearised momentum equation in only one dimension, $\rho\frac{\partial u(x,t)}{\partial x} = -\frac{\partial p}{\partial t}$, this velocity is

$$u(x) = \frac{-i}{\Gamma\rho_0 c_0}(\overline{p}_A e^{\Gamma kx} - \overline{p}_B e^{-\Gamma kx})e^{i\omega t} \tag{5}$$

By applying the boundary conditions at $x = 0$ and $x = l$, the amplitudes of the forward and backward travelling waves, $\overline{p}_A$ and $\overline{p}_B$, can be derived. These boundary conditions are (ζ is the dimensionless acoustic impedance)

$$\begin{aligned} p(0) &= p_0 \qquad\qquad for \quad x = 0, \\ \zeta(L) &= \frac{1}{\rho_0 c_0}\frac{p(l)}{u(l)} \quad for \quad x = l. \end{aligned} \tag{6}$$

The required transfer function is therefore

$$\frac{u}{p_{ref}} = \frac{1}{\rho_0 c_0}\left(\frac{\cosh\Gamma kL}{\zeta} + \frac{i\sinh\Gamma kL}{\Gamma}\right) \tag{7}$$

where L is the distance between the sensor at place x and the reference probe at the end of the tube; $L = x - l$.

For an acoustically hard wall at the end $x = l$ of the tube, (which is the case) $\zeta(l) = \infty$. If additionally the viscous and thermal effects can be neglected, the function is simplified to $\frac{u}{p_{ref}} = \frac{i}{\rho_0 c_0}\sin(kL)$. In this situation, the measured transfer function will only be determined by the sensitivity characteristics of the Microflown and yields the required information about its frequency dependent behaviour. This behaviour is rigorously analysed in [1,3]. So, if viscothermal effects are neglected, the ratio $\frac{u}{p_{ref}}$ equals $\frac{i}{\rho_0 c_0}\sin(kL)$, and since the effects will turn out to be not very large, this ratio will approximately behave as a sine function of (kL), i.e. of $\frac{\omega}{c}(x - l)$. The phase shift between the velocity u and the reference pressure p_{ref} then equals plus or minus 90 degrees.

Measuring the transfer function as a function of frequency, results as in Fig.2 are found. By varying the distance $L = x - l$, the distance between maxima and minima is altered.

In a standing wave tube of length l=98cm and with a 9cm diameter, a Microflown is placed at a position $x = 0.5l$.

To avoid a mechanical transfer function between the loudspeaker and the sound probes, the loudspeaker was not mounted rigidly to the tube.

At the minima it is not possible to calibrate the Microflown. To overcome this problem the tube is used in two ways. The reference microphone can be mounted on both sides of the tube. Furthermore, the entrance-mounting in the tube for the probe is made not symmetrical. So by altering the position of the loudspeaker and the microphone, the distance $(x - l)$ is varied from $20cm$ to $50cm$. The results of the altering are shown in Fig 2.

In Fig.3 the phase difference between u and p_{ref} is shown. As stated, idealiter this phase difference shifts between $+90^0$ and-90^0. The extra phase difference that occurs because of the Microflown itself and the phase lag of the preamplifier, is obvious from the measurements in the anechoic room.

2.2 Influence of the viscothermal effects

If subsequently the influence of the viscothermal effects is taken into consideration, for Γ should be written $\Gamma = \Gamma_{re} + i\Gamma_{im}$, and if the approximation of Kirchhoff, eq. 4, is applied, for the real and imaginary part of the propagation coefficient can be written

$$\Gamma_{re} = \frac{\sqrt{2}}{2}\left(\frac{\gamma - 1 + \sigma}{R_e\sqrt{\frac{\rho_0 C_p}{\lambda}}}\right)\frac{1}{\sqrt{\omega}}, \qquad (8)$$

$$\Gamma_{im} = 1 + \frac{\sqrt{2}}{2}\left(\frac{\gamma - 1 + \sigma}{R_e\sqrt{\frac{\rho_0 C_p}{\lambda}}}\right)\frac{1}{\sqrt{\omega}}.$$

For convenience, the factor $\frac{\sqrt{2}}{2}\left(\frac{\gamma-1+\sigma}{R_e\sqrt{\frac{\rho_0 C_p}{\lambda}}}\right)$ is replaced by a coefficient, e.g. by a, so that the transfer function $\frac{1}{\rho_0 c_0}\frac{i\sinh\Gamma kL}{\Gamma}$ becomes

$$\frac{u}{p_{ref}} = \frac{1}{\rho_0 c_0}\frac{i\sinh\Gamma kL}{\Gamma} = \frac{1}{\rho_0 c_0}\frac{\cos((1+\frac{a}{\sqrt{\omega}})kL)\sinh(\frac{a}{\sqrt{\omega}}kL) + i\sin((1+\frac{a}{\sqrt{\omega}})kL)\cosh(\frac{a}{\sqrt{\omega}}kL)}{\frac{a}{\sqrt{\omega}} - i(1+\frac{a}{\sqrt{\omega}})} \qquad (9)$$

of which the modulus is

$$\left|\frac{u}{p_{ref}}\right|(\omega) = \frac{1}{\rho_0 c_0}\sqrt{\frac{\cos^2((1+\frac{a}{\sqrt{\omega}})kL)\sinh^2(\frac{a}{\sqrt{\omega}}kL) + \sin^2((1+\frac{a}{\sqrt{\omega}})kL)\cosh^2(\frac{a}{\sqrt{\omega}}kL)}{(1+\frac{a}{\sqrt{\omega}})^2 + \frac{a^2}{\omega}}} \qquad (10)$$

where it should be realised that $k = \frac{\omega}{c}$.

As stated above, for negligibly small thermal and viscous effects, the transfer function is simply $\frac{u}{p_{ref}} = \frac{i}{\rho_0 c_0} \sin(kL)$. The modulus of this function, which is one of the measured quantities in the experiment, equals $\frac{1}{\rho_0 c_0}$, so if the sensitivity of the flow sensor is plotted as a function of frequency in a calibrating experiment, the maxima are found at frequencies for which $kL = n\frac{\pi}{2}$ (with n integer).

These maxima constitute an envelopping function that yields the required information about the frequeny dependence of the sensitivity.

It is therefore useful to compare these maxima with the maxima of the corresponding transfer function, eq. 10, for which the viscothermal effects are not negligible. This transfer function, eq. 10, divided by $\frac{1}{\rho_0 c_0}$, so actually normalised to 1, can be calculated for various geometries of the standing wave tube and different gas parameters.

Assuming the realistic values of $\gamma = 1.4$, $\sigma = 0.845$, $C_p = 1.0\ 10^3 J\ kg^{-1} K^{-1}$ and a gas density $\rho_0 = 1.2\ kg\ m^{-3}$, thermal conductivity $\lambda = 24\ 10^{-3} W\ m^{-1} K^{-1}$ and dynamic viscosity $\mu = 17.1\ 10^{-6} Pa\ s$, and subsequently taking a circular tube with a diameter of 12 mm, the correction or attenuation in dB of the transfer function, eq. 10, is plotted as a function of frequency f ($\omega = 2\pi f$) in figure 4. It is seen that only for low frequencies, ω below ca.$100Hz$, and these dimensions of the tube, the viscothermal effects can be of significant influence on the measured sensitivity of the sensor and can reduce this with about 10 or 20 %; ca. 1 or 2 dB deviance (i.e. an attenuation).

The viscothermal influence occurs particularly for low frequencies, and the assumed small diameter (and therefore small equivalent radius R_e) of $12mm$. For larger R_e, and higher frequencies, the real component of Γ becomes very small, and the imaginary component approaches 1, see eq. 8. It can be argued, that for the parameter values for which there is a significant effect, i.e. small R_e, small ω and large Prandtl number (σ^2), the condition $s >> 1$ is not satisfied and therefore the Kirchhoff approximation, eq. 4, can not be applied. It would be better to analyse the problem for small s by applying eq. 2 and eq. 3. Calculating in this way Γ_{re} and Γ_{im} for the same parameter values as mentioned above, for Γ_{re} and Γ_{im} the functions plotted in figure 5a are found. They are plotted as a function of the logarithmically represented frequency.

In figure 5b, the value of Γ_{im} is plotted for the mentioned air parameters and a tube radius $R_e = 1.0$, 5.0 and $10cm$ respectively. It is seen that for small radii, this value differs significantly from 1. This means that, in the low-frequency region, for small tube diameters the influence of viscothermal effects can be considerable; from ca.1 to 4% for $40Hz$.

It can therefore be concluded that for the calibration experiments of the Microflown in the standing wave tube, the amplitude of the transfer function is only slightly influenced by viscous and thermal effects. In particularly in the low-frequency calibration however, the viscothermal effects can play a role if small tube diameters are used.

2.3 The influence on the phase of the transfer function

The phase of the transfer function is influenced as well. This is illustrated by figure 5. If the propagation coefficient is purely imaginary, $\Gamma = i$, so that $\frac{u}{p_{ref}} = \frac{i}{\rho_0 c_0} \sin(kL)$, the phase between u and p_{ref} is always $\pm\frac{\pi}{2}$. If Γ has a small real part, determined by the parameters s and σ, see eq.4, this phase of the transfer function is not exactly $\pm\frac{\pi}{2}$ anymore. The phase is then calculated from

$$\phi_{u,p} = \arctan \frac{\mathrm{Im}(\frac{i}{\Gamma} \sinh \Gamma k L)}{\mathrm{Re}(\frac{i}{\Gamma} \sinh \Gamma k L)} \tag{11}$$

where for 'wide' tubes the term $\frac{i}{\Gamma} \sinh \Gamma k L$ is obtained from eq.9. For a value of the propagation factor of $\Gamma = \frac{0.001}{\sqrt{\omega}} + i(1 + \frac{0.001}{\sqrt{\omega}})$, i.e. a small value of $a = 0.001$ is assumed (see eq.8 and9), this phase is plotted is figure 5. For a larger value of a, the function behaves more like the dotted function. This behaviour is partly explained if the Kirchhoff approximation is used to assume a propagation factor of the form $\Gamma = \frac{a}{\sqrt{\omega}} + i(1 + \frac{a}{\sqrt{\omega}})$ in eq.11 and the phase is evaluated.

The ideal phase characteristic of $\frac{i}{\rho_0 c_0} \sin(kL)$, when no viscothermal effects are present, shifts between $\frac{\pi}{2}$ and $-\frac{\pi}{2}$at the frequency values $\omega = n\pi\frac{c}{L}$ $(n \in \mathbf{N})$.

Since small values of a are assumed, a first order approximation can be used to see that these frequencies are slightly shifted to values of ω for which $(1 + \frac{a}{\sqrt{\omega}})\frac{\omega}{c} L = n\pi$. Evaluating these ω, the frequency shift is calculated as $\Delta\omega = \frac{a^2}{2} - \frac{a}{2}\sqrt{a^2 + 4\pi\frac{c}{L}}$, which can even further be simplified, for very small values of a, to $\Delta\omega = -a\sqrt{\pi\frac{c}{L}}$. It is seen in figure 6, that for increasing a, the function not only shifts more to the left, but also differs more from the ideal phase characteristic, by its 'rounded' angles.

Comparing Fig.6 to Fig. 3, the influence of viscothermal effects on the phase of the transfer function are, contrary to its amplitude, considerable and can be experienced in measurements.

This also implies that if the distance L can be measured accurately, so that $\omega = \pi\frac{c}{L}$ is well known, a value of a and therefore of Γ_{re} and Γ_{im} can in principal be deduced from the shift of the function.

Concluding, implications for an accurate calibration measurements of the Microflown are requirements for the dimensions of the calibration set-up. To reduce the influence of thermodynamic and viscous effects in the standing wave tube, s and σ should be increased: Γ_{re} should be made as small as possible and Γ_{im} close to 1. Since many material parameters, like ρ_0, λ,and γ, can hardly be influenced, one should increase the (equivalent) radius R_e of the tube. For low frequencies in particular, this gives a minimum limitation for the radius.

Since the propagation factor Γ appears in the form ΓkL in the numerator of the transfer function, it is also important to choose the length L not to large. On the other hand, L should be larger than about 1/8 wavelength of the acoustical wave for an accurate measurement, and as a rule of thumb, the total length of the tube must be twice as large. In other words: $\frac{L}{c} \geq \frac{1}{8f}$.

These conclusions lead to practical implications for the possible dimensions of the calibration set-up.

Since the cut-off frequency for a tube of diameter d ($d = 2R_e$) equals $f_c = \frac{0.59c}{d}$, the chosen diameter determines the maximum frequency that can be measured as well. A diameter of $15cm$, for example, implies that this maximum frequency is about $1.3kHz$.

Practical values that can be opted for are: a tube of 8 meters in length (in which L is about $4m$) and $15cm$ in diameter, for a bandwidth of $10Hz$ up to ca. $1kHz$. In a bandwidth of $100Hz$ to $4kHz$, a tube of 1 meter length and a diameter of $4cm$ is used; and for the calibration of the Microflown element itself a tube of $40cm$ and a diameter of $12mm$ is preferable. The usable bandwidth of the latter tube is $500Hz$ up to more than $10kHz$.

3. Measurements

Calibration measurements of the Microflown were performed in standing wave tubes of different lengths and diameters. These results were compared to measurements of the sensor in the anechoic room. In the latter measurements, viscothermal effects can be practically neglected, and indeed the results of the measurements in the anechoic room can be described very well by the model of the behaviour of the Microflown, which is almost a first-order frequency behaviour [6].

For a bandwidth of 10Hz to 1KhZ, a standing wave tube of 8 meter length and 15 cm diameter was used; for the frequency range from 100Hz to 4kHz length and diameter were respectively 90cm and 4cm; for 500Hz-16kHz the length was 60cm and the diameter 12mm. In Fig.7 all these results were combined to one transfer function that could be compared to the anechoic function. It is seen that for relatively high frequencies, more than about 2kHz, the results in the standing wave tube give a small overestimation, which seems to be caused by the viscothermal effects, and agree with the theory both qualitatively as quantitatively (see Fig.4). The underestimation for low frequencies occurs indeed, although it is rather small. It is therefore debatable of this deviance is certainly due to the viscothermal effects. Over all, the theory about the viscothermal effects seems to describe the results rather accurately; the deviance does not become large in the measured bandwitdhs.

4. Conclusions

It was shown that for calibration measurements of the Microflown in a standing wave tube, viscothermal effects due to fluid properties and the used set-up, have to be considered. Generally, these effects turn out to be relatively small, even for low frequencies. In particular for these low frequencies the Microflown is an accurate and low-cost flow sensor, that can be calibrated well in such a tube.

The transfer function of the sensor was deduced, in which the propagation factor Γ, representing the viscothermal effects, was included. To reduce the influence of thermodynamic and viscous effects in the standing wave tube, the shear wave number s and the Prandtl number (σ^2) should be increased, which implies that the radius of the tube should be chosen not to small. This is mainly important for the low frequency spectrum. On the other hand, the length of the tube L should be taken as small as possible, provided that the quotient L/c remains larger than the frequency ω to be measured.

Although the viscothermal effects on the modulus of the transfer function are small, they are well visible in the shape of the phase characteristic.

In the experiments it is shown that reducing the radius of the tube for very low frequencies may increase the viscothermal influences, but that over all they are small. In the measured phase characteristic of the transfer function these influences are well observed. In a large frequency spectrum (ca. 40 to $4 \cdot 10^3 Hz$) theory is verified by measurements.

For the broad frequency spectrum, from ca. $10Hz$ to $5kHz$, calibration of the sensor using the tube is very well possible.

Acknowledgements

The author would like to thank the Dutch Technology Foundation STW for its financial support.

References

[1] H -E. de Bree et al., Realisation and calibration of a novel half inch p-u sound intensity probe, 106th AES Munchen, 1999.

[2] Chung, J.Y., Blaser, D.A. Transfer function method of measuring in- duct acoustic properties. I. Theory, J. of the Acoust. Society
of America, 68(3), 907-913 (1980)
[3] Bree, H-E. de, Leussink, P.J., Korthorst, M.T., Jansen, H.V., Lammerink, T., Elwenspoek, M. The Microflown: a novel device
measuring acoustical flows, Sensors and Actuators A, 54, 552-557 (1996)
[4] Tijdeman, H. On the propagation of sound waves in cylindrical tubes, J. of Sound and Vibration, 39(1), 1-33 (1975)
[5] Beltman, W.M. Viscothermal wave propagation including acousto- elastic interaction, part I: theory, J. of Sound and Vibration
227(3), 555-586 (1999)

[6] V.B. Svetovoy, I.A. Winter, 'Model of the $\mu-$flown microphone', Yaroslavl State University, Yaroslavl 150007 and 150000, Russia (1999)

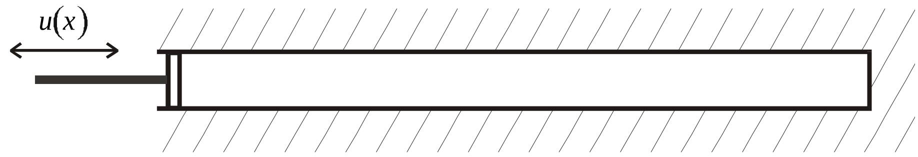
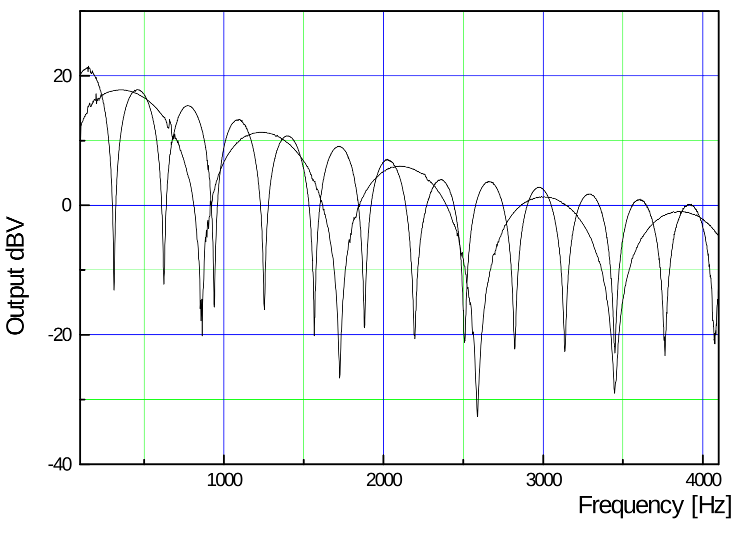


Fig.1 A tube that is rigidly terminated at x=l and in which the fluid is driven by a vibrating piston at x=0.



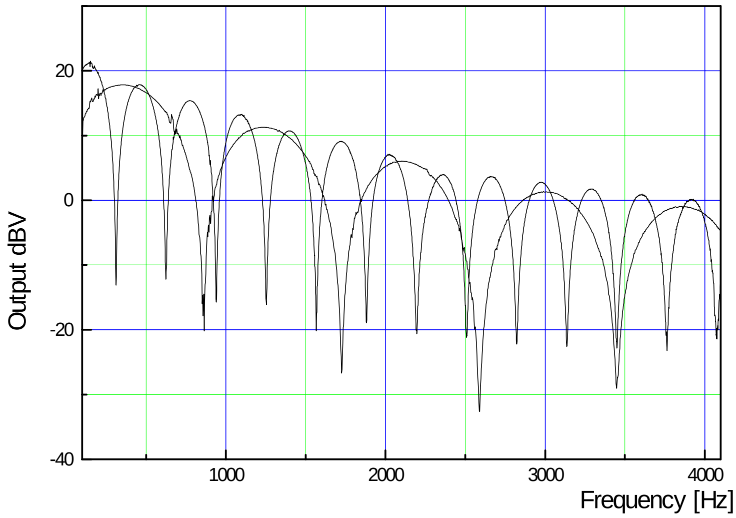


Fig.2 Measurement in the tube. By turning the tube (the reference microphone and the loudspeaker change in position) the distance $L = x - l$ is varied.

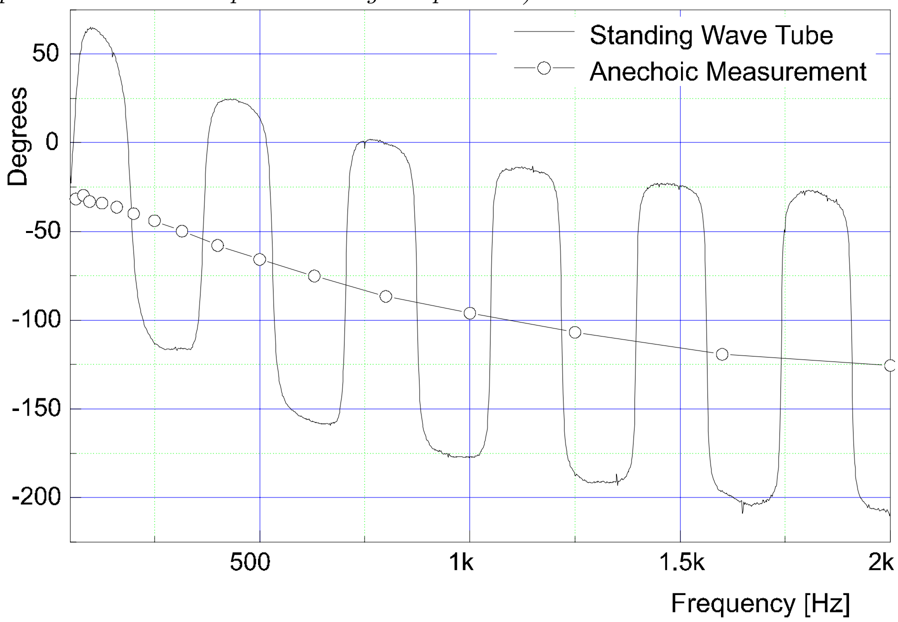


Fig. 3: Phase difference between u and p_{ref}. Solid line corresponds to measurements in the standing wave tube; line with circles is measured in the anechoic room.

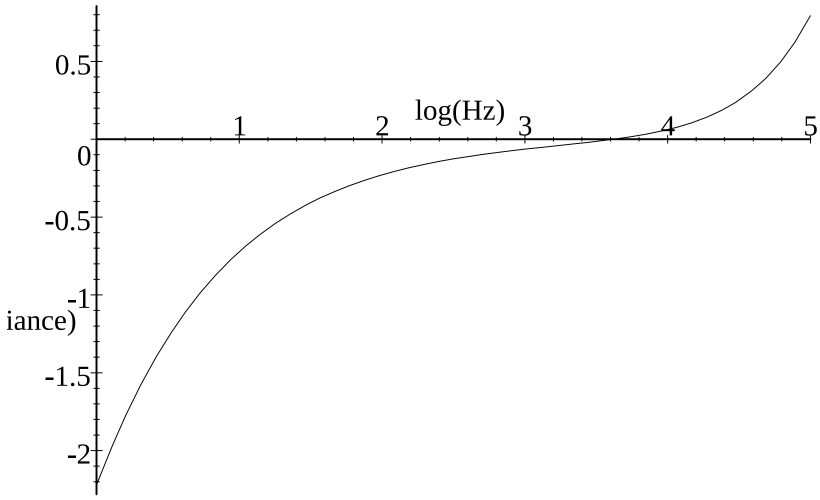


Fig.4 Deviance of the transfer function $\frac{u}{p_{ref}}$*, in dB, as a function of frequency f. (Assumed parameter values mentioned in the text)*

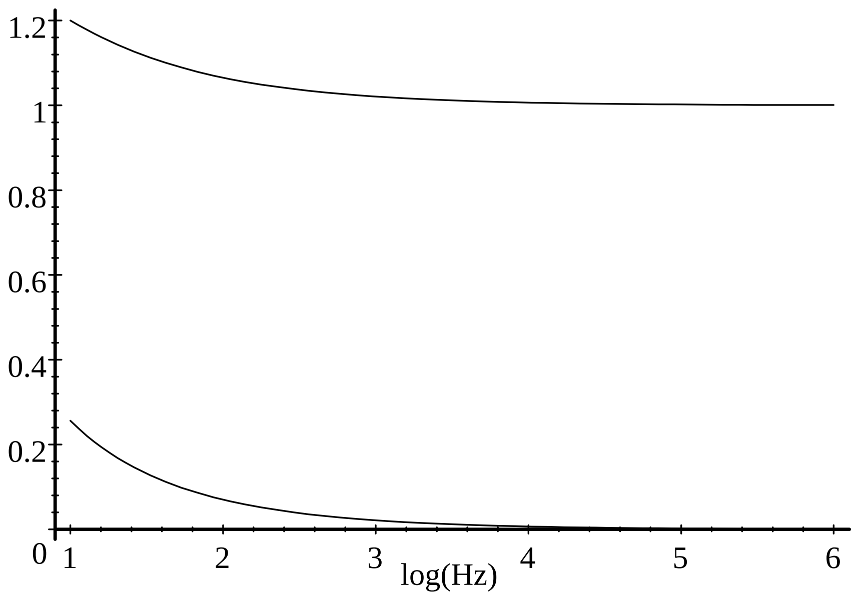


Fig.5a Value of Γ_{im} *(dotted) and* Γ_{re}*for a tube radius of 12 cm.*

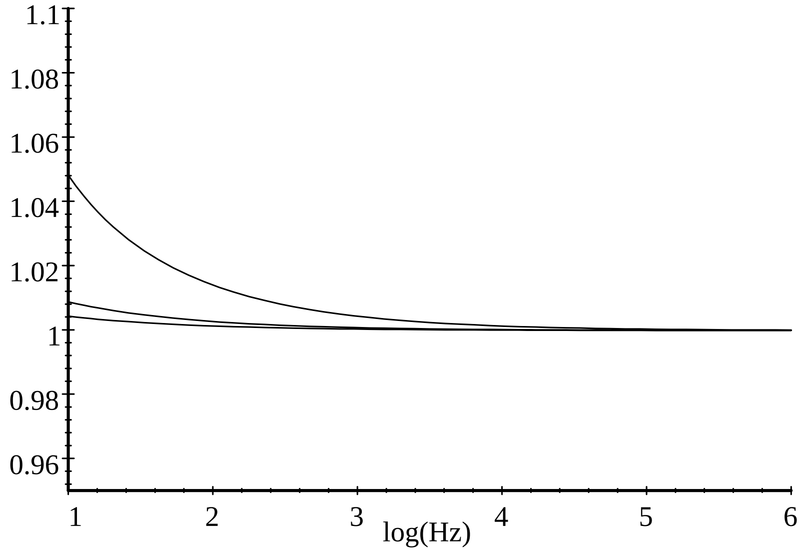


Fig.5b Value of Γ_{im}*for a tube radius of 1(upper curve),5 and 10 cm.*

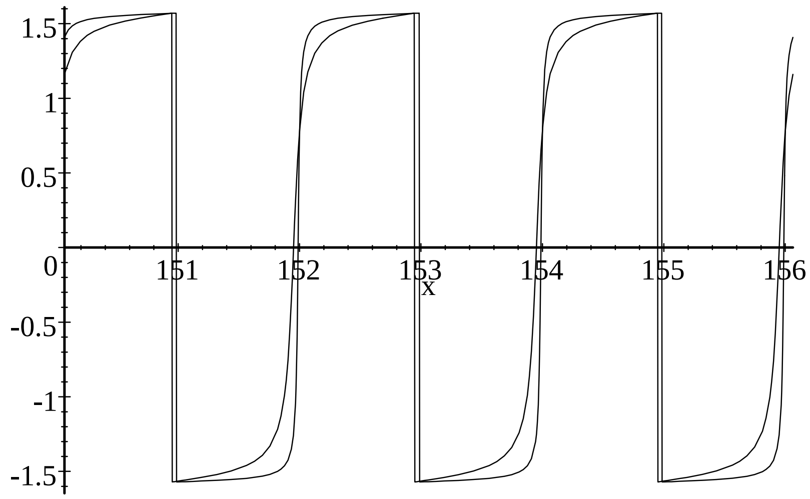


Fig.6 Phase characteristic of the transfer function $\frac{u}{p_{ref}}$*, for increasing values of the parameter a (see text).*

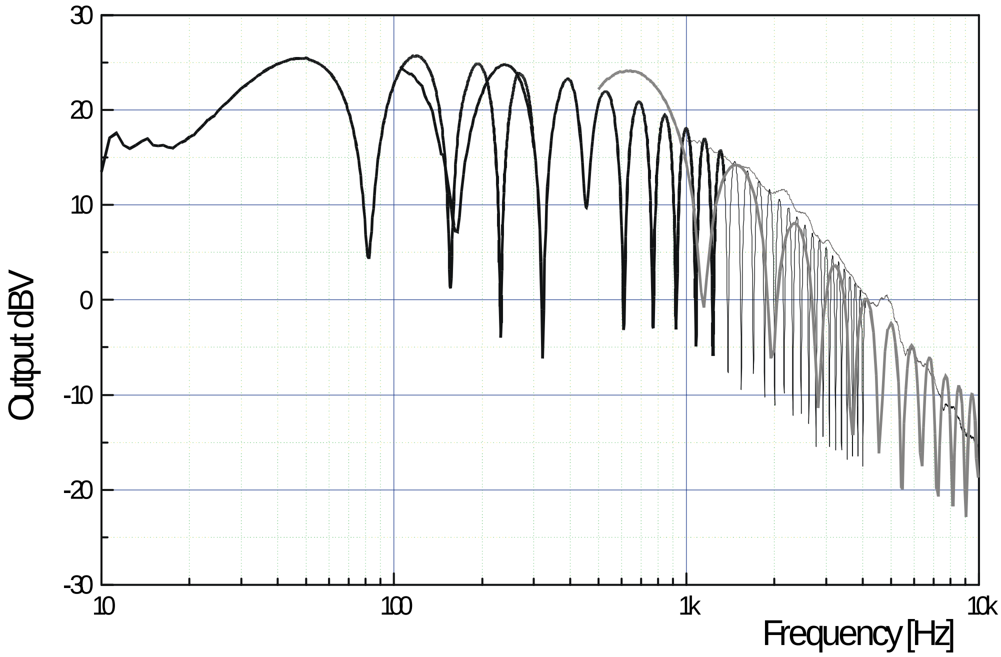


Fig.7 Measured transfer function in different tubes, compared to measurements performed in the anechoic room (see text)