Robustness of ToF estimators – an Empirical Evaluation

F. van der Heijden[1], P.P.L. Regtien[1] and T.S.Beneder[1]

[1]University of Twente, Faculty EEMI, Laboratory for Measurement and Instrumentation, PO Box 217, 7500 AE Enschede, The Netherlands
email: F.vanderHeijden@el.utwente.nl http://www.mi.el.utwente.nl/

Summary: *The performance of ToF estimators for acoustic tone bursts is empirically evaluated. In indoor applications, the observed waveform is likely to be disrupted by multiple echoes. These echoes can cause complex interference patterns. The paper presents the results of a comparison study of the robustness of various ToF estimators against such type of disruptions.*

Keywords: *ultrasonic position measurement, time-of-flight, performance evaluation, multiple reflections.*
Category: *1 (General, theoretical and modeling)*

1 Introduction

The determination of the time of flight (ToF) of an acoustic tone burst is a key issue in position and distance measurement systems. In reflective environments, e.g. indoor applications, the ToF measurement is often difficult. Multiple echoes cause complex interference patterns. In the example of Fig 1, the second peak of the interference pattern is larger than the first peak [1]. Most methods for ToF estimation rely on simple models where the shape of the observed waveform is assumed to be known in advance. The only parameters that are considered as unknown are the magnitude and the Tof. However, the occurrence of interfering echoes breaks down the validness of these models, and the methods may suffer from performance degradation.

This paper studies the robustness of ToF estimators, i.e. their ability to cope with waveforms that are disrupted by multiple echoes. The aim of this paper is to empirically evaluate this quality aspect of the various methods.

Some literature exists that addresses the performance evaluation of ToF estimators [2], [3]. The usual performance criterion is the RMS plotted against the SNR of the observed waveform, thus providing information about the noise sensitivity of the estimators. However, these evaluations do not address the problem of having multiple echoes, while the errors caused by multiple echoes can be much larger than the error caused by noise.

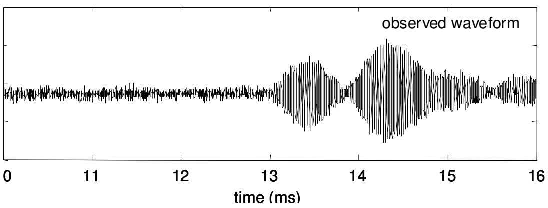

Fig. 1. An example of an observed waveform.

2 ToF estimators considered

The tone burst that we consider consists of a few cycles of a sinoidal wave. The frequency of the sine wave is the carrier frequency. The envelope of the observed waveform, in the absence of multiple echoes and noise, is a smooth function (for which different models have been proposed in the literature).

The ToF estimators that we evaluate are tabulated in Table 1. Some methods use the envelope of the waveform. We use quadruple filtering and Rice's representation to calculate it. This method is considered as optimal [4]. The covariance model based method is a new method presented in [5]. It uses a filter bank of 7 correlators whose outputs are combined so as to form the log-likelihood of the ToF. As such, it is a generalization of the well-known matched filter (also described in [5])

The other methods are discussed in [2] and [6]. The threshold method can either be applied directly to the observed waveform, or be applied to the envelope. The threshold itself can be either fixed, or taken relative with respect to the maximum amplitude of the waveform.

We have implemented two curve fitting method. The first method uses a one-sided parabola as a model for the feet of the envelope. See Fig 2. The algorithm consists of two parts. The first part, determines the time interval which is used for the fitting. For that purpose, a low pass filtered version $w_{lpf}^{env}(t)$ of the envelope $w^{env}(t)$ is used. See Fig 3. First, the noise level σ_n is estimated using a part of the observed waveform that is guaranteed to contain only noise, e.g. the first few 100 μs of the waveform. Then, the first point t_1 where $w_{lpf}^{env}(t)$ crosses the $2\sigma_n$-level is determined. The point t_p is defined as the time point at which $w_{lpf}^{env}(t)$ takes a (possibly local) maximum just after t_1. Next, the end point t_e of the feet is defined as the point between t_1 and t_p where $w_{lpf}^{env}(t)$ crosses the level

$2\sigma_n + \alpha\left(w_{lpf}^{env}(t_p) - 2\sigma_n\right)$ where α is some constant between zero and one. The begin point t_b of the feet is defined as $t_b = t_1 - \beta(t_e - t_1)$ where β is a constant. The interval (t_b, t_e) thus obtained appeared to be stable during all experiments. Suitable values of α and β are 0.3 and 2.4.

The second part of the curve fitting is the determination of the parameters of the parabola. We use the LSE criterion to fit the model to the (unfiltered) envelope $w^{env}(t)$. Since the model is nonlinear in its parameters an iterative minimalization procedure (using MatLab's implementation of the Nelder-Mead simplex method) is applied.

The simple implementation of the curve fitting only uses the two points t_1 and t_e defined above. Using the model $w_{lpf}^{env}(t) = \sigma_n + b(t - ToF)^2$ these two points suffices to solve for *ToF*.

The deconvolution method [7] models the observed waveform as:

$$w(t) = a\,h(t - ToF) + \sum_i d_i h(t - ToF - \tau_i) + n(t) \quad (1)$$

The first term is the direct response whose ToF should be estimated. a is the corresponding amplitude. The second term represents the multiple echoes with amplitudes d_i and delays τ_i (relative to *ToF*). $n(t)$ is the noise. $w(t)$ can be regarded as a convolution $w(t) = z(t) * h(t)$ with:

$$z(t) = a\,\delta(t - ToF) + \sum_i d_i \delta(t - ToF - \tau_i) + n(t) \quad (2)$$

A deconvolution operator aims at reconstructing the point process $z(t)$ from $w(t)$. Our implementation uses the pseudo Wiener filter with transfer function:

$$H_{Wiener}(f) = H^*(f) \Big/ \left(|H(f)|^2 + 1/SNR\right) \quad (3)$$

where $H(f)$ is the Fourier transform of $h(t)$ and *SNR* is the signal-to-noise ratio.

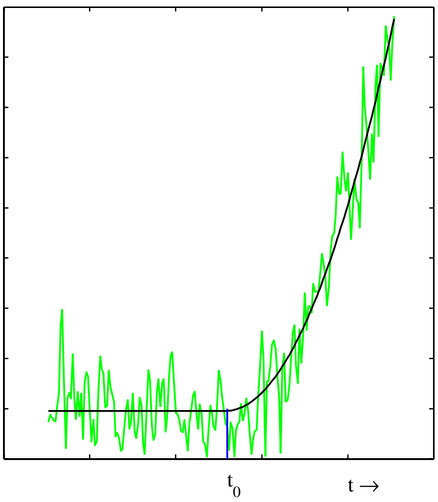

Fig. 2. A one-sided parabola fitted to the envelope.

3 Evaluation Procedure

The first step of the empirical evaluation is the tuning of the adjustment parameters of all the estimators. We recorded 150 waveforms acquired under different conditions (different rooms, different distances, and different heights above the floor, etc.). Fig. 1 is an example of a waveform obtained in this way.

For each record, we manually determined the *ToF*. For that, we used knowledge usually not available, e.g. the geometry of the setup. But in this experimental situation, such knowledge can be exploited. These manually obtained *ToF*s are considered as the conventional true values. As a criterion for the tuning of the adjustment parameters we used the sample standard deviation of the estimation errors calculated over the 150 records. A bias compensation guarantees that the bias (calculated as the mean error over all 150 records) of all operators is zero. Table 1 shows the performance of the various operators in terms of sample standard deviation and bias.

Eq. (1) provides a simple model of the occurrence of multiple echoes in a waveform. Usually, only one of these echoes is dominant. We silently ignore the existence of the others.

In order to assess the robustness of the operators, we selected one record whose multiple echoes were small relative to the direct response. The model for this record is: $a\,h(t - ToF) + n(t)$. We also recorded a waveform obtained in an anechoic room. Since the SNR for this record is large, we model this record with $h(t)$. Using these two records we are now able to simulate the occurrence of a second echo in a controlled fashion:

$$z(t) = a\,h(t - ToF) + D\,h(t - ToF - T) + n(t) \quad (4)$$

D and T are the parameters that control the

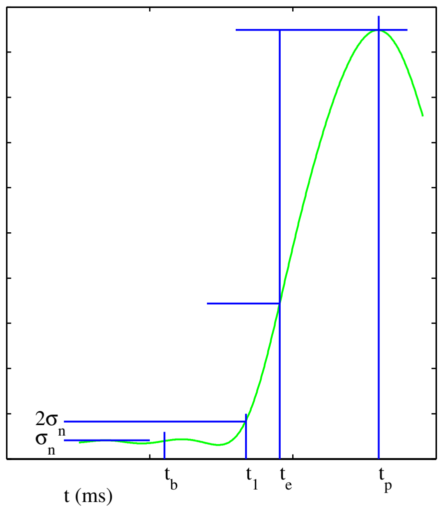

Fig. 3. Determination of the interval (t_b, t_e) using a low pass filtered version of the envelop.

Table 1.
Standard deviation and bias of the estimation errors

method	**overall bias** (ms)	**std. dev.** (ms)
Covariance model. based filtering	-0.010	0.027
Adaptive envelope thresholding	0.254	0.034
Adaptive direct thresholding	0.240	0.038
Envelope thresholding	0.090	0.044
Direct thresholding	0.108	0.053
Matched filtering on the envelope	0.024	0.107
Matched filtering	0.008	0.120
Iterative curve fitting	-0.031	0.022
Simple curve fitting	-0.030	0.021
Deconvolution	-0.009	0.026

simulation. The simulated waveforms are used to measure the influence of the second echo on the accuracies of the various ToF-estimators.

4 Results

We determined the estimation errors of the simulated waveform either with fixed delay T and varying amplitude D, or vice verse. Results are shown in fig 4 and fig. 5. The first figure shows the errors with varying D with T fixed to 0.5 ms. The second figure shows the errors with varying T and with D fixed to a.

5 Conclusion and discussion

Usually, the quality of a ToF estimator is assessed in terms of RMS involving both standard deviation and bias. However, the bias (given in Table 1) is a systematic error, and if known, it can be compensated. In our case, the bias can be estimated since we have a representative ensemble of records whose conventional true values of the ToFs are available. If only the standard deviation is considered, then the curve fitting methods prevail. They are closely followed by the covariance based method and the deconvolution method.

However, the purpose of this paper is to study the robustness of the operators.

From fig 4 and 5 and table 1 we conclude the following:

- The non-adaptive threshold methods are robust except when the second echo occurs almost directly after the first. However, these methods have a large standard deviation.
- The adaptive threshold methods, and the deconvolution method are robust except when the amplitude of the second echo becomes large (influencing the maximum of the wave).
- Matched filtering in either form is not robust.

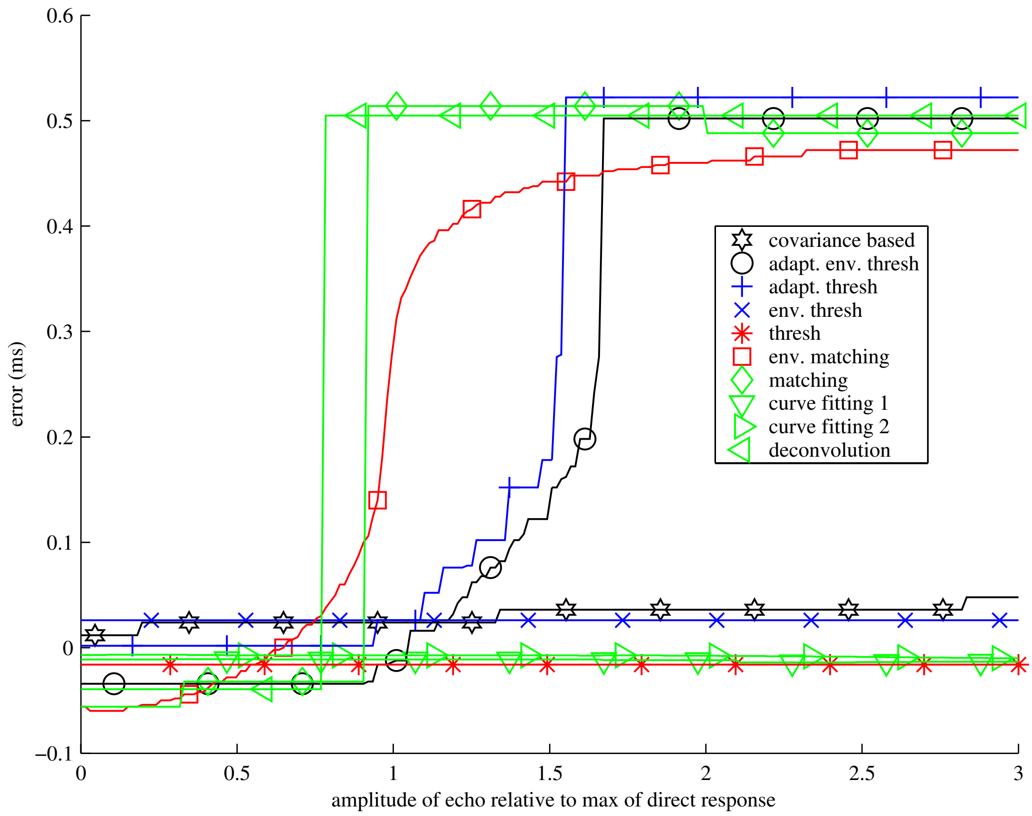

Fig. 4. Error versus relative amplitude of a second echo at a fixed delay.

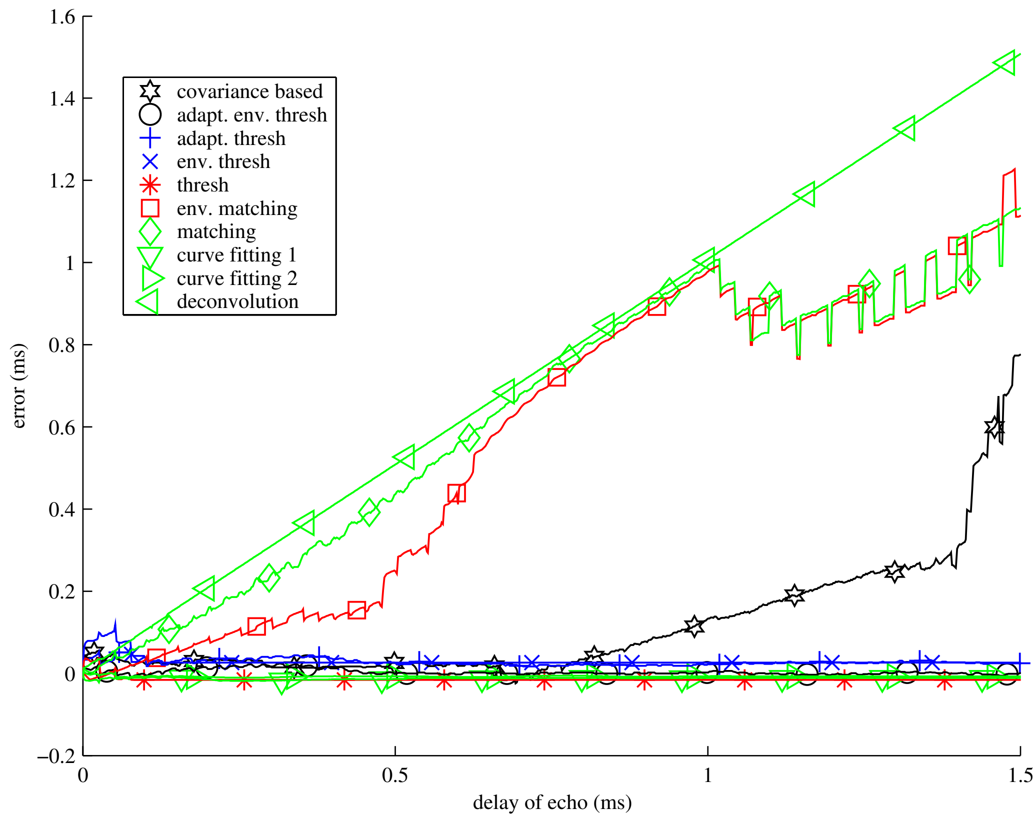

Fig. 5. Error versus delay of a second echo with a fixed amplitude.

- The curve fitting methods and the covariance model based method combines a small standard deviation with good robustness. The adaptive envelope threshold method performs slightly worse, but is much easier to implement.

The overall conclusion is that the curve fitting methods and the covariance based method combine a small standard deviation of the estimation error with a large robustness against multiple reflections. The reason for the robustness of the curve fitting methods is that they only use the feet of the observed waveform which is hardly influenced by multiple reflections. The reason for the robustness of the covariance based method is that it is based on a model where the multiple reflections are explicitly modeled.

The computational complexity is another aspect of a method. The adaptive envelope threshold method is least complex and has the advantage that it can be realized electronically. If DSP hardware is available, then the simple curve fitting method is preferable because the algorithm can be implemented fast, the standard deviation is lowest and the robustness is fine.

References

[1] F. van der Heijden, G. Túquerres, P.P.L. Regtien, “Time-of-flight estimation based on covariance models”, Meas. Sci. Technol. 14 (2003).

[2] B. Barshan, “Fast processing techniques for accurate ultrasonic range measurements”, Meas. Sci. Technol. 11 (2000) 45-50.

[3] H.W. When, P.R. Bélanger, “Ultrasound-Based Robot Position Estimation”, IEEE Trans on Robotics and Automation, Vol. 13, No. 5, p. 682-692, October 1997.

[4] A. Papoulis, Probability, Random Variables, and Stochastic Processes – Third Edition”, MacGraw-Hill, 1991.

[5] F. van der Heijden, G. Túquerres, P.P.L. Regtien, “Acoustic Time-of-flight measurements in a reflective room”, to appear in Proc. XVII IMEKO World Congress, June 2003.

[6] M. Parrilla, J.J. Anaya, C. Fritsch, Digital Signal Processing Techniques for High Accuracy Ultrasonic Range Measurements, IEEE Trans on Instrumentation and Measurement, Vol 40, No. 4, pp 759-763, August 1991.

[7] G. Benet, F. Blanes, M. Martínez, J. Simó, "Map Building Intelligent Sensor for an Autonomous Vehicle" IFAC Workshop on Intelligent Components for Vehicles. Sevilla, España. 1998