FREAK WAVE
Prediction and its Generation from Phase Coherence

Arnida L. Latifah
FREAK WAVE
PREDICTION AND ITS GENERATION FROM PHASE
COHERENCE

Arnida Lailatul Latifah
Samenstelling promotiecommissie:

Voorzitter en secretaris:
prof. dr. P. M. G. Apers University of Twente

Promotor
prof. dr. ir. E. W. C. van Groesen University of Twente

Leden
prof. dr. ir. J. J. W. van der Vegt University of Twente
prof. dr. ir. A. E. P. Veldman University of Twente
prof. dr. C. Kharif IRPHE, France
prof. dr. ir. R. H. M. Huijsmans Delft University of Technology
dr. G. van Vledder Delft University of Technology
dr. ir. T. Bunnik MARIN, Wageningen

The research presented in this dissertation was carried out at the group of Applied Analysis, Department of Applied Mathematics, Faculty of Electrical Engineering, Mathematics and Computer Science (EEMCS) of the University of Twente, PO Box 217, 7500 AE Enschede, The Netherlands.
FREAK WAVE
PREDICTION AND ITS GENERATION FROM PHASE COHERENCE

DISSERTATION

to obtain
the degree of doctor at the University of Twente,
on the authority of the rector magnificus,
prof. dr. H. Brinksma,
on account of the decision of the graduation committee,
to be publicly defended
on Wednesday the 13rd of October 2016 at 12:45

by

Arnida Lailatul Latifah
born on 22nd March 1987
in Bantul, Indonesia
Dit proefschrift is goedgekeurd door de promotor prof. dr. ir. E. W. C. van Groesen
To my family
Contents

Summary xi
Samenvatting xiii

1 Introduction 1
   1.1 Definition of extreme waves .................................. 3
   1.2 Mechanisms of extreme waves .................................. 5
   1.3 Contribution of the thesis ..................................... 6
   1.4 Outline of the dissertation .................................... 8

2 Unidirectional dispersive wave model 11
   2.1 AB model ......................................................... 11
   2.2 Influx signal ..................................................... 12
   2.3 Numerical Implementation ....................................... 13
      2.3.1 Solving AB model ........................................ 14
      2.3.2 Damping zone ............................................. 14
      2.3.3 Grid size ................................................... 15
   2.4 Test Case ....................................................... 15
   2.5 Conclusions ..................................................... 19

3 Extreme waves in a laboratory wave tank 21
   3.1 Introduction ...................................................... 21
   3.2 Experimental set-up and conditions ............................ 22
   3.3 Design versus experiments of extreme waves ................ 25
   3.4 Experiments versus a-posteriori simulations .................. 26
      3.4.1 Dispersive focussing wave (TUD 102) .................... 28
      3.4.2 New Year wave (TUD 201) ................................ 31
4 Coherence and Predictability of Extreme Events in Irregular Waves

4.1 Introduction .................................................. 38

4.2 Signal coherence: from maximal to irregular signals ...

4.2.1 Notation .................................................. 40

4.2.2 Maximal signal ........................................... 41

4.2.3 Phase effects ............................................. 43

4.3 Wave coherence and pm-waves ................................

4.3.1 Pseudo-max waves ....................................... 45

4.3.2 Nonlinear effects ......................................... 46

4.4 Freak wave prediction method and study cases ............

4.4.1 Linear Prediction method ................................ 51

4.4.2 Study cases ............................................... 52

4.5 Conclusion .................................................... 64

5 Localized Coherence of Freak Waves ......................

5.1 Introduction .................................................. 66

5.2 Coherence and Wavelet Transform ....................

5.2.1 Signal coherence ......................................... 69

5.2.2 Wavelet transform ....................................... 72

5.3 Characterizing Freak Waves ..............................

5.3.1 Critical group events ................................... 77

5.3.2 Most energetic waves ................................... 78

5.3.3 Local coherence .......................................... 79

5.4 Case Studies .................................................. 80

5.4.1 Focussing wave (202002) .............................

5.4.2 Synthetic signals ......................................... 84

5.4.3 Experimental signal: Irregular Wave (IW12) ....

5.5 Conclusions .................................................... 92

6 Conclusions and Recommendations .....................

6.1 Conclusions .................................................. 95

6.2 Recommendations ............................................ 96

A Supplementary of experiment results ....................

97
CONTENTS

B Stokes corrections 103
References 105
Acknowledgments 115
About the author 117
Summary

The processes that lead to the appearance of an extreme wave are not unique: one extreme wave may occur due to different mechanisms than another extreme wave. This gives challenges in the study of extreme waves, which are also called 'freak' waves, or 'rogue' waves when they satisfy certain conditions on the wave height compared to (the average of) neighbouring waves. After a freak wave with 18.5 m crest height hit the Draupner oil platform in the North sea on 1 January 1995, the investigation in the topic of freak wave has become more intense. It has been widely recognized that freak waves in the ocean are an important cause of accidents, and that they occur more frequent than expected. It is therefore important to understand the freak wave appearance.

This dissertation is intended to understand the development of irregular waves into freak waves, restricting ourselves to waves travelling in one horizontal direction, corresponding to long-crested waves in the ocean. It contains new concepts that can explain the mechanism that can lead to a freak wave. The mechanism for freak wave appearance that is investigated is that of phase coherence: the more coherent the phases of waves contributing to the freak wave, the higher the crest of the freak wave will be. A freak wave for which all waves contribute to the highest possible crest is the so-called maximal wave. Although this concept is used in hydrodynamic laboratories to generate high waves in a tank, much higher than can be achieved with a single stroke of the wave flap, it is extremely unlikely to occur in the real open seas so that the occurrence of freak waves in a random wave field has to be investigated. It turns out that the more flexible notion of pseudo-maximal wave as a description of an extreme wave with less coherent phase is more applicable for extreme wave occurrence in the ocean. Even less restrictive, a weak pseudo-maximal wave that only takes into account the most energy carrying waves can be used to describe an extreme wave as well. These proposed concepts are based on linear wave theory, while nonlinear contributions
are added by the Stokes correction. By understanding that an extreme wave may occur as a consequence of linear coherence, a linear prediction method based on minimizing the total wave phase can estimate the time and position of an extreme wave. The description and the prediction of the extreme wave can then be given using the power spectrum and the phase of the signal at a certain position.

Eventhough signal coherence can describe and predict the appearance of freak waves, the concepts are not fine enough for a more detailed local investigation applicable for irregular waves. Therefore a further contribution of this dissertation investigates the local energy propagation that leads to a freak wave. A freak wave is mostly developed from a localised wave group that contains a considerable amount of energy that evolves into successive states with even higher coherence. The wavelet transformation is used effectively for identifying the spectral energy distribution of the group events and its evolution. The local energy of waves in a wave group interact and build a larger amplitude. This interaction is based on local dispersive effects within the wave group. A high correlation between the local coherence and the wave amplitude showed that the local coherence can be a good indicator of the appearance of freak waves.

Various study cases are investigated in this dissertation to test the applicability of the description, the prediction, and the local mechanism of freak waves. The studies are limited to unidirectional dispersive waves above flat bottom. Hence, for the numerical simulation of the wave evolution, a unidirectional dispersive wave equation is used, the so-called AB-equation, that is accurate in second order and can be adjusted for any water depth. The AB model is part of the software package under the name HAWASSI-AB. More information of the software can be found on [http://hawassi.labmath-indonesia.org](http://hawassi.labmath-indonesia.org).
Samenvatting

Er zijn verschillende processen die kunnen leiden tot extreem hoge golven: de ene hoge golf kan door andere mechanismen ontstaan dan een andere. Dit is de uitdaging in de studie van extreme golven, die ook wel 'freak' waves, of 'rogue' waves worden genoemd als de golfhoogte groot is ten opzichte van de gemiddelde golfhoogte in de omgeving. Nadat op nieuwsjaardag 1995 een extreme golf met een tophoogte van 18.5m het Draupner olieplatform in de Noordzee had geraakt, is het onderzoek naar extreme golven veel intensiever geworden. Het wordt nu veel breder erkend dat deze golven een belangrijke oorzaak kunnen zijn van ongelukken en dat ze veel vaker voorkomen dan eerder verwacht.

Dit proefschrift is bedoeld om bij te dragen aan het begrip over het ontstaan van freak waves, waarbij we ons beperken tot golven in één richting, zogenaamd langkammige golven in de oceaan. Het proefschrift bevat nieuwe concepten die het ontstaan van freak waves kunnen verklaren. Het mechanisme voor het ontstaan van freak waves dat onderzocht zal worden is dat van fase-coherence: hoe meer coherent de fases zijn van de erbij betrokken golven, des te hoger de golf zal zijn. De extreme golf waarvan alle golven bijdragen aan de hoogst mogelijke golftop is de zogenaamde maximale golf. Ofschoon dit concept gebruikt wordt in hydrodynamische laboratoria om hoge golven te genereren in een golftank, veel hoger dan met één slag van de golfopwekker kan worden bereikt, is het zeer onwaarschijnlijk dat zo’n golf op open zee zal voorkomen. De freak waves daar zullen moeten ontstaan uit een interactie van veel willekeurig optredende golven, waarvoor de minder strikte begrippen van een pseudo-maximale en zwak-pseudo-maximale golf dan beter toepasbaar zijn om extreme golven te beschrijven. Deze ingevoerde begrippen zijn gebaseerd op de aannemer van lineariteit, waaraan dan de kleine niet-lineaire Stokes correcties kunnen worden toegevoegd. Het feit dat een extreme golf kan ontstaan door lineaire coherentie leidt tot een methode om de tijd en positie van een freak wave te voorspellen door de totale golf fase te
minimaliseren. De beschrijving en voorspelling kan dan gebaseerd worden op het spectrum en de fasen van het golfsignaal op een bepaalde plaats.

Ofschoon coherentie het ontstaan en voorspellen van freak waves kan beschrijven, zijn deze concepten nog onvoldoende nauwkeurig voor een gedetailleerd lokaal onderzoek van een onregelmatige golf. Daarom is een verdere bijdrage van dit proefschrift het onderzoek naar de lokale energieverdeling dat leidt tot een freak wave. Een extreme golf ontwikkelt zich gewoonlijk uit een gelokaliseerde golfgroep met voldoende energie die zich vervolgens in meerdere stadia verder ontwikkelt tot hogere coherentie. De zogenoemde wavelet-transformatie is gebruikt om de spectrale energieverdeling in dat groepsproces te identificeren. De golven in een groep zijn in interactie ten gevolge van dispersieve effecten, en bouwen daardoor een hogere energie op. Een hoge mate van correlatie tussen de lokale coherentie en de golfamplitude heeft aangetoond dat de lokale coherentie een goede aanwijzing is voor het ontstaan van freak waves.

Meerdere testcases zijn onderzocht om de toepasbaarheid en kwaliteit van de beschrijving, de voorspelling en het lokale golfgedrag te testen. Alle onderzochte gevallen zijn beperkt tot éénrichtings-golven boven vlakke bodem. Voor numerieke berekeningen is het zogenaamde AB-model gebruikt, dat tweede orde nauwkeurig is en bruikbaar voor elke diepte. Het AB model is onderdeel van een software pakket onder de naam HAWASSI-AB; meer informatie kan gevonden worden op http://hawassi.labmath-indonesia.org.
Chapter 1

Introduction

For centuries scientists thought that the occurrence of extreme waves in the ocean is a sailor’s myth. In the recent years they revealed that it is in fact a very real phenomenon, particularly after data from offshore platforms and satellite tracking confirmed their existence. One photograph of an extreme wave by NOAA also confirmed the existence of extreme waves in the open ocean (see Figure 1.1). In 2001 European Space Agency (ESA) satellites captured more than 10 individual extreme waves of more than 25 m wave height around the globe during three weeks of data collection (BBC News UK, July 2004). This is quite a high number for such a relatively short time interval. National Geographic News by Cameron Walker (August 2004) also published an article entitled ”Monster” Waves Suprisingly Common, Satellites Show.

The importance to study extreme waves is motivated by the significant damage they can cause to marine, offshore, or coastal structures. In the report of ESA News 21 July 2004, extreme waves are believed to be the major cause in the sinking more than 200 supertankers and container ships in the last two decades. Extreme wave accidents reported in media are becoming more often. Most media reported the damages caused by extreme waves. Occurrence of extreme waves around the world were collected and analyzed in various catalogues such as by Didenkulova et al. [2006], Nikolkina and Didenkulova [2012], Liu [2007]. The recent catalogue of Nikolkina and Didenkulova [2012] presented a collection of extreme wave events reported in the mass media during 2006-2010. There are 106 events, which are classified by their validity as true (78) and possible (28). Bilyay et al. [2011] presented wave measurements in Filyos, Western Black Sea, Turkey, for a period of two years; above a depth of 12.5 m there were 209 extreme waves. Pelinovsky and Kharif [2011] and Nikolkina and Didenkulova [2011] reported that the Indonesian region has the largest number of fatalities caused by
extreme waves, including several casualties in August 2010 when a ship carrying 60 people (of which only 21 were rescued) capsized and sank following a collision with an extreme wave. Over 70 people were missing and three had been confirmed dead after the passenger vessel Marina Baru capsized on December 19, 2015 off southern Sulawesi, Indonesia, due to rough weather [World Maritime News, 21 December 2015]. On 1 January 2016, local media reported a high wave came to the shore and dragged five people; one witness described that the wave reached up to 9 m high [Pitaloka 2016, Arifin 2016].

An annual review of extreme waves by Dysthe et al. [2008] mentioned two most well-studied extreme waves which were recorded on oil platforms. The first one is exceptional wave from the Gorm field in the North Sea on November 17, 1984. The extreme wave stands out with a crest height of 11 m in a sea with significant wave height of 5 m. The second one is the Draupner or New Year wave, that was recorded by a laser instrument at the Draupner platform in the North Sea on January 1, 1995 (see Figure 1.2). The wave is above 70 m water depth and has a crest height of approximately 18.5 m in a sea with a significant wave height of 11.8 m. Only minor damage was inflicted on the platform during the extreme wave event, confirming the validity of the reading made by the downward-pointing laser sensor. Since then the Draupner wave has attracted serious attention of scientists.

Besides from measurements on oil platforms, extreme waves are also possibly recorded from marine radar or satellite data. Satellite data should help scientists to understand more reliably the statistics of extreme waves. The potential of using satellites for global surveillance of the worlds oceans is considerable, and
1.1 Definition of extreme waves

The phenomenon of extreme waves was for the first time introduced to the scientific community by Draper [1964], but the scientific community has studied the topic of extreme waves more intensely since 2000 [Dysthe et al., 2008]. Mariners, have a lot of experience observing extreme waves in the ocean, and define an extreme wave as a great wall of water appearing from nowhere. Extreme waves are also known as freak waves, rogue waves, giant waves, or monster waves [Pelinovsky and Kharif, 2008]. Until now there is no unique definition of extreme wave. Many definitions of extreme wave are proposed by researchers, namely

"an event that represents an outlier when seen in view of the popu-
lation of events generated by a piecewise stationary and homogeneous second order model of the surface process”, [Haver, 2004]

"extraordinarily larger water wave with potentially devastating effects on offshore structures and ships” [Fedele, 2007]

"a particular kind of ocean wave that displays a singular, unexpected wave profile characterized by an extraordinary large and steep crest or trough”, [Pinho et al., 2004]

"a wave whose wave height at least two times as high as the significant wave height”, [Kharif and Pelinovsky, 2003]

"a wave with maximum height is much greater than two times the significant wave height (Hs), and the ratio of the crest height of the maximum crest height to significant wave height is greater than 1.25” [Olagnon and van Iseghem, 2000]

The first definition mentioned above is based on statistical theory, while the last two give a mathematical definition, which is the most common used by scientists. From this definition, the New Year wave is categorized as a freak wave since its crest height is $1.57Hs$, while the Gorm wave is categorized as a freak wave since the wave height exceeds $2Hs$. Tsunamis can also result in large waves and two tsunamis events, the Indian Ocean tsunami 2004 and the Japanese tsunami 2011, are well described in the media. Different from unpredictable extreme waves that we will deal with in this dissertation, the origin of a tsunami is already well explained, and will therefore not be considered in our study.

C. Kharif and E. Pelinovsky in [Ruban et al., 2010] revealed that the mathematical definition of extreme waves is too restrictive to characterize an extreme wave. They described an extreme wave which consisted of three large waves (so-called three sisters) with their height approximately 8 m and the significant wave height of 5 m. From the mathematical definition it is not a freak wave, but the waves may give a larger distraction than a single high wave. Therefore, they suggested that instead of a geometric definition, a definition based on wave energy seems to be more relevant to include the large population of different rogue waves appearing in the ocean. Additionally, [Forristall, 2005] claimed that the mathematical criteria of extreme waves gives very small exceedance probabilistically, for a linear Gaussian sea model. Eventhough the model always underpredict the occurrence of extreme waves, it gives a reasonable prediction for the wave height, therefore the linear Gaussian model is still widely used as the basis of either probabilistic or deterministic investigations. This is also discussed in [Baxevani
1.2 Mechanisms of extreme waves

How such an extreme wave occurs had been an open question for decades. Now the mechanisms of the appearance of extreme waves becomes more explainable. There is no single general mechanism for the formation of extreme waves. Gemmrich and Garrett [2008] argued that an extreme wave is generated by a simple consequence of linear superposition of waves. Another mechanism was proposed by Kharif and Pelinovsky [2006] that the interactions between envelope solitons provide a satisfactory explanation for the formation of extreme waves. Fedele [2007] gives an overview that the occurrence of extreme wave is highlighted into two scenarios: a nonlinear mechanism of second order bound waves (non-resonant interactions) and third order four-wave resonance interaction of free waves. In the annual review of Dysthe et al. [2008], a number of physical mechanisms of extreme waves are discussed: spatial focusing, dispersive focusing, and nonlinear focusing. Besides these three mechanisms, Slunyaev et al. [2011] discussed also other possible mechanisms in the appearance of extreme waves in shallow water, deep water, and even in the shore area, such as nonlinear wave interaction and wave-current interaction.

Spatial focusing can be achieved by the refraction of waves in varying bottom or in variable currents. As waves propagate into shallower water and their wavelength becomes comparable to the water depth, the waves get refracted, and they align their crests with the topography and steepen. Such extreme waves often occur near the shore. As the wave approaches the shore and the water depth decreases, the wave height increases. Wave velocity and wave length decrease with the decreasing depth, which by conservation of energy leads to an increasing amplitude as the kinetic energy is converted to potential energy. This can result in huge waves at the coast [Hincks et al., 2013]. Current focusing can also explain the appearance of extreme waves in the open sea. A wave colliding against a current going in the opposite direction gathers energy and builds up wave height. Even though the current velocities in the open seas are small, they can give small deflection of waves when they act over long distances and it increases or decreases wave energy intensity [Dysthe et al., 2008], [White and Fornberg, 1998]. Recently, Cousins and Sapsis [2014, 2016] perform a spatially localized analysis of the energy distribution along different wave numbers to understand the energy transfer during the development of an extreme wave. Their studies underlined that the appearance of extreme events can be triggered by
focussing energy in localized wave groups.

Dispersive focusing is a linear effect and occurs even in linear Gaussian seas. It had been proposed in the eighties by Boccotti [1981] that the mechanism of the occurrence of extreme waves in a Gaussian sea are particular realizations of the evolution of a well defined wave group. The dispersive focussing can be explained by creating a long wave group with decreasing frequencies, then the dispersion forces the group to contract to a few wavelengths at a given position. The emergence of a single high wave can be seen as resulting from an essentially linear constructive interference of numerous harmonics and the role of nonlinearity is limited to modification of the initial spectrum to the prescribed shape at the focusing locations identical phases of all harmonics [Shemer, 2016]. This mechanism is also suggested in Kharif and Pelinovsky [2003], Gemmrich and Garrett [2008], Pelinovsky and Kharif [2011] and Longuet-Higgins [1974]. Fedele [2007] confirmed that extreme waves most likely occur due to the dynamics of a single wave group in agreement with the dynamics imposed by the Zakharov equation [Zakharov, 1999].

Nonlinear focusing is the most active research topic as the mechanism for extreme wave occurrence [Dysthe et al., 2008]. It concerns focusing due to a modulational instability of nonlinear waves, the so-called Benjamin-Feir (BF) instability after T.B. Benjamin and J.E. Feir discovered it in a laboratory tank in 1967. The essence of this phenomenon is in an unstable growth of weak wave modulations, which evolve into short groups of steep waves [Slunyaev et al., 2011]. Alber [1978] showed that the BF instability occurs in a narrow band random wave field and it can be neglected for a broad wave spectrum. While the instability develops, the population of rogue waves increases, but this only occurs for very long-crested waves [Gramstad and Trulsen, 2007]. The extreme wave governed by this instability is formed as a breather wave, an anomaly in a series of waves that gathers in the energy of its neighbors and builds itself up to a great height [Chabchoub et al., 2010, Chabchoub and Fink, 2014]. Another example of a freak wave that occurs due to strong nonlinear effects is the case of three long waves (see Figure 1.3 that shows an initial wave and its spectrum evolution at various time). This type of wave was discussed in detail by Viotti et al. [2014]. The development of high wave amplitude comes from the growth of the nonlinear waves.

1.3 Contribution of the thesis

The dissertation is concerned with the investigation of extreme wave in the ocean. The main purpose is to give a scientific contribution in understanding the generation of extreme waves. We restricted the extreme waves in our study by the
1.3 Contribution of the thesis

Figure 1.3: The left plot shows a wave profile at initial time $t = 0$ [s]. The right plot presents the evolution of the spectrum at various times that shows the development of the nonlinearity during the wave evolution.

definition that an extreme wave is a wave with a wave height that exceeds approximately two times the significant wave height ($H_s$) or exceeds the crest height by $1.25H_s$. We studied extreme waves with mostly broad spectrum and a weak modulational instability, i.e. low degree of nonlinearity. In this dissertation the extreme waves are mostly generated from the linearly dominated constructive interference of waves.

Our study primarily focused on (1) introducing new mathematical concepts that can describe a freak wave and model the maximal crest of the freak wave, (2) predicting the time and the position of a freak wave for given an initial signal at a certain position, (3) investigating the local energy propagation of the freak wave, and (4) analyzing the origin of the freak wave through the wave energy propagation. All theoretical chapters in this dissertation not only discuss the conceptual background, but also present various study cases of how the new approaches can be implemented. Most of the study cases are taken from experimental results in a laboratory wave tank.

The new mathematical concepts to describe an extreme wave, the so-called maximal and (weak) pseudo-maximal wave, may serve as the first contribution of this dissertation. For a given power spectrum, a maximal wave is defined as a wave with zero phases that has the highest possible crest height. This concept is also mentioned in [Shemer, 2016] that the crest height of waves cannot exceed the value attained when the phases of all harmonics coincide (perfect coherence). This is actually based on the simple linear superposition of harmonics. Due to the change of phases of the harmonics the surface shape can vary fast. Therefore, phase coherence is essential in the appearance of an extreme wave. The zero phases of all waves is unlikely to occur in the ocean, thus a (weak) pseudo-
maximal wave is introduced as a wave with less coherent phase. This concept is then more applicable for extreme ocean waves. Besides the description of extreme waves, this dissertation presents a linear prediction method of extreme waves based on phase coherence. The prediction gives an estimation of the time and the position of the extreme wave occurrence.

Further, a detailed investigation of a local coherence mechanism of a wave group as one possible mechanism of extreme wave appearance is discussed. An extreme wave can be developed by local interactions of waves in a wave group for which the effect of waves that are not in the immediate vicinity is minimal. Then, local coherence of the wave group measures the appearance of the extreme wave quite well.

1.4 Outline of the dissertation

This thesis is divided into six chapters, starting with an introduction, followed by four main chapters, then a chapter of conclusions and recommendation. An appendix is given at the end.

Chapter 2 presents the unidirectional dispersive wave model, the so-called AB wave model that is used in this dissertation. It includes the numerical implementation of the model for practical use. An example to apply this model with measurement verification is also given here.

Chapter 3 provides the experimental results of extreme wave generation that have been executed in the laboratory wave tank of Delft University of Technology. Four cases of extreme waves were conducted in the experiment: a dispersive focussing wave, a New Year wave, a dispersive focussing wave with harmonic background, and bi-chromatic waves. The generated extreme waves are designed with the help of the AB wave model. To see the performance of the wave tank we compare the designed and the reconstructed extreme waves in the laboratory.

In Chapter 4, a published paper of Latifah and van Groesen [2012] is presented. The paper discusses the description and the predictability of extreme waves. (Pseudo)-maximal waves introduced in the paper describe extreme waves very well. Meanwhile, the prediction of the extreme wave position could be done by minimizing the total phase that evolves from the given influx signal. Four case studies from measurements, a dispersive focussing wave and irregular waves are dealt with in this chapter.

Chapter 5 proposes a local coherence mechanism within a wave group that can be one mechanism leading to a freak wave. This chapter discusses that the localized energy may trigger a freak wave appearance. In four study cases presented in this chapter, it is shown that a high correlation exists between the local coherence and the appearance of a freak wave, that the freak waves are
developed by local interactions of waves in a wave group and that the effect of waves that are not in the immediate vicinity is minimal. This chapter has been submitted for publication as [Latifah and van Groesen, 2016]. Conclusions and recommendations for future work are covered in the final chapter. Some additional details are presented in the Appendix.
Summary

This chapter describes concisely the so-called AB wave model that we used in the research for numerical simulations. It also discusses how we deal with the influxing problem for which the surface wave elevation at one position as a function of time is given and the wave elevation at downstream positions for each time has to be calculated. The numerical implementation to solve the model is given and illustrated for a relevant test case.

2.1 AB model

We used the nonlinear AB equation for the numerical simulations. The AB equation proposed by van Groesen and Andonowati [2007] is a unidirectional wave equation above a flat bottom describing the surface wave elevation. This equation is derived by exploiting the variational formulation of surface water waves. It is accurate in second order in the wave height, has exact dispersion properties and is applicable for finite and for infinite depth, but here we only present the equation for the finite depth. For waves above finite depth, the AB equation can be interpreted as a higher-order KdV equations; in lowest order it is the classical KdV equation.

We describe the dynamics by the surface elevation, $\eta(x,t)$. The nonlinear AB
equation can be written as:

$$\partial_t \eta = \pm \sqrt{g} A \left[ \eta + \frac{1}{4} (B \eta)^2 + \frac{1}{2} B (\eta B \eta) - \frac{1}{4} (A \eta)^2 + \frac{1}{2} A (\eta A \eta) \right]$$  \hspace{1cm} (2.1)

where $A$ and $B$ are pseudo differential operators which depend on the dispersion relation, see also van Groesen et al. [2010]. The linear AB equation is only the first term within the brackets of (2.1). The minus sign in the equation (2.1) is for the wave evolution travelling to the right and the plus sign is for the wave evolution travelling to the left. We consider dispersive wave evolution and apply the exact dispersion relation for water waves. In one space dimension, water waves on a layer of depth $h$ in a constant gravity field $g$, have dispersion given by the relation,

$$\omega = \Omega(k) = \text{sign}(k) \sqrt{g k \tanh(kh)}$$  \hspace{1cm} (2.2)

where $k$ is the wave number. The skew-symmetric operator $A$ and the symmetric operator $B$ are defined by:

$$A = C \frac{\partial_x}{\sqrt{g}} \quad B = \frac{\sqrt{g}}{C}$$  \hspace{1cm} (2.3)

Here $C$ is the phase velocity operator, i.e. the symmetric pseudo differential operator with symbol the phase velocity $\hat{C}$, defined by $\hat{C} = \frac{\Omega(k)}{k}$. Hence the symbols of the operator $A$ and $B$ as:

$$\hat{A} = i \ \text{sign}(k) \sqrt{k \tanh(kh)} = \frac{i\Omega(k)}{\sqrt{g}}$$  \hspace{1cm} (2.4)

$$\hat{B} = \sqrt{\frac{k}{\tanh kh}}$$  \hspace{1cm} (2.5)

The quadratic operators in the nonlinear terms of the AB equation cannot be easily approximated by ordinary differential operators. Thus, instead of solving the AB equation (2.1) in physical space, the AB equation is solved by a pseudo-spectral method as described in Section 2.3.1.

### 2.2 Influx signal

Given an influx or initial condition at one position or one time respectively, the AB equation can compute the surface wave elevation at every time and at every position. The initial value problem does not cause much problems, since
2.3 Numerical Implementation

the description of the state variables in the spatial domain at an initial time is independent of the specifics of the evolution model. In the influxing problem, the waves are generated by influx-boundary conditions, or by some embedded, internal, forcing [Liam et al., 2014]. The influxing case has been applied widely by [van Groesen et al., 2010], [van Groesen and van der Kroon, 2012], [Latifah and van Groesen, 2012] for the case of one dimensional unidirectional equation and is also applied in [Adytia, 2010], [Adytia and van Groesen, 2012] for the variational Boussinesq model.

To generate waves travelling to the right with surface elevation \(s(t)\) at specific position \(x_0\) as influx, we use a source term \(S(x, t)\) as an embedded influx. The influxing problem with the nonlinear AB model for \(x_0 = 0\) is then described by:

\[
\partial_t \eta = - \sqrt{gA} \left[ \eta + \frac{1}{4} (B\eta)^2 + \frac{1}{2} B(\eta B\eta) - \frac{1}{4} (A\eta)^2 + \frac{1}{2} A(\eta A\eta) \right] + S(x, t) \\
\eta(0, t) = s(t)
\]  

(2.6)

The condition for the source term in 1D and 2D wave propagations has been discussed in detail by [Liam et al., 2014]. In this study, we implement the embedded point generation to the influx in the nonlinear AB equation for the simulation. For a source concentrated at \(x_0 = 0\) we take \(S(x, t) = \delta(x)f(t)\) in which \(\delta(x)\) is the Dirac-delta function and the source function \(S(x, t)\) will be of the form

\[
\check{S}(K(\omega), \omega) = \frac{1}{2\pi} V_g(K(\omega))\check{s}(\omega)
\]

in Fourier space. The check notation represents a temporal Fourier transform and the notation \(V_g\) is the group velocity which brings the dispersive information to the influxing model with \(K(\omega) = \Omega^{-1}(\omega)\). The function \(f\) is the so-called modified influx signal which is the convolution between the original source \(s(t)\) and the inverse temporal Fourier transform of the group velocity \(\omega \rightarrow V_g(K(\omega))\) which depends on the water depth.

2.3 Numerical Implementation

Different from the linear AB equation, it is impossible to find the analytic solution of the nonlinear AB equation. This section will describe how to solve the nonlinear AB model numerically by pseudo-spectral methods and some numerical issues that we have to deal with in the numerical simulation. We will also give an example of the simulation and analyze its performance.
2.3.1 Solving AB model

In the numerical implementation, we solve the AB model by transforming the equation into Fourier space and it becomes:

\[ \partial_t \hat{\eta} = -\sqrt{gA} \left[ \hat{\eta} + \frac{1}{4} (B\hat{\eta})^2 + \frac{1}{2} B(\hat{\eta}\hat{B}) - \frac{1}{4} (A\hat{\eta})^2 + \frac{1}{2} A(\hat{\eta}\hat{A}) \right] \]  \hspace{1cm} (2.7)

The hat notation represents a spatial Fourier transform. The nonlinear terms can be computed by the pseudo-spectral procedure, i.e.

\[ (B\hat{\eta})^2 = FFT \left[ IFFT \left( \hat{B} \cdot \hat{\eta} \right) \right]^2 \]

\[ B(\hat{\eta}\hat{B}) = \hat{B} \cdot FFT \left[ IFFT (\hat{\eta}) \cdot IFFT (\hat{B} \cdot \hat{\eta}) \right] \]

These are also applied for the operator \( A \) in the rest of the terms of equation (2.7). The notation \( FFT \) and \( IFFT \) represent the Fast Fourier Transform and its Inverse Fast Fourier Transform in MatLab. The time integration procedure is done by ODE45-solver in MatLab. For each time step, we apply anti-aliasing in order to prevent aliasing in linear and non-linear terms during the simulation.

2.3.2 Damping zone

The spectral implementation in a restricted domain leads to a periodic solution, i.e. the wave propagating to the right approaching the right boundary will come back to the left boundary and can disturb the influxing signal. To avoid this looping, we add a damping zone at the left and right near the boundary of the analysis domain so that the wave in the damping zone will be vanishing smoothly. Therefore, we define a smoothed characteristic function \( \chi(x) \) which has value one in the damping zone and vanishes inside the domain of interest. The illustration of the characteristic function in a domain is presented in Figure 2.1. We damp the waves by adding term \( \alpha \chi(x)\hat{\eta} \) with \( \alpha > 0 \) in the right hand side (RHS) of the AB equation:

\[ \partial_t \eta = RHS - \alpha \chi(x)\eta \]

Then in the numerical implementation, the equation in Fourier space becomes:

\[ \partial_t \hat{\eta} = RHS - \alpha FFT (\chi(x) \cdot IFFT (\hat{\eta})) \]

The value of the (positive) damping coefficient \( \alpha \) determines how fast the waves will decrease to zero. The larger the value of \( \alpha \) the faster the decaying. The solution of \( \eta \) is decaying exponentially with the order of \( e^{-\alpha T_l} \) in the damping zone, where \( T_l \) is the travel time of the wave in the damping zone \[^{[2013]}\]. Since the longest wave has speed \( c_0 = \sqrt{gh} \), a damping zone with length \( L = dc_0/\alpha \) will give an exponential decay approximately \( e^{-d} \).
2.3.3 Grid size

For the numerical simulation, we need to determine the spatial and temporal grid size, $dx$ and $dt$. We can take any reasonable time step for the calculation output since the time integrator from matlab has automatic time step so that the accuracy of the numerical output does not depend on $dt$, but it depends on $dx$. We should choose the grid size $dx$ so that the shortest wave will be represented by enough points, let say 10 points for one wave. The length of the shortest wave can be observed from the spectrum of the initial signal. Suppose the spectrum start vanishing at $\omega_m$, which is related to $k_m$ by the dispersion relation, the minimum wave length will be $\frac{2\pi}{k_m}$ and we should take $dx$ smaller than $\frac{2\pi}{10k_m}$ to obtain at least 10 points in one wave for the numerical computation.

2.4 Test Case

We will show the performance of the nonlinear AB equation by implementing it in the laboratory version of the New Year wave (MARIN case 204001). We compare the simulation results with the measurements as performed by MARIN hydrodynamic laboratory. Comparison with the analytic solution which is actually the solution of the linear AB equation is also given.

Influx signal and numerical parameters

The influx signal for the simulation is taken from the measurement data 204001 at the first wave probe (W1), 10 meters after the wave flap. The wave was generated above a water depth of $h = 1m$. We prefer to use the measured surface elevation at W1 to the wave flap motion as influx signal since it avoids errors that may arise when translating the flap motion to a surface elevation. Measurements at five other wave probes will be used for comparison with the AB simulations: W2 at 20m, W3 at 30m, W4 at 49.5m, W5 at 50m, and W6 at 54m from the wave flap.

In Figure 2.1 the plots at the left show the influx signal and its normalized amplitude spectrum. In the simulation, we took a domain of $x \in [-40, 140]m$ including the damping zone. At the right Figure 2.1 the layout of the simulation domain is shown with the smoothed characteristic function. We took $2^{12}$ modes in the numerical computation domain, so that the grid size was approximately $dx = 0.04m$. From the amplitude spectrum of the influx signal in Figure 2.1 it follows that the wave energy is almost vanishing for frequencies larger than $\omega_m = 10$. This value corresponds to a wave length $\lambda_m = 0.62m$. Then the choice of $dx$ is sufficient since it leads to more than 14 grid points per wavelength for waves with $\omega_m \leq 10$. For the wave at peak frequency ($\omega_p = 4$), the grid points
Figure 2.1: Plots at the left show the time signal at W1 and its normalized amplitude spectrum. At the right the layout of the simulation domain is given: the shaded areas represent the domain for damping the waves with the smoothed characteristic function shown by the dotted line; the dashed line shows the function of the inverse temporal Fourier transform of the group velocity which depends on the water depth.

will be more than 80 per wave length. We used the same time step as defined by the measurement at the influx position for the time integration. In order to make the wave sufficiently damped, we chose the value of the damping parameter $\alpha = 4$.

Simulation results

Figure 2.2: The left plot shows the density of the surface wave elevation computed by the nonlinear AB equation in the domain $x \in [10, 70]m$ and $t \in [100, 190]s$. The right plot shows the maximal temporal amplitude (MTA) and the minimal temporal trough (MTT) for the nonlinear AB simulation (dashed) and for the linear solution (dotted). The bullets show the MTA and MTT from the measurements.

Figure 2.2 shows the density of the surface wave elevation and the maximal
temporal amplitude (MTA) and minimal temporal trough over the time at each position for $x \in [10, 70]$m. We observed that each bullet matches with both MTA and MTT of the nonlinear AB simulation, while the linear solution underestimates the maximal amplitude of the measurements and the nonlinear AB simulation. If we look at the MTA of the nonlinear AB simulation, we can see that there is still a more extreme amplitude at $x = 50.5$, which is 1cm higher than the maximal amplitude at 49.5m. Unfortunately, there was no wave probe installed at $x = 50.5$.

![Figure 2.3: Snapshots of the time signal at measurement points W2, W3, W4, W5, and W6 for the measurement (solid), for the nonlinear AB simulation (dashed), and for the linear solution (dotted).](image-url)

Figure 2.3 show the time signals of the nonlinear AB simulations, measurements, and also the linear solutions at five measurement positions. In order to evaluate the performance of the AB simulation, we compared the time signal and the spectrum with the measurements data. At all measurement positions, the nonlinear AB simulation agrees very well with the measurements, see Figure 2.3-2.4. The small differences might be caused by various model and numerical errors. The performance of the nonlinear AB simulation can also be measured quantitatively by its correlation coefficient with the measurements. The corre-
ulation is defined as the inner product between the normalized time signals from the simulations and from the measurements. Deviations from the maximal value 1 of the correlation measures mainly the error in phase, a time shift of the signal. For the minimal value -1 the simulation signal is in counter phase with the measurement. Table 2.1 shows that the correlation of the time signals computed with the nonlinear AB simulation are the measurements is more than 92%. The linear solution also gives a good approximation of the measurement at W2, but after W2 the correlation drops under 80%. This could be explained by the fact that position W2 is still close to the influx position, so that the nonlinearity is not yet developed much.

Figure 2.4: The solid lines show the normalized amplitude spectrum for the measurements. The dashed and the dotted lines show the differences with the nonlinear AB simulation (dashed) and for the linear solution (dotted).

For a numerical simulation, we measured the relative computation time as the cpu-time for computing the time integration divided by the length of the time interval of the initial signal of the experiment. The numerical computations were performed on a computer with CPU i7, 1.8 GHz processor with 4 GB memory. As a result, the relative computation time is 45%. In this case the computation using $2^{11}$ modes is actually already sufficient as the correlation between the sim-
ulation results and the measurements will only be 1% less than using \(2^{12}\) modes. Additionally, the relative computation time will reduce much when the simulation uses less modes: it takes only 16% for \(2^{11}\) modes and 6% for \(2^{10}\) modes.

**Table 2.1:** Correlations between measurements and simulations with the nonlinear AB model and with the linear solution.

<table>
<thead>
<tr>
<th></th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>W2</td>
</tr>
<tr>
<td>AB</td>
<td>0.95</td>
</tr>
<tr>
<td>Linear</td>
<td>0.94</td>
</tr>
</tbody>
</table>

### 2.5 Conclusions

The nonlinear AB equation models unidirectional dispersive surface wave elevation quite well. The nonlinear terms of the AB equation give substantial contributions to the waves evolution. Different from the linear equation that has a well-defined analytic solution, the nonlinear equation is solved numerically by the pseudo-spectral method. For practical use in a hydrodynamic laboratory, the waves are generated in timely manner. Therefore, we presented the numerical implementation of the AB equation with an influxing signal. One test case, i.e. the laboratory version of New Year wave has been performed to show the capability and the accuracy of the nonlinear AB model. The numerical simulations by the AB equation showed qualitatively and quantitatively good correlation with the experimental results. The time signals at the measurement positions give more than 92% correlation with the measurement data from the laboratory. The relative computation time is also efficient: for the simulation with \(2^{12}\) modes it takes 45% and for the simulation with \(2^{11}\) modes it takes 16%.

It should be noted that further developments after the design of the above AB equation, improved versions of the model, and their numerical implementation have been developed by other authors; these results are assembled in software called HAWASSI (Hamiltonian Wave-Ship-Structure Interaction), see [www.hawassi.labmath-indonesia.org](http://www.hawassi.labmath-indonesia.org) for details and references.
Chapter 3

Extreme waves in a laboratory wave tank

Summary

This chapter provides the experimental results of generating extreme waves in the laboratory wave tank of Delft University of Technology (TU Delft). The main purpose of the experiment is to see the capability and the performance of the wave tank to generate extreme waves. A brief review about the previous experiments of extreme wave in many other laboratories is given in Section 3.1. The experimental set-up and the designed-extreme waves are presented in Section 3.2. Section 3.3 presents the comparison between the designed and the reconstructed extreme waves in the laboratory. Section 3.4 describes the reconstructions in the laboratory and the simulation after the experiment using surface elevations of a first measurement as input signals for each case. The last section gives conclusions and some remarks about the experiments.

3.1 Introduction

Researchers have long used well-controlled experiments in wave tanks to study the development of regular and irregular waves [Dysthe et al., 2008]. A number of experiments in laboratory have been performed to investigate extreme waves, in the subject of existence, nonlinearity, instability, soliton, statistics, ship testing, etc. Most of the experiments are for unidirectional wave propagation. Modern laboratory facilities can reconstruct accurately surface water wave profiles from field observations [Clauss and Klein, 2009, Clauss, 2002]. Of course, a controlled
laboratory environment is still limited to explain what really happens in the open ocean. Nevertheless, the wave tank experiments help a lot to better understand extreme wave appearance and verify the theoretical studies of extreme waves [Onorato et al., 2006a].

Extreme waves are often related to a breather-soliton which is already more than 30 years found [Akhmediev et al., 2011, Chabchoub et al., 2010]. One specific soliton is called the Peregrine soliton that is the solution of the nonlinear Schrodinger equation with coherent state condition. [Chabchoub et al., 2010] performed a miniature version of an extreme wave in terms of the Peregrine soliton in a laboratory water tank. They generated 1 cm harmonic waves and then gave a slight disturbance by modulating the wave. As a result, the wave suddenly appeared traveling half as fast as the others, then it grew up to three times higher values, which is exactly what was expected. [Shemer et al., 2010]. Shemer and Sergeeva [2009] built an experiment for extreme waves generated by an initial wave with a narrow banded Gaussian power spectrum with random phases. The well-known New Year wave has been successfully reproduced with a certain scaling in a seakeeping basin by [Clauss and Klein, 2009, 2011]. Extreme waves were also generated in laboratory from dispersive focussing waves showing that the maximum amplitude can grow up to eight times higher than the initial signal [Hennig and Schmittner, 2009, Shemer et al., 2007b]. MARIN hydrodynamic laboratory also reproduced the New Year wave and generated dispersive focusing wave in wave tank. The data were used for a benchmark workshop on numerical wave modelling in 2012.

This chapter will complement experimental studies about the generation of extreme waves in a laboratory wave tank with the additional aim to test the capability and the accuracy of the TU Delft Towing tank 1 to build extreme waves. We designed experiments such that an extreme wave occurs at a certain position using the nonlinear AB model as described in Chapter 2. The experiments were executed in collaboration with TU Delft. Recently, there were follow-up experiments including breaking waves in the same wave tank with much success by [Kurnia et al., 2015].

3.2 Experimental set-up and conditions

The experiments were executed in the Towing tank 1 at Ship Hydromechanics Laboratory, TU Delft. The tank is mostly used to generate regular or irregular waves, a mainy used to test properties of ship behaviour, such as resistance in calm water and waves, motions and accelerations of fast ships in waves, slamming phenomena, etc. For more detailed information about the Towing tank, see the official website of TU Delft [http://www.3me.tudelft.nl].
3.2 Experimental set-up and conditions

Figure 3.1: Photograph of Towing tank 1 at TU Delft.

The length of the tank is 150m, the width is 4m, and the maximal water height is 2.41m. An artificial beach for absorbing the waves and for minimizing wave reflection starts at 142m. The waves are generated by a wave maker consisting of one flap at the beginning of the tank. All experiments were carried out with a water depth 2.12m and frequency 50Hz. The chosen water depth is reasonably enough for the experiment since it is suitable for the wave tank and leaves enough free board in the tank to generate a large wave without damaging equipments inside or outside the tank.

Figure 3.2: Scheme of the experiments and the position of the wave probes. The wave probe 5 is the position of the expected extreme wave.

The set-up of the experiment is illustrated in Figure 3.2. Alongside the tank, eight wave probes were installed at certain positions. All wave probes were of wire-type connected to the processing equipment. To avoid the distortion of the wave maker into the measurements, we cannot position the wave probe too close
to the wave flap, so we installed the first wave probe at 9.95m from the wave flap. If the wave probes are positioned further away from the wave flap, we will also obtain the disturbance from the reflection of the artificial beach. Therefore the last wave probe was installed at 90.1m which is still far enough from the artificial beach. The detailed positions of all wave probes are shown in Table 3.1.

**Table 3.1: Position of wave probes from the wave flap**

<table>
<thead>
<tr>
<th>Wave probes</th>
<th>WP1</th>
<th>WP2</th>
<th>WP3</th>
<th>WP4</th>
<th>WP5</th>
<th>WP6</th>
<th>WP7</th>
<th>WP8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position [m]</td>
<td>9.95</td>
<td>19.90</td>
<td>41</td>
<td>67</td>
<td>70</td>
<td>73</td>
<td>80</td>
<td>90.1</td>
</tr>
</tbody>
</table>

The procedure of the experiments consists of four steps: installation, calibration, runs with waiting time between each run, and cleaning and removal. The mechanics of the wave flap, the properties of the waves, and the dimensions of the basin provide limitations to the wave generation. The limitations of the wave maker and the accuracy of the measurement gauges are shortly described in Table 3.2. The generated wave with frequency larger than six may damage the equipments. Due to the restricted frequencies, we removed the frequencies of the wave input which are smaller than one or larger than six.

**Table 3.2: Capability of the wave maker and the accuracy of the measurement gauges**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Capability</th>
<th>Parameter</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>1.0 - 6.0 [rad/s]</td>
<td>Amplitude</td>
<td>0.001 [m]</td>
</tr>
<tr>
<td>Frequency step size</td>
<td>1.0 [rad/s]</td>
<td>Frequency</td>
<td>0.001 [rad/s]</td>
</tr>
<tr>
<td>Amplitude</td>
<td>0.05 - 0.2 [m]</td>
<td>Electric potential</td>
<td>0.001 [V]</td>
</tr>
<tr>
<td>Amplitude step size</td>
<td>0.05 [m]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum stroke</td>
<td>6 [V]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The input for the experiments are the motion of the wave maker, therefore we use a transfer function between the surface wave elevation and the motion of the wave flap as described in Ooms [1996]. Then the transfer function will give the stroke of the wave maker as the input. For the equipments safety, we check and smooth the stroke data, especially for the beginning and the ending time of the input to make sure that there is no jump stroke which may damage the equipment. In addition, the data obtained from the experiments are still full of noise, therefore data filtering to remove the noises is needed to extract the measurement data.
3.3 Design versus experiments of extreme waves

There are four different types of extreme wave to be generated in the wave tank: a dispersive focusing wave, a scaled New Year wave, a focussing wave with harmonic background, and a bi-chromatic wave. The dispersive focusing wave and the New year wave experiments were adopted from the MARIN experiment 203001 and 204001 respectively, which we have scaled based on the capability of the wave tank at TU Delft. The focussing wave with harmonic background is motivated to investigate the effect of the background into the extreme wave generation. The bichromatic wave was motivated by the experiment of Westhuis [2001].

We executed seven successful runs as presented in Table 3.3: two dispersive focussing waves with different amplitudes, one scaled New Year wave, three focussing waves with harmonic background with increasing amplitudes, and one bi-chromatic wave. For each case, we will present and discuss the largest extreme wave. The results for other cases will be presented in Appendix.

Table 3.3: Type of wave

<table>
<thead>
<tr>
<th>No</th>
<th>Case</th>
<th>Name of run</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Dispersive focussing wave</td>
<td>TUD 101</td>
</tr>
<tr>
<td></td>
<td></td>
<td>TUD 102</td>
</tr>
<tr>
<td>2.</td>
<td>New Year wave</td>
<td>TUD 201</td>
</tr>
<tr>
<td>3.</td>
<td>Focussing waves with harmonic background</td>
<td>TUD 301</td>
</tr>
<tr>
<td></td>
<td></td>
<td>TUD 302</td>
</tr>
<tr>
<td></td>
<td></td>
<td>TUD 303</td>
</tr>
<tr>
<td>4.</td>
<td>Bi-chromatic wave</td>
<td>TUD 401</td>
</tr>
</tbody>
</table>

We first designed the extreme waves at one position about 70m from the wave flap. To obtain the wave signal at the wave flap as the experimental influx, we simulated a wave evolution in the direction to the wave flap by the nonlinear AB equation using the designed-extreme wave as the initial signal. Then, the experiments were generated according to the wave tank capability, i.e. the limited frequencies in [1, 6].

Figures 3.3-3.6 show the snapshots of the designed extreme waves and the reconstrated waves in the laboratory for the case TUD 102, TUD 201, TUD 303, and TUD 401. It turns out that the cut-off frequencies in the influx signal decreases the wave elevation and the amplitude spectrum of the reconstructed waves. In all cases we observe that the limitations of the wave tank give some differences of extreme waves from the designed extreme waves. Since the wave contributions from the small and high frequencies were off in the influx signals,
the amplitude of the reconstructed waves are less than originally designed. Neverthe-
less, the correlation coefficient between the designs and the experiments are mostly still high, see 3.4. This gives us confidence that the wave tank is capable to generate extreme waves.

Figure 3.3: The left plots show the time signals of the measurement (solid line) and the design (dashed-line) at WP2-WP8 for TUD 102 case. The right plots show the corresponding amplitude spectrum.

3.4 Experiments versus a-posteriori simulations

In this section we present and analyze the experimental results of four different extreme waves. We compare the measurements with the numerical simulation by nonlinear AB simulation after the experiments. We use the measurement data at WP1 as the influx signal.

The conclusive Table 3.4 shows the correlation coefficient of the time signals between the measurements and the AB simulation at each wave probe for four cases. Table 3.5 presents the correlation coefficient of the amplitude spectra. The AB simulation models all the cases very well, except for the bi-chromatic waves. This is explainable as in the bi-chromatic wave experiment many breaking waves occur that are not accounted for in the nonlinear AB equation. The model and
3.4 Experiments versus a-posteriori simulations

Figure 3.4: Same as Figure 3.3 now for TUD 201 case.

Figure 3.5: Same as Figure 3.3 now for TUD 303 case.
Figure 3.6: Same as Figure 3.3 now for TUD 401 case.

laboratory experiments about breaking waves are discussed in detail by Kurnia [2016], Kurnia et al. [2015].

In the experiments we can only measure the temporal signals at some positions, therefore we cannot measure the extreme wave in the whole spatial domain. Nevertheless, we will use the nonlinear AB equation to describe the generated extreme wave in the spatial domain.

3.4.1 Dispersive focussing wave (TUD 102)

The measurement results are presented in Figure 3.7. It shows a good agreement between the measurements and the nonlinear AB simulations.

In the experiment of TUD 102, the peak of the wave amplitude broke a few second after the focussing (approximately at 72 m). The wave steepness of the maximum amplitude is approximately 0.07, which is still less than the criterion of breaking wave 0.1. The existence of the breaking phenomena in the experiment TUD 102 cannot be modeled by AB-equation. This phenomena can be observed from the measurement at WP6 (see Figure 3.7). The right Figure 3.7 shows that the amplitude spectrum of the time signal measurement at WP6 is less than the AB-simulation, especially around the peak frequency. This indicates a dissipation
### Table 3.4: Correlation coefficient of the signals between the designs and the measurements (I) and between the measurements and a-posteriori AB-simulations using WP1 as influx signal (II) at wave probes.

<table>
<thead>
<tr>
<th>Positions</th>
<th>TUD 102</th>
<th>TUD 201</th>
<th>TUD 303</th>
<th>TUD 401</th>
</tr>
</thead>
<tbody>
<tr>
<td>WP1</td>
<td>0.93</td>
<td>0.92</td>
<td>0.97</td>
<td>0.96</td>
</tr>
<tr>
<td>WP2</td>
<td>0.92</td>
<td>0.94</td>
<td>0.95</td>
<td>0.96</td>
</tr>
<tr>
<td>WP3</td>
<td>0.88</td>
<td>0.96</td>
<td>0.91</td>
<td>0.96</td>
</tr>
<tr>
<td>WP4</td>
<td>0.89</td>
<td>0.95</td>
<td>0.90</td>
<td>0.95</td>
</tr>
<tr>
<td>WP5</td>
<td>0.82</td>
<td>0.92</td>
<td>0.89</td>
<td>0.94</td>
</tr>
<tr>
<td>WP6</td>
<td>0.82</td>
<td>0.91</td>
<td>0.90</td>
<td>0.96</td>
</tr>
<tr>
<td>WP7</td>
<td>0.81</td>
<td>0.89</td>
<td>0.89</td>
<td>0.95</td>
</tr>
<tr>
<td>WP8</td>
<td>0.89</td>
<td>0.89</td>
<td>0.89</td>
<td>0.93</td>
</tr>
</tbody>
</table>

### Table 3.5: Correlation coefficient of the amplitude spectra between the designs and the measurements (I) and between the measurements and a-posteriori AB-simulations using WP1 as influx signal (II) at wave probes.

<table>
<thead>
<tr>
<th>Positions</th>
<th>TUD 102</th>
<th>TUD 201</th>
<th>TUD 303</th>
<th>TUD 401</th>
</tr>
</thead>
<tbody>
<tr>
<td>WP1</td>
<td>0.95</td>
<td>0.95</td>
<td>0.93</td>
<td>0.86</td>
</tr>
<tr>
<td>WP2</td>
<td>0.95</td>
<td>0.99</td>
<td>0.94</td>
<td>0.98</td>
</tr>
<tr>
<td>WP3</td>
<td>0.96</td>
<td>0.99</td>
<td>0.95</td>
<td>0.98</td>
</tr>
<tr>
<td>WP4</td>
<td>0.97</td>
<td>0.99</td>
<td>0.94</td>
<td>0.98</td>
</tr>
<tr>
<td>WP5</td>
<td>0.94</td>
<td>0.99</td>
<td>0.94</td>
<td>0.98</td>
</tr>
<tr>
<td>WP6</td>
<td>0.95</td>
<td>0.99</td>
<td>0.94</td>
<td>0.98</td>
</tr>
<tr>
<td>WP7</td>
<td>0.96</td>
<td>0.99</td>
<td>0.94</td>
<td>0.98</td>
</tr>
<tr>
<td>WP8</td>
<td>0.96</td>
<td>0.99</td>
<td>0.95</td>
<td>0.98</td>
</tr>
</tbody>
</table>
Figure 3.7: The left plots show the time signals from the measurement (solid line) and a-posteriori AB simulation (dashed line) at WP2-WP8 for TUD 102 case. The right plots show the corresponding amplitude spectrum.

Figure 3.8: Maximal and minimal temporal amplitude of TUD 102, the AB-simulation (dashed-line) and the measurements (+). The solid line presents the extreme wave occurred in the tank computed from the nonlinear AB-simulation.
of the energy. The breaking phenomena explains the lower value of the correlation coefficient at WP7 and WP8 compared to the earlier positions (see Table 3.4).

Besides the interesting time signal at WP6, we present Figure 3.8 showing the maximum and minimum temporal amplitude of TUD 102 simulated by nonlinear AB equation. The maximum and minimum measurement at the wave probes are also displayed in the same figure. It shows that in the experiment TUD 102 the simulation by the nonlinear AB-equation agrees very well with the measurement.

### 3.4.2 New Year wave (TUD 201)

We present the measurement results and the AB-simulations of the experiment TUD 201 in Figure 3.9. We also present the MTA of TUD 201 compared to the measurement data at various positions in Figure 3.10. If we look at the MTA, the increase of the maximal amplitude is not so extreme: at the initial position there is already quite large amplitude, then it becomes larger around $x = 30 \text{m}$ and again at $x = 70 \text{m}$.

![Figure 3.9: Same as Figure 3.7, now for TUD 201 case.](image)

According to the nonlinear AB simulation the maximum amplitude occurs at $x = 70.25 \text{m}$. This experiment is adopted from the New Year wave, therefore we will compare the generated extreme wave and the New Year wave in the original scale. Figure 3.11 presents that the period of both time signals are well matched. The maximum amplitude of the generated extreme wave is less and
the troughs are also deeper than the New Year wave’s. The amplitude of the original New Year wave is steeper than in the experiment. This shows that the experiment gives a more linear extreme wave than the New Year wave. This might be a consequence of the restricted frequencies of the influx signal and the filtering of waves in the post-processing after the experiment. Eventhough there are some differences between the generated and the original New Year wave, this experiment confirmed that such extreme wave may occur in the ocean.

3.4.3 Dispersive focussing wave with harmonic background (TUD 303)

The experiment of the focussing wave with harmonic background was designed by adding a harmonic background into a focussing wave. The frequency of the
harmonic wave is chosen approximately the peak frequency of the focusing wave. The snapshots of the measurement results and the AB-simulations are presented in Figure 3.12. It shows that the AB-simulation models the wave propagation of TUD303 very well.

Figure 3.12: Same as Figure 3.7 now for TUD 303 case.

The MTA presented in Figure 3.13 demonstrates that the wave amplitude grows up three times higher than the initial. This shows that the existence of the harmonic background does not change the development of the focusing wave. The generated extreme wave has a wave height 40 cm and the significant wave height 12 cm. Their ratio is very large (3.33), much more than the ratio of the extreme wave definition.

3.4.4 Bi-chromatic wave (TUD 401)

The bi-chromatic wave in the experiment TUD 401 was designed with frequencies 4.9 and 5.1, which was expected to build a very high amplitude at the position 70 m. According to the prior AB-simulation, the bi-chromatic wave with these frequencies had been expected to break as it builds a very steep wave. In the experiment we observed that before arriving at WP3 the waves started breaking and many breaking waves appeared further. Consequently, the differences between the measurements and the AB simulations are significant since the AB-equation does not take into account the breaking waves (see Figure 3.14). This
is also shown in Figure 3.15: the maximum and minimum temporal amplitude of
the measurements at WP4-WP8 are much less than the simulated. Even though
the measured and simulated amplitude signal at WP3 agree quite well, Figure
3.14 at WP3 indicates that the measured time signal is slightly out of phase com-
pared to the AB-simulation. The correlation of the time signal at WP3 between
measurement and AB-simulation also starts decreasing very much.

3.5 Conclusion

Four different types of extreme waves were successfully generated in the wave
tank at TU Delft. There were seven experiments while two of them were break-
ing: TUD 102 and TUD 401. Even though the largest amplitude of the generated
extreme waves are less than the originally expected for all cases, the correlation
between the reconstruction signals and the designs are still high ($\geq 88\%$, ex-
cluding the breaking signals). The differences between the prior design and the
experiments might be caused by (1) cutting-off frequencies, (2) smoothing influx
signal at the beginning and at the end of time, and (3) filtering noises when
extracting the measurements data. The measurements at seven wave probes are
well verified by the nonlinear AB simulation. For non-breaking waves, the AB
simulations give correlation to the measurements more than 91%. In general, we
remark that the wave tank is capable to generate extreme waves. It can recon-
struct and follow the designed wave quite well with some limitations. Therefore,
the follow-up experiments by Kurnia et al. [2015] could be done successfully after
considering the capability of the equipment.
3.5 Conclusion

Figure 3.14: Same as Figure 3.7, now for TUD 401 case.

Figure 3.15: Same as Figure 3.8, now for TUD 401 case.
Coherence and Predictability of Extreme Events in Irregular Waves

Abstract

This paper concerns the description and the predictability of a freak event when at a certain position information in the form of a time signal is given. The prediction will use the phase information for an estimate of the position and time of the occurrence of a large wave, and to predict the measure of phase coherence at the estimated focussing position. The coherence and the spectrum will determine an estimate for the amplitude. After adjusting for second order nonlinear effects, together this then provides an estimate of the form of a possible freak wave in the time signal, which will be described by a pseudo-maximal signal. In the exceptional case of a fully coherent signal, it can be described well by a so-called maximal signal.

We give four cases of freak waves for which we compare results of predictions with available measured (and simulated) results by nonlinear AB-equation [van Groesen et al., 2010, van Groesen and Andonowati, 2007]. The first case deals with dispersive focussing, for which all phases are (designed to be) very coherent at position and time of focussing; this wave is nearly a maximal wave. The second case is the Draupner wave, for which the signal turns out to be recorded very close to its maximal wave height. It is less coherent but can be described

in a good approximation as a pseudo-maximal wave. The last two cases are irregular waves which were measured at MARIN (Maritime Research Institute Netherlands); in a time trace of more than 1000 waves freak-like waves appeared accidentally. Although the highest wave is less coherent than the other two cases, this maximal crest can still be approximated by a pseudo-maximal wave.

4.1 Introduction

In this paper we consider extreme waves that can 'accidentally' appear in irregular, uni-directional wave fields with very broad spectrum and relatively low value of the Benjamin-Feir index (BFI). These waves satisfy the common definition [Dysthe et al., 2008, Shunyaev et al., 2005, Kharif and Pelinovsky, 2003] of rogue, or freak, wave that the wave height exceeds two times the significant wave height. However, different from much current research on rogue waves, the modulational instability does not play a (dominant) role. Instead of nonlinearity dominated waves, the extreme waves here will appear at position and time of a high degree of coherence, in the sense that many wave components contribute to a linearly dominated constructive interference phenomenon. This agrees with Gemmrich and Garrett [2008] that many extreme waves are merely the simple consequence of linear superposition. For realistic wind waves, this coherence may be just as important as nonlinear effects (which may have played a role to obtain the coherence, and may enforce the linear converging of group lines near the extreme event). In fact, we will show that the well-known Draupner (or New Year) wave [Haver, 2004], measured in the North-Sea, shows a high degree of coherence while its BFI of approximately 0.55 [Janssen, 2003, Adcock and Taylor, 2009] is below the critical threshold value 1. In addition, we will show similar extreme waves that were generated accidentally in experiments on irregular waves in a wave tank at MARIN hydrodynamic laboratory. In two experiments and successive numerical calculations, with in total more than 2300 waves that were observed evolving downstream above a flat bottom over a distance of at least 30 wavelengths, 3 of such rogue events could be identified. Measurements and numerical simulations show a relatively gradual growth and decay before and after the rogue event. This long-life character does not satisfy the other characterization of rogue waves that these should appear suddenly and disappear quickly. Also, as has also been shown for four other measured freak events in the North Sea [Shunyaev et al., 2005], the linear and nonlinear simulations show remarkable little difference in shape and wave height, although with some difference of position and time due to nonlinear effects in propagation speed.

Referring to wave tank experiments by Shemer et al. [2010], Shemer and Sergeeva [2009], it should be noticed that these experiments were designed to
study BFI-dominated rogue waves. In these experiments, narrow band Jonswap spectra with $\gamma = 7$ and some narrow Gaussian spectra are considered. The coherence reported in Onorato et al. [2006b, Shukla et al. 2006] refers to the phase coupling between free wave and the higher order bound waves due to nonlinear wave generation. But since the free waves have random phase, these experiments can be seen as a bridge between the pure 'soliton' (Akhmediev-breather [Akhmediev et al., 2011]) rogue waves that generate a triangle spectrum shape from an initially very narrow spectrum, and the low valued BFI irregular waves (obtained for broad band Jonswap spectra with $\gamma = 3$) as will be described here.

In this paper we will characterize and discuss in various ways the appearance of extreme events and the role played by coherence as a constructive interference property. By this is meant that the phase nearly vanishes for waves in a considerable interval around the peak frequency. Together with the almost linear evolution property, this fact makes it possible to design a prediction method for this type of rogue waves. We will show that from a given elevation signal measured at some observation point, the position, time and profile of the rogue event can be rather well predicted over distances of 30 or more wave lengths. The method searches for the freak event by looking for the position and time such that the total phase, obtained by linear evolution of the observed phase, has minimal variance. Supported by linear and nonlinear numerical simulations and experimental observations, the predictions of rogue events for the Draupner and for the MARIN waves will be investigated and compared.

The contents of the paper can be described as follows. Sections 2 and 3 deal with the effect of partial or complete constructive interference. In Section 2 we consider time signals, obtained when vanishing phases in a so-called maximal wave create fully constructive interference at a certain instant. For a Jonswap spectrum as example, the effect of partial interference is investigated. For fixed random phase $\theta(\omega) \in (-\pi, \pi]$, signals with a fraction of that phase, so phase $\alpha \theta(\omega) \in \alpha(-\pi, \pi]$, are investigated for increasing $\alpha \in [0, 1]$. Upon increasing $\alpha$ until for $\alpha = 1$ the irregular signal (fully random) is obtained, the highest elevation in the maximal signal will decrease while the background grows, with clusters of larger and smaller waves depending on $\alpha$. The details of the full signal depend on the choice of the initial phase function $\theta$, but the average over random phases for fixed $\alpha$, produces a so-called pseudo-maximal wave, which is shown to be a scaled version of the maximal wave, with scaling amplitude tending to zero for $\alpha \to 1$.

In Section 3 we show the corresponding process for linear waves, and investigate the effects of 2nd order nonlinear Stokes contributions (detailed formulas are given in Appendix A). The linear propagation modifies the phase with $K(\omega) x$ where $x$ is the displacement, and $K(\omega)$ the wave number related to $\omega$. The non-
linear contributions, for realistic cases of wind waves in a coastal area, change the spectrum. But the changes are mainly in the long-wave components (leading to wave set-down) and slightly in the higher components but mainly in a neighbourhood of the double peak frequency, as expected. The nonlinear effects on the maximal wave are small, and just as well for the irregular wave, except for some different propagation speed.

In line with these observations, we formulate in Section 4 the prediction method based on the minimization of the phase variance over time and space. And we describe in detail 4 study cases; after a specially designed experiment for dispersive focussing, we investigate the Draupner wave and two irregular MARIN waves. The prediction method is shown to be capable to detect the extreme waves reasonably well.

In the final Section 5 we conclude with some additional remarks and conclusions.

4.2 Signal coherence: from maximal to irregular signals

4.2.1 Notation

Since waves in the ocean are described at each point by a time signal, we first consider real valued signals with zero mean defined on a time interval $[0,T]$. We introduce some notation, and then derive a priori estimates for the highest possible wave heights. In the following we describe the relation between a function $s(t)$ and its Fourier transform $\hat{s}(\omega)$ using notation with integrals as

$$s(t) = \int_{-\omega_{\text{max}}}^{\omega_{\text{max}}} \hat{s}(\omega)e^{-i\omega t}d\omega \text{ and } \hat{s}(\omega) = \frac{1}{2\pi} \int_{0}^{T} s(t)e^{i\omega t}dt$$

Here $\omega_{\text{max}} = 2\pi/\Delta t$ and $2\pi/T = \Delta \omega$ will be used because in practical situations we deal with discrete signals sampled with some time step $\Delta t$. From the real-valuedness of the signal we have $\hat{s}(\omega) = \overline{\hat{s}(-\omega)}$ (the bar denoting complex conjugation) and for the phase $\theta(\omega) = -\theta(-\omega)$. Hence

$$s(t) = \int_{-\omega_{\text{max}}}^{\omega_{\text{max}}} \hat{s}(\omega)e^{-i\omega t}d\omega = \int_{-\omega_{\text{max}}}^{\omega_{\text{max}}} |\hat{s}(\omega)|e^{i\theta(\omega)}e^{-i\omega t}d\omega$$

$$= 2 \int_{0}^{\omega_{\text{max}}} |\hat{s}(\omega)|\cos(\theta(\omega) - \omega t)d\omega$$

Parceval’s identity links the $L_2$-norms of the signal and its FT:

$$\int_{0}^{T} s^2(t)dt = 2\pi \int_{-\omega_{\text{max}}}^{\omega_{\text{max}}} |\hat{s}(\omega)|^2d\omega = 4\pi \int_{0}^{\omega_{\text{max}}} |\hat{s}(\omega)|^2d\omega$$
We define the variance and standard deviation $\sigma$ of the signal,

$$\sigma^2 = Var(s) = \frac{1}{T} \int_0^T s^2(t) dt$$

$$= \frac{4\pi}{T} \int_0^{\omega_{\text{max}}} |\hat{s}(\omega)|^2 d\omega = 2 \Delta \omega \int_0^{\omega_{\text{max}}} |\hat{s}(\omega)|^2 d\omega$$

the significant wave height $H_s$ as $H_s = 4\sigma$, the (one-sided) spectrum $E(\omega)$ such that

$$\int_0^{\omega_{\text{max}}} E(\omega)d\omega = \text{var}(s),$$

so $E(\omega) = 2 \Delta \omega |\hat{s}(\omega)|^2 = \frac{4\pi}{T} |\hat{s}(\omega)|^2$, and higher order moments

$$m_n = \int_0^{\omega_{\text{max}}} \omega^n E(\omega)d\omega$$

### 4.2.2 Maximal signal

From

$$|s(t)| = 2 \left| \int_0^{\omega_{\text{max}}} |\hat{s}(\omega)| \cos(\theta(\omega) - \omega t) d\omega \right|$$

$$\leq \int_{-\omega_{\text{max}}}^{\omega_{\text{max}}} |\hat{s}(\omega)| d\omega$$

it is seen that the inequality is actually an equality if at some time the cosine is identically 1. This can happen (only) if the total phase $\phi(\omega) = \theta(\omega) - \omega t$ vanishes for all $\omega$ at that time, say at $t = T_{\text{foc}}$. Then the signal has its maximal possible value:

$$\max_t s(t) = s(T_{\text{foc}}) = \int |\hat{s}(\omega)| d\omega \text{ if } \theta(\omega) - \omega T_{\text{foc}} = 0$$

For this reason we will call a signal with all phases zero at some time a **maximal signal**, $s_{\text{max}} = \int_{-\omega_{\text{max}}}^{\omega_{\text{max}}} |\hat{s}(\omega)| \cos(\omega(t - T_{\text{foc}})) d\omega$; (4.4)

at $T_{foc}$ all wave components contribute to a constructive interference. We will show maximal signals for a spectrum given by a Jonswap spectrum that is commonly used to describe developing wind wave fields. The specific expression is
given by

\[ E_{Jon}(\omega) = A g^2 \left( \frac{\omega_p}{\omega} \right)^5 \exp \left( -\frac{5}{4} \left( \frac{\omega_p}{\omega} \right)^4 \right) \gamma^r, \]

\[ r = \exp \left[ -\frac{1}{2\varsigma^2} \left( \frac{\omega}{\omega_p} - 1 \right)^2 \right] \]

(4.5)

The parameter \( \gamma \) specifies the narrow bandedness of the spectrum; the choice \( \gamma = 3 \) is taken for most realistic coastal situations and provides a broad band spectrum. Meanwhile the parameter \( A \) is related to the wave amplitude. We took as illustration \( A = 0.0408 \); these values are motivated by study cases of irregular MARIN waves in Section 4.4.2. The \( \varsigma \) is defined as \( \varsigma = 0.007 \) if \( \omega \leq \omega_p \) and \( \varsigma = 0.009 \) if \( \omega > \omega_p \). In Fig. 4.1 the dotted, solid and dashed line corresponds with \( \gamma = 1.5, \gamma = 3, \) and \( \gamma = 7 \) respectively (the narrow spectrum for \( \gamma = 7 \) was used in Shemer et al. [2010]).

\[ \text{Figure 4.1: The Jonswap spectrum, } E_{Jon}(\omega), \text{ where } A = 0.0408 \text{ and } \gamma = 1.5 \text{ (dotted), } \gamma = 3 \text{ (solid), } \gamma = 7 \text{ (dashed)} \]

\[ \text{Figure 4.2: The maximal signal corresponding to Jonswap spectrum with } \gamma = 3 \]

In order to see the maximal crest height of a wave with the Jonswap spectrum, we give an example for \( \gamma = 3 \). The plot of the maximal signal of Jonswap
4.2 Signal coherence: from maximal to irregular signals

spectrum for $\gamma = 3$ is given in Fig. 4.2. This maximal signal has significant wave height about 3.4m and the maximal possible amplitude is approximately 19m. From Fig. 4.2 we can see that the wave is confined to an interval of length equal to 8 peak periods. Outside the interval the wave nearly vanishes.

4.2.3 Phase effects

For the maximal signal above, all phases vanish at one position. In this section we investigate the effect of non-vanishing phases which may be partly coherent or random.

An irregular signal is obtained in case the phases are uniformly distributed in $(-\pi, \pi]$. To investigate cases in between a completely random signal and a fully coherent maximal signal, we will consider signals with ’cut-off’ phases. That is, for given random function $\theta(\omega) \in (-\pi, \pi]$, we consider for $\alpha \in [0, 1]$ signals with phase $\theta_\alpha = \alpha \theta$. Although not much can be said about an individual signal, the ensemble averaged signal at fixed $\alpha$, denoted by

$$[s]_\alpha = \text{Average} \left| \hat{s}(\omega) \cos (\theta_\alpha (\omega) - \omega (t - T_{foc})) \right|, \quad \theta_\alpha \in \alpha U (-\pi, \pi)$$

is interesting. Using the Strong Law Large Number [Ott and Longnecker, 2001] it can be shown that this average is a scaled version of the maximal signal. We will call this average a pseudo-maximal signal; it can be written as

$$[s]_\alpha = \rho(\alpha)s_{max}$$

where the scaling factor is

$$\rho(\alpha) = \frac{\sin(\alpha \pi)}{\alpha \pi}$$

For identically vanishing phases, the maximal signal was already shown in Fig. 4.2. In the plots of Fig. 4.3 we show for a given Jonswap spectrum $E_{Jon}(\omega)$ with $\gamma = 3$, the effect of phases. For a fixed random phase $\theta(\omega) \in (-\pi, \pi]$, we illustrate the effect of adding a fraction of that phase $\alpha \theta(\omega) \in \alpha (-\pi, \pi]$, for increasing $\alpha \in [0, 1]$. Upon increasing $\alpha$, the extreme wave is decreasing while the background grows. The original extreme wave may disappear completely, while in the background clusters of larger and smaller waves are formed, depending on $\alpha$ and on the specific random function $\theta(\omega)$; characteristic effects are visible in Fig. 4.3.
Figure 4.3: Jonswap signal with significant wave height of 3.4[m] and random phases in $\alpha(-\pi, \pi]$ in which $\alpha = 0.6$, $\alpha = 0.8$, and $\alpha = 1$. 
4.3 Wave coherence and pm-waves

In this section we illustrate for synthetic cases that wave coherence plays an important role in the appearance of extreme events in irregular wave trains. Extreme events will appear at instants and positions of a high degree of coherence, to be defined precisely in the following. This will prepare for the examples in the next section, and will motivate the prediction method of freak waves.

Furthermore, we will show by investigating the evolution over longer periods and positions, that away from the focussing area, the wave has still a considerable amplitude over a long range. Stated differently, the extreme wave is not an isolated phenomenon on an almost flat sea, but builds up gradually and disappears gradually back into the background. Since these phenomena are observed in linear as well as in nonlinear irregular waves, we will investigate effects of nonlinearity, effects on the spectrum as well on the wave evolution.

4.3.1 Pseudo-max waves

A wave evolution in 1D denoted by the surface elevation $\eta(x,t)$ describes at each position $x$ the signal $t \to \eta(x,t)$. In fact, for a given elevation signal $s_{obs}(t)$ at one observation point $X_{obs}$, we can describe the uni-directional linear evolution as

$$\eta(x,t) = \int |s_{obs}(\omega)| \cos(\Phi(t,x,\omega)) \, d\omega \quad (4.7)$$

where $\Phi(t,x,\omega) = K(\omega)(x - X_{obs}) + \theta_{obs}(\omega) - \omega t$ is the total wave phase, $K(\omega)$ is the wave number related to the frequency by the dispersion relation. For exact dispersion of infinitesimal waves, $K$ is the inverse of $\Omega$ given by

$$\Omega(k) = \text{sign}(k) \sqrt{gk \tanh(kD)}$$

where $g$ is the gravitational acceleration and $D$ is the water depth.

We determine the focussing position and time $(X_{foc},T_{foc})$ at which the phase variance

$$PV(x,t) = \text{Var}(\Phi(t,x,\omega))$$

$$= \int_{0}^{\omega_{\text{max}}} |\Phi(t,x,\omega)|^2 d\omega$$

is minimal, so-called $PV_{foc}$. In practice we compute the phase variance over an interval of the dominant frequencies $[\omega_{\text{min}},\omega_{\text{max}}]$. We define $\Gamma$ as the degree of coherence,

$$\Gamma = 1 - PV_{foc}.$$
For given random phase \( \theta_\alpha = \alpha \theta \) as described in the pseudo-maximal signal from Section 4.2.3, the phase variance can be computed to be \( PV(\theta_\alpha) = (\alpha \pi)^2/3 \).

Conversely, for arbitrary phases, we will take this relation to define \( \alpha \) to correspond with the phase variance. In particular at focussing we define

\[
\alpha^2_{foc} = \frac{3}{\pi^2} PV_{foc}
\]

and then define a pseudo-maximal (pm) wave as

\[
\eta_{pm}(x, t) = \rho(\alpha_{foc}) \int |\tilde{s}_{obs}(\omega)| \cos(\Phi(t, x, \omega)) \, d\omega
\]

(4.8)

with total wave phase \( \Phi(t, x, \omega) = K(\omega)(x - X_{foc}) - \omega(t - T_{foc}) \). This pseudo-max wave will model the neighbourhood of the extreme event.

Fig. 4.4 shows the density plots for the linear evolution of Jonswap waves with a restricted random phase for \( \alpha = 0, 0.6, 0.8, \) and 1. Besides that, we present the density of the variance of the total wave phase. Those density plots are shown in a frame moving with the group velocity. From both densities we can observe the position and the propagation of the wave with \( \alpha = 0 \) or \( \alpha = 0.6 \); the development of the high wave into the focussing wave is noticeable and the position of the minimal phase variance (PV) which shows the focussing position is prominent. The case with \( \alpha = 0.8 \) does not show the high waves clearly and the position of the focussing is hardly visible. For \( \alpha = 1 \), the Jonswap signal is purely random and there is no clear extreme wave.

### 4.3.2 Nonlinear effects

In this section we will take into account the nonlinear wave contributions, therefore we can investigate the importance of nonlinearity, especially in some cases we study here. From laboratory observation, a focussing signal has nonzero phase at low frequencies; there is a generated nonlinear interaction which causes a nonlinear set-down contribution. Moreover, a second order set-up contribution might appear. The effect of the nonlinear interaction should be involved as suggested by Clamond and Grue [2002], especially for highly-nonlinear phenomenon of freak wave. Therefore a nonlinear pm-wave needs to be designed to describe an extremal wave profile more precisely.

A nonlinear pm-wave will now be defined by adding the second-order contributions to the linear pm-wave; we neglect the higher order contributions. The quadratic nonlinear interaction of two waves with frequencies \( \omega_1 \) and \( \omega_2 \) produces higher-order waves with possible frequencies of 0, \( 2\omega_1, 2\omega_1, \omega_1 + \omega_2, \) and \( \omega_1 - \omega_2 \). The general interactions for pair of waves have been given by Dalzell [1999]. For
4.3 Wave coherence and pm-waves

Figure 4.4: In successive rows we show plots of the linear evolution of Jonswap waves with a random phase restricted for $\alpha = 0, 0.6, 0.8$ and 1 respectively. At the left density plots are shown of the evolution in a frame moving with the group velocity (horizontal axis, time vertical axis with normalized units). At the right, with the same axis the evolution of the density of the phase variance is shown.
the irregular waves we are dealing with, we sum up all the two waves interactions. The full expression of the nonlinear pm-wave is then given by:

\[ \eta_{pm}(x,t) = \rho(\alpha_{foc}) (I_{01} + I_{02} + I_p + I_m) \]  

(4.9)

\[ I_{01} = \int |\hat{s}(\omega)| \cos(\Phi(x,t)) d\omega \]

\[ I_{02} = \int |\hat{s}(\omega)|^2 (B_0(k) + B_2(k) \cos(2\Phi(x,t))) d\omega \]

\[ I_p = \int \int |\hat{s}(\omega_1)\hat{s}(\omega_2)| B_p(k_1, k_2) \cos(\Phi_1 + \Phi_2) d\omega_1 d\omega_2 \]

\[ I_m = \int \int |\hat{s}(\omega_1)\hat{s}(\omega_2)| B_m(k_1, k_2) \cos(\Phi_1 - \Phi_2) d\omega_1 d\omega_2, \]

where \( \Phi(x,t) = K(\omega)(x - X_{foc}) - \omega(t - T_{foc}) \) and the coefficients \( B_0, B_2, B_p, \) and \( B_m \) are symmetric functions of \( K(\omega) \) defined in Appendix A. The first term is the linear pm-wave defined in (4.8). \( I_{02} \) is the contribution generated by two identical frequencies. \( I_p \) and \( I_m \) are the contributions of two different frequencies; \( I_m \) gives a set-down contribution. According to [Chen, 2006] this set-down contribution is actually much more significant than the classical Stokes term. This set-down makes it possible for a nonlinear wave to have a lower amplitude than the linear wave. The effect of the second-order contributions will be shown in Jonswap spectrum case in Section 4.3.2. The linear part is a so-called 'free' wave, meaning that the wave number and frequency satisfy the dispersion relation. The other quadratic waves are so-called bound waves: the sum or difference of the wave numbers and the corresponding frequencies do not satisfy the dispersion relation and hence would not satisfy individually the wave equations, but their 'bounded' connection with the constituent free waves does satisfy the law of wave propagation.

The maximal signal corresponding to a given time signal is symmetric in time around the time of focussing \( T_{foc} \). Since a pseudo-maximal signal is just a scaled version of a maximal signal, the same holds true for a pseudo-maximal signal. Similarly, a (linear) maximal and a pm-wave is symmetric in time around \( T_{foc} \), and just as well symmetric in space around position \( X_{foc} \). From the nonlinear interactions shown above, it follows that even nonlinear corrections will respect these symmetry properties.

**Effects on spectrum**

A Jonswap spectrum is a spectrum that is supposed to describe realistic random sea waves, thereby neglecting long waves. Consequently the spectrum contains contributions from free and bound waves. To see the contribution of the bound
waves, we plot in Fig. 4.5 an example of a Jonswap spectrum (solid). Then we construct and plot the spectrum obtained by removing the second order bound waves and long waves (dashed line), so-called free-wave Jonswap. The subtraction of bound waves changes the original Jonswap spectrum by short wave removal that starts at 1.65 times the peak frequency, and annihilates practically all waves above $2\omega_p$. If we then add second order nonlinear contributions to this free-wave spectrum, we almost precisely recover the original Jonswap spectrum; a small overshoot near $2.3\omega_p$ is the only difference. This is visible in the enlarged lower plot in Fig. 4.5.

![Graph](image)

**Figure 4.5:** Top: The original Jonswap spectrum (solid), the free-wave Jonswap spectrum without bound waves (dashed), and the free-wave spectrum with nonlinear contributions (dotted). Bottom: The zoomed version.

To investigate the nonlinear effects on the signals, we consider the maximal signals corresponding to the 3 spectra above. In Fig. 4.6 we plotted the maximal signal of the original Jonswap (solid), the maximal signal of the free-wave Jonswap (dashed) and the maximal signal of the free-wave spectrum with the nonlinear additions (dotted, invisible behind the solid line). The plots show that the high frequency contributions make only little difference for the maximal signal.
Figure 4.6: The maximal signals corresponding to the original Jonswap spectrum (solid), the free-wave spectrum (dashed) and corresponding to the free-wave spectrum with nonlinear contributions (dotted, behind the solid line).

Figure 4.7: Similar as Fig. 4.4 left column, but now for nonlinear evolution, the waves with Jonswap spectrum and restricted phase $\alpha(-\pi, \pi]$ with $\alpha = 0, 0.6, 0.8$ and 1.
Effects on wave evolution

In this section we present the nonlinear wave evolution of the four cases of Jonswap signal as shown in Fig. 4.7. In the extreme case of the Jonswap signal with zero phases, the difference of the linear (see Fig. 4.4) and nonlinear evolution shows itself mainly in the propagation speed and wave height. The highest amplitude in the nonlinear evolution appears earlier than in the linear evolution; a similar behaviour is seen for the case of the Jonswap signal with random phases in $0.6(-\pi, \pi]$.

For Jonswap signal with $\alpha = 0$ which is a maximal wave, the density plot of the nonlinear evolution is not as smooth as the linear evolution. Around the focussing position and for $\alpha = 0.6$ we can observe the symmetry of the waves. The amplitude of the nonlinear wave is higher than the linear wave, as can be observed from the color bar in Fig. 4.7. For Jonswap signal with $\alpha = 0.8$ or $\alpha = 1$ which is mostly random, the difference between linear and nonlinear evolution is more difficult to see.

4.4 Freak wave prediction method and study cases

The description in the previous section leads to a simple and direct strategy to make predictions of the highest wave that will occur during the wave evolution. In the first subsection we describe the method for linear dispersive evolution to which we will restrict. This strategy will then be applied in four study cases in Section 4.4.2.

4.4.1 Linear Prediction method

Starting point is a given time signal $s_{obs}(t)$ at a given position $X_{obs}$. The length of the time interval of the signal is essential; despite some dispersive broadening of that interval while evolving away from $X_{obs}$, predictions can only be made within this (with distance shifted) time interval.

From the phase information of $s_{obs}(t)$ we determine the variance of the total wave phase, and look at its minimal value in space and time, finding $(X_{foc}, T_{foc})$, $PV_{foc}$ and the coherence $\Gamma_{foc}$. Using the spectrum of $s_{obs}(t)$, and calculating the phase band $\alpha_{foc}$ related to $\Gamma_{foc}$, we obtain the pseudo-maximal wave with parameter $\alpha_{foc}$.

It will be shown in the study cases that this pm-wave will approximate the highest wave that occurs in the linear wave evolution from the observed signal $\eta(x, t)$ in a neighbourhood of $(X_{foc}, T_{foc})$; in particular $(X_{foc}, T_{foc})$ estimates the position and time of the appearance of this highest wave. But also the shape of the time signal at $X_{foc} : t \rightarrow \eta(X_{foc}, t)$ for times near $T_{foc}$ will be well approximated.
Coherence and Predictability of Extreme Events in Irregular Waves

by the pseudo-maximal signal. We can actually reconstruct a more reliable signal prediction $\eta_{foc}$ which is the signal of the linear wave evolution at position $X_{foc}$:

$$
\eta_{foc}(t) = \int \left| \hat{s}(\omega) \right| \cos(\Phi(t - T_{foc}, X_{foc}, \omega)) d\omega
$$

(4.10)

In the following we will compare predictions with numerical simulations. Although the numerical results are not crucial for the main results presented here, we use linear and nonlinear simulations to compare with the linear-based predictions. When we talk about linear simulations in the following, this refers to simulations for the linear evolution with the exact dispersion relation (using a spectral method). Simulations with the AB-model refer to a nonlinear spectral code that has been described in various publications \cite{vanGroesenAndonowati,2007,vanGroesenetal.,2010,vanGroesenAndonowati,2011}. Specifically, in \cite{vanGroesenAndonowati,2011} the freak wave of the study case IW12 and in \cite{vanGroesenetal.,2011} the freak wave of the study case IW9 have been described in detail.

4.4.2 Study cases

The four study cases, for which measurements are available to test our descriptions and predictions of appearance of freak waves, are a dispersive focusing wave in a wave tank, MARIN experiment 202002, the Draupner wave with elevation measurement obtained from Dr. Sverre Haver of Statoil, and two irregular waves of Jonswap type, which were measured at MARIN but scaled (1 : 50 in space) to geophysical dimensions. For the irregular waves, the first case is IW12, Marin experiment 103001 with peak period 12[s] and the second case is IW9, Marin experiment 102003 with peak period 9[s].

For each case we follow the same strategy and show plots to illustrate the findings, which are summarized in a conclusive table at the end. From elevation heights at a certain position $X_{obs}$ we predict the pseudo-maximal wave: its coherence $\Gamma$, the position $X_{foc}$ and time $T_{foc}$ of focussing, and determine from that the maximal crest height and the maximal wave height at the moment of focussing. For the Draupner wave and the irregular waves we also provide the significant wave height and the Benjamin Feir index BFI as calculated at $X_{foc}$. The BFI is a measure of the quotient of nonlinearity and spectrum width. Various versions were described in the literature \cite{Shemer,2010}; we will use the definition from \cite{Janssen,2003}, $\text{BFI} = \sqrt{2\epsilon / \Delta \omega / \omega_p}$. Since we are dealing with deterministic waves, we define the nonlinearity $\epsilon$ by $ka_{rms}$ ($a_{rms}$ is the root mean square amplitude) as suggested in \cite{Kharifetal.,2009,Osborneetal.,2005,Shunyaev,2006}. The
spectral width $\Delta \omega$ is defined according to the energy level that corresponds to half of the spectral peak value [Shemer, 2010] and $\omega_p$ is the peak frequency.

**Dispersive focussing**

This first study case is a designed wave at MARIN based on the principle of dispersive wave focussing. The maximal wave height is more than five times the highest waves at the generation position. As we will show, at time and position of focussing, the wave is almost perfectly a maximal wave that can be accurately predicted from the initial signal at the observation position. In this case, the observation position is $X_{obs} = 10$ m. The initial signal is shown in Fig. 4.8.

![Figure 4.8: Measured time signal at $X_{obs} = 10$ m of the focussing wave](image)

First we compute the coherence by minimizing the variance of the total wave phase. Using spectrum and phase information of the initial signal, the variance of the total wave phase by choosing $\omega_{\text{min}} = 1.26$ and $\omega_{\text{max}} = 8.85$ is found to be minimal for $T_{foc} = 109.3$ and $X_{foc} = 50.05$; the minimal value is $PV_{foc} = 0.001$; the value of coherence $\Gamma$ is 0.999 (nearly fully coherent). Thus the extremal wave profile at focussing can be described well by a maximal signal.

The left Fig. 4.9 shows the linear and nonlinear maximal signal; the right one compares the time signal of the AB-simulation result at focussing and the nonlinear maximal signal (the highest crests are fitted at $t = 109.4$ s), including the spectrum and the phase. We use the spectrum and the phase to show the differences caused by nonlinear effect. In this case the nonlinear correction does not significantly affect the amplitude of the linear maximal signal. The effect of adding the second-order nonlinear corrections is almost invisible. The right Fig. 4.9 shows that the nonlinear maximal signal follows the actual focussing behaviour and perfectly models the extremal wave profile at focussing. For both low frequencies and high frequencies the spectrum is lifted up similar to the actual
Figure 4.9: We show in the left column the comparison between properties for the linear (solid) and the nonlinear maximal signal (dashed), and in the right column a comparison between properties of the nonlinear maximal signal (solid) and AB-simulations started at X = 10 m of the signal at X = 50.2 m (dashed). In the upper row for the signals, in the middle row for the spectrum, and in the lower row for the phase.

Figure 4.10: The zoomed density plot of the variance of the total wave phase PV(x,t). The minimal value PV_{foc} is shown in black
evolution by AB-simulation, although a bit lower. By observing the phase, we know the nonlinear correction in maximal signal yields long waves set down with phase $\pi$ or $-\pi$ in about $\omega < 1$ as in the actual evolution.

![Figure 4.11](image.png)

**Figure 4.11:** *Top:* Maximal Temporal Amplitude of linear (dashed) and nonlinear (solid) AB simulation; *Bottom:* Density plot of the nonlinear AB simulation

The minimal variance of the total wave phase is obtained at $(50.05; 109.3)$: the linear prediction leads to a focussing point at $x = 50.05$ m and focussing time of $t = 109.3$ s. This can be seen from Fig. 4.10 showing the density plot of the phase variance as a function of $x$ and $t$: the minimal phase variance is quite well visible since the area of minimal phase variance is very small and is surrounded by much higher values. To validate the predicted focussing position, the actual evolution is calculated by nonlinear AB simulation. The validation at the precise focussing position could not be done by measurement because of
the limited number of the measured positions, but the nearby measurement at 50m confirms the simulation results. Some other results of the AB-simulation are shown in Fig. 4.11. In the nonlinear AB simulation for which the density of the evolution is shown in the lower plot of Fig. 4.11 the extreme wave occurs at \( x = 50.2 \) m with the time focussing at \( t = 109.4 \) s. The upper plot of Fig. 4.11 presents the maximal temporal amplitude (the highest amplitude at each position during the time evolution) of the linear and nonlinear evolution by AB model, showing that the linear and nonlinear AB simulations do not differ significantly. The detailed comparison is presented in Table 4.1 confirming that the linear prediction agrees very well with the nonlinear evolution.

<table>
<thead>
<tr>
<th>CASE</th>
<th>Focussing wave</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth</td>
<td>1[m]</td>
</tr>
<tr>
<td>Position ( X_{obs} )</td>
<td>10[m]</td>
</tr>
<tr>
<td>Meas position</td>
<td>Prediction</td>
</tr>
<tr>
<td>( X_{foc} )</td>
<td>50.05</td>
</tr>
<tr>
<td>( T_{foc} )</td>
<td>109.3</td>
</tr>
<tr>
<td>( \Gamma ) (coherence)</td>
<td>1</td>
</tr>
<tr>
<td>Max Crest height</td>
<td>0.061</td>
</tr>
<tr>
<td>Max Waveheight</td>
<td>0.086</td>
</tr>
</tbody>
</table>

**Draupner wave**

The Draupner wave (also called New Year wave) is a point measurement at approximately 70 m depth under the Draupner platform (16/11-E) in the North Sea off the coast of Norway. The measurement of this time signal is 20 minutes long. We will first show that the wave shape at the Draupner position \( X_{Dr} \) is well approximated by a pm-signal by adjusting the height to the observed crest height.

To compare the Draupner wave with a pm-wave, we observe that the maximal wave corresponding to the spectrum would have crest height 37.5 m, instead of the actual height of 18.5 m. The ratio 18.5/37.5=0.49 is taken as multiplication factor of the maximal wave, which is precisely a pm-wave with \( \alpha = 0.6 \) and coherence \( \Gamma = 0.88 \). The plots of the Draupner wave (solid) and the shifted pm-wave (dashed) are shown in Fig. 4.13.

Using the observed signal at \( X_{Dr} \) we predict that actually an even higher wave has occurred at a few meters distance. In the following we take for conve-
4.4 Freak wave prediction method and study cases

Figure 4.12: The Draupner signal, with time of highest wave crest put at $t = 0$

Figure 4.13: Draupner Wave (solid), $P_m$-signal (dashed), and Signal prediction (dotted line) which has been shifted so that the highest peak is at $t = 0$

Figure 4.14: The density of the elevation of Draupner wave using initial signal at $X_{synth} = -400 \text{ m}$
nience $X_{Dr} = 0$. To test prediction capacity over longer distances, we simulate a backward (nonlinear) evolution to a synthetic observation position $X_{synth} = X_{Dr} - 400$, and use the (nonlinearly corrected) linear prediction method to determine the pm-wave from the signal information at $X_{synth}$.

To predict from $X_{synth}$ the position of the extreme wave, the minimal value of the phase variance is computed; Fig. 4.15 shows the density plot of the phase variance. In this case, we restrict the frequencies to calculate the phase variance to the interval $\omega \in (0.25; 1)$ as the linear wave contribution seems to be dominant in this interval. Then the minimal value of the phase variance is $PV_{foc} = 0.12$, which leads to $\alpha = 0.6$. This value of $\alpha$ is related to a pseudo-maximal wave with scaling factor of 0.5.

With the linear prediction, the most coherent wave is found at $x = 9$ m and $t = 1.1$ s which is shown in Fig. 4.13 approximately the position of the Draupner wave with 1.1 s shifted. The plot of the signal prediction from $X_{synth}$ is also shown in Fig. 4.13 Table 4.2 provides parameters of the prediction and the AB evolution using the initial time signal at $X_{synth}$, and of the measurement.

**Irregular waves**

The last two study cases of irregular waves provide a more realistic situation than the Jonswap example treated in Section 3. Although there are measurement positions more or less close to the highest wave appearance, we used simulations to compare the prediction results and compare wave shapes at the focussing.
4.4 Freak wave prediction method and study cases

Table 4.2: Parameters for the Draupner wave

<table>
<thead>
<tr>
<th>CASE</th>
<th>Draupner wave</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth</td>
<td>70[m]</td>
</tr>
<tr>
<td>Data position $X_{\text{synth}}$</td>
<td>-400[m]</td>
</tr>
<tr>
<td>$H_s$ at $X_{\text{synth}}$</td>
<td>12</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Meas position</th>
<th>Prediction</th>
<th>Simulation</th>
<th>Meas</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_{\text{foc}}$</td>
<td>9</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>$T_{\text{foc}}$</td>
<td>1.1</td>
<td>0.47</td>
<td>1.3</td>
</tr>
<tr>
<td>$\Gamma$ (coherence)</td>
<td>0.88</td>
<td>0.87</td>
<td>0.81</td>
</tr>
<tr>
<td>Max Crest height</td>
<td>20.22</td>
<td>19.09</td>
<td>19.32</td>
</tr>
<tr>
<td>Max Waveheight</td>
<td>27.42</td>
<td>26.5</td>
<td>28.28</td>
</tr>
<tr>
<td>$H_s$ at $X_{\text{foc}}$</td>
<td>11.64</td>
<td>11.9</td>
<td>13.5</td>
</tr>
<tr>
<td>BFI</td>
<td>0.49</td>
<td>0.48</td>
<td>0.61</td>
</tr>
</tbody>
</table>

The examples presented here show that irregular wave can generate freak events. In the laboratory experiment the freak wave appeared accidentally in a time record of about 30 minutes. Actually, at the end of the tank there was a 1:20 slope to study coastal effects, but we will restrict here to the waves above the flat part of the tank; reflections from the slope (and tank boundaries) were small and not relevant for our considerations.

In our description below, the dimensions and results are scaled to a geophysical situation with a spatial factor of 50, and corresponding temporal factor of $\sqrt{50}$.

Irregular wave IW12

We use as initial time signal the surface elevation of an irregular wave as measured 39.15 m from the wave maker in the wave tank, similar to 1957.7 m in geophysical dimension. Further on we always use the geophysical dimension. The initial time signal has total length of more than 3 hours, with significant wave height of 3.14 m and is shown in Fig. 4.16. We will predict the position, time and characteristics of the most extreme wave downstream and describe the extremal wave profile.

To predict the extreme wave, we compute the minimal value of the variance of the total wave phase; the density plot of the phase variance, $PV(x,t)$ is shown in Fig. 4.17. In this case we computed the phase variance for frequencies between $\omega_{\text{min}} = 0.3$ and $\omega_{\text{max}} = 0.7$, and obtain $PV_{\text{foc}} = 0.22$ at (5089.5; 1994). The calculated value of coherence is $\Gamma = 0.78$, and the corresponding pseudo-maximal signal has $\alpha = 0.81$ (scaling factor of 0.22). The pseudo-maximal signal
has linear maximal amplitude of 4.30 m. Adding the second order contributions, the maximal amplitude becomes 4.32 m. For validation, we performed nonlinear simulations with the linear and the (nonlinear) AB-model of the complete time signal at positions downstream the observation point. For these numerical simulations we predicted the position and time of the highest wave as listed in Table 4.3.

![Graph](image)

**Figure 4.16:** Time signal of the irregular wave IW12 at 1957.5 m. This is a measured signal that will be used to forecast the freak wave downstream.

![Graph](image)

**Figure 4.17:** The density plot of the variance of the total wave phase for IW12

The plots in Fig. 4.18 show the time signal of the elevation at the point of maximal amplitude as calculated by the nonlinear AB-model (solid), with superimposed on it (dashed) the profile of the pseudo-maximal wave as predicted by
our prediction method but shifted in time some 5 second to let the crests coincide. The predicted signal is also plotted in Fig. 4.18. Even though the maximal crest height of the pseudo-maximal signal is higher than the AB-simulation, it still describes the freak wave well around the highest crest. The results of the predicted position of the pseudo-maximal wave and the numerically simulated highest wave are presented in Table 4.3.

![Time signal as calculated by nonlinear AB at focussing position (solid), nonlinear pm-wave predicted from time signal at $X_{obs}$ (dashed), and Signal prediction (dotted line) for IW12](image)

**Figure 4.18:** Time signal as calculated by nonlinear AB at focussing position (solid), nonlinear pm-wave predicted from time signal at $X_{obs}$ (dashed), and Signal prediction (dotted line) for IW12

**Table 4.3:** Parameters for irregular wave IW12

<table>
<thead>
<tr>
<th>CASE</th>
<th>IW12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth</td>
<td>30[m]</td>
</tr>
<tr>
<td>Data position $X_{obs}$</td>
<td>1957.5[m]</td>
</tr>
<tr>
<td>$H_s$ at $X_{obs}$</td>
<td>3.14</td>
</tr>
<tr>
<td>Meas position</td>
<td></td>
</tr>
<tr>
<td>$X_{foc}$</td>
<td>5089.5</td>
</tr>
<tr>
<td>$T_{foc}$</td>
<td>1994</td>
</tr>
<tr>
<td>$\Gamma$ (coherence)</td>
<td>0.78</td>
</tr>
<tr>
<td>Max Crest height</td>
<td>4.32</td>
</tr>
<tr>
<td>Max Waveheight</td>
<td>6.26</td>
</tr>
<tr>
<td>$H_s$ at $X_{foc}$</td>
<td>3.14</td>
</tr>
<tr>
<td>BFI</td>
<td>0.17</td>
</tr>
</tbody>
</table>
Irregular wave IW9

The second case of irregular wave has smaller period; $T_p \approx 9$ s. In this case we also use the time signal at $X_{obs} = 1957.5$ m as initial signal for both prediction and AB-simulation. This initial signal is shown in Fig. 4.19.

![Time signal of the irregular wave IW9 at 1957.5 m from the wave maker.](image1)

**Figure 4.19:** Time signal of the irregular wave IW9 at 1957.5 m from the wave maker.

The same strategy is executed to this initial signal to get the description and the prediction of a freak wave. Similar to the irregular wave IW12, this case is also approximated well by pseudo-maximal wave. According to the prediction, the coherence of the irregular wave IW9 is less than IW12. The maximal crest height of IW9 at focusing is a bit higher than the IW12, even though their significant wave heights at the observation point $X_{obs}$ are almost the same.

![The density plot of the variance of the total wave phase for IW9](image2)

**Figure 4.20:** The density plot of the variance of the total wave phase for IW9
In the minimization of the phase variance, we chose $\omega \in [0.5; 1]$ for integration. Then the coherence of IW9 is $\Gamma \approx 0.72$ at $x = 2618.5$ and $t = 8562$. The focussing position is difficult to be identified in the density plot of the phase variance Fig. 4.20. The maximal wave at focussing by a nonlinear pseudo-maximal signal with scaling factor about 0.1 is shown in Fig. 4.21. We also compare it by the time signal at focussing computed by nonlinear AB-model. The parameters of the prediction and the AB-simulation are presented in Table 4.4. We do not have measurement data close to the focussing position, so for this case we only compare the prediction and the AB-simulation.

**Table 4.4: Parameters for irregular wave IW9**

<table>
<thead>
<tr>
<th>CASE</th>
<th>IW9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth</td>
<td>30[m]</td>
</tr>
<tr>
<td>Data position $X_{obs}$</td>
<td>1957.5[m]</td>
</tr>
<tr>
<td>$H_s$ at $X_{obs}$</td>
<td>3.1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Prediction</th>
<th>Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_{foc}$</td>
<td>2618.5</td>
<td>2478.5</td>
</tr>
<tr>
<td>$T_{foc}$</td>
<td>8562</td>
<td>8542</td>
</tr>
<tr>
<td>$\Gamma$ (coherence)</td>
<td>0.72</td>
<td>0.71</td>
</tr>
<tr>
<td>Max Crest height</td>
<td>4.67(2nd order)</td>
<td>4.3</td>
</tr>
<tr>
<td>Max Waveheight</td>
<td>6.62(2nd order)</td>
<td>7.78</td>
</tr>
<tr>
<td>$H_s$ at $X_{foc}$</td>
<td>3.10</td>
<td>3.18</td>
</tr>
<tr>
<td>BFI</td>
<td>0.27</td>
<td>0.18</td>
</tr>
</tbody>
</table>
4.5 Conclusion

This paper has discussed the description and the predictability of extreme waves by investigating the phase coherence using the power spectrum and the phase information at a certain position. The extreme profile can be described in a small neighbourhood by a (pseudo-)maximal wave. Moreover, we have shown that the position and time of an extreme wave is predicted well by minimizing the variance of the total wave phase. It should be noted that this minimization requires the choice of a suitable frequency interval to which the variance is restricted, but that the precise choice is not yet well motivated.

Because of the symmetry in both linear and nonlinear evolution, extreme waves (in the linear and nonlinear maximal signal wave) appear at approximately the same position; except for some shift (in time and consistently in space) the linear prediction gives a good estimation for the nonlinear evolution. In the four different applications, the focussing signal for which the phases are highly or moderately coherent could very well be modeled by a nonlinear maximal signal or by a pseudo-maximal signal; the parameters of the waves could be predicted to a good degree of accuracy from measurement data at a position upstream.

A final remark concerns the difference of the concept of pseudo-maximal wave with the concept of the New Wave model proposed by Walker et al. [2004]; the (pseudo)-maximal wave can be designed completely by knowledge of the spectrum, without the necessity as for the New Wave to determine the amplitude based on the probability of appearance.
Chapter 5

Localized Coherence of Freak Waves

Abstract

This paper investigates in detail a possible mechanism of energy convergence leading to freak waves. We give examples of a freak wave as a (weak) pseudo-maximal wave to illustrate the importance of phase coherence. Given a time signal at a certain position, we identify parts of the time signal with successive high amplitudes, so-called group events, that may lead to a freak wave using wavelet transform analysis. The local coherence of the critical group event is measured by its time spreading of the most energetic waves. Four types of signals have been investigated: dispersive focusing, normal sea condition, thunderstorm condition, and an experimental irregular wave. In all cases presented in this paper, it is shown that a high correlation exists between the local coherence and the appearance of a freak wave. This makes it plausible that freak waves can be developed by local interactions of waves in a wave group and that the effect of waves that are not in the immediate vicinity is minimal. This indicates that a local coherence mechanism within a wave group can be one mechanism that leads to the appearance of a freak wave.

5.1 Introduction

Understanding the mechanism of the freak wave phenomenon is intriguing for scientists, engineers, and mariners. The mechanisms that lead to freak waves are understandably diverse and it is not surprising that different freak waves exhibit different qualitative features [Liu and Mori, 2000]. A review of the existing mechanisms of freak waves was presented by Pelinovsky and Kharif [2008], Slunyaev et al. [2011].

We consider freak waves in unidirectional wave fields which satisfy the common definition of a freak wave, namely that the wave height exceeds approximately 2 times the significant wave height ($H_s$) or that the crest height exceeds $1.25H_s$ [Kharif and Pelinovsky, 2006, Olagnon and van Iseghem, 2000, Dysthe et al., 2008, Kharif and Pelinovsky, 2003]. Freak waves that are dominantly generated from wave energy convergence as a consequence of the random superposition of many wave components with not necessarily strong nonlinearity is still under discussion [Wang et al., 2015, Onorato et al., 2013, Garrett and Gemmrich, 2009, Gemmrich and Garrett, 2008, Slunyaev et al., 2005, Muller et al., 2005]. Different from some papers [Haver, 2004, Kharif and Pelinovsky, 2003, Pelinovsky et al., 2000], in which a freak wave is discussed as an accidental event from nowhere that appears and disappears suddenly, we discuss freak waves in (mainly) random wave fields that exhibit long-life gradual growth and decay. Latifah and van Groesen [2012] described and predicted freak waves by measuring the degree of phase coherence from a given time series at one position. It is the phase variance over an interval of the dominant wave frequencies. In this paper, we investigate the local coherence computed from the local time spreading of the most energetic waves, which is determined by wavelet transform. Nowadays, wavelet transformation is widely applied to analyze freak waves [Hu et al., 2015, Kwon et al., 2015, Cherneva and Soares, 2014, Bai et al., 2015, Wang et al., 2015, Wu et al., 2010] as it has wider applicability than Fourier techniques [Lin and Liu, 2004].

In the study of Slunyaev et al. [2005], the calculation of the first derivative of the local group velocity in the time series shows the presence of regions of strong wave convergence or divergence near freak events where strong modulations occurs. However, the question about the origin of the freak wave, whether it is naturally contained in the wave trains or induced by Benjamin Feir instability, is still open. Pelinovsky et al. [2011] discussed a freak wave of the solitary-like shape that is originated from the wave packet and is based on the dispersive focusing of unidirectional wave packets. In addition to the references cited above, we will contribute in understanding the process and the origin of freak wave appearance in random wave fields that is mainly based on dispersive effects. In realistic sea states a directional spreading could possibly influence dispersive focusing effects.
5.1 Introduction

Also Johannessen and Swan [1998] concluded that the introduction of directionality significantly reduces the nonlinearity of wave groups. That nonlinearity gives little or no extra amplitude compared to linear extreme events, but the changing shape of the extreme crest was also observed by Adcock et al. [2015]. In this paper, we will not take directional spreading into account, but will restrict to long-crested, unidirectional waves.

In unidirectional linear waves, the focusing due to dispersion is one mechanism that causes a freak wave [Porubov et al., 2005, Shunyaev et al., 2005, Kharif et al., 2001, Brown and Jensen, 2001, Pelinovsky et al., 2000, Baldock et al., 1996]. If short waves with small group velocities are initially located in front of long waves having large group velocities, the long waves will overtake the short waves with increasing time, and large-amplitude waves can appear. Afterwards, the long waves will be in front of the short waves, and the amplitude of the wave train will decrease [Kharif and Pelinovsky, 2003]. This mechanism is observed in the type of dispersive focussing waves which are often used in hydrodynamic laboratories [Merkoune et al., 2013, Brown and Jensen, 2001, Claus, 2002, Shemer et al., 2007a, 2005, Grue et al., 2003]. In random waves, this mechanism could also trigger a freak wave, but it is not as clear as in the dispersive focussing case. In the study of Wang et al. [2015], they presented a freak wave in a random wave field that was generated from two successive wave groups with different main frequencies and the higher frequency waves are in front of the others.

According to the study of Sergeeva et al. [2014] and Sergeeva and Shunyaev [2013], most of the long-living freak waves often occur on the background of intense wave groups. The evolution of modulated wave groups over large spatial and temporal scales were also a concern in the study of Viotti et al. [2013] and Grimshaw et al. [2001]. Recently Cousins and Sapsis [2014, 2016] and Ruban [2013] underlined that the appearance of extreme events can be triggered by focusing energy in localized wave groups. Therefore, to identify the group profiles that can be the origin of freak waves appearance, they used envelope equations and identified the envelope of the dominant groups associated with the length scale and amplitude by a group detection algorithm. Further, they computed the probability of the group to develop an extreme event. The evolution of the freak waves is summarized into focusing-defocusing process of energy. During the generation, a single wave absorbs energy from neighboring waves, increases its amplitude, reaches a maximum and then returns its energy back to other waves [Xia et al., 2015, Shunyaev et al., 2005]. According to Kharif et al. [2009], the transient change of the local energy of wave groups can be caught by wavelet analysis better than Fourier analysis.

In this paper, we will consider the appearance of freak waves in evolving wave groups in space and time. The waves are generated from a signalling problem: at
the influx position, say \( x_0 \), a given time signal \( \eta(x_0, t) \) is forced in one direction, the positive \( x \)-axis. The resulting waves \( \eta(x, t) \) may show a freak wave at certain time and space \( (x_f, t_f) \) at which the amplitude is larger than \( 1.25H_s \), which is taken as the definition of a freak wave in the rest of this paper. We will investigate this appearance by concentrating on successive high amplitudes in the initial signal, which will be called critical group events. We will apply the wavelet analysis for the identification of the energy spectral distribution in the group events.

This paper is organized into five sections starting with this introduction. Section 2 starts with a motivation to investigate the local coherence by showing the rapid decrease of the maximal amplitude when the coherence is decreased. Wavelet transformation is then described and shown to be better capable than Fourier methods to analyze the local phase of a wave. Section 3 starts with the selection of possible freak waves by estimating the critical group events from the influx signal that can lead to freak waves further downstream. The propagation of the most energetic group is then simulated to show the successive local energy convergence. We introduce quantitative measures of local coherence as one tool to predict the freak wave appearance. Using numerical simulations of linear and nonlinear waves with the AB equation described in Chapter 2 [van Groesen and Andonowati, 2007; van Groesen et al., 2010], we compute the wave evolution and measure the local coherence of the time signal at several positions. We consider various wave types, a dispersive focusing wave and irregular waves, synthetic and experimental signals from the MARIN hydrodynamic laboratory in Sect. 4. Conclusions are formulated in the final section.

5.2 Coherence and Wavelet Transform

In this section, we will start to motivate and illustrate the role of coherence by considering maximal, pseudo-maximal (pm) and weak pseudo-maximal (wpm) signals that can describe freak waves. In Latifah and van Groesen [2012], the notion of a “pseudo-maximal” signal was introduced for which the phases of all frequencies were band limited. Below, we also consider a less restrictive notion of weak pseudo-maximal signal, by restricting the phase only for the most energy-carrying modes. The measure of phase coherence in these concepts uses Fourier transform that represents the energy and the phase as function of the frequency. In Sect. 2.2, we describe the wavelet transform that is used in this paper to extract the local energy spectral distribution and the local phase as the time-frequency information of a given signal. Plots of the energy distribution over the frequencies will show that the wavelet transform improves results obtained with Fourier transform.
5.2.1 Signal coherence

Waves in the ocean at a specific position are described by a time signal. An irregular signal will have phases that are commonly understood to be uniformly distributed in \((-\pi, \pi]\). Previous study [Latifah and van Groesen, 2012] defined maximal waves and pseudo-maximal waves. A maximal wave is a wave with all phases zero and has maximal amplitude equal to the integration of its two-sided absolute spectrum. Thus, we call a signal with all phases zero at some time (say \(t = 0\)) a maximal signal, as

\[
MS(t) = \int |\tilde{\eta}_0(\omega)| \cos (\omega t) \, d\omega.
\]

At \(t = 0\), all wave components contribute to a constructive interference, hence

\[
MS(0) = \int |\tilde{\eta}_0(\omega)| \, d\omega.
\]

This is the highest amplitude that is possible for given spectrum, \(\tilde{\eta}_0(\omega)\). In view of the assumption of uniform distribution of the phases, the chance for such a maximal wave vanishes.

A pseudo-maximal (pm) wave is a partly coherent wave, that is in between a completely irregular wave and a fully coherent maximal wave. For a given signal with random phase \(\theta(\omega) \in (-\pi, \pi]\) as a function of wave frequencies with \(\theta(\omega) = -\theta(-\omega)\), we consider a pm signal as the signal for which the phases are restricted for certain \(\alpha \in (0, 1)\) to the phases \(\theta_\alpha(\omega) = \alpha \theta(\omega)\), as

\[
[\eta_{pm}(t)]_\alpha = \int |\tilde{\eta}_0(\omega)| \cos (\theta_\alpha(\omega) - \omega t) \, d\omega.
\] (5.1)

By taking a fraction \(\alpha\) of the random phase, the maximal amplitude decreases and the background increases for increasing \(\alpha\). For \(\alpha = 0\), it is a maximal wave with coherent phases while for \(\alpha = 1\) it is an irregular wave and the freak wave may disappear completely.

The phases of all frequencies in a pm signal are constrained as \(|\theta(\omega)| < \alpha \pi\). We now define a weak pseudo-maximal (wpm) signal, \(\eta_r(t)\), by restricting the phases of only the frequencies of large energy-carrying modes (see Fig. 5.1 for an illustration). We also illustrate the importance of such restrictions for coherence by plotting the maximal, pm and wpm signals of a given Jonswap spectrum in Fig. 5.2.

The restriction of wpm signal is typically for frequencies within one (or a half) standard deviation (SD) around the mean frequency, \(|\omega - \omega_m| < \sigma_\omega\) or less. Then
Figure 5.1: A JONSWAP spectrum with restricted random phases, $\theta_\alpha(\omega)$. The shaded area represents the energy-carrying modes (restricted by a half standard deviation).

Figure 5.2: Shown are plots from up to down of a maximal, pseudo-maximal, and weak pseudo-maximal signal corresponding to the same random signal at the bottom. The random signal corresponds to a JONSWAP spectrum with $H_s = 6.3$ m and $\gamma = 1.9$. The pm and wpm signals correspond to the value $\alpha = 0.7$. 
we consider a signal for $\alpha = [0, 1]$ and define $\theta_\alpha$ as

$$\theta_\alpha(\omega) = \begin{cases} \alpha \theta(\omega), & \text{for } |\omega - \omega_m| < \sigma_\omega \\ \theta(\omega), & \text{elsewhere} \end{cases} \quad (5.2)$$

and get a signal that has maximal amplitude less than the maximal amplitude of the pm signal:

$$[\eta_r(t)]_\alpha = \int_{-\infty}^{\infty} |\tilde{\eta}(\omega)| \cos(\theta_\alpha(\omega) - \omega t) d\omega \leq [\eta_{pm}(0)]_\alpha. \quad (5.3)$$

In general the mean frequency is not necessarily equal to the peak frequency because the spectrum of waves, that is usually of Jonswap shape, is not symmetric around the peak frequency.

Figure 5.3 illustrates that the value of $\alpha$ significantly affects the maximal crest height and the wave evolution along the $x-$axis. The smaller the value of $\alpha$, the higher the value of the generated crest. On the other hand, variations in $\sigma_\omega$ influence much less the maximum elevation of the influx signal. In any case, the wave evolution is tremendously affected and the maximum amplitude during the evolution can be much higher for larger $\sigma_\omega$. In Fig. 5.4 it is shown that at an influx position ($x \approx -3600$ m), the maximum amplitudes are quite the same for various fractions of SD, but near the focussing position a larger fraction of SD produces a higher maximum amplitude. This is the consequence of the fact that the larger fraction of SD gives more wave components with coherent phases.
Figure 5.4: The maximal temporal amplitude of the linear evolution of the wpm signal with $\alpha = 0.5$ for various fractions of the standard deviation $\sigma_\omega$.

Although the signal coherence can describe and measure the appearance of freak waves, the concepts use the whole interval of the time signal. However, not the whole interval will contribute in generating a freak wave since the waves propagate with their own group and phase velocity. The freak wave will be generated from local waves’ interaction. Therefore, we will investigate the local energy propagation using wavelet transformation. This is expected to give a more refined measure of the appearance of the freak wave.

5.2.2 Wavelet transform

In Fourier analysis we transform a function that depends on time into a function that depends on the frequency as a single variable. Given a time signal $\eta(t)$, Fourier transformation gives the relations

$$\tilde{\eta}(\omega) = \frac{1}{2\pi} \int \eta(t)e^{i\omega t} dt$$

$$\eta(t) = \int \tilde{\eta}(\omega)e^{-i\omega t} d\omega$$

$$= 2 \int_0^\infty |\tilde{\eta}(\omega)| \cos(\omega t + \theta(\omega)) d\omega.$$

The Fourier transform of $\eta(t)$ is the complex valued function $\tilde{\eta}(\omega) = |\tilde{\eta}(\omega)|e^{i\theta(\omega)}$, in which $|\tilde{\eta}(\omega)|$ is the amplitude spectrum and $\theta(\omega)$ is the phase of the signal. The spectral energy density of the signal is defined by $|\tilde{\eta}(\omega)|^2$ that describes how the energy of the signal is distributed with frequency. Any local (time) information is not directly contained in Fourier transform, but is hidden in the spectrum and
phase. At a certain local time, \( t = t_0 \), we have

\[
\eta(t_0) = 2 \int_0^\infty |\hat{\eta}(\omega)| \cos(\omega t_0 + \theta(\omega)) d\omega. \quad (5.4)
\]

The term inside the integral represents the amplitude spectrum and phase distribution with the frequency at a single time. Then we may define a local energy spectrum, \( E(t_0, \omega) \), as

\[
E(t_0, \omega) = (|\hat{\eta}(\omega)| \cos(\omega t_0 + \theta(\omega)))^2, \quad (5.5)
\]

presenting the local information of the signal directly. More generally, we will not only consider the energy at a single instant but will also analyze the energy in the neighborhood. Therefore, we will use wavelet transformation for the local energy analysis since it will show the distribution of the local energy spectrum better because it includes energy contributions from neighboring times instead of only one local time. Figures 5.5 and 5.6 illustrate the local energy distribution computed by Fourier and wavelet transform for a dispersive focusing wave and an irregular wave that will be used as study cases in Sect. 5.4.1 and 5.4.3. The plots show that the wavelet transform gives a more refined description of the local energy distribution as a function of time and frequency.

The wavelet transform is an extension of Fourier transformation. The basis function in Fourier transform is a sinusoidal of a specific frequency, and the \( L^2 \)-inner product with the signal leads to the Fourier coefficient of that frequency only. A wavelet is composed of a mixture of frequencies (which is indicated by its own Fourier transform). As a consequence, the wavelet coefficients refer to this mixture of frequencies, not a single frequency. We will now provide a summary of the main notions needed in the following sections.

**Definition 5.2.1.** A mother wavelet is a zero average function, \( \psi \), as

\[
\psi \in L^2(\mathbb{R}) : \int \psi(t) dt = 0.
\]

**Definition 5.2.2.** A wavelet family is family of functions generated from any type of mother wavelet, \( \psi \), through dilatation \((s > 0)\) and translation \((u \in \mathbb{R})\):

\[
\psi_{u,s}(t) = \frac{1}{\sqrt{s}} \psi \left( \frac{t - u}{s} \right).
\]

There are many types of mother wavelets: Morlet, Haar, Daubechies, Meyer, etc. (see Vialar [2009]). In this paper, we use the Morlet wavelet consisting of a plane wave modulated by a Gaussian, \( \psi(t) = e^{-t^2/2}e^{-i\omega_0 t} \), which is given in the Fourier domain by \( \hat{\psi}(\omega) = \sqrt{2\pi}e^{-(\omega - \omega_0)^2/2} \) with the central frequency \( \omega_0 \).
Figure 5.5: Shown is the distribution of the local energy of the dispersive focusing wave at some positions before the focusing point. The left plots are computed by Fourier transform and the right plots are by wavelet transform. The upper plots are in 3-D view, while the lower plots are in 2-D view.

Figure 5.6: The same as Figure 5.5, now for the irregular wave IW12 at some positions before the freak wave.
Definition 5.2.3. The continuous wavelet transform of a signal $\eta(t)$ at the scale $s$ and at the time $u$ is calculated by correlating $\eta$ with the wavelet family, $\psi_{u,s}$:

$$\mathcal{W}\eta(u, s) = \langle \eta, \psi_{u,s} \rangle = \int \eta(t) \frac{1}{\sqrt{s}} \psi^* \left( \frac{t-u}{s} \right) dt,$$  \hspace{1cm} (5.6)

where $\psi^*$ is the complex conjugate of $\psi$.

From Definition 5.2.3 the wavelet transform of a time signal $\eta(t)$ gives a complex valued function $\mathcal{W}\eta(u, s)$. For the Morlet wavelet, we obtain

$$\mathcal{W}\eta(u, s) = \int \eta(t) \frac{1}{\sqrt{s}} e^{-(t-u)^2/2s^2} e^{i\omega_0(t-u)/s} dt.$$  

By substituting $s = \omega_0/\omega$ and writing $F(t; u, \omega) = \frac{1}{\sqrt{s(\omega)}} e^{-(t-u)^2/2s(\omega)^2}$, the equation above gives

$$\mathcal{W}\eta(u, \omega) = \int \eta(t) F(t; u, \omega) e^{i\omega(t-u)} dt.$$  

The function $F(t; u, \omega)$ applies as a Gaussian window function to the signal $\eta(t)$. This shows that the wavelet transform can be interpreted as the Fourier transform of a windowed signal in the neighborhood of $t = u$. The magnitude of the wavelet transform, $|\mathcal{W}\eta(u, \omega)|$, represents the energy distribution of the signal over frequency and time and its angle, $\arg(\mathcal{W}\eta(u, \omega))$, represents the local phase of the signal.

Similar to Fourier transform, it is possible to rebuild the signal from the wavelet transform, the so-called inverse wavelet transform. It is given by

$$\eta(t) = \frac{1}{C_\psi} \int_0^{\infty} \left[ \frac{1}{\omega_0 C_\psi} \int \mathcal{W}\eta(u, \omega) F(t; u, \omega) e^{-i\omega(t-u)} du \right] d\omega,$$  \hspace{1cm} (5.7)

with

$$C_\psi = \int_0^{\infty} \frac{|\hat{\psi}(\omega)|^2}{\omega} d\omega.$$  

As an example, for $\omega_0 = 6$ the Morlet wavelet above produces $C_\psi \approx 1.883$. Different from equation (5.4) that gives the local energy spectrum computed at one time, equation (5.7) shows that the local energy spectrum from the wavelet transform is not only computed at the local time but it also includes the contribution of the signal surrounding that time.
The choice of the central frequency $\omega_0$ should be such that the Morlet wavelet satisfies the admissibility condition, $C_{\psi} < \infty$, which is equivalent to $\hat{\psi}(0) = 0$. Then the (real) wavelet transform is complete and preserves the quantity of energy:

$$\int |\eta(t)|^2 dt \approx \frac{1}{\omega_0 C_{\psi}} \int_0^\infty \int |W\eta(u, \omega)|^2 dud\omega.$$ 

Actually, the Morlet wavelet satisfies the condition only approximately because $\hat{\psi}(0) = \sqrt{2\pi}e^{-\omega_0^2/2}$ does not vanish exactly. A proper choice of $\omega_0$ can make the wavelet at least practically admissible and allows one to apply it widely to the signal decomposition [Lebedeva and Postnikov, 2014, Mertins, 1999]. The defined Morlet wavelet is sufficiently admissible if we choose $\omega_0 > 5$ (see Mertins [1999]), hence $\omega_0 = 6$ is taken to be sufficient since then $\hat{\psi}(\omega) \leq 3.8 \times 10^{-8}$, for $\omega \leq 0$. Parseval’s identity gives a relation between the signal and its Fourier transform as

$$\int |\eta(t)|^2 dt = 2\pi \int |\hat{\eta}(\omega)|^2 d\omega \approx \int \left[ \frac{1}{\omega_0 C_{\psi}} \int |W\eta(u, \omega)|^2 du \right] d\omega.$$ 

Therefore, the spectral energy density of a signal can be computed through the wavelet transform, i.e,

$$|\hat{\eta}(\omega)|^2 \approx \frac{1}{2\pi \omega_0 C_{\psi}} \int |W\eta(u, \omega)|^2 du. \tag{5.8}$$

This equation shows that the energy distribution from the wavelet transform behaves locally, and its integration over the time shift $u$ is approximately the spectral energy density obtained by Fourier transform.

### 5.3 Characterizing Freak Waves

The capability of the wavelet transform to represent a signal in time and frequency domain motivates us to investigate a freak wave locally. For a given signal, we identify group events which are parts of the time signal that may develop into propagating wave groups, i.e. that contain an amount of energy larger than a certain threshold. This threshold is determined such that the group event can build a freak wave if additional conditions are satisfied. We then determine the most energetic waves from each group event to see how the energy is distributed in both time and frequency. The most energetic waves will determine the evolution of the group event, and whether its energy will converge or diverge. With
5.3 Characterizing Freak Waves

these elements, we will be able to define the local coherence which will describe quantitatively the process of freak wave formation from a critical group event.

5.3.1 Critical group events

Holthuijsen [2007] defines a wave group as an uninterrupted sequence of waves with wave heights higher than an arbitrarily chosen, but usually high, threshold value. Instead of a wave group, we define a group event based on a chosen local energy level as threshold, which is determined by the contour level of the spectral energy determined by the wavelet transform. A group event of a time signal $\eta(t)$ is part of the time signal with $|W\eta(u, \omega)|$ higher than a threshold value. We denote the set of group events with respect to the threshold value $\epsilon$, by $WG_\epsilon$:

$$WG_\epsilon = \{ \eta_i(t) : i = 1, 2, \ldots, N_g \}$$
$$\eta_i(t) = \{ \eta(t) : t = [t_1, t_2] \subset [0, T] \mid |W\eta(u, \omega)| \geq \epsilon \}$$ (5.9)

$WG_\epsilon$ is the assembly of $N_g$ group events; each group is determined by the time interval during which the wavelet transform is larger than a specified value $\epsilon$. The selection of the group events depends on the chosen threshold value $\epsilon$. In practice, we normalize the value of $|W\eta|$ with its maximum, so that the value of $\epsilon$ is chosen in $(0, 1)$. The choice depends on the background waves since it aims to ignore the waves that do not contribute to the evolution of the group under consideration. When the background waves are high, we should choose a large $\epsilon$, but when the background waves are small, we can choose a small value of $\epsilon$. In this paper, we choose $\epsilon \approx 0.65$ for the random signals and $\epsilon \approx 0.2$ for the maximal signal.

From all the group events determined in this way, we characterize the groups that may lead to a freak wave. For a given time signal, $\eta(t), t \in [0, T]$, we define a total energy signal, $E_T$, as

$$E_T = \int_0^T |\eta(t)|^2 dt \approx \frac{1}{\omega_0 C_\psi} \int_0^T \int |W\eta(u, \omega)|^2 d\omega du.$$ (5.10)

For each value of the total energy signal, there can be a maximal wave with a coherent state.

Next, we define the total energy threshold to eliminate group events which unlikely generate a freak wave. The remaining groups are so-called critical group events.

$$WG_{crit} = WG_\epsilon \cap \left\{ \eta_i(t) \left| \int_{t_1}^{t_2} \eta_i(t)^2 dt \geq \rho^2 E_T \right. \right\}$$ (5.11)
Localized Coherence of Freak Waves

\[ \rho = \frac{1.25H_s}{\int |\hat{\eta}(\omega)|d\omega} \]

is a freak wave threshold normalized by the amplitude of a maximal signal. Based on their local energy, these critical group events could generate a freak wave forward or backward, but the probability depends on the phases.

### 5.3.2 Most energetic waves

We start from the complex value of the wavelet transform of \( \eta(t) \),

\[ \mathcal{W}\eta(u, \omega) = |\mathcal{W}\eta(u, \omega)|e^{i\Theta(u, \omega)} \]

It gives the spectral energy distribution \( |\mathcal{W}\eta(u, \omega)| \) and the phase information \( \Theta(u, \omega) \) as a function of time and frequency. From this we may look at the frequencies that carry most energy as a function of time denoted by \( \omega_m(u) \) from the critical group event:

\[ \omega_m(u) = \{ \omega \mid |\mathcal{W}\eta(u, \omega)| = \max_{\omega} |\mathcal{W}\eta(u, \omega)| \} \]

Convergence of waves will occur when long waves catch up with shorter waves. Hence, when the local wave length increases, i.e., when the wave frequency decreases, the waves will converge at a later time and vice versa. Therefore, the distinction is determined by the frequency in the time interval: when decreasing in forward time, this leads to a focussing energy, and an increase leads to defocussing energy. Since continuity of the local wave frequency in the random waves cannot be guaranteed, we approximate the local wave frequency by a linear interpolation, \( \omega_m(u) \approx \tilde{\omega}(u) \), so that we can distinguish the two cases:

\[ \tilde{\omega}(u) = \tilde{A}u + \tilde{B} \begin{cases} \tilde{A} > 0, & \text{defocussing/ diverging energy} \\ \tilde{A} \leq 0, & \text{focussing/ converging energy} \end{cases} \quad (5.12) \]

Moreover, we can also look at the most energetic waves as a function of wave frequency. This leads to a local time of each wave contribution. In the case of a dispersive focussing wave, focussing of the energy occurs when all wave contributions are in phase at one local time.

Motivated by this, for each critical group event in a local time interval \([t_1, t_2]\), we define a function \( \tau_m(\omega) \) representing the local time of the maximal energy, \( \tau_m(\omega) \in [t_1, t_2] \), as

\[ \tau_m(\omega) = \left\{ u \mid |\mathcal{W}\eta(u, \omega)| = \max_u |\mathcal{W}\eta(u, \omega)| \right\} \quad (5.13) \]
5.3 Characterizing Freak Waves

Hence, if the critical group event gives a constant $\tau_m(\omega)$, all frequencies contribute at the same time, which leads to local coherence at that time. If the frequencies are decreasing over the local time interval, it may indicate a local focussing at a later time.

5.3.3 Local coherence

The observations of the most energetic waves in either time or frequency can be used to see whether a freak wave may appear in forward or backward time, but the generation of a freak wave is still not assured, since the amplitude is not determined yet. The local information of the energy and phase gives a method to investigate locally the relation between the local coherence and freak wave occurrence. In this subsection, we measure the local coherence of the group event along its evolution and we will show that the highest amplitude occurs when the local coherence is maximum in the restricted frequency interval. As the wavelet transformation gives a function of frequency and time, we define a time spreading of the most energetic waves $[\varphi(\omega)]_\tau \in [-\pi, \pi]$ for each time $\tau \in (t_1, t_2)$ as follows:

$$[\varphi(\omega)]_\tau = [\tau_m(\omega) - \tau] \mod 2\pi$$

that is taken at the time at which the absolute mean is minimal.

$$\varphi(\omega) = \left\{ [\varphi(\omega)]_\tau \mid |\varphi(\omega)|_\tau = \min_{\tau} |\varphi(\omega)|_\tau \right\}$$

(5.14)

The time spreading is exactly zero at a certain frequency interval when $\tau_m(\omega)$ is constant at that interval. To investigate the local coherence, we determine the maximum ($M$), the mean ($\mu$), and the standard deviation ($\sigma$) of the absolute value of the time spreading normalized by $\pi$. Accordingly, we define three quantities depending on position that can represent local coherence, $\Gamma_{M,\mu,\sigma}(x) \in [0, 1]$, depending on the choice for the parameters, $M, \mu$ or $\sigma$:

$$\Gamma_M(x) = 1 - M \quad \Gamma_\mu(x) = 1 - 2\mu \quad \Gamma_\sigma(x) = 1 - \sqrt{3}\sigma$$

(5.15)

These values represent a somewhat different measure of local coherence. Note that the extreme case ($\Gamma_M = \Gamma_\mu = \Gamma_\sigma = 1$) occurs for the maximal signal, when all the phases are zero. Note also that this measure is different from the degree of phase coherence defined in Latifah and van Groesen [2012], as it corresponds to the local time spreading of the most energetic waves of a group event. To investigate the dependence between the local coherence and the occurrence of freak waves, we compute the correlation between the local coherence and the maximum amplitude normalized by its time-averaged local energy, $\text{Corr}(\Gamma_{M,\mu,\sigma}, A_m)$. 

For $N$ number of time signals at the positions $(x_1, x_2, \cdots, x_N)$, the correlation is computed by

$$
\text{Corr}(\Gamma, A_m) = \frac{\sum_{i=1}^{N} (\Gamma(x_i) - \mu_\Gamma) (A_m(x_i) - \mu_A)}{(N-1)\sigma_\Gamma \sigma_A}
$$

with

$$
A_m(x_i) = \frac{\max_{t \in (t_1, t_2)} |\eta(x_i, t)|}{\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \eta(x_i, t)^2 dt}
$$

and $\Gamma(x_i)$ is the local coherence of the time signal at $x_i$.

### 5.4 Case Studies

This section presents the investigations of four study cases: an experimental dispersive focussing wave, a synthetic normal wave condition (W100), a synthetic thunderstorm condition (TS10000) and an experimental irregular wave (IW12). For each case, we start to characterize the critical group events, then we investigate the local features of these groups, namely the most energetic wave and its time spreading. We investigate the evolution of the local energy and the time spreading of each case, particularly around the critical group events, and measure the local coherence. Furthermore, we compute the correlation between the local coherence and the maximum amplitude of the group event that generates a freak wave. It will give an impression of the relevance of the parameters $\Gamma$ for measuring a freak wave.

#### 5.4.1 Focussing wave (202002)

The case is a focussing wave that will lead to a maximal wave. We consider a dispersive focussing wave with significant wave height 0.013m, for which measurements at several positions are available from an experiment at MARIN (Case 202002). The experiment was executed at a water depth of 1m. Here, we use the elevation at the first measurement position after the wave flap as the influx signal for the numerical simulation by both the linear and nonlinear AB equation. The spectral shape of the influx signal with peak frequency of approximately 5rad/s is shown in Fig. 5.7. The result of the numerical simulation of a dispersive focussing wave using both the linear and nonlinear AB equation have been
previously verified with the measurements [Liam et al., 2014, Lakhturov et al., 2012].

Referring to Fig. 5.5, the influx signal only consists of one group event with almost zero background, which is therefore the only critical group event. This is an idealized case as the freak wave turns out to be a maximal wave that is generated from all wave components in the initial signal. This can be observed from the evolution of the influx signal; the shorter (slower) waves are followed by longer (faster) waves such that at the focussing point at 50.2m all waves have vanishing phase. See Fig. 5.8 for various plots of snapshots of the dynamics at successive measurement positions.

During the evolution, the changes of the distribution of the local energy in the time-frequency frame are described well by the filled contour plot of the local energy. The local energy distribution from one group event is squeezed into a maximal wave. This is also shown by the decreasing width of the time intervals towards the focussing point in Fig. 5.9. We can see at $x = 20\text{m}$ that the energy is distributed in 20s, at $x = 40\text{m}$ it is distributed approximately in 10s and at the focussing point the energy is only distributed in 3s. Moreover, the pure maximal wave is shown by the zeroes of the time spreading at $x = 50.2\text{m}$ in Fig. 5.8b. The profile of the maximal wave can be seen in Fig. 5.10.

In order to show that the occurrence of the freak wave is related to a local coherence, and to illustrate the three different measures of coherence introduced above, we show the evolution of these coherence measures for the linear and nonlinear evolution in Table 5.1. It can be observed from this table that the correlation of each of the three measures of coherence and the occurrence of the maximum amplitude at $x = 50.2\text{m}$ is very strong ($\approx 0.95$), although outside the focussing position the values of the three $\Gamma$’s can be rather different. $\Gamma_M$ and $\Gamma_\mu$ seem to be much better indicators for the focussing than $\Gamma_\sigma$. 

Figure 5.7: The normalized spectral shape of the influx signal for the case 202002.
Figure 5.8: Case 202002. (a) Time signals at various positions of the evolution of the critical group event with the filled contour plot of wavelet spectra. The vertical axis at the left represents the wave frequency $\omega$ and the vertical axis at the right represents the surface elevation in meters. (b) The corresponding time-averaged wavelet spectra (solid line) and the time spreading (dotted line). Observe that at $x = 50.2$ m the time spreading vanishes identically in the shaded area. The shaded areas show the chosen frequency interval of the most energy-carrying modes.

Table 5.1: Measure of the local coherence of the dispersive focussing wave

<table>
<thead>
<tr>
<th>$x$</th>
<th>Linear</th>
<th>Nonlinear</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Gamma_M$ $\Gamma_\mu$ $\Gamma_\sigma$ $A_m$ $A$</td>
<td>$\Gamma_M$ $\Gamma_\mu$ $\Gamma_\sigma$ $A_m$ $A$</td>
</tr>
<tr>
<td>20</td>
<td>0.009 0.044 0.504 0.012 -</td>
<td>0.002 0.009 0.04 0.49 -</td>
</tr>
<tr>
<td>30</td>
<td>0.002 0.116 0.516 0.019 -</td>
<td>0.001 0.002 0.12 0.52 -</td>
</tr>
<tr>
<td>40</td>
<td>0.001 0.231 0.507 0.046 -</td>
<td>0.001 0.001 0.23 0.51 -</td>
</tr>
<tr>
<td>45</td>
<td>0.312 0.285 0.659 0.113 -</td>
<td>0.293 0.29 0.27 0.65 -</td>
</tr>
<tr>
<td>50.05/</td>
<td>0.987 0.996 0.994 0.692 +</td>
<td>0.975 0.98 0.97 0.99 +</td>
</tr>
<tr>
<td>50.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>56</td>
<td>0.230 0.208 0.627 0.097 + 0.281 0.28 0.24 0.64 +</td>
<td></td>
</tr>
<tr>
<td>Corr($A_m(x), \Gamma$)</td>
<td>0.95 0.96 0.94 1</td>
<td>0.93 0.94 0.92 1</td>
</tr>
</tbody>
</table>
Figure 5.9: Case 202002. A filled contour plot of the energy distribution of the critical group event at position $x = 20, 30, 40, 50.2, 56\text{m}$. At each position, the red solid lines show the time of maximal energy at each wave frequency. The ++ lines show the wave frequency as function of time. Both are estimated by the most energetic waves in time and frequency, respectively. Before $x = 50.2\text{m}$, both solid and ++ lines show decreasing frequencies (increasing wave length) in time; then it leads to energy convergence.

Figure 5.10: Case 202002. Zoomed version of the maximal wave; the crest height is $4.65\text{m}$ and the wave height is $6.56$ times the significant wave height.
5.4.2 Synthetic signals

The second and third case are synthetic signals of irregular waves that are generated from a Jonswap spectrum with normal and thunderstorm sea conditions at a water depth of 480m (deep water). The wave evolutions are computed linearly by the AB equation as the nonlinear effect for these cases is not significant. However, a freak wave is still found in both cases.

Normal sea (W100)

The initial time signal is generated from a Jonswap spectrum with time period $11.3\text{s}$, $\gamma = 1.9$, and significant wave height $6.3\text{m}$ [van ’t Veer and Vlasveld 2014]. The spectral shape of the initial signal is shown in Fig. 5.11. The duration of the time signal is approximately 3 hours. From the initial time signal, there are nine critical group events, of which the two largest groups will be investigated. We do not investigate the other critical group events since their amount of the local energy signal is slightly equal to the threshold such that they are unlikely to develop a freak wave.

![Figure 5.11: The same as Fig. 5.7; now for the case W100.](image)

Figure 5.12 shows the two critical groups of the influx signal with approximately the same amount of local energy signal; one is around $t = 3200\text{s}$ and the other is at $t \approx 3600\text{s}$. Those are the most probable group events that can develop a freak wave. In the observation of the contour energy distribution, the preceding group event gives a positive $\tilde{A}$ while the other one gives a negative value. Therefore, the critical group event around $t = 3600\text{s}$ is the candidate to generate a larger amplitude in forward time. The evolution of this critical group together with its energy distribution is shown in Fig. 5.13a and the changes of its time spreading are in Fig. 5.13b. We observe that at the freak wave position ($x = 1420\text{m}$), the time spreading is almost zero for the wave-carrying modes. Outside the freak wave position, the time spreading of the critical group event is distributed in $[0, \pi]$. The freak wave is shown in Fig. 5.14.
Figure 5.12: Case W100. Initial time signal in the interval \( t \in [2500, 4500] \) s. The critical group events are shown in the shaded areas of the upper plot. The lower plot presents the amount of local energy signal of the recognized group events compared to the local energy threshold (dashed line). The local energy signal of the critical group events is above the threshold.

In this case, the occurrence of the freak wave can also be observed from the most energetic wave in either time or frequency (see Fig. 5.15). Before the freak wave, the most energetic waves give a decreasing wave frequency and after the freak wave, an increasing wave frequency occurs. At \( x = 1420\text{m} \), the local time of the maximal energy is almost constant for the carrying wave modes \( (\omega \in [0.5; 0.7]) \), therefore its time spreading is nearly coherent and it generates a freak wave.

Table 5.2: Measure of the local coherence of the normal sea condition wave

<table>
<thead>
<tr>
<th>( x )</th>
<th>( \Gamma_M )</th>
<th>( \Gamma_\mu )</th>
<th>( \Gamma_\sigma )</th>
<th>( A_m )</th>
<th>( \hat{A} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>0.05</td>
<td>0.10</td>
<td>0.35</td>
<td>0.01</td>
<td>-</td>
</tr>
<tr>
<td>1420</td>
<td>0.68</td>
<td>0.90</td>
<td>0.76</td>
<td>0.018</td>
<td>-</td>
</tr>
<tr>
<td>2000</td>
<td>0.045</td>
<td>0.167</td>
<td>0.397</td>
<td>0.010</td>
<td>-</td>
</tr>
<tr>
<td>2500</td>
<td>0.045</td>
<td>0.175</td>
<td>0.306</td>
<td>0.008</td>
<td>+</td>
</tr>
<tr>
<td>3000</td>
<td>0.045</td>
<td>0.028</td>
<td>0.341</td>
<td>0.007</td>
<td>+</td>
</tr>
</tbody>
</table>

Corr\((A_m(x), \Gamma)\) 0.78 0.82 0.74 1

Furthermore, we investigate the change of the local coherence of the critical
Figure 5.13: Case W100. (a) Time signals at various positions of the evolution of the critical group event with the filled contour plot of wavelet spectra. (b) The corresponding time-averaged wavelet spectra (solid line) and the time spreading (dotted line). Observe that at $x = 1420$ m the time spreading is zero in the shaded area. The shaded areas show the chosen frequency interval of the most energy-carrying modes.

Figure 5.14: Case W100. Zoomed version of the freak wave; the crest height is 1.35 m and the wave height is 2.37 times the significant wave height.
group event during its 3km linear wave evolution. The measure of coherence at various positions is shown in Table 5.2 and the correlation between the local coherence and the maximum amplitude along the evolution is presented in the lowest row. All three $\Gamma$’s show a quite high correlation ($\geq 0.74$) between the local coherence and the maximum amplitude. According to the correlation value, $\Gamma_M$ and $\Gamma_\mu$ seem to be better indicators for the freak wave appearance than $\Gamma_\sigma$.

**Thunderstorm sea (TS10000)**

The other synthetic signal is generated from a Jonsswap spectrum with time period 13.6s, $\gamma = 2$, and significant wave height 15.2m [van ’t Veer and Vlasveld, 2014]. A snapshot of the initial time signal is shown in Fig. 5.17 and its spectral shape is presented in Fig. 5.16. This type of wave is categorized as thunderstorm sea condition, in which the appearance of a freak wave is more probable than in a normal sea condition. The duration of the initial time signal is approximately 3 hours. There are five critical group events found from the influx signal, but the two unlikely ones do not generate a freak wave since their local energy signal is not so high compared to the threshold. The largest local energy signal of the group events appears around $t \approx 5400s$ and its maximum crest is already quite high at the initial time. Then in forward time it still develops to a higher crest and generates a freak wave.

Figure 5.18a presents the snapshots of the time signals at various positions. Also shown is the local energy distribution of the critical group event that leads
Figure 5.16: The same as Fig. 5.7 now for the case TS10000.

Figure 5.17: Case TS10000. Initial time signal in the interval \( t \in [3500, 7000] \) s. The critical group events are shown in the shaded areas of the upper plot. The lower plot presents the amount of local energy signal of the recognized group events compared to the local energy threshold (dashed line). The local energy signal of the critical group events are above the threshold.
5.4 Case Studies

Figure 5.18: Case TS10000. (a) Time signals at various positions of the evolution of the critical group event with the filled contour plot of wavelet spectra. (b) The corresponding time-averaged wavelet spectra (solid line) and the time spreading (dotted line). The shaded areas show the chosen frequency interval of the most energy-carrying modes.

Figure 5.19: Case TS10000. Zoomed version of the freak wave; the crest height is 1.22m and the wave height is 2.23 times the significant wave height.
Localized Coherence of Freak Waves

to a freak wave. In Fig. 5.18b, the time spreading of the critical group event shows the chosen carrying wave modes ($\omega \in [0.45; 0.52]$). A freak wave appears at $x = 2985$ m (see Fig. 5.19). If we observe the time spreading at $x = 2000$ m, it seems that the local time is more coherent than at the freak wave position. This can also be seen from the measure of the local coherence in Table 5.3. The larger amplitude of the freak wave compared to the group event at $x = 2000$ m can be explained from its local energy distribution. The width in time of the energy spectral distribution is a bit squeezed and there is some higher wave frequency contribution which does not appear at $x = 2000$ m. Figure 5.20 shows the filled contour plot of the local energy distribution for the most energetic waves at several positions as function of time and frequency. It can be observed that there is a change of the wave frequency order. Before the freak wave, the short waves run ahead the long waves and after the freak wave, the short waves are behind, just as in focussing waves.

Figure 5.20: Case TS10000. A filled contour plot of the energy distribution of the critical group event at various positions. At each position, the red solid lines show the time of maximal energy at each wave frequency. The +++ lines show the wave frequency as function of time. Both are estimated by the most energetic waves in time and frequency, respectively.

We measure the local coherences of the critical group event along its linear evolution and the results are presented in Table 5.3. The correlation for each local coherence $\Gamma$ and the maximum amplitude normalized by the local energy signal is quite high ($\geq 0.78$). This shows that the appearance of the freak wave is mostly caused by the local coherence of the critical group event from the influx signal.
### Table 5.3: Measure of the local coherence of the thunderstorm condition wave

<table>
<thead>
<tr>
<th>x</th>
<th>$\Gamma_M$</th>
<th>$\Gamma_\mu$</th>
<th>$\Gamma_\sigma$</th>
<th>$A_m$</th>
<th>$\tilde{A}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1500</td>
<td>0.19</td>
<td>0.17</td>
<td>0.52</td>
<td>0.01</td>
<td>-</td>
</tr>
<tr>
<td>2000</td>
<td>0.53</td>
<td>0.66</td>
<td>0.77</td>
<td>0.018</td>
<td>-</td>
</tr>
<tr>
<td>2985</td>
<td>0.37</td>
<td>0.39</td>
<td>0.69</td>
<td>0.010</td>
<td>-</td>
</tr>
<tr>
<td>3500</td>
<td>0.05</td>
<td>0.03</td>
<td>0.47</td>
<td>0.008</td>
<td>+</td>
</tr>
<tr>
<td>3800</td>
<td>0.11</td>
<td>0.16</td>
<td>0.50</td>
<td>0.007</td>
<td>+</td>
</tr>
<tr>
<td>Corr($A_m(x), \Gamma$)</td>
<td>0.80</td>
<td>0.78</td>
<td>0.81</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

### 5.4.3 Experimental signal: Irregular Wave (IW12)

The fourth case is an irregular wave, for which measurements at several positions are available from MARIN experiment with a water depth of 0.6m (Case 103001). It has 1.697s peak period and significant wave height of approximately 0.06m. We use the time signal from the first measurement position after the wave flap as the influx signal. The spectral shape of the influx signal is shown in Fig. 5.21. The local energy distribution of the signal is presented in Fig. 5.6. There are six critical group events from the influx signal as shown in Fig. 5.22. The largest local energy signal of the wave groups is found around $t \approx 240s$ and it develops a freak wave.

![Figure 5.21](image-url)  
*Figure 5.21: The same as Fig. 5.7; now for the case IW12.*

The evolution of the time signal around the critical group event and its energy distribution at several positions are shown in Fig. 5.23. Even though the energy spectral distribution does not show clearly the development of the critical group event into a freak wave, the change of the time spreading shows the development of its local coherence (see Fig. 5.23). A freak wave occurs at $x = 103.7m$ when its time spreading is near coherent for a short carrying wave mode. The freak wave is shown in Fig. 5.24. From Fig. 5.25 we can also see that there
Figure 5.22: Case IW12. The upper plot shows the influx signal. Four critical group events are shown in the shaded areas. The lower plot shows the local energy signal of group events compared to the local energy threshold (dashed line). The local energy signal of the critical group events are above the threshold.

is unclear increasing or decreasing wave frequencies of the most energetic wave. The local coherences are measured and presented in Table 5.4. The three values of $\Gamma$’s present quite high correlation ($\geq 0.75$) between the local coherences and the maximum amplitude in both the linear and nonlinear evolution. In this case, $\Gamma_\mu$ performs as the best indicator for the freak wave appearance.

<table>
<thead>
<tr>
<th>$x$</th>
<th>Linear</th>
<th>Nonlinear</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Gamma_M$</td>
<td>$\Gamma_\mu$</td>
</tr>
<tr>
<td>80</td>
<td>0.19</td>
<td>0.31</td>
</tr>
<tr>
<td>90</td>
<td>0.15</td>
<td>0.46</td>
</tr>
<tr>
<td>102.2/103.7</td>
<td>0.82</td>
<td>0.86</td>
</tr>
<tr>
<td>110</td>
<td>0.55</td>
<td>0.66</td>
</tr>
<tr>
<td>120</td>
<td>0.10</td>
<td>0.22</td>
</tr>
<tr>
<td>Corr($A_m, \Gamma$)</td>
<td>0.78</td>
<td>0.86</td>
</tr>
</tbody>
</table>

5.5 Conclusions

In this paper, we showed the relevance of phase coherence by illustrations of signals with increasingly less restrictions on the phase function. Then, the wavelet
Figure 5.23: Case IW12. (a) Time signals at various positions of the evolution of the critical group event with the filled contour plot of wavelet spectra. (b) The corresponding time-averaged wavelet spectra (solid line) and the time spreading (dotted line). The shaded areas show the chosen frequency interval of the most energy-carrying modes.

Figure 5.24: Case IW12. Zoomed version of the freak wave; the crest height is 1.31m and the wave height is 2.15 times the significant wave height.
transform was used to determine the time-frequency spectrum of a time signal. We used the wavelet transform to identify critical group events of the influx signal and it is shown that the group event with the largest local energy signal is the most probable group to generate a freak wave. We remarked that the identification of a group event is dependent on the choice of the threshold value ($\epsilon$). For irregular waves, we suggested to choose $\epsilon \approx 0.65$ and for waves with vanishing background we could choose a smaller value $\epsilon \approx 0.2$. We defined local coherence by three parameters (the mean, maximum or standard deviation) of the time spreading of the most energetic waves from the critical group events. We investigated the change of the local coherence along its evolution and showed that all three values of the local coherence are strong indicators for the appearance of a freak wave. This indicates a local mechanism of a freak wave appearance: the freak wave is mostly developed by a local coherence of a group event. At the influx signal, the group event already contains a considerable amount of energy, which evolves into successive states with even higher coherence. Four study cases illustrate the usefulness of the introduced concepts to describe and predict the appearance of freak waves.
Conclusions and Recommendations

6.1 Conclusions

This dissertation serves as a contribution in understanding extreme wave generation. We tried to understand the mechanism of extreme wave appearance in a deterministic fashion. Besides from radar, satellite, or oil-platform measurements, the appearance of extreme waves could be confirmed by generating extreme waves in a laboratory wave tank, including the experiments presented in this dissertation. The conducted experiments at the laboratory wave tank of TU Delft discussed in Chapter 3 presented the 7 measurements data of extreme waves that were designed using the AB models. The results showed that interaction of the moderate waves could build up a large amplitude, the so-called extreme wave.

We introduced new concepts based on the phase coherence to describe extreme waves. Given the power spectrum and the phase information at a certain position, the extreme profile can be described in a small neighbourhood by a (pseudo)-maximal wave. We stressed that a maximal wave describes an extreme wave with all zero phases in which the maximal crest is the maximum possible amplitude that can occur. A maximal wave is presented very well by a focusing dispersive wave which develop a coherent phase, while a pseudo-maximal wave describes extreme waves with less coherent phases, for instance the New Year wave or other extreme waves in irregular waves. Less restrictive, a weak pseudo-maximal wave can also be defined, by restricting the phase only for the most energy carrying modes. Different from the concept of the New Wave model proposed by Walker et al. [2004] which is based on the probability of the amplitude apperance, the
(pseudo)-maximal wave is designed completely by knowledge of the given power spectrum. Further, we have investigated that the extreme wave occurs at the time and the position where the phase is most coherent. Therefore, our proposed linear prediction method by minimizing the variance of the total wave phase gives a good estimation of the time and the position of the extreme wave.

We also investigated in detail one possible mechanism of freak waves. We showed that an extreme wave may occur due to a local coherence mechanism of a wave group. The extreme wave is mostly developed from a wave group containing a considerable amount of energy which evolves into successive states with even higher coherence. This revealed the importance of local coherence in extreme wave occurrence. Given an influx signal, we identified the group that contains enough local energy to build an extreme wave with the help of wavelet transform. Our investigation showed that the group with the largest local energy is the most probable group to generate a freak wave. The possibility of the group to generate a freak wave in forward time was measured by its local coherence. We observed that a high correlation exists between the local coherence and the appearance of a freak wave.

6.2 Recommendations

The description and the prediction of extreme waves presented in Chapter 4 of this dissertation is mostly limited to extreme waves with low BFI for which the nonlinearity does not give much contribution to the wave evolution. Therefore, further study in extreme waves with high BFI may improve the prediction method. For the description of extreme waves, the nonlinear terms have been added in the concepts of maximal and pseudo-maximal wave by Stokes corrections, but the nonlinear contribution seems to be very small. The nonlinear terms might be significant if we implement it in the extreme waves with high BFI.

Furthermore, if we look at the proposed prediction method, the minimization of the total wave phases is restricted in a chosen frequency interval. The choice is not definite and is not yet well motivated. In this dissertation we estimate the interval by looking at the amplitude spectrum and we choose the frequency interval for which the linear waves mainly contribute. Therefore, further investigation to improve the prediction is still desired.

In Chapter 5, we proposed a local coherence mechanism within a wave group that can be one mechanism leading to the appearance of a freak wave. The approach is mainly based on the local dispersive effect and focused on unidirectional dispersive waves. It would be very interesting to broaden the research to multidirectional waves: the directional spreading in realistic sea states could influence a dispersive focusing effect in various ways.
Supplementary of experiment results

Experiment results of TUD 101, 301, and 302

Figure A.1: The left plots show the time signals of the measurement (solid line) and the design (dashed-line) at WP2-WP8 for TUD 101 case. The right plots show the corresponding amplitude spectrum.
Figure A.2: The left plots show the time signals from the measurement (solid line) and a-posteriori AB simulation (dashed-line) at WP2-WP8 for TUD 101 case. The right plots show the corresponding amplitude spectrum.

Figure A.3: Maximal and minimal temporal amplitude of TUD 101, the AB-simulation (dashed-line) and the measurements (+). The solid line presents the extreme wave occurred in the tank computed from the nonlinear AB-simulation.
Figure A.4: Same as Figure A.1, now for TUD 301 case.

Table A.1: Correlation coefficient of the signals between the designs and the measurements (I) and between the measurements and a-posteriori AB-simulations using WP1 as influx signal (II) at wave probes.

<table>
<thead>
<tr>
<th>Positions</th>
<th>TUD 101</th>
<th>TUD 301</th>
<th>TUD 302</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
<td>II</td>
<td>I</td>
</tr>
<tr>
<td>WP1</td>
<td>0.95</td>
<td>0.95</td>
<td>0.98</td>
</tr>
<tr>
<td>WP2</td>
<td>0.94</td>
<td>0.98</td>
<td>0.96</td>
</tr>
<tr>
<td>WP3</td>
<td>0.94</td>
<td>0.99</td>
<td>0.95</td>
</tr>
<tr>
<td>WP4</td>
<td>0.93</td>
<td>0.99</td>
<td>0.93</td>
</tr>
<tr>
<td>WP5</td>
<td>0.89</td>
<td>0.99</td>
<td>0.94</td>
</tr>
<tr>
<td>WP6</td>
<td>0.89</td>
<td>0.99</td>
<td>0.94</td>
</tr>
<tr>
<td>WP7</td>
<td>0.89</td>
<td>0.99</td>
<td>0.94</td>
</tr>
<tr>
<td>WP8</td>
<td>0.90</td>
<td>0.98</td>
<td>0.93</td>
</tr>
</tbody>
</table>
Figure A.5: Same as Figure A.2 now for TUD 301 case.

Figure A.6: Same as Figure A.3 now for TUD 301 case.
Table A.2: Correlation coefficient of the amplitude spectra between the designs and the measurements (I) and between the measurements and a-posteriori AB-simulations using WP1 as influx signal (II) at wave probes.

<table>
<thead>
<tr>
<th>Positions</th>
<th>TUD 101 I</th>
<th>TUD 101 II</th>
<th>TUD 301 I</th>
<th>TUD 301 II</th>
<th>TUD 302 I</th>
<th>TUD 302 II</th>
</tr>
</thead>
<tbody>
<tr>
<td>WP1</td>
<td>0.97</td>
<td>0.88</td>
<td>0.81</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WP2</td>
<td>0.97</td>
<td>0.99</td>
<td>0.87</td>
<td>0.99</td>
<td>0.81</td>
<td>0.99</td>
</tr>
<tr>
<td>WP3</td>
<td>0.97</td>
<td>0.99</td>
<td>0.88</td>
<td>0.99</td>
<td>0.77</td>
<td>0.99</td>
</tr>
<tr>
<td>WP4</td>
<td>0.98</td>
<td>0.99</td>
<td>0.89</td>
<td>0.99</td>
<td>0.76</td>
<td>0.99</td>
</tr>
<tr>
<td>WP5</td>
<td>0.97</td>
<td>0.99</td>
<td>0.89</td>
<td>0.99</td>
<td>0.77</td>
<td>0.99</td>
</tr>
<tr>
<td>WP6</td>
<td>0.94</td>
<td>0.99</td>
<td>0.89</td>
<td>0.98</td>
<td>0.77</td>
<td>0.99</td>
</tr>
<tr>
<td>WP7</td>
<td>0.96</td>
<td>0.99</td>
<td>0.88</td>
<td>0.99</td>
<td>0.78</td>
<td>0.99</td>
</tr>
<tr>
<td>WP8</td>
<td>0.97</td>
<td>0.99</td>
<td>0.88</td>
<td>0.99</td>
<td>0.78</td>
<td>0.99</td>
</tr>
</tbody>
</table>
Figure A.8: Same as Figure A.2 now for TUD 302 case.

Figure A.9: Same as Figure A.3 now for TUD 302 case.
Stokes corrections

Second order wave-wave interaction leads to nonlinear contribution as derived by [Dalzell 1999]. The form of second order solution is applied to define the nonlinear wave profile here. The final solution for the wave elevation, up to second order, for the superposition of two waves is given by:

\[
\eta(x,t) = \sum_{j=1}^{2} a_j \cos(\varphi_j) + \sum_{j=1}^{2} a_j^2 B_0(k_j) \\
+ \sum_{j=1}^{2} a_j^2 B_2(k_j) \cos(2\varphi_j) + a_1 a_2 B_p(k_1, k_2) \cos(\varphi_1 + \varphi_2) + a_1 a_2 B_m(k_1, k_2) \cos(\varphi_1 - \varphi_2) 
\] (B.1)

The first term of (B.1) is the linear contribution. The rest are the second order contributions. The coefficients of the second order contributions depend on the wave number and frequency. These are defined by:

\[
B_0(k_j) = \frac{|k_j|}{4 \tanh(|k_j|h)} \left[ 2 + \frac{3}{\sinh^2(|k_j|h)} \right] \\
B_2(k_j) = -\frac{|k_j|}{2 \sinh(2|k_j|h)} 
\]
\[ B_p(k_1, k_2) = \frac{1}{2g} \left[ \omega_1^2 + \omega_2^2 - \omega_1 \omega_2 (1 - P_1) \right. \]
\[ \cdot \frac{(\omega_1 + \omega_2)^2 + \Omega^2(|k_1 + k_2|)}{(\omega_1 + \omega_2)^2 - \Omega^2(|k_1 + k_2|)} + \frac{(\omega_1 + \omega_2)P_2}{(\omega_1 + \omega_2)^2 - \Omega^2(|k_1 + k_2|)} \]
\[ B_m(k_1, k_2) = \frac{1}{2g} \left[ \omega_1^2 + \omega_2^2 + \omega_1 \omega_2 (1 + P_1) \right. \]
\[ \cdot \frac{(\omega_1 - \omega_2)^2 + \Omega^2(|k_1 - k_2|)}{(\omega_1 - \omega_2)^2 - \Omega^2(|k_1 - k_2|)} + \frac{(\omega_1 + \omega_2)P_2}{(\omega_1 - \omega_2)^2 - \Omega^2(|k_1 - k_2|)} \]

in which \( j = 1, 2 \), \( \varphi_j = k_j x - \omega_j t \) is the phase, \( a_j \) is the amplitude, \( k_j = K(\omega_j) \) is wave number, \( \omega_j \) is the frequency, and \( h \) is water depth. For simplification we write
\[ P_1 = \frac{1}{\tanh(|k_1|h) \tanh(|k_2|h)} \]
\[ P_2 = \left[ \frac{\omega_1^3}{\sinh^2(|k_1|h)} + \frac{\omega_2^3}{\sinh^2(|k_2|h)} \right] . \]

The dispersion relation between \( \omega_j \) and \( k_j \) is given by
\[ \omega_j^2 = \Omega^2(k_j) = g|k_j| \tanh(|k_j|h) \]


Ooms, J.: Wavemaker Capabilities of the Nr. 1 Basin at the Delft Shiphydromechanics Laboratory, Delft University of Technology, Faculty of Mechanical Engineering and Marine Technology, Ship Hydromechanics Laboratory, 1996.


van Groesen, E., Bunnik, T., and Andonowati: Surface wave modelling and simulation for wave tanks and coastal areas, 2011.


Acknowledgments

This dissertation would never been impossible without contributions and supports from a great many people. I owe my gratitude to all of those people who made my graduate happens as one of my moment I will cherish forever.

My deepest gratitude is to my supervisor, Prof. E. van Groesen, for his excellent guidance in doing research. I have been fortunate to have a supervisor who always encourages me in exploring my thought and developing my own idea. I am also thankful to him for patiently and carefully reading and commenting on countless revisions of this manuscript.

I appreciate the financial support from STW that funded the research discussed in this dissertation. Further, I would like to thank to all members of my graduation committee: Prof. René Huijsmans, Prof. Jaap van der Vegt, Prof. Arthur Veldman, dr. Gerbrant van Vledder, dr. ir. T. Bunnik, and Prof. Christian Kharif who come from France. I also would thank to Prof. dr. J. L. Hurink as the chairperson and secretary of my graduation committee.

Special thanks for MARIN hydrodynamic Laboratory, who provides me experiment data from which I used in most of my study cases. Thanks also to Dr. Sverre Haver for providing the original data of the Draupner Wave.

I would also like to acknowledge Prof. Stephan van Gils and Dr. Gerard Jeurnink for giving me opportunity to be involved as a teaching assistant. It was a valuable experience for me. I would like to thank again Prof. Rene Huijsman, who made the collaboration experiments in the laboratory wave tank at TU Delft possible. I am also indebted to the collaboration experiments team: Peter Naaijen, Sander Dragt, Arno Dubois with whom I generated the extreme waves in the laboratory tank.

I am thankful to the secretaries of AACS: Mariëlle Slotboom-Plekenpol and Linda Wychgel - van Dalm for all their help during my stay in Enshede, particularly in administration matter. Special thanks to Mariëlle who help me a lot in
preparation of my graduation.

I would also like to thank all my teachers, who throughout my educational career have supported and encouraged me to believe in my abilities. They have directed me through various situations, allowing me to reach this accomplishment. I acknowledge all AACS members for numerous discussions on related topics that helped me improve my knowledge.

I am also grateful to the Indonesian families, especially PPIE for their fruitful friendship. Many friends have helped me stay sane through the difficult years. Their support and care helped me overcome setbacks and stay focused on my study. I greatly value their friendship and I deeply appreciate their belief in me.

Most importantly, none of this would have been possible without the love, support, and patience of my family, to whom this dissertation is dedicated to. I am eternally grateful to my parents, who always support and encourage me with their best wishes. To my husband and my wonderful daughters, my abundance of gratitude for your love, support and sacrifice.
Arnida Lailatul Latifah was born in Bantul, Yogyakarta, Indonesia, on 22 March 1987. After finishing her high school in two years, she decided to study mathematics at Institute of Technology Bandung in 2004. In March 2008, she received her Bachelor of Science degree with honor. Then she worked at Laboratorium Matematika (LabMath) Indonesia as a junior researcher for five months before she received a scholarship from University Twente and started to study Master in September 2008. She joined the combined MSc-PhD program in the group of Applied Analysis and Computational Science, Department of Applied Mathematics. In May 2011 she earned her Master of Science degree and then directly continued the research as a PhD student in the STW research project entitled ”Extreme Surface Waves, models, simulations and experiments” under supervision of Prof. E. van Groesen. In July 2013, she came back to her country and then employed as a researcher in Indonesian Institute of Sciences until now. Finally, she finished her doctoral research and presented the results in this dissertation. She received her Doctoral degree in 13 October 2016.