VALIDATION OF THE GRANULAR TEMPERATURE PREDICTION OF THE KINETIC THEORY OF GRANULAR FLOW BY PARTICLE IMAGE VELOCIMETRY AND A DISCRETE PARTICLE MODEL

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ABSTRACT
In order to give a detailed description of the hydrodynamics in large industrial scale fluidized beds, continuum models are required. Continuum models often use the kinetic theory of granular flow (KTGF) to provide closure equations for the internal momentum transport in the particulate phase. In this work the outcome of the continuum model is compared with both an experimental technique and detailed simulations, i.e. particle image velocimetry (PIV) and the discrete particle model (DPM).

PIV is used for the measurement of an instantaneous velocity field of the flow in the front plane of a fluid bed. The classical PIV analysis is extended to enable the measurement of the local velocity fluctuations in the interrogation area, i.e. the granular temperature. In the DPM, each particle is tracked individually. In this model detailed collision models can be incorporated, rendering the DPM a valuable research tool to validate the underlying assumptions in the KTGF concerning the particle-particle interactions and the particle velocity distribution functions.

The aforementioned experimental and numerical techniques are used to measure the granular temperature distribution around a single bubble rising in a gas-fluidized bed. It was found that the results of PIV and the DPM are very similar. Although the initial bubble shape and size are well predicted by the continuum model, it fails once the bubble has detached from the bottom plate. Further research in the area of KTGF closures is needed to improve the predictions of the TFM.

INTRODUCTION

Large industrial scale fluidized beds are frequently used in industry. Continuum models are required to describe the hydrodynamic phenomena prevailing in such systems in sufficient detail. In the continuum model both the gas and particulate phases are described as interpenetrating fluids. Often the kinetic theory of granular flow (KTGF) is used to provide closure equations for the internal momentum transport in the particulate phase in the continuum model.

In this work the validity of the KTGF is tested with experiments and detailed simulation. The first technique used to validate the KTGF, is particle image velocimetry (PIV), which is a non-intrusive technique that can be used for the measurement of an instantaneous velocity field of the flow in the front plane of a fluid bed.

The KTGF is also tested with the discrete particle model (DPM), in which each particle is tracked individually. In this model detailed collision models can be incorporated, rendering the DPM a valuable research tool to validate the underlying assumptions in the KTGF concerning the particle-particle interactions and the particle velocity distribution functions.

In this work, the classical PIV analysis is extended to enable the measurement of the ensemble average of the squared particle fluctuation velocity, i.e. the granular temperature. Both the extended PIV technique and the DPM are used to measure the granular temperature distribution around a single bubble rising in a gas-fluidized bed. The results of the PIV and DPM are compared with predictions of an Euler-Euler model employing the KTGF.

THEORETICAL BACKGROUND

Discrete particle model

The discrete particle model used in this work is based on the hard-sphere model developed by Hoomans et al. [1,2]. A short description of the model is given in this section, for details the interested reader is referred to the work of Hoomans et al. [1,2].

Particle collision dynamics are described by collision laws, which account for energy dissipation due to non-ideal particle interaction by means of the empirical coefficients of normal and tangential restitution and the coefficient of friction.

The particle collision characteristics play an important role in the overall bed behaviour as was shown by Goldschmidt et al. [3]. For this reason the collision properties of the particles used for the experimental validation were accurately determined by detailed impact experiments and supplied to the model.

The motion of every individual particle in the system is calculated from the Newtonian equation of motion:

\[ m_p \frac{d \mathbf{v}_p}{dt} = -V_p \nabla p_p + \frac{V_p \beta}{1 - \epsilon} (\mathbf{u}_p - \mathbf{v}_p) + m_p g \]

(1)

where \( \beta \) represents the inter-phase momentum transfer coefficient, which in this work is modeled using the drag relation of Koch and Hill [4] (see Table 1). Bokkers et al. [5] have shown that for relatively large particles the relation of Koch and Hill yields much better results than the
conventional combination of drag relations of Ergun, and Wen and Yu as was suggested by Gidaspow [6].

The gas phase hydrodynamics are calculated from the volume-averaged Navier-Stokes equations:

\[
\frac{\partial}{\partial t} (\rho \mathbf{v}) + \nabla \cdot (\rho \mathbf{v} \mathbf{v}) = -\nabla p + \nabla \cdot (\mathbf{T}) + \mathbf{g}
\]

(2)

\[
\frac{\partial}{\partial t} (\rho \mathbf{v}) + \nabla \cdot (\rho \mathbf{v} \mathbf{v}) =
-\nabla p + \nabla \cdot (\rho \mathbf{v} \mathbf{v}) - \nabla \cdot (\rho \mathbf{v} \mathbf{v}) - \rho \mathbf{g}
\]

(3)

The two-way coupling between the gas-phase and the particles is achieved via the sink term \( S_p \), which is computed from:

\[
S_p = \frac{1}{V_{cell}} \sum_{V_{cell}} V \beta_{p} (v_{cell} - v_{p}) D(r - r_{p}) dV
\]

(4)

The distribution function \( D \) distributes the reaction force acting on the gas phase to the Eulerian grid.

**Two-fluid model**

When a two-fluid model is used instead of the discrete particle model, the particulate phase is treated as a continuum, similar to the gas phase. For the gas phase Eqs. (2) and (3) are used, where \( S_p \) is given by:

\[
S_p = \beta ((v_{cell} - v_{p})
\]

(5)

The volume-averaged Navier Stokes equations of the solid phase take a similar form:

\[
\frac{\partial}{\partial t} (\rho_{p} \mathbf{v}_{p}) + \nabla \cdot (\rho_{p} \mathbf{v}_{p} \mathbf{v}_{p}) = -\nabla p_{p} + \nabla \cdot (\mathbf{T}_{p})
\]

(6)

This equation contains an extra contribution for the solids pressure. Both the solids pressure and the apparent particle viscosity in the particle stress term depend on the fluctuating motion of the particles owing to particle-particle collisions, i.e. the granular temperature. In this work, this dependence is described according to the Kinetic Theory of Granular Flow (KTGF) [6,7]. The variation of the particle velocity fluctuations is described with a separate transport equation for the granular temperature:

\[
3 \left[ \frac{\partial}{\partial t} (\rho_{p} \mathbf{v}_{p}) + (\nabla \cdot \rho_{p} \mathbf{v}_{p}) \mathbf{v}_{p} \right] = - (p_{p} \mathbf{I} + \rho_{p} \mathbf{v}_{p}) : (\nabla \mathbf{v}_{p})
\]

(7)

\[
= (\nabla \cdot \rho_{p} \mathbf{v}_{p} \mathbf{v}_{p}) + \beta (\mathbf{v}_{p} \cdot \mathbf{v}_{p} - \mathbf{3}) - \gamma
\]

(8)

The constitutive equations that were used in this work to model the rheology of the particulate phase for the KTGF model can be found in the work of Nieuwland et al. [7].

<table>
<thead>
<tr>
<th>Table 1. Equations for the calculation of the gas-particle drag.</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \text{Re} = \frac{\rho_{g} d_{p} (\mathbf{v}<em>{cell} - \mathbf{v}</em>{p})}{\mu_{g}} ]</td>
</tr>
<tr>
<td>[ \beta = 18 \mu_{g} \rho_{g} \mathbf{v}<em>{p} \left( F</em>{o} (\mathbf{v}<em>{p}) + \frac{1}{2} F</em>{T}(\mathbf{v}<em>{p}) \mathbf{v}</em>{p}^{2} \right) ; \text{Re} &lt; 40 ]</td>
</tr>
<tr>
<td>[ \beta = \frac{18 \mu_{g} \rho_{g} \mathbf{v}<em>{p} \left( F</em>{o} (\mathbf{v}<em>{p}) + \frac{1}{2} F</em>{T}(\mathbf{v}<em>{p}) \mathbf{v}</em>{p}^{2} \right) ; \text{Re} &gt; 40 ]</td>
</tr>
<tr>
<td>[ F_{o} (\mathbf{v}<em>{p}) = \frac{1 + 3 (\mathbf{v}</em>{p} / 2)^{1/2} + (135 / 64) \mathbf{v}<em>{p} \ln (\mathbf{v}</em>{p} + 17.14 \mathbf{v}<em>{p}) ; \mathbf{v}</em>{p} &lt; 0.4 }{1 + 0.681 \mathbf{v}<em>{p} - 8.48 \mathbf{v}</em>{p}^{2} + 8.16 \mathbf{v}_{p}^{3}} ]</td>
</tr>
<tr>
<td>[ F_{T}(\mathbf{v}<em>{p}) = \frac{10 \mathbf{v}</em>{p}}{\mathbf{v}<em>{p}^{3}} ; \mathbf{v}</em>{p} &gt; 0.4 ]</td>
</tr>
<tr>
<td>[ F_{o} (\mathbf{v}<em>{p}) = 0.110 + 5.10 \cdot 10^{-4} \mathbf{v}</em>{p}^{1.64} ]</td>
</tr>
<tr>
<td>[ F_{T}(\mathbf{v}<em>{p}) = 0.0673 + 0.212 \mathbf{v}</em>{p} + \frac{0.0232}{\mathbf{v}_{p}^{3}} ]</td>
</tr>
</tbody>
</table>

**Particle image velocimetry**

Particle image velocimetry (PIV) is a non-intrusive technique for the measurement of an instantaneous velocity field in one plane of a flow. In traditional PIV the flow is visualized by seeding it with small tracer particles that closely follow the flow. In gas-particle flows, the discrete particles can readily be distinguished, so no additional tracer particles are needed to visualize the particle movement. The flow in the front of the bed is illuminated with the use of halogen lamps. A CCD camera is used to record images of the particles in the illuminated plane. Two subsequent images of the flow, separated by a short time delay, \( \Delta t \), are divided into small interrogation areas. Cross-correlation analysis is used to determine the volume-averaged displacement, \( s(x,t) \), of the particle images between the interrogation areas in the first and second image. When the interrogation areas contain a sufficient number of particle images, the cross-correlation consists of a dominant correlation peak embedded in a background of noise peaks. The location of the tall peak, referred to as the displacement-correlation peak, corresponds to the particle-image displacement. The velocity within the interrogation area is then easily determined by dividing the measured displacement by image magnification, \( M \), and the time delay:

\[
\mathbf{v}_{p}(x,t) = \frac{s(x,t)}{M \Delta t}
\]

provided that \( \Delta t \) is sufficiently small. The shape of the displacement-correlation peak can be described with the following equation [8]:

\[
R_{o}(s) \propto N_{f} F_{o} F_{T} * F_{o}
\]

(10)

Here, \( N_{f} \) represents the total number of particles in the interrogation area. \( F_{o} \) and \( F_{T} \) are correction factors for loss of correlation due to respectively in-plane and out-of-plane motion of particle images. \( F_{T} \) is a function, which describes the shape of the peak due to the intensity distribution of the
particle image. $F_p$ is a function that describes the shape of the peak due to the particle velocity distribution.

When the intensity distribution of the particle image and the particle velocity distribution are assumed to be Gaussian, the convolution of these terms will also be Gaussian. Given the latter assumptions the displacement-correlation peak can be written as follows:

$$R_p(s) = h \cdot \exp[-\frac{s^2}{2\sigma^2}] \exp[-\frac{s_p^2}{2\sigma_p^2}] \tag{11}$$

The height of the peak, $h$, is proportional to $N_F F_{C,0}$, while the standard deviation, i.e. the correlation peak width $\sigma$, depends on the convolution of $F_T$ and $F_p$.

$$\sigma^2 = \sigma_{v,x}^2 + \sigma_{p,x}^2, \quad \sigma_{y}^2 = \sigma_{v,y}^2 + \sigma_{p,y}^2 \tag{12}$$

where $\sigma_v$ and $\sigma_p$ are the standard deviations of the Gaussian functions describing the particle velocity distribution and the particle image intensity respectively.

The overall standard deviation can be determined by fitting a Gaussian curve to the measured correlation signal. Usually a three-point curve-fit is used for this purpose. While this is very accurate for the determination of the sub-pixel displacement, the determination of the peak width is very sensitive to noise, especially for large particles (> 4 px). In order to improve the accuracy in the width calculation, we used all points in the vicinity of the peak, for which $R > 0.55h$ is valid.

The standard deviation of the particle image intensity distribution is obtained in a similar fashion: to eliminate the influence of the particle velocity distribution the recorded images are auto-correlated.

Finally the standard deviation of the particle velocity distribution is obtained through Eq. 12, by subtracting the particle image intensity contribution from the overall standard deviation.

Finally the granular temperature is obtained as follows:

$$\Theta = \frac{m_e}{\Delta \tau} \Theta_{v,x}, \quad \Theta_y = \frac{m_p}{\Delta \tau} \Theta_{v,y} \tag{13}$$

where $m_e$ is the mass of the particle and $\Delta \tau$ the time delay between the two subsequent images used in the measurement.

The overall granular temperature – based on measurements in 2 dimensions – can now be calculated from the following equation:

$$\Theta = \frac{\Theta_x + \Theta_y}{2} \tag{14}$$

It was verified with the use of artificially generated particle images, that the error in the determination of the granular temperature is less than 10%.

It is noted that to our knowledge PIV has not been used to measure the granular temperature. Wildman et al. [9,10] have used particle tracking velocimetry (PTV) to measure the granular temperature in a vibro-fluidised bed. In this technique the velocity of every single particle is calculated. From this data the velocity distribution and the granular temperature can be calculated. The downside of this method however is that a very high frame rate is required in order to keep track of every single particle. Furthermore, the particle tracking technique demands for a high spatial resolution in order to yield precise velocity and granular temperature measurements. When PIV is used, neither a high frame rate nor a high resolution is needed to yield precise measurements.

**EXPERIMENTAL SETUP**

Experiments were carried out in a pseudo 2D lab-scale fluidized bed, which is represented in Figure 1. The front plate of the small bed is made of glass and the back-panel of polycarbonate (Lexan). The sidewalls are made of aluminium and the bottom consists of a sintered porous plate. A jet region with dimensions of 10 x 15 mm (W x D) is positioned in the center of the bottom plate. On either side of the jet region gas is entered for the background velocity.

The bed is filled with approximately 30 000 glass particles with a size of 2.5 mm and a density of 2525 kg/m$^3$. The particles are fluidized with air. Steam is added to the air to obtain a humidity of about 60%, in order to diminish the effect of static electricity.

The bed was illuminated with two 500 W halogen lamps, which are positioned under an angle, in order to prevent reflections from the front wall of the bed to the camera. Images of the flow were recorded with a DALSA Motion Vision CA-D6-0512W camera. The camera has a resolution of 544 x 516 pixels and runs at a frame rate of 262 Hz.

The particle images in the experiments are about 7.5 pixels large. In the experiments interrogation areas of 50 x 50 pixels were used, which yields a maximum number of particles in the interrogation area of around 50. Although accurate velocity measurements can be carried out with much smaller interrogation areas and consequently much less particles, we used these settings to ensure a precise measurement of the granular temperature.

![Figure 1. Schematic representation of the experimental setup. Left: top view, right: front view.](image-url)
RESULTS

The principle of the granular temperature measurement is illustrated in Figure 2. In this figure a snapshot of a single bubble in a fluidized bed can clearly be seen. The roof of the bubble is just outside the image, however particles raining down from the roof can clearly be identified. At the bottom of the bubble a particle jet can be seen, which occurs when the two streams of particles meet each other in the wake of the bubble. Around the snapshot, four graphs of the correlation function are shown for different positions in the bed. In the corner of the bed, where the particles are almost quiescent, the correlation peak is rather narrow and positioned in the middle of the correlation plane (i.e. no movement). Its width is not determined by the particle velocity fluctuations, but by the size of the particles. In the downflow region near the wall, the peak has the same shape as in the corner of the bed. However, this time it is positioned away from the center of the correlation plane, indicating that the particles are flowing downwards. In the wake of the bubble and in the jet the particle velocity fluctuations are relatively large, which results in a broader correlation peak.

Both PIV measurements of the velocity and granular temperature as well as DPM and KTGF simulations were carried out for a single bubble injected in a mono-disperse fluidized bed at incipient fluidization conditions. Details of the experimental and numerical settings are given in Table 2.

Both experimental and numerical snapshots of the particle positions (or particle volume fraction), particle velocity maps and granular temperature plots are shown in Figures 3-5. Results obtained from PIV experiments are shown in Figure 2. From this figure, the evolution of the bubble growth and movement through the bed can be observed. The injection of the bubble leads to an expansion of the bed, along with downflow of particles along the walls, moving into the wake of the bubble. The largest granular temperatures are observed in the direct vicinity of the bubble, especially at the roof of the bubble, where particles are raining down and in the jet region. It is noted that inside the bubble too few particles are present to perform reliable measurements of the particle velocity and granular temperature. For this reason, no measurement data is shown inside the bubble. 300 ms after injection it is observed that below the bubble wake, the granular temperature vanishes.

In Figure 4, equivalent results are shown for the DPM simulation. It is seen that the evolution of the bubble shape is very similar to the experimental results. The raining of particles from the roof of the bubble is also observed in the DPM. Furthermore, the widening of the jet, once the bubble detaches (between 200 and 300 ms) is very similar in the experiments and the DPM simulation.

Finally, the predictions of the KTGF can be inferred from Figure 5. The first column in this figure shows porosity plots instead of particle positions, due to the fact that in the two-fluid model the particulate phase is modeled as a continuous phase. It is seen that the initial shape (at 100 and 200 ms) is well predicted. In the direct vicinity of the bubble the granular temperature reaches the same level as in the experiments and the DPM simulation. After the bubble has detached from the bottom plate (after 200 ms), the granular temperature below the bubble is overpredicted. Furthermore, it is observed that after about 300 ms the bubble dissolves through the roof of the bubble into the emulsion phase. This is not in agreement with the experiment or the DPM simulation. A possible explanation for the deficits in the TFM may be the lack of frictional particle stress in the KTGF closure. This stress is experienced in regions with long-term multiple particle contacts in combination with frictional work. This type of momentum transport is for example found in the wake of the bubble. Although several frictional particle stress models have been proposed in literature, Bokkers et al. [11] have shown that none of these models gives satisfactory results for the case studied in this work. The DPM may be used to develop better closure models for the frictional particle stress.

Table 2. Simulation settings.

<table>
<thead>
<tr>
<th></th>
<th>PIV</th>
<th>DPM</th>
<th>TFM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width (m)</td>
<td>0.15</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>Height (m)</td>
<td>1.0</td>
<td>0.45</td>
<td>0.45</td>
</tr>
<tr>
<td>Depth (m)</td>
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<td>0.015</td>
<td>0.015</td>
</tr>
<tr>
<td>Grid cells in x-direction (-)</td>
<td>17</td>
<td>15</td>
<td>30</td>
</tr>
<tr>
<td>Grid cells in y-direction (-)</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Grid cells in z-direction (-)</td>
<td>19</td>
<td>45</td>
<td>90</td>
</tr>
<tr>
<td>Time step (s)</td>
<td>$3.8 \times 10^{-3}$</td>
<td>$1.0 \times 10^{-4}$</td>
<td>$1.0 \times 10^{-5}$</td>
</tr>
<tr>
<td>Number of particles (-)</td>
<td>~30 000</td>
<td>30 000</td>
<td>30 000</td>
</tr>
<tr>
<td>Background velocity (m/s)</td>
<td>1.2</td>
<td>1.2</td>
<td>1.2</td>
</tr>
<tr>
<td>Jet pulse velocity (m/s)</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Jet pulse duration (s)</td>
<td>0.15</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>Coefficient of restitution (-)</td>
<td>0.97</td>
<td>0.97</td>
<td>0.97</td>
</tr>
</tbody>
</table>

Figure 2. Snapshot of a single bubble rising in a fluidized bed at incipient fluidization conditions, along with sample correlation signals at selected areas around the bubble.
Figure 3. Experimental PIV results at different times, from left to right: snapshot bubble, particle velocity field and granular temperature field. From bottom to top: 100, 150, 200 and 300 ms after bubble injection.
Figure 4. DPM simulation results at different times, from left to right: snapshot particle positions, particle velocity field and granular temperature field. From bottom to top: 100, 200 and 300 ms after bubble injection.
Figure 5. TFM simulation results at different times, from left to right: porosity map, particle velocity field and granular temperature field. From bottom to top: 100, 200 and 300 ms after bubble injection.
CONCLUSIONS

In this work the fluid dynamics of a single bubble rising in a gas fluidized bed were studied with the use of a two-fluid model (TFM). The kinetic theory of granular flow (KTGF) was used to provide closure equations for the internal momentum transport in the particulate phase. In order to verify the KTGF, the simulations were compared with a particle image velocimetry (PIV) measurement and a discrete particle model (DPM) simulation. The PIV technique was extended to facilitate the measurement of the granular temperature, which is a key parameter in the KTGF. It was shown that the granular temperature could be measured by determining the peak width of the cross correlation displacement peak. The PIV measurement and the DPM simulation yielded very similar results. Although the initial bubble shape and size are well predicted by the TFM, the model fails once the bubble has detached from the bottom plate. Further research in the area of KTGF closures is needed to improve the predictions of the TFM.

NOMENCLATURE

\[ \begin{align*}
\tau & \quad \text{Stress tensor} \\
\mu_g & \quad \text{Gas-phase viscosity} \\
\Delta t & \quad \text{Time delay in PIV measurement}
\end{align*} \]

\[ \begin{align*}
\sigma & \quad \text{Standard deviation} \\
\varepsilon & \quad \text{Volume fraction} \\
\theta & \quad \text{Granular temperature} \\
\rho & \quad \text{Density} \\
\end{align*} \]

\[ \begin{align*}
c_p & \quad \text{Fluctuation particle velocity} \\
c_v & \quad \text{Fluctuation gas velocity} \\
d_p & \quad \text{Particle diameter} \\
F_t & \quad \text{In-plane particle loss correction factor} \\
F_O & \quad \text{Out-of-plane particle loss correction factor} \\
F_r & \quad \text{Shape of the correlation peak, due to the particle diameter} \\
F_\theta & \quad \text{Shape of the correlation peak, due to the particle velocity distribution} \\
g & \quad \text{Gravitational acceleration} \\
I & \quad \text{Unit tensor} \\
M & \quad \text{Camera magnification} \\
m_p & \quad \text{Particle mass} \\
N_p & \quad \text{Number of particles in an grid cell} \\
N_i & \quad \text{Number of particles in an interrogation area} \\
p & \quad \text{Pressure} \\
r & \quad \text{Radius} \\
R_0 & \quad \text{Displacement correlation peak} \\
S_p & \quad \text{Source term (reactive force to the drag force)} \\
s & \quad \text{Displacement} \\
t & \quad \text{Time} \\
u_p & \quad \text{Gas velocity} \\
v_p & \quad \text{Particle velocity} \\
v_\alpha & \quad \text{Velocity of particle } \alpha \\
V_{cell} & \quad \text{Volume of a grid cell} \\
x & \quad \text{Position in the fluidised bed} \\
\beta & \quad \text{Inter-phase momentum transfer coefficient} \\
\varepsilon & \quad \text{Volume fraction} \\
\theta & \quad \text{Granular temperature} \\
\rho & \quad \text{Density} \\
\end{align*} \]

\[ \begin{align*}
\text{[m/s]} & \quad \text{[m/s]} \\
\text{[m]} & \quad \text{[-]} \\
\text{[-]} & \quad \text{[-]} \\
\text{[m]} & \quad \text{[m]} \\
\text{[m]} & \quad \text{[N/m^2]} \\
\text{[m]} & \quad \text{[kg/m^2]} \\
\text{[kg/(m^2)]} & \quad \text{[J]} \\
\text{[kg/m^2]} & \quad \text{[m]} \\
\text{[kg/m^2]} & \quad \text{[kg/m^2]} \\
\text{[m/s]} & \quad \text{[m/s]} \\
\text{[m/s]} & \quad \text{[m/s]} \\
\text{[m^3]} & \quad \text{[m^2]} \\
\text{[kg/(m^2)]} & \quad \text{[kg/(m^2)]} \\
\text{[J]} & \quad \text{[J]} \\
\text{[kg/m^2]} & \quad \text{[kg/m^2]} \\
\text{[m]} & \quad \text{[m]} \\
\end{align*} \]

REFERENCES