BER Analysis of a DPSK Phase Diversity Receiver for Coherence Multiplexing

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Abstract. Coherence multiplexing (CM) is an optical multiplexing technique that is particularly suitable for optical access networks and optical LANs, because of its low costs. In most previously reported CM systems, either single-ended or balanced detection is used, which requires either phase-locking (for homodyne detection) or frequency-shifting (for heterodyne detection) to avoid fading of the demodulator output signal. Recently, we proposed a scheme in which the receiver output signal is stabilized by means of a phase diversity network. To perform bit-by-bit detection using this scheme, OOK modulation should be used, which introduces some performance degradation with respect to PSK modulation in conventional systems. Moreover, using OOK puts very severe conditions on the CMRR of the balanced photodiode pairs in the receiver, in order to avoid crosstalk. In this paper, a phase diversity receiver based on DPSK demodulation is introduced. It is shown that this receiver results both in an improved performance with respect to OOK modulation, and moreover, that it is less sensitive to crosstalk.

1 Introduction

Increasing bandwidth demands of end users call for a conversion of the present communication networks, from copper cables to optical fibers. This conversion has already taken place in core networks, and is currently drifting towards the access parts, where the end users are connected. To minimize fiber costs, passive optical network (PON) architectures seem suitable. To enable multiple access on this network, a multiplexing scheme is required that minimizes the costs per end user. A well-developed and relatively bandwidth efficient technique is wavelength division multiplexing (WDM). This technique is therefore often used in core networks, both for effectively utilizing the capacity of the optical fiber and for enabling optically transparant wavelength routing. WDM is a very expensive technique, however, since it requires very narrowband lasers and selection filters. Moreover, temperature control is needed, both for stabilizing the laser wavelength and stabilizing the selection range of the filters. Although this is not much of a problem in core networks (where the costs are being shared by a vast number of users), it seems to make WDM too expensive for access networks. An alternative solution is optical time division multiplexing (OTDM), where each user is assigned a particular timeslot for transmitting one or more bits or packets. This is particularly effective in the downstream transmission path of a PON, as the transmission of the downstream channels can be performed by a single transmission node. In the upstream, however, the distributed transmitters are not localized to a node. Therefore, a synchronization scheme is required to avoid crosstalk, resulting in inefficient bandwidth utilization. For smaller networks, optical code division multiplexing (OCDM) schemes seem to be more suitable, as they can operate asynchronously in the same wavelength band, thereby enabling low access delay suited to bursty traffic. Another advantage is the so-called soft capacity, which means that the performance of the network degrades only gradually when the number of users is increased; adding a user does not require reconfiguring the network [1]. In our paper, we will consider a very simple form of OCDM, called coherence multiplexing.

2 Coherence Multiplexing (CM)

Coherence multiplexing (CM) [2] is based on the principle of distinguishing between coherent and incoherent mixing of lightwaves, as illustrated in Fig. 1. The idea is that each transmitter launches two versions of a broadband carrier into the common fiber. These two versions are made mutually incoherent (uncorrelated) by delaying them with respect to each other by a timeshift T_{tr} which is much larger than the coherence time τ_{c} of the source. Moreover, one of these two carriers is BPSK modulated by the information signal $m(t)$. (The other carrier is transmitted unmodulated.) In the receiver, the correct channel can be selected by mixing the received signal with the same signal being delayed by a timeshift T_{re}. Mixing is performed by a balanced detector, which consists of a 2×2 coupler and two photodiodes. If $T_{\mathrm{re}} = T_{\mathrm{tr}}$ then the lightwave taking the upper path in the transmitter and the lower path in the receiver mixes coherently with the lightwave taking the lower path in the transmitter and the upper path in the receiver, as their mutual time delay is zero. Since one of these lightwaves is modulated, the mixing product is an antipodal baseband signal proportional to the modulating signal $m(t)$. If the delays of all the transmitters are spaced apart much more than the coherence time of the sources, then it can be simply verified that all the lightwaves from interfering transmitters mix incoherently, resulting in broadband interferometric noise. Moreover, the coherent mixing term suffers from source intensity noise. Therefore, the output signal has to be matched filtered before thresholding is performed. For large received powers, interferometric noise and intensity noise dominate over other noise sources like shot noise and thermal receiver noise. Assuming that the complex envelope of the field emitted by the broadband source can be modelled as a circular complex Gaussian process with a Gaussian spectral profile, and that all the fields in the receiver have matched polarization states, the signal-to-noise ratio after matched filtering can be shown to be [2]

$$SNR = \frac{2}{4M^2 + 2M + 1} \frac{T_{\mathrm{b}}}{\tau_{\mathrm{c}}} = \frac{\sqrt{\frac{2\pi}{\ln 2}}}{4M^2 + 2M + 1} \frac{\Delta f}{R_{\mathrm{b}}} \ , \tag{1}$$

where M is the number of active users, T_{b} is the bit-time of the modulating signal $m(t)$, Δf is the 3 dB linewidth of the source and R_{b} is the bitrate.

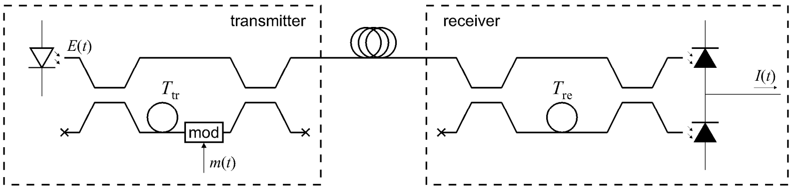

Fig. 1. A coherence multiplex system with one transmitter and one (balanced) receiver

As the matched filter bandwidth will generally be much smaller than the bandwidth of the noise, the decision samples can be assumed to be Gaussian distributed, so the bit error rate is given by

$$\mathrm{P_e} \approx \frac{1}{\sqrt{2\pi}} \int_{\sqrt{SNR}}^{\infty} \exp\left(-\frac{x^2}{2}\right) \, \mathrm{d}x \equiv \mathrm{Q}\left(\sqrt{SNR}\right) \quad . \tag{2}$$

Although CM cannot compete with WDM as far as bandwidth efficiency is concerned, it might be favourable from a cost point of view, as broadband light sources (for instance LEDs) and only simple components are required for crosstalk-free transmission. Therefore, CM is particularly suitable for small-scale networks like LANs and access networks.

3 Stability Issue

The problem with the balanced receiver in Fig. 1 is that the coherent lightwaves have to be mixed exactly in phase in order to achieve constructive interference. Therefore, even a small drift in the delay in either the transmitter or receiver will introduce a drop in the amplitude of the output signal, by a factor $\cos\bigl(2\pi f_\mathrm{c}(T_\mathrm{tr} - T_\mathrm{re})\bigr)$, where f_c is the carrier frequency of the light [3]. Three solutions are known to solve this problem. The most straightforward solution is to lock the coherent lightwaves in phase, by applying a feedback loop from the detected signal to the delay T_re, which adapts the refractive index, for example by means of the thermo-optic or electro-optic effect [4]. A second solution is to perform heterodyne detection instead of homodyne detection, by shifting one of the two coherently mixed lightwaves in frequency with respect to the other. This can for instance be done by means of an acousto-optic modulator in one of the Mach-Zehnder branches, either in the transmitter or the receiver [5]. Finally, the output signal of a coherence multiplex receiver can be stabilized by means of a phase diversity network, which will get some more attention in the next section.

4 Phase Diversity Detection of Coherence Multiplexed OOK Signals

Recently, we proposed how stabilized detection without phase-locking can be performed using either a 3×3 [3] or a 4×4 [6] phase diversity scheme. Phase diversity is a detection method which has been successfully applied in numerous types of coherent optical systems [7]. Consider the coherence multiplex receiver based on a 4×4 phase diversity network, as given in Fig. 2. In this receiver, the balanced mixer is replaced by a 4×4 optical hybrid (for instance a multimode interference coupler) and two differential pairs of photodiodes.

The geometry of the hybrid is assumed to be such that the power of the input signals is equally divided over the output ports, and moreover, that the mixing phases are given by ϕ, $\phi + \frac{\pi}{2}$, $\phi - \frac{\pi}{2}$ and $\phi + \pi$, where ϕ is an arbitrary phase that depends both on the phase difference between the input signals (which changes with $|T_{\mathrm{tr}} - T_{\mathrm{re}}|$) and on the absolute phase transfer of the hybrid. As a result, the output currents of the differential photodiode pairs $I_{\mathrm{I}}(t)$ and $I_{\mathrm{Q}}(t)$ both consist of desired signals $I_{\mathrm{I,s}}(t)$ and $I_{\mathrm{Q,s}}(t)$, respectively, and broadband noise $I_{\mathrm{I,n}}(t)$ and $I_{\mathrm{Q,n}}(t)$, respectively. It can be shown that the desired signals are given by $m(t)A\cos\phi$ and $m(t)A\sin\phi$, respectively, where the amplitude A incorporates transmitted power, splitting and coupling losses, and the responsivity of the photodiodes. To minimize the degradation due to the noise, both signals are first matched filtered by the low-pass filters, before they are squared and added. As the difference in mixing phase is $\frac{\pi}{2}$ radians, the output signal does not depend on ϕ. Therefore, this receiver is not able to demodulate BPSK modulated signals like in Fig. 1. This problem can be solved by replacing BPSK modulation by on-off keying (OOK), which can be performed by directly modulating the source. (In the next section, a different solution will be discussed.) Since OOK modulation incorporates modulating the power of the source, the variance of the interferometric noise depends on the number of users that are simultaneously transmitting a '1'-bit; therefore the signal-to-noise ratio can be considered as a random process $\gamma(t)$ which is varying with time. It is assumed (for simplicity) that the received signals from both the matched transmitter and the interfering transmitters are bit-synchronized and have the same power level. Hence, for a particular bit k, the signal-to-noise ratio is a discrete random variable γ_k that can take one out of M possible values. Moreover, all transmitters are assumed to transmit '0' or '1' bits independently, with an equal probability of $\frac{1}{2}$. Then it can be proven [6] that the average bit error probability of the phase diversity receiver can be approximated by

$$\mathrm{P_e} \approx \left(\tfrac{1}{2}\right)^M \cdot \sum_{n=0}^{M-1} \binom{M-1}{n} \left(\begin{array}{l} \exp\left(-\dfrac{T_{\mathrm{b}}}{4\left(4n^2+2n\right)\tau_{\mathrm{c}}}\right) \\ +\mathrm{Q}\left(\sqrt{\dfrac{T_{\mathrm{b}}}{2\left(4n^2+10n+7\right)\tau_{\mathrm{c}}}}\right) \end{array} \right), \quad (3)$$

whereas the average bit error probability for balanced detection of OOK is

$$\mathrm{P_e} \approx \left(\tfrac{1}{2}\right)^M \cdot \sum_{n=0}^{M-1} \binom{M-1}{n} \left(\begin{array}{l} \mathrm{Q}\left(\sqrt{\dfrac{T_{\mathrm{b}}}{2\left(4n^2+2n\right)\tau_{\mathrm{c}}}}\right) \\ +\mathrm{Q}\left(\sqrt{\dfrac{T_{\mathrm{b}}}{2\left(4n^2+10n+7\right)\tau_{\mathrm{c}}}}\right) \end{array} \right). \quad (4)$$

Figure 3 shows the network capacities one can obtain for a given number of users M at an average bit-error rate of 10^{-9}.

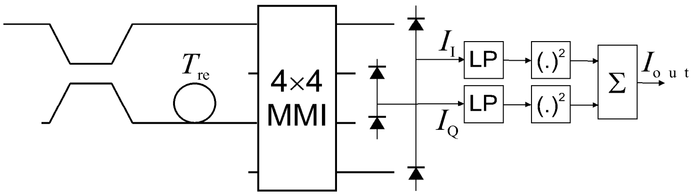

Fig. 2. A coherence multiplex received based on a 4x4 phase diversity network

From this figure it is obvious that phase diversity detection of OOK modulated signals results in a lower capacity than balanced detection of PSK modulated signals. The results for balanced detection of OOK modulated signals suggest, however, that this degradation is mainly caused by the change of the modulation format rather than the change of the detection method. In the next section, it will be proven that this is indeed the case. A second disadvantage of using OOK modulation concerns the subtraction of the photodiode currents. As the output signals of the differential pairs are no longer antipodal, the subtraction operations should be DC-coupled. Moreover, canceling the common mode signals is very difficult as they are no longer DC. As a result, the splitting ratio of the 4×4 coupler should be strictly uniform and the responsivities of the photodiodes need to be identical, in order to avoid crosstalk from interfering users. Finally, a disadvantage of using OOK modulation is that the detection threshold needs to be adapted to the received power in order to minimize the bit error probability.

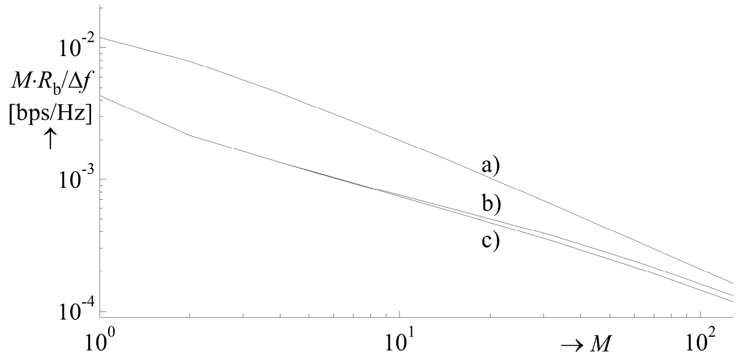

Fig. 3. Network capacity over linewidth at a bit error rate $\mathrm{P_e} = 10^{-9}$
a) BPSK modulation and balanced detection
b) OOK modulation and balanced detection
c) OOK modulation and phase diversity detection

5 Phase Diversity Detection of Coherence Multiplexed DPSK Signals

The two main disadvantages of using OOK modulation can be cleared away by performing differential phase shift keying (DPSK) modulation instead of OOK modulation. This means that, in the phase modulator in the transmitter, a transition of π radians is made when a binary '1' is transmitted, and no transition is made when a binary '0' is transmitted. In the receiver, the squaring operation should be replaced by a delay-and-multiply operation, as illustrated in Fig. 4. Now let the output samples of the matched filters in the I- and Q-branches for a particular bit k be denoted by I_k and Q_k. Then it can be shown that I_k and Q_k contain information terms $I_{\mathrm{s},k} = \mathrm{E}[I_k] = m_k A \cdot T_\mathrm{b} \cos\phi$

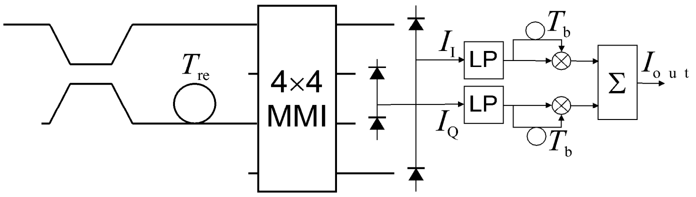

Fig. 4. A DPSK coherence multiplex received based on a 4x4 phase diversity network

and $Q_{\mathrm{s},k} = \mathrm{E}[Q_k] = m_k A \cdot T_{\mathrm{b}} \sin\phi$, respectively, source intensity noise terms $I_{\mathrm{sin},k} = A_{\mathrm{sin},k}\cos\phi$ and $Q_{\mathrm{sin},k} = A_{\mathrm{sin},k}\sin\phi$, respectively, and interferometric noise terms $I_{\mathrm{in},k}$ and $Q_{\mathrm{in},k}$, respectively, where $m_k = m_{k-1}$ when a binary '0' is transmitted, and $m_k = -m_{k-1}$ when a binary '1' is transmitted. $A_{\mathrm{sin},k}$ is zero mean Gaussian with variance $\sigma^2_{\mathrm{sin}} = A^2 T_{\mathrm{b}}\tau_{\mathrm{c}}$, and $I_{\mathrm{in},k}$ and $Q_{\mathrm{in},k}$ are both zero mean Gaussian with variance $\sigma^2_{\mathrm{in}} = \frac{1}{2}(4M^2 + 2M - 1)A^2 T_{\mathrm{b}}\tau_{\mathrm{c}}$. Because the complex envelopes of the lightwaves are assumed to be circular complex Gaussian distributed with a symmetrical spectrum, the corresponding quadrature components are independent. One can prove that this implies that the interferometric noise currents are uncorrelated, such that $I_{\mathrm{in},k}$ and $Q_{\mathrm{in},k}$ are uncorrelated and therefore independent, as they are jointly Gaussian. The output signal of the receiver can be written as

$$S_k = I_k I_{k-1} + Q_k Q_{k-1} \ . \tag{5}$$

By assuming that the phase ϕ does not change significantly during two bit-times, and by defining

$$I_{\mathrm{n},k} \equiv A_{\mathrm{sin},k} + I_{\mathrm{in},k}\cos\phi + Q_{\mathrm{in},k}\sin\phi \ , \tag{6}$$

$$Q_{\mathrm{n},k} \equiv I_{\mathrm{in},k}\sin\phi - Q_{\mathrm{in},k}\cos\phi \ , \tag{7}$$

this can be written as

$$S_k = \Big(m_k A \cdot T_{\mathrm{b}} + I_{\mathrm{n},k}\Big)\Big(m_{k-1} A \cdot T_{\mathrm{b}} + I_{\mathrm{n},k-1}\Big) + Q_{\mathrm{n},k} Q_{\mathrm{n},k-1} \ . \tag{8}$$

$I_{\mathrm{n},k}$, $I_{\mathrm{n},k-1}$, $Q_{\mathrm{n},k}$ and $Q_{\mathrm{n},k-1}$ are zero mean Gaussian distributed and mutually independent. Their variances do not depend on k and are given by $\sigma^2_{I_{\mathrm{n}}} = \frac{1}{2}(4M^2 + 2M + 1)A^2 T_{\mathrm{b}}\tau_{\mathrm{c}}$ and $\sigma^2_{Q_{\mathrm{n}}} = \frac{1}{2}(4M^2 + 2M - 1)A^2 T_{\mathrm{b}}\tau_{\mathrm{c}}$. From (8), it is obvious that the conditional probability density functions $f_{S_k|m_k=m_{k-1}}$ and $f_{S_k|m_k\neq m_{k-1}}$ of S_k under the conditions that m_k and m_{k-1} are the same or not, respectively, are related as $f_{S_k|m_k=m_{k-1}}(s) = f_{S_k|m_k\neq m_{k-1}}(-s)$. As a result, when equal bit probabilities are assumed, the probability of bit error $\mathrm{P_e}$ can be minimized by setting the threshold to $s_{\mathrm{th}} = 0$. Moreover, the probability of error does not depend on the transmitted bit, so $\mathrm{P_e}$ is given by

$$\mathrm{P_e} = \mathrm{P}_{\mathrm{e}|m_k=m_{k-1}=1} = \mathrm{P}\big[S_k < 0 | m_k = m_{k-1} = 1\big] \ . \tag{9}$$

By defining

$$\alpha^2 \equiv \left(A \cdot T_{\mathrm{b}} + \frac{I_{\mathrm{n},k} + I_{\mathrm{n},k-1}}{2}\right)^2 + \left(\frac{Q_{\mathrm{n},k} + Q_{\mathrm{n},k-1}}{2}\right)^2 \ , \tag{10}$$

$$\beta^2 \equiv \left(\frac{I_{\mathrm{n},k} - I_{\mathrm{n},k-1}}{2}\right)^2 + \left(\frac{Q_{\mathrm{n},k} - Q_{\mathrm{n},k-1}}{2}\right)^2 \ , \tag{11}$$

we can write

$$S_k|_{m_k=m_{k-1}=1} = \alpha^2 - \beta^2 \ . \tag{12}$$

Since α and β can be proven to be mutually independent, we get

$$\mathrm{P_e} = \mathrm{P}\big[\beta > \alpha\big] = \int_0^\infty f_\alpha(a) \int_a^\infty f_\beta(b)\ \mathrm{d}b\ \mathrm{d}a \ . \tag{13}$$

Unfortunately, it is very hard (if not impossible) to find analytical expressions for the probability density functions $f_\alpha(a)$ and $f_\beta(b)$ of α and β, respectively, as the variances $\sigma^2_{I_\mathrm{n}}$ and $\sigma^2_{Q_\mathrm{n}}$ are not the same. For large M (say $M > 3$), however, one can easily verify that a very close upper bound on $\mathrm{P_e}$ is obtained by assuming that

$$\sigma^2_{Q_\mathrm{n}} \approx \sigma^2_{I_\mathrm{n}} = \tfrac{1}{2}\left(4M^2 + 2M + 1\right) A^2 T_\mathrm{b} \tau_\mathrm{c} \ . \tag{14}$$

With that assumption, α is approximately Rician distributed

$$f_\alpha(a) \approx \frac{2a}{\sigma^2_{I_\mathrm{n}}} \exp\left(-\frac{a^2 + (A\cdot T_\mathrm{b})^2}{\sigma^2_{I_\mathrm{n}}}\right) \mathrm{I}_0\left(\frac{2a\cdot A\cdot T_\mathrm{b}}{\sigma^2_{I_\mathrm{n}}}\right) \ , \tag{15}$$

whereas β is approximately Rayleigh distributed

$$f_\beta(b) \approx \frac{2a}{\sigma^2_{I_\mathrm{n}}} \exp\left(-\frac{a^2}{\sigma^2_{I_\mathrm{n}}}\right) \ , \tag{16}$$

where $\mathrm{I}_0(x)$ is the modified Bessel function of the first kind and order zero:

$$\mathrm{I}_0(x) \equiv \frac{1}{2\pi} \int_0^{2\pi} \exp(x \cos\theta)\ \mathrm{d}\theta \ . \tag{17}$$

Subsituting this in (13) gives

$$\begin{aligned} \mathrm{P_e} &\approx \tfrac{1}{2} \exp\left(-\frac{(A\cdot T_\mathrm{b})^2}{2\sigma^2_{I_\mathrm{n}}}\right) \\ &= \tfrac{1}{2} \exp\left(-\frac{1}{4M^2 + 2M + 1} \frac{T_\mathrm{b}}{\tau_\mathrm{c}}\right) = \tfrac{1}{2} \exp\left(-\frac{\sqrt{\frac{\pi}{2\ln 2}}}{4M^2 + 2M + 1} \frac{\Delta f}{R_\mathrm{b}}\right) \ . \end{aligned} \tag{18}$$

The resulting network capacities at a bit error rate of 10^{-9} are shown in Fig. 5, together with the previously obtained results. The results for the phase diversity receivers apply both to the 3×3 as well as the 4×4 solution.

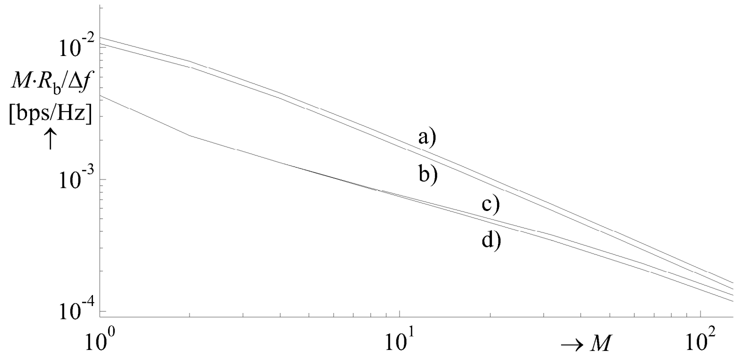

Fig. 5. Network capacity over linewidth at a bit error rate $P_e = 10^{-9}$
a) BPSK modulation and balanced detection
b) DPSK modulation and phase diversity detection
c) OOK modulation and balanced detection
d) OOK modulation and phase diversity detection

6 Conclusion

Stabilization of a coherence multiplex receiver by means of a phase diversity network introduces only a minor capacity degradation with respect to a phase-locked balanced detector. This holds for amplitude modulation as well as phase modulation; phase modulation result in the best performance, however. When phase modulation is used, the bits should be differentially encoded prior to modulation, and moreover, the demodulation in the diversity branches should be done using a delay-and-multiply circuit instead of a square law device. A second advantage of using phase modulation is that a constant power is transmitted by all the users, such that the common mode terms in the photocurrents are DC; this makes the subtraction of the photocurrents easier. A disadvantage of phase modulation is that performing the modulation is more difficult than amplitude modulation, which can simply be performed by directly modulating the source.

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