Modelling and Control of Systems with Flow

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Cover picture: Three dimensional schematic representation of the steps that lead to controller design. The fish is cloned for scientific purposes only, without permission of the Dutch government.


MODELLING AND CONTROL OF SYSTEMS WITH FLOW

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1

Introduction

1.1 Motivation and goal

In the automation process of modern industry, the development of automatic feedback control is a key element. For over half a century, mathematically based control algorithms have successfully provided the industry with effective and efficient control strategies for a wide range of applications. Especially controllers that are based on mathematical models can give a high performance.

The feedback control of systems with air or fluid flow plays a major role in various industrial applications. Examples are HVAC (heating, ventilation, air conditioning, cooling) control, process control in chemical reactors, motion control of air vehicles, and motion control of liquid. The dynamics of these systems vary with time and space, and they are commonly referred to as distributed parameter systems. Motivated by this, the subject of this thesis is the modelling and feedback controller design of systems with flows. In many cases, the spatial variations are assumed to be negligible, which results in a system described by ordinary differential equations. However, for many systems this assumption cannot be applied without losing some essential dynamics. Incorporating the spatial variations leads to (nonlinear) partial differential equations (pde’s), which in turns leads to distributed parameter systems. Although there is yet no standard control design algorithm for this type of systems, control of distributed parameter systems has received considerable attention over the years, from a purely mathematical as well as from an engineering point of view. The openness of the field is further illustrated by the various approaches and applications of modelling and control.

The most common design tools in control are $H_{\infty}$ and LQG control. Some common control objectives are tracking of a reference signal, stabilization and minimization of costs. However, these techniques apply only to systems that can be represented in linear state space form in the time domain,
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or equivalently, rational transfer functions in the frequency domain. Unfortunately, a distributed parameter system does not have the form that is directly suitable for these types of control, since it is infinite dimensional and possibly nonlinear.

In practice, control design of a distributed parameter system consists of three steps. First, a model has to be derived that is as simple as possible, but sufficiently accurate. This model is calibrated with, and validated against, experimental results. Secondly, model reduction or approximation can be applied to provide a model that is suitable for control design, without losing the essential dynamics. Third, the control objective is defined and a controller is designed.

Although modelling, model reduction and control design are often regarded as separate research fields, they are strongly connected by their simultaneous application to real life systems. In particular, it is the combination of these fields that produces practical solutions with respect to controller design. Focusing on one particular step can therefore seriously endanger practicality. For example, a highly sophisticated model that is extremely accurate, but too complex for effective model reduction and controller design, is useless. Moreover, oversimplified models lead to oversimplified controllers with insufficient performance on the real life system, whereas controllers with extreme complexity can be undesirable for practical and economic reasons. It is this tradeoff that makes it extremely difficult, if not impossible, to come up with a general strategy.

The first goal of this thesis is to investigate the possibility of practical controller design for distributed parameter systems, by finding a suitable combination of modelling, model reduction and controller design. Since developing a general strategy is not a realistic option, a modest effort is made by focusing on two realistic cases that are supposed to represent two classes of real life control problems. These are the temperature control of bulk stored harvested foods, and the concentration control of microorganisms inside a UV disinfection reactor for fluid purification. Figure 1.1 gives an impression of what these systems look like. The second goal is to preserve the physical insight of the model as much as possible during the model reduction process and the controller design. Motivated by the results in [83], the research challenge that integrates systems design and control was posed in [27]. An application to bulk storage is found in [57]. In this thesis, the preservation of physical insight is shown to have advantages for this field.

The organization of this chapter is as follows. Section 1.2 gives a literature survey of the modelling, model reduction and controller design that are relevant to this work. Section 1.3 gives an overview of the research methods that

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1 We use the terms system and model loosely. By system we mean a model with an input and an output.
1.2 Literature survey

1.2.1 Modelling approaches for analysis and controller design

There is a vast amount of literature on the modelling of distributed parameter systems in general, see for example [21] and [46] ch. 28. Here we restrict ourselves to the modelling approaches on (i) bulk storage rooms and (ii) UV disinfection reactors. This restriction allows us to maintain a clear but representative overview of the common approaches. Most of the modelling approaches in these areas are intended for analysis rather than control design.

**Bulk storage room**

In a bulk storage room for agricultural produce the air quality is controlled in order to maintain an optimal product temperature. Since the air and product temperatures are temporally and spatially distributed, this type of systems normally leads to distributed parameter systems. The main control inputs are air velocity, for example induced by a fan, and air temperature, for example from a heat exchanger. In case of natural ventilation, the valve opening for the flow of ambient air into the storage room is another control input. Normally, the ventilation inputs enter the system equations in...
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a nonlinear way (from a control systems point of view), which results in a nonlinear distributed parameter system.

Let us start by mentioning modelling approaches for a bulk storage room. For each approach we mention which controller was designed. In [58] the main goal was to derive a model for the dynamics of the humidity and temperature of the air, for which standard control design is applicable. In [103] a nonlinear lumped multi-scale model was derived to enable model predictive control of temperature, moist-, and sugar content of potatoes in a bulk storage room with outside air ventilation. In [44] a nonlinear lumped model was used, and a model predictive control (MPC) algorithm was designed for the temperature and humidity control of a bulk storage room with outside air ventilation, and additionally in [59] the resulting controller was applied to a real life plant. In [22] the uncertainty of weather forecasts is investigated. For analysis purposes, extensive distributed parameter systems were proposed in [15, 16, 66, 109, 110], which were analyzed by computational fluid dynamics (CFD) simulations. Experimental validation studies were done in [15, 66, 109, 110]. In [102] a review of different CFD modelling approaches is given. In none of the above mentioned CFD modelling studies a controller was designed.

UV disinfection reactor

In a UV disinfection reactor, fluid flows through the reactor and is irradiated with UV light. By this irradiation, the concentration of harmful microorganisms is reduced to an acceptable level. In the most general case the fluid flow is represented by the Navier-Stokes equations and a mass balance equation for the microorganisms, which results in a distributed parameter system. The main control inputs are flow rate, which determines the production rate, and light intensity, which steers the output concentration of microorganisms. The inputs enter the system in a bilinear way. Hence, a general model of a disinfection reactor is a bilinear set of pde’s.

A great deal of experimental research is conducted to the effect of UV radiation on different types of microorganisms [78, 79]. In [54, 55, 76, 77, 112] the UV dosage is linked to the inactivation rate of microorganisms in agriculture and horticulture. In [23] the required UV dosage for apple cider pasteurization was examined. In [68, 82] the reactor performance was analyzed by CFD simulations using a Reynolds averaged Navier-Stokes model for the flow, combined with first order inactivation kinetics. In [42] a three dimensional CFD model with nonlinear inactivation kinetics was validated successfully by using experimental results on a pilot plant for air purification. In [13] an extensive review is given on UV reactor modelling for homogeneous and heterogenous media, with different types of lamps, including effects such as reflecting surfaces. The model that was used throughout for illustration,
was an annular reactor with first order reaction kinetics, which is similar to the reactor model that is examined in this thesis. For none of these models a feedback controller was designed. So far, only in [45] an effort was made. In that paper, the physical model was bilinear and had a plug flow. The reaction kinetics of the attenuation of the microorganisms due to UV irradiation were of first order. A linear feedback-feedforward controller with time delay was designed after linearization of the model.

1.2.2 System identification

A method to circumvent the physical modelling procedure is to obtain experimental data and apply system identification. This thesis is part of a combined STW funded Ph.D. project with D. Vries (Wageningen University). A part of his research concerns system identification that is conducted on an experimental bulk storage room, which was reported in [104]. In that paper, parameter estimation was conducted using an approximate physical model that was analytically derived from the full physical model, in order to retrieve physical information from the system.

A special case of system identification is model realization from the impulse response of the system. In chemical engineering there exist standard measurement techniques for the measured impulse response, see [34]. In [49] realization theory is developed for systems of which the impulse response is known, and in [1, 80] numerical algorithms are developed for realization of finite dimensional linear systems. For systems without a chemical reaction or production, the impulse response equals the retention time distribution. The linear state space form of the realized system is suitable for standard controller design. However, it was only since recently that this feature was exploited. In [53] the step response was used to develop various controllers for chemical reactors with linear inputs. In this thesis, a controller for a reactor with nonlinear dynamics and time delay is designed by realization theory, inspired by, and applied to, a UV disinfection reactor.

1.2.3 Model reduction and approximation

Of all the model reduction and approximation methods that are available, we will discuss the methods that are used in the fields that are represented by our two applications, i.e. control in food storage and control of chemical reactors.

Linearization

Perhaps the most common model simplification technique in control engineering is linearization. In some region around the linearization point (or
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profile for infinite dimensional systems), the system dynamics is approxi-
mated accurately. The result is an approximated linear system that is suit-
able for standard control. However, for large deviations and for systems
with extreme nonlinear dynamics, the linearized system dynamics can de-
viate considerably from the original model. Consequently, linearization can
result in loss of controller performance. For details, see for example [70].

Singular perturbation theory

Another common technique is singular perturbation theory, where the model
with dominant slow dynamics is reduced in complexity by discarding the
dynamics of fast model components, see for example [47] for theoretical
background, and [44] for application to bulk storage. This technique is
applied in chapter 2.

Input/output balancing

Another standard model reduction technique is input/output balanced trunc-
cation. This procedure was introduced in [63] and is now a textbook subject
(see for example [4], [113], chapter 7, and [32] for applications). The method
consists of truncating a balanced realization. A balanced realization (also
called Lyapunov- or internally balanced) is a realization for which the con-
trollability and observability gramians are equal and diagonal. The system
reduction consists of discarding the least controllable and observable states.

Padé approximation

Transfer functions for linear pde systems are obtained by transformation
to the Laplace domain w.r.t. time. After that, the remaining differential
equations are solved. The solution is the transfer function, and it represents
the signal transfer between the input and the output, as a function of the
signal frequency. The transfer function can be quite general, for example
the signal transfer between two points in space. This expression of the
transformed time variable is generally transcendental, and this prohibits
controller design. Moment matching, or Padé approximation, is a technique
that consists of the approximation of transcendental or high order transfer
functions by low order rational ones. These low order transfer functions can
be transformed back to low order linear time invariant systems in the time
domain, whereafter a standard linear controller can be designed. In [5] an
overview is given on numerical procedures for Padé approximation.
1.2 Literature survey

1.2.4 Control design

In this subsection, an overview of different control algorithms is given. The control fields are here divided into three classes, and for each class the applicability for real-life systems is discussed.

Linear control for finite dimensional systems

Since standard control design plays such an important role in this thesis, a short description of the different fields is given to provide a basic overview. The field of classical feedback control started in the early 1930’s with H. Nyquist’s stability criterion [67], and the works of Bode and Black at Western Electric. Classical control design consists of a wide variety of graphical tools, such as Bode plots, Nyquist plots, M-, N- and root loci. Furthermore, classical control design comprises methods like loop shaping, lead and/or lag compensation, and quantitative feedback design [11]. The tools are in particular designed for linear single-input-single-output (SISO) systems, but they also provide insight into design issues of more complex systems, such as multiple-input-multiple-output (MIMO) systems [11]. A well known example is proportional, integral, and derivative (PID) control, which is widely applied in industry. The success of PID control is the simplicity and effectiveness, since the controller is derived from mathematically well founded design principles, and is easy in application due to its simple mathematical structure. In the past, more ad hoc PID design was developed, for example the Ziegler-Nichols, and the Cohen-Coon tuning rules. These rules are based on second order linear systems, and are widely applied to industrial processes with unmodelled or complex dynamics. For an overview of classical control, see for example [24]. In the second half of the twentieth century, the design for SISO systems was extended to MIMO systems, with various techniques to examine and design the stability robustness and performance of systems with parametric uncertainty. For an overview of the different control design methods, see [11]. Examples of design methods for linear MIMO systems are LQ, LQG, $H_2$, and $H_{\infty}$ control. Since these types of control are all textbook subjects, we call them standard. The most important advantages of standard linear control is the solid mathematical basis that results in a variety of design tools, and the reliable robustness analysis. The main drawback of standard control design is that the theory is based on linear finite-dimensional systems. Furthermore, the number of system dimensions is in practice limited by the computational capacity that is needed to solve the Riccati equations for high order systems, and the complexity of the resulting controller.
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Linear control for infinite dimensional systems

Before model reduction or numerical discretization is applied, a distributed parameter system is infinite dimensional. In [56] the LQ control problem was solved for systems described by pde’s, and in [19] this was solved using semigroup theory. This resulted in a controller in operator form, i.e., infinite dimensional. Although a rigorous robustness analysis is available in [20], applications to real life industrial problems that make use of this operator are very limited. Instead, there is, besides the methods mentioned in section 1.2.3, a considerable amount of research on approximation techniques, such as proper orthogonal decomposition, to come to a finite dimensional model [12, 50, 52, 60]. For an overview of research directions and applications, see [81].

Nonlinear control for distributed parameter systems

In the last decade there has been a great progress in mathematically based control design for nonlinear distributed parameter systems, [25, 26, 28, 29]. In [18] an overview of stabilizing control algorithms is given for a wide range of systems described by hyperbolic and parabolic pde’s, with applications to a variety of models of mostly chemical processes. Bounds on stability robustness have been established, which indicates the practical possibilities of nonlinear pde control. Hence, the results that are obtained so far are promising. However, some obstacles to real life application are that (i) the control algorithms are complex, and (ii) the design is rather laborious since extensive numerical algorithms have to be employed. Further, there is still a poor predictability of performance and performance robustness. In [39] feedback control theory for flow systems has been developed using computational methods, but no robustness guarantee is given.

Model predictive control (MPC) is a numerically based control strategy that is widely applied in industry [33, 60, 72]. The basic idea is the following. The time is divided into discrete intervals. After each time interval an optimal open loop control input is calculated for a certain time horizon via a numerical search algorithm, and applied to the system. To incorporate feedback, the open loop controller is used only until the end of the time interval, whereafter a new controller is computed, and the time horizon is updated. The main advantages of MPC is that this strategy covers a very wide range of systems, and that the online optimization accounts for model mismatch, disturbances and constraints. The main drawback is that the online optimization is often not practical since the disturbances have to be predicted. Also, the real time solution of the open loop controller is required each time interval, which generally requires extra software for the high computational demands. This is in contrast to a standard controller.
1.3 Research description

The combination of modelling, model reduction and control design for distributed parameter systems is applied to two case studies that represent areas in which system dynamics and control are mainly governed by flow and reaction. The literature on these areas can be divided into analytical mathematical research, with eventual some textbook examples of systems, and into numerical research on more complex systems. In this thesis, the developed models are realistic and more sophisticated than most textbook examples. Nevertheless, they are simple enough to allow a clear analysis and methodology. The applications give a good impression of what problems arise in model based control design for distributed parameter systems in practice. The complicating factors that stand in the way of a simple reduction or control design are the flow, nonlinear input, switching input, and the extremely high number of states that is needed for an accurate state space discretisation. It is shown that combining model reduction and approximation techniques with control design methods can be a powerful tool to come to a practical design. The models are modelled by parabolic pde’s. According to [86] the dynamics are mainly linear and of low order. Hence, the models can be approximated by a small number of ode’s, or equivalently, a linear system with a small number of system states. The main idea in this thesis is to extract the main dynamics from the original models and formulate it as a reduced system in the form of sets of linear ode’s that are suitable for standard or even classical linear control design. Subsequently, the controller is connected to both models. The dynamics of both controlled systems are compared under disturbances. The differences in dynamics indicate whether any essential system dynamics was discarded in the reduction process. Figure 1.2 shows the interaction of the steps that lead to controller design. The arrows indicate the relation between the five objects in the figure. By each arrow, the keywords of the processes that connect the objects, are given. The relation between the chapters and the structure in Figure 1.2 is explained in the next section.

1.4 Summary

In this section a summary of the chapters is given, together with some keywords that apply to Figure 1.2. To make the chapters self-contained, an introduction and (where necessary) a list of notation is given in each chapter.

Chapter 2 A bulk of agricultural products, such as potatoes, onions, or fruits is normally stored in a climate controlled room. The products produce heat due to respiration, and a fan blows cooled air around to keep the
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Figure 1.2: Structure of the steps that lead to controller design

products at a steady temperature to prevent spoilage. The climate in this room, while neglecting heat transfer through the walls, is modelled by a set of nonlinear pde’s. The model is validated against experimental results and analyzed. The extension of the model to more complex dynamics, and the effect on the analysis and control design, is discussed. To make the model suitable for standard control design and clear analysis, it is approximated by means of timescale separation and transfer function approximation. The control input vector consists of the forced convection by the fan, and the temperature of the cooling element inside the heat exchanger. The input is switched regularly, which corresponds to the realistic case where the cooling installation is switched on and off on a regular basis. The open loop control problem, consisting of the determination of the switching moment, can be solved explicitly, with the advantage that all the physical knowledge of the system is preserved. Also an explicit expression is derived for the time that is needed to cool down the bulk, right after the products are stored. This chapter is based on results in [94–96]

Keywords: modelling, numerical simulation, open loop controller, model reduction, validation

Chapter 3 In this chapter we extend the model for a bulk storage room of chapter 2, incorporating the heat transfer through the outside walls. The costs associated with this system are defined. Model relations that could not be found in the literature are obtained via experiments. An open loop
controller is derived. All expressions contain all the physical information of the system. This gives numerical advantages, especially when the system is undisturbed and in steady state. For a realistic choice of parameters, an example is worked out, which results in realistic tradeoffs. For feedback control, the model is approximated by a system that allows linear optimal control, and an associated cost criterion is defined. This chapter is based on results in [91–93].

Keywords: calibration, numerical simulation, feedback controller

Chapter 4 Motivated by, and applied to, a temperature model of a bulk storage room for food, a controller is designed for a class of scalar bilinear systems with switching input, using design theory for linear systems. This is done by approximating the model by a linear system with the switching time as the input. A stability criterion that contains all the physical system parameters is derived by standard Lyapunov theory, allowing a stability analysis without the need for numerical algorithms. For this model the stability robustness was analyzed mathematically. The performance robustness was tested by numerical simulations. It is shown that the approximation methods do not discard any essential system dynamics for control purposes. This chapter is based on results in [97–99].

Keywords: model reduction numerical simulation, feedback controller

Chapter 5 A mathematical model describing fluid flow and concentration dynamics of microorganisms inside a UV reactor is developed. Using physical arguments and techniques from system theory, we approximate this model by a first order linear one. For this reduced model, a controller is designed. The employed techniques are linearization, Padé approximation, and input/output balanced truncation. The developed controller is tested on the original model as well as on the reduced model by numerical simulation. This showed only very small differences in dynamics, which indicates that for the original model a standard controller with excellent properties can be designed. This chapter is based on results in [89,90].

Keywords: modelling, numerical simulation, feedback controller, model reduction

Chapter 6 To circumvent the complex modelling and model reduction procedure, a linear model realization is proposed for chemical reactors with nonlinear input/output dynamics. The realization makes use of a first order reaction equation, and the residence time distribution of the fluid particles inside the reactor. Also dead time is incorporated in the modelling. The inputs are the ingoing concentration of a certain component in the fluid, and the reaction rate. The output is the outgoing concentration. The method is tested on two models with nonlinear input. The first model is a series of mixed tanks, and it is shown by simulation that the realization method gives
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an accurate approximation of the original model. The second model is a UV disinfection reactor, which has a dead time. For this model, the residence time distribution is first fitted by a form that is suitable for our realization method. Simulations show that for realistic disturbances a high performance linear controller can be designed. After that, the residence time distribution of a real life UV reactor (for which we have no model) is fitted by a suitable form. The fit is of the same quality as for the UV reactor model. This indicates that also for the real life UV reactor a high performance controller can be designed.

Keywords: realization, feedback controller, numerical simulation
2

Analytic control law for a food storage room

2.1 Introduction

A large volume of stored perishable foods consists of bulk stored harvested foods, such as potatoes and onions. The storage time ranges from a month to almost a year. Harvested products are living organisms that produce heat, transpire, and produce ethylene and CO₂. Therefore, the control parameters for maintaining the food quality are temperature, humidity, ethylene concentration, and CO₂ concentration. Too much ethylene and CO₂ accelerates the spoilage [87]. This is prevented by ventilation with outside air, most often once a day. In general, the temperature control is done in two ways; ventilation with outside air, or by means of a heat exchanger. Furthermore, a fan enforces the air circulation. For most harvested products, the optimal relative humidity is high, to prevent weight losses, and since the products themselves transpire, this condition is usually satisfied in closed storage rooms. Therefore, daily ventilation with air slightly cooler than the product temperature not only reduces the ethylene and CO₂ concentrations, but also avoids too high relative humidities and condensation. However, the optimal air temperature inside is less easy to realize, since the outside temperature is often much higher than the product temperature (especially in autumn and spring). Moreover, as mentioned before, the products respirate, i.e., produce heat.

The temperature of the products in the bulk varies spatially. Usually, cold air flows upwards through the bulk. Inside the bulk, the air warms up and consequently the products at the top will be somewhat warmer than those at the bottom. Therefore the dynamics is described by a distributed parameter system, and hence it is not a trivial task to develop a control algorithm that keeps the products in the bulk at a constant, desired temperature. For detailed information, see [74].

There is extensive literature available about the modelling aspects of bulk
storage rooms. In [58, 103] the main goal is to derive a model describing
the dynamics of the enthalpy that is suitable for control design. Extensive
CFD models are proposed in [15, 16, 66, 109, 110], and experimental valida-
tion studies were done in [15, 66, 109, 110]. In [102] a review of different CFD
modelling approaches is given. There is a considerable amount of literature
concerning control of nonlinear distributed parameter systems with applica-
tions to chemical and process engineering, for example [2, 27] and the review
article of [17]. However, there is not a vast amount of literature on control
design for bulk storage rooms. In [35] a fuzzy controller was tested on a
mathematical model. Gottschalk proposed in [37] a sensor based control law
for a bulk storage room ventilated with outside air, and constructed in [36] a
fuzzy controller. The controller was tested experimentally. For sensor based
control design, the control algorithms are relatively easy implementable in
practice. However, since these types of control laws are not model based,
less physical interpretation and insight of the controlled process is obtained
than for model based controllers. Keesman et al [44] used a model predic-
tive control (MPC) algorithm for the temperature and humidity control of
a bulk storage room with outside air ventilation. Verdijck [103] proposed
an MPC algorithm for the control of temperature, moist-, and sugar con-
tent of potatoes in a bulk storage room with outside air ventilation. Both
proposed algorithms are model based and were tested by simulation stud-
ies using real weather conditions. The aim of the algorithms was to keep
temperature and humidity within bounds at low economic costs. Due to the
rather high complexity of the models, model based control design requires
computer simulation studies. While the outcomes and predictions are often
accurate enough, this method has some considerable drawbacks. Complex
numerical simulations are always very time consuming, due to both software
programming and demand on computer capacity. The information that sim-
ulations give, can be detailed and accurate, but always hold for a particular
parameter choice. Consequently, a large part of the physical interpretation
and insight is lost. Also, the controllers themselves require complex control
technology and software. It is therefore desirable to reduce these models, or
model components, in complexity.

The main goal of this chapter is to derive a simplified model that preserves
the prior physical knowledge and which is suitable for standard linear con-
trol design. This opens the door for deriving guidelines regarding storage
room design in a computationally easy way, as is shown in chapter 3. The
approximation technique that is proposed here could also help with the sim-
plication of the complex models used for MPC. Further, in chapter 4 a
PI controller that is based on the approximated model in this chapter, is
designed. It should also be noted that this research on bulk stored food
could be a starting point for more advanced storage of vulnerable agricul-
2.1 Introduction

The structure of this chapter consists of the following three steps, see Figure 2.1. First, we derive a basic model for the heat transport inside the storage room. We make some assumptions to keep the model as simple as possible without losing the essential physical properties. The resulting distributed parameter system is validated by experimental results.

Secondly, the inputs are assumed to be piecewise constant. The control problem consists now of the determination of the moment to switch between the constant values of the inputs. Between two input switches, the system is linear, and the system dynamics is mathematically approximated by means of a transfer function approximation, and a timescale decomposition. This ultimately leads to a first order linear model. Third, an open loop control law is derived analytically. Also, the time that the products need to cool down after the harvest (the settle time for the bulk temperature) is derived analytically.

The organization of this chapter is as follows. In Subsection 2.2.1 some physical assumptions are made, and a model is derived. In Subsection 2.2.2 a model simplification is done by fixing the input and approximating a pde with a first order ordinary differential equation (ode). The resulting model is referred to as the nominal model. In section 2.3 the nominal system is validated experimentally. The nominal model is shown to be reasonably ac-
2 Analytic control law for a food storage room

accurate. In Section 2.5, the fast dynamics of the air temperatures is neglected. Using this simplification, the settle time for the bulk temperature after the harvest is calculated. The timescale separation and the transfer function approximation result in a first order, linear model. In Section 2.6 an open-loop control law, that consists of the optimal switching time between the discrete inputs, is based on this model and tested on the nominal model. Simulations show that the control law results in the same output dynamics for both systems, which validates the model simplifications. In Section 2.7 we briefly discuss the results.

2.2 The model

2.2.1 Physical model

We consider a closed storage room with a bulk of products, as depicted in Figure 2.2. We consider potato as specific product, given the large amount of data available in literature. The air temperatures $T_a(x, t)$ and $T_0(t)$ are regulated by a fan that blows the air through the shaft and through the bulk. A cooling element, with variable temperature $T_c(t)$, is placed right below the fan. The air flow rate induced by the fan, and the temperature of the cooling element are the control inputs. The controlled variable is the product temperature $T_p(x, t)$. The aim is now to design a control law such that $T_p(x, t)$ at $x = L$ is kept at a constant, desired level. The following assumptions were made:

![Figure 2.2: Schematic representation of a bulk storage room](image)
2.2 The model

1. The air- and product temperature in the bulk, $T_a$ and $T_p$, only vary with the height of the bulk, so they are uniform w.r.t. the width.

2. The walls are perfectly insulated.

3. The air temperature in the shaft and under the floor, $T_0(t)$, is well-mixed and therefore spatially uniform.

4. The temperature dynamics of the air between the top of the bulk and the fan is neglected.

5. No effects of moisture transport are incorporated. However, the heat capacity of air is adjusted for a high humidity.

6. The products are spherical.

7. The product skin has the same heat conduction as the product interior.

8. The whole product surface is exposed to air.

9. There is no bulk conduction, i.e. there is no heat exchange between the products.

10. Diffusion in the air is neglected.

The motivation and the restrictiveness of each assumption is discussed below.

1. This is a restrictive assumption, since incorporating temperature gradients in more than one direction would require a far more complex model, due to a nonuniform airflow.

2. In [93] the model is extended with heat transport through the walls, which makes the analysis not much more laborious.

3. A spatial model for $T_0(t)$ would not alter the analysis, but the expressions would get more involved.

4. This can be accounted for by adding an extra equation for the state variable $T_{in}$, see Figure 2.2.

5. Including moisture transport complicates the model drastically. To make the analysis below possible, a linearization seems necessary.

6. This assumption makes the derivation of transfer functions easier. However, since the product temperature dynamics are approximated by a lumped model, other shapes with a typical hydraulic diameter are also allowed, see [88].
2 Analytic control law for a food storage room

7. This simplifies the analysis in the next section, and since the product temperatures are lumped later on, this has no further influence.

8. The contact with other product surfaces can easily be accounted for, by reducing the total product surface. Since it is small, it will have negligible influence on the heat transport. This is checked by simulations while varying the total product surface.

9. The ratio of heat exchange between air and products, and heat exchange between products by means of conduction, is

\[
\frac{hA_1(T_p(R) - T_a)}{\lambda_a A_2 \frac{\partial T_p}{\partial x}} \approx \frac{hA_1 2R(T_p(R) - T_a)}{\lambda_a A_2(T_{p,1} - T_{p,2})} \gg 1 .
\]

Here, \( A_1 \) is the product surface per bulk volume that contacts the air, \( A_2 \) the product surface per bulk volume not contacting the air, \( T_p(R) \) the product surface temperature, \( R \) the product radius, and \( T_{p,1} \) and \( T_{p,2} \) temperatures of two products that lie next to each other. \( T_{p,1} - T_{p,2} \) and \( T_p(R) - T_a \), are supposed to be of the same order, as well as \( A_1 2R \) and \( A_2 \). Because for forced convection \( h \) is 10 to 100 times larger than \( \lambda \), heat transport inside the bulk will be convection-dominated. With bulk conduction it was not possible to derive an analytical expression for the open loop controller. A list of all symbols is given in the Appendix.

10. This is justified by the fact that the Péclet number (that indicates the ratio of convection over diffusion)

\[
Pe = \frac{v d \rho_a c_a}{\lambda_a} \gg 1 .
\]

Here \( v \) is the air velocity, \( d \) the diameter of the pores, \( \rho_a \) the air density, \( c_a \) the heat capacity of air, and \( \lambda_a \) the heat conduction of the air.

The assumptions lead to the following energy balance. The energy inflow of the air in the shaft under the heat exchanger is modelled in a basic way: \( \rho_a c_a \Phi(t)(\alpha T_c(t) + (1 - \alpha)T_{in}(t)) \), with \( c_a \) the heat capacity of air, and \( \Phi(t) \) the flow rate of air through the shaft. The dimensionless \( \alpha \) denotes the effectiveness of the cooling device: \( \alpha = 1 \) implies that the incoming air \( T_{in}(t) \) is totally cooled down (or heated up) to the temperature of the cooling element, \( T_c(t) \), while \( \alpha = 0 \) implies that the incoming air is not cooled at all. In chapter 3 the relation between \( \alpha \) and \( \Phi(t) \) was experimentally determined. Without loss of generality we assume that \( \alpha \) is constant. Because of the perfect insulation of the walls, we have that \( T_{in}(t) = T_a(L, t) \). The energy outflow equals \( -\rho_a c_a \Phi(t)T_0(t) \). The dynamic energy balance for \( T_0(t) \)
therefore becomes
\[ \rho_a c_a V \frac{\partial T_0(t)}{\partial t} = -\rho_a c_a \Phi(t) \alpha \left( T_a(L, t) - T_c(t) \right) + \rho_a c_a \Phi(t) T_a(L, t) - \rho_a c_a \Phi(t) T_0(t), \] (2.3)

with \( V \) the volume of the shaft.

The energy balance for \( T_a(x, t) \) is, with \( x \in (0, L) \),
\[ \rho_a c_a \gamma \frac{\partial T_a(x, t)}{\partial t} = -\gamma \rho_a c_a v(t) \frac{\partial T_a(x, t)}{\partial x} + h(v(t)) A_{ps} \left( T_p(R, x, t) - T_a(x, t) \right), \] (2.4)

with boundary condition
\[ T_a(0, t) = T_0(t). \] (2.5)

Here, \( A_{ps} \) is the product surface area that is exposed to air per bulk volume, and \( T_p(R, x, t) \) is the product surface temperature at \( x \). The two r.h.s. terms in (2.4) denote the convection of heat and the heat exchange between product surface and air, respectively. The heat transfer coefficient \( h \) depends on \( v \) via the implicit relation (see [106])
\[ \text{Nu} = (0.5 Re^{1/2} + 0.2 Re^{2/3}) Pr^{1/3}, \quad 10 < Re < 10^4 \] (2.6)

with Nu, Re and Pr the Nusselt, Reynolds and Prandtl number respectively, which are functions of \( v \) and \( h \), see Appendix 2.8.2. The heat transport inside a product at height \( x \) is modelled by diffusion in a sphere with radius \( R \).
\[ \rho_p c_p \frac{\partial T_p(r, x, t)}{\partial t} = \lambda_p \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T_p(r, x, t)}{\partial r} \right) + \rho_p \tilde{a} T_p(r, x, t) + \rho_p b, \] (2.7)

where \( \rho_p, c_p, \) and \( \lambda_p \) are the product density, heat capacity, and conductivity, respectively. The last two terms in equation (2.7) denote the heat production, see [110] and the references therein. The boundary conditions are
\[ \frac{\partial T_p}{\partial r}(0, x, t) = 0 \text{ by symmetry at the origin} \] (2.8)
\[ \lambda_p \frac{\partial T_p}{\partial r}(R, x, t) = h(v(t))(T_a(x, t) - T_p(R, x, t)) \] (2.9)

The second equation denotes the heat flux through the product surface. The values of \( \tilde{a} \) and \( b \) in equation (2.7) are used in [109], and fit the experimental data in [74] well for \( T_p > 278 K \). To simplify the analysis in the next sections, the heat production is approximated as \( \tilde{a} T_p + b \approx a T_p \), with equality in \( T_p = 280 K \). We impose the following additional assumption
The heat flux between air and products at $x$ is modelled as the flux between a sphere with air temperature $T_a(x,t)$ along its surface. The equations are nonlinear since $v$ enters equation (2.3) by $Φ(t) = A_f v(t) γ$ (with $A_f$ the surface of the bulk floor, and $γ$ the bulk porosity), equation (2.4) via the implicit relation (2.6), and equation (2.7) via equation (2.9). The full system dynamics together with boundary conditions is described by

\[
\frac{∂T_0(t)}{∂t} = -\frac{Φ(t)}{V} α(T_a(L,t) - T_c(t)) + \frac{Φ(t)}{V} T_a(L,t) - \frac{Φ(t)}{V} T_0(t) \quad (2.10)
\]

\[
\frac{∂T_a(x,t)}{∂t} = -v(t) \frac{∂T_a(x,t)}{∂x} + M_4 (T_p(R,x,t) - T_a(x,t)) \quad (2.11)
\]

\[
T_a(0,t) = T_0(t) \quad (2.12)
\]

\[
\frac{∂T_p(r,x,t)}{∂t} = M_1 \frac{1}{r^2} \frac{∂}{∂r} \left( r^2 \frac{∂T_p(r,x,t)}{∂r} \right) + M_2 T_p(r,x,t) \quad (2.13)
\]

\[
\frac{∂T_p}{∂r}(0,x,t) = 0 \quad (2.14)
\]

\[
\frac{∂T_p}{∂r}(R,x,t) = \frac{h(v(t))}{λ_p} \left( T_a(x,t) - T_p(R,x,t) \right), \quad (2.15)
\]

with $Φ(t) = A_f v(t) γ$, $M_1 = \frac{λ_p}{ρ_p c_p}$, $M_2 = \frac{α}{c_p}$, $M_4 = \frac{h(v)A_p s}{γρ_a c_a}$, and appropriate initial conditions. This model will be referred to as the basic model. The controlled variable is $T_p(R,L,t)$, and the control inputs are $v(t)$ and $T_c(t)$.

### 2.2.2 Model approximation

In what follows, we assume the inputs to be piecewise constant. For fixed $v$, the system described by equations (2.10)–(2.15) becomes linear. In the frequency or Laplace domain the heat transfer between the air and the product surface is solved with respect to $r$, using equations (2.13)–(2.15), and then it can be written as

\[
\hat{T}_p(R,x,s) = G_p^R(s) \hat{T}_a(x,s). \quad (2.16)
\]

The transcendental transfer function $G_p^R(s)$ is approximated by a rational one. As a rational approximation we choose the Padé[0,1] approximation in $s = 0$, so

\[
\hat{G}_p^R(s) = \frac{a_1}{a_2 + a_3 s}. \quad (2.17)
\]
2.2 The model

Because of the rational form we can transform (2.16)–(2.17) back into the time domain

$$\frac{\partial T_p(R, x, t)}{\partial t} = -\frac{a_2}{a_3} T_p(R, x, t) + \frac{a_1}{a_3} T_a(x, t)$$

$$= A_p T_p(R, x, t) + B_p T_a(x, t).$$

(2.18)

We found that

$$a_1 = Bi$$

$$a_2 = 2M_3 \cot(M_3) - 2 + Bi$$

$$a_3 = \frac{R^2}{M_1} \cot^2(M_3) + \frac{R^2}{M_1} - \frac{M_3}{M_2} \cot(M_3).$$

$$M_1 = \frac{\lambda_p}{\rho_p c_p}; \text{ diffusion coefficient } (m^2/s)$$

$$M_2 = \frac{a}{c_p}; \text{ reaction constant } (1/s)$$

$$M_3 = \sqrt{M_2/M_1 R}. \quad (2.19)$$

The dimensionless parameter $M_3$, which indicates the heat production rate over the diffusive heat transfer rate, is analogous to the Thiele modulus

$$Th = \frac{\text{chemical reaction rate}}{\text{diffusive mass transfer rate}},$$

(2.20)

and $Bi = \frac{2h(v)R}{\lambda_p}$ is the Biot number, which implicitly depends on $v$. Using the Padé approximation, the approximated system becomes

$$\frac{\partial T_0(t)}{\partial t} = -\frac{\Phi(t)}{V} \alpha(T_a(L, t) - T_0(t))$$

$$+ \frac{\Phi(t)}{V} T_a(L, t) - \frac{\Phi(t)}{V} T_0(t)$$

(2.21)

$$\frac{\partial T_a(x, t)}{\partial t} = -v(t) \frac{\partial T_a(x, t)}{\partial x} + M_4 \left( T_p(x, t) - T_a(x, t) \right)$$

(2.22)

$$\frac{\partial T_p(R, x, t)}{\partial t} = A_p T_p(R, x, t) + B_p T_a(x, t)$$

(2.23)

$$T_a(0, t) = T_0(t),$$

(2.24)

with $\Phi(t) = A_f v(t) \gamma$. This approximated model resembles a heat transfer model inside a porous medium, which is done for example in [44,58]. The difference is that here there is no bulk conduction. From now on we refer to system (2.21)–(2.24) as the nominal model.
2.3 Model validation by experiment

In [62] and [110] experimental data was reported from an experiment in which a forced laminar airflow cools down a column that was filled with a bulk of potatoes of 15.5 °C. A constant laminar airflow of 6.7 °C was forced through the column for 92 hours. The physical properties of that experiment are listed in Table 2.1. The specific area of potato was not reported. From the average size (length 95 mm, diameter 51 mm), the potato density (1014 kg/m\(^3\) according to [58]), and the total weight of the bulk (360 kg), it follows that the column was filled with approximately 3000 potatoes. The total product surface exposed to air, assuming this is 95 % of the total product surface, is 41 m\(^2\) per m\(^3\) bulk. In [110] a model that incorporates heat transfer by moisture transport is used, and this model predicts the experimental bulk temperatures very well. The experiment is simulated by our model using \(T_c(t) = 6.7 °C\), \(\alpha = 1\), and \(V = 0\), such that \(T_0(t) = 6.7 °C\) at all times. Table 2.2 shows the experimental air temperatures at the top, the middle, and the bottom of the bulk, and the temperatures predicted by system (2.21)–(2.24) at different times. According to [110], the complex temperature dynamics between the bottom and middle of the bulk are caused by evaporation effects. Our model predictions match the experimental data reasonably well. In the middle of the bulk the predictions are least accurate. Our model predicts higher temperatures compared to the experiment, and the model proposed in [110]. This is probably due to the heat loss by evaporation that was neglected in our model. We note that only nine data points cannot completely prove model accuracy, but nevertheless we have some confidence that our model is reasonably accurate for control purposes.

| \(c_a\) | \(1.7 \times 10^3\) J/kg K | \(A_f\) | 0.3848 m\(^2\) |
| \(c_p\) | \(3.52 \times 10^3\) J/kg K | \(R\) | 0.0325 m |
| \(\Phi\) | \(2.6 \times 10^{-3}\) m\(^3\)/s | \(L\) | 2.4 m |
| \(v\) | 0.0109 m/s | \(T_c\) | 6.7°C |
| \(\gamma\) | 0.6 | \(RH\) | 60 % |

Table 2.1: Physical properties of the experiment.

2.4 Separation of timescales

The nominal model is approximated by neglecting the dynamics of the fast model states. To justify this, the timescales of the different states are analyzed. For this analysis, we assume that \(v\) is constant.
2.4 Separation of timescales

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Table 2.2: Comparison of predicted and measured air temperatures (°C) in the bulk at different times. The experimental data are taken from [110]. Here m and e denote the model and experiment.

2.4.1 Timescale of the product temperature

Since the experimental setup in the previous section is a specific one without interaction between \( T_a(L, t) \) and \( T_0(t) \), we will use a more general parameter choice in the following (Table 2.3). The transfer function \( G_p^R(s) \) connects the input \( \hat{T}_a(x, s) \) to \( \hat{T}_p(R, x, s) \) (the Laplace transformed product surface temperature), and is approximated with a first order Padé method (as explained in subsection 2.2.2). Plot (a) in Figure 2.3 shows the Bode magnitude plots of \( G_p^R(s) \) (with \( s = j\omega \)) from equation (2.16) and its first order approximation \( \hat{G}_p^R(s) \). The static gains are (per definition, see Appendix 2.8.1) equal, and the time constant of \( \hat{G}_p^R(s) \) is accurate. The crossover frequency is of the order of \( \omega = 10^{-3} - 10^{-4} \) Hertz, which indicates a time constant of order \( 10^3 - 10^4 \) seconds. A widely used definition of the time constant of a state is 1/6 of the time that it needs to reach its equilibrium value after a step input, [70] pp. 292-293. Therefore, the product temperature settles in hours. \( G_p^0(s) \) is the transfer function that connects the input \( \hat{T}_a(x, s) \) to \( \hat{T}_p(0, x, s) \) (the Laplace transformed core temperature). Plot (b) in Figure 2.3 shows \( G_p^0(s) \) and \( G_p^R(s) \). The orders of their time constants are equal, and we conclude that the time constant of \( T_p(0, x, t) \) is of the same order as that of \( T_p(R, x, t) \). This is also not surprising, since the skin temperature will not settle before the core temperature does. The strong descent of \( G_p^0(s) \) for high \( s \) indicates that the core temperature \( T_p(0, x, t) \) barely responds to high frequency fluctuations in \( T_a(x, t) \). Intuitively, this is not surprising, but analytically this is hard to see because of the complex form of \( G_p^0(s) \). The difference in gain is less than 10 % for all frequencies, so the variation in \( T_p \) is always less than 10% of the variation in \( T_a \). Further, the static gains of \( G_p^0(s) \) and \( G_p^R(s) \) are almost identical. This can be seen analytically from
Figure 2.3: Plot (a): the static gain and the time constant of the first order approximation of $G_p^R(s)$ are accurate. Plot (b): the time constants of $G_p^R(s)$ and $G_p^0(s)$ are of the same order.

respectively. Since $M_3$ is of the order of $10^{-2}$ we have that $\sin(M_3) \approx M_3$. The static gains are approximately equal, and therefore the spatial temperature differences inside a product due to respiration will be negligible
2.4 Separation of timescales

in the equilibrium situation. Altogether, we conclude that $T_p(R, x, t)$ and $T_p(0, x, t)$ will practically never have large differences. Therefore, we will only look at $T_p(R, x, t)$ from now on, and denote it with $T_p(x, t)$.

2.4.2 Timescales of the air temperatures

After focusing on the product temperature, we will analyze the behavior of the air temperatures in the different compartments of the storage room.

The time constants of $T_0(t)$ are obtained from the transfer functions that correspond to equation (2.21)

$$
\hat{T}_0(s) = G_3(s)\hat{T}_a(L, s) + G_4(s)\hat{T}_c(s)
= \frac{1 - \alpha}{1 + \frac{\alpha}{\Phi} s} \hat{T}_a(L, s) + \frac{\alpha}{1 + \frac{\alpha}{\Phi} s} \hat{T}_c(s).
$$

(2.26)

The time constants are both equal to $\frac{\Phi}{\Phi}$. Therefore, the time constant of this subsystem equals $\frac{\Phi}{\Phi}$, which will be in the order of $10^0$–$10^1$. The transfer function corresponding to equation (2.21) is

$$
\hat{T}_a(x, s) = \exp\left(-\frac{s + M_4}{v}x\right) \left( \int_0^x \frac{M_4}{v} \exp\left(-\frac{s + M_4}{v}z\right) \hat{T}_p(z, s)dz + \hat{T}_0(s) \right).
$$

(2.27)

From this expression it is hard to derive a time constant. Therefore, we temporarily assume that $T_p(x, t)$ is uniform in $x$. This is justified by the fact that the uniformity of $T_p(x, t)$ will not change the settle time of the air temperature significantly. The approximated transfer functions corresponding to equation (2.21) now become

$$
\hat{T}_a(x, s) = G_1(s, x)\hat{T}_0(s) + G_2(s, x)\hat{T}_p(x, s),
$$

(2.28)

with

$$
G_1(s, x) = \exp\left(-\frac{s + M_4}{v}x\right),
$$

$$
G_2(s, x) = \frac{M_4(1 - \exp\left(-\frac{s + M_4}{v}x\right))}{s + M_4}.
$$

(2.29)

They are not rational since they contain a time delay $x/v$. Hence, we cannot see their time constants directly. The Padé[0,1] approximations in $s = 0$ are of first order and do not alter the time constant. They are given by

$$
G_1(s, x) \approx \frac{1}{\exp\left(\frac{M_4 x}{v}\right) + \frac{x}{v} \exp\left(\frac{M_4 x}{v}\right) s},
$$

$$
G_2(s, x) \approx \frac{2(\cosh(\frac{M_4 x}{v}) - 1)}{\exp\left(\frac{M_4 x}{v}\right) - 1 + \frac{1}{M_4}(\exp\left(\frac{M_4 x}{v}\right) - 1 - \frac{M_4 x}{v})s}.
$$

(2.30)
2 Analytic control law for a food storage room

with $M_4 = \frac{h A_{ps}}{\gamma \rho_a c_a}$ the reaction constant of the product heat production (1/s).
The corresponding time constants are

$$\frac{x}{v} \quad \text{and} \quad \frac{1}{M_4} \left( \exp \left( \frac{M_4 x}{v} \right) - 1 - \frac{M_4 x}{v} \right) \exp \left( \frac{M_4 x}{v} \right) - 1,$$

(2.31)

which are typically of order $10^0 - 10^1$. We note that the dimensionless number

$$M_5 = \frac{M_4 L}{v}; \quad \frac{\text{chemical reaction rate}}{\text{convective heat transfer rate}},$$

(2.32)

where the chemical reaction corresponds to the heat production inside the products, is analogous to the Damköhler I number

$$\text{Da I} = \frac{\text{chemical reaction rate}}{\text{convective mass transfer rate}}.$$

(2.33)

The settle times for air temperatures depend on a transport delay $x/v$. The static gain of $G_1(s, x)$ decreases exponentially with $x$ and $M_4$, and it increases exponentially with $v$. For realistic parameter values, changes in $x M_4/v$ barely influence the static gain of $G_2(s, x)$. This implies that for higher values of $x M_4/v$, $T_a(x, t)$ is coupled stronger to $T_p(x, t)$ and less strong to $T_c(t)$. So $T_a(0, t)$ will respond much stronger to variations in $T_c(t)$ than $T_a(L, t)$. In Subsection 2.4.4 this is further confirmed by a model simulation.

The time constants of $T_a(x, t)$ and $T_0(t)$ are typically three orders lower than that of $T_p(x, t)$. We expect that after a switch in $T_c(t)$, $T_a(x, t)$ and $T_0(t)$ will settle quickly, whereafter it will move with the same rate as $T_p(x, t)$. When $v$ is switched (and not $T_c(t)$), the system is piecewise linear between two switches. This means that there is not one single transfer function that connects the input $v$ to the output $T_p(x, t)$, so the analysis above does not apply. Nevertheless we expect that the difference between the time constants of the air and product temperatures will remain large. The assumption of a spatially uniform $T_p(x, t)$ in (2.27)–(2.29) seems strong. However, it is not a physically crucial assumption; it only simplifies the analysis mathematically. This analysis is carried out per subsystem, i.e., for $T_a$, $T_0$, and $T_p$ individually. In reality, these subsystems are coupled. This complicates the analysis considerably. However, we expect no dramatic effects due to the coupling and the non-uniformity in $T_p(x, t)$, but for confidence in the next sections numerical analysis that supports these expectations, is carried out on the nominal system with a realistic choice of parameters.

### 2.4.3 State space analysis

Let us consider the dynamics of the nominal system, described by equations (2.21)–(2.24), by means of eigenvalue analysis and time simulation.
2.4 Separation of timescales

The equations are made discrete as follows. The term \( v \frac{\partial T_a(x,t)}{\partial x} \) in equation (2.22) is upwind approximated like \( v T_{a,n} - T_{a,n-1} \), where the second subscript denotes the discrete space, starting from the bulk bottom. The spatially discretised full system is of the form

\[
\frac{\partial T}{\partial t} = A_{full}(v)T + B_{full}(v)T_c,
\]

with \( T = [T_{a,1}..T_{a,n} T_0 T_{p,1}..T_{p,n}]^T \). This differential equation is simulated in Simulink with the ode45 Dormand-Prince algorithm. The physical parameters of the chosen configuration are listed in Table 2.3. The heat capacity of the air is adjusted for humidity, by relating heat capacity to enthalpy change for air between 7 °C and 10 °C with a 90-95% relative humidity. The obtained value, 2 \( 10^3 \), is also suggested in [74], chapter 13. For our parameter choice, \( Re = 2.17 \times 10^2 \), so that we may use equation (2.6). For clarity, the system is discretised in only two spatial components: \( n = 2 \). For a system of the form (2.34), the negative inverses of the (real parts of the) eigenvalues of \( A_{full} \) represent the time constants of the system. The (real parts of the) eigenvalues of \( A_{full} \) are shown in the top row of Table 2.4. These eigenvalues contain small imaginary parts. These are probably numerical errors due to the high condition number of \( A_{full}(v) \) of \( O(10^7) \). Simulations therefore show small oscillations for any number of \( n \), but when the state vectors \( [T_{p,1},..,T_{p,n}]^T, [T_0], \) and \( [T_{a,1},..,T_{a,n}]^T \) are simulated in parallel, the oscillations do not show up. The (real parts of the) eigenvectors of the full system are shown as the columns below the eigenvalues in Table 2.4. The two left eigenvectors correspond to the small eigenvalues and thus to the slow dynamics, and the three right eigenvectors correspond to the large eigenvalues and thus to the fast dynamics. The only eigenvectors that have substantial components in the directions \( T_{p,1}(t) \) and \( T_{p,2}(t) \), are the slow ones, so \( T_p \) has only slow dynamics. This is in agreement with the analysis of the uncoupled subsystems from the previous section. 

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>0.4</td>
</tr>
<tr>
<td>( R )</td>
<td>3.25 ( 10^{-2} ) m</td>
</tr>
<tr>
<td>( L )</td>
<td>4 m</td>
</tr>
<tr>
<td>( \lambda_p )</td>
<td>0.55 J/s m K</td>
</tr>
<tr>
<td>( a )</td>
<td>3.1 ( 10^{-5} ) J/s kg K</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>0.31</td>
</tr>
<tr>
<td>( T_c )</td>
<td>275 K</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_f )</td>
<td>5 m²</td>
</tr>
<tr>
<td>( V )</td>
<td>10 m³</td>
</tr>
<tr>
<td>( v )</td>
<td>0.2 m/s</td>
</tr>
<tr>
<td>( \rho_p )</td>
<td>1014 kg/m³</td>
</tr>
<tr>
<td>( A_{ps} )</td>
<td>40 m²/m³</td>
</tr>
<tr>
<td>( c_p )</td>
<td>3.6 ( 10^3 ) J/kg K</td>
</tr>
<tr>
<td>( c_a )</td>
<td>2 ( 10^3 ) J/kg K</td>
</tr>
</tbody>
</table>

Table 2.3: Physical parameters of a bulk with potatoes. The data specific for potato were taken from [44,110].
Table 2.4: The eigenvalues are listed in the top row. The columns below the eigenvalues are the eigenvectors. The state space is $[T_{a,1} T_{a,2} T_0 T_{p,1} T_{p,2}]$.

Slow eigenvectors also have substantial components in all the other directions, so the states $T_0$, $T_{a,1}$, and $T_{a,2}$ also have slow dynamics. The three fast eigenvectors have large components in the directions $T_0$, $T_{a,1}$, and $T_{a,2}$, so these states have also fast dynamics. We conclude that the dynamics of $T_a$, $T_0$ and $T_p$ are coupled to each other: After a change of input, $T_a$ and $T_0$ will settle quickly and then slowly move together with $T_p$. From the inverse eigenvalues we see that the time constants of the slow states are $O(10^3)$, and that the time constants of the fast states are $O(10^0)$. The rate of heat transfer, and therefore the rate of change of the system states depends strongly on $v$. However, for different choices of $v$, namely 0.1 and 10, similar results were obtained. The minus sign in the fourth row of the second eigenvector in Table 2.4 indicates that the temperature profile of $T_p(x, t)$ is spatially not uniform since the $T_{p,1}$ and $T_{p,2}$ move in opposite directions according to the second eigenvector. The time simulation in the next section shows that the air and product temperatures move together in one direction mainly, which implies that the first eigenvector is dominant.

### 2.4.4 Time simulation

The difference in timescales is visualized by a time simulation of $T_a(L, t)$, $T_a(0, t)$, $T_p(L, t)$, and $T_p(0, t)$. $T_c(t)$ is switched once every fifteen minutes between 275 $K$ and 285 $K$. In all the simulations $n = 20$ layers was used, which was found to be quite accurate. The rest of the parameters are listed in Table 2.3. Figure 2.4 shows the fast and slow dynamics of $T_a(L, t)$ and $T_a(0, t)$, and the slow dynamics of $T_p(L, t)$ and $T_p(0, t)$: after a switch $T_a(L, t)$ and $T_a(0, t)$ settle quickly, whereafter they slowly move with $T_p(L, t)$ and $T_p(0, t)$. Since $T_p(L, t)$ and $T_p(0, t)$ are both at their equilibrium values, they hardly move and only the fast dynamics of $T_a(L, t)$ is visible. $T_p(L, t)$ is a bit higher than $T_p(0, t)$ due to the warming up of $T_a(x, t)$ inside the bulk. We observe that the fast dynamics of $T_a$ and $T_0$ (not shown) are negligible on a timescale of fifteen minutes. This allows us to further simplify the model.
2.5 Settle time of the bulk temperature

Figure 2.4: From top to bottom: the dynamics of \( T_p(L,t) \), \( T_a(L,t) \), \( T_p(0,t) \), and \( T_a(0,t) \). The input \( T_c(t) \) is switched every fifteen minutes.

in the next section. We also observe that \( T_a(0,t) \) responds much stronger on changes in \( T_c(t) \) than \( T_a(L,t) \), as was predicted in Subsection 2.4.2. As a consequence, \( T_p(0,t) \) moves more than \( T_p(L,t) \) during a switching interval.

2.5 Settle time of the bulk temperature

Given the results from the previous sections we will now focus on the behavior of the product temperature in the bulk under cooling and ventilation. Notice therefore that if a system is dominated by first order dynamics, the settle time can be predicted accurately by the time constant of a first order approximated system. We look at the time constant of the product surface rather than the product core, since in Subsection 2.4.1 we have shown that these two time constants are of the same order. We exploit the differences in timescales. The cooling down of the bulk is a slow process. Due to their fast dynamics, \( T_a \) and \( T_0 \) settle quickly to their equilibrium values, and slowly move along with \( T_p \). Under quasi-steady state conditions the time derivative in (2.21) and (2.22) are set to zero. This is equivalent to neglecting the fast dynamics of \( T_a \) and \( T_0 \), which are only apparent at high frequencies or at
short timescales. This leads to the approximation
\begin{align}
0 &= -\alpha(T_a(L,t) - T_c(t)) + T_a(L,t) - T_0(t) \quad (2.35) \\
0 &= -\frac{\partial T_a(x,t)}{\partial x} + \frac{M_4}{v} (T_p(x,t) - T_a(x,t)) \quad (2.36) \\
T_a(0,t) &= T_0(t) \quad (2.37) \\
\frac{\partial T_p(x,t)}{\partial t} &= A_p T_p(x,t) + B_p T_a(x,t). \quad (2.38)
\end{align}

Laplace transformation of (2.38), and substituting this in (2.36) gives
\begin{align}
\frac{\partial \hat{T}_a(x,s)}{\partial x} &= \frac{M_4}{v} \left( \frac{B_p \hat{T}_a(x,s)}{-A_p + s} - \hat{T}_a(x,s) \right) \\
\hat{T}_a(0,s) &= \hat{T}_0(s) \\
\Rightarrow \hat{T}_a(L,s) &= \hat{T}_0 \exp \left( \frac{M_4 L}{v} \left( \frac{B_p}{-A_p + s} - 1 \right) \right) \\
&= \left( -\alpha(\hat{T}_a(L,s) - \hat{T}_c(s)) + \hat{T}_a(L,s) \right) \cdot \exp \left( M_5 \left( \frac{B_p}{-A_p + s} - 1 \right) \right), \quad (2.39)
\end{align}
where we have used (2.35) and (2.32). Consequently,
\begin{align}
\hat{T}_a(L,s) &\approx \frac{\alpha \exp \left( M_5 \left( \frac{B_p + A_p - s}{-A_p + s} \right) \right)}{1 - (1 - \alpha) \exp \left( M_5 \left( \frac{B_p + A_p - s}{-A_p + s} \right) \right)} \hat{T}_c(s). \quad (2.40)
\end{align}

A Pade[0,1] approximation results in a first order system
\begin{align}
\hat{T}_a(L,s) &= \alpha \frac{A_p^2 \exp \left( M_5 \left( \frac{B_p + A_p}{-A_p + s} \right) \right)}{M_5 B_p - \frac{A_p^2 (1 - \alpha)}{M_5 B_p} \exp \left( M_5 \left( \frac{B_p + A_p}{-A_p + s} \right) \right)} + s \\
&= \frac{\hat{B}_p}{-A_p + s} \hat{T}_c(s). \quad (2.41)
\end{align}

Laplace transformation of equation (2.38) gives
\begin{align}
\hat{T}_p(L,s) &= \frac{B_p}{-A_p + s} \hat{T}_a(L,s). \quad (2.42)
\end{align}

So the approximated transfer function from \( \hat{T}_c(s) \) to \( \hat{T}_p(L,s) \) equals
\begin{align}
\frac{\hat{B}_p B_p}{A_p A_p - (A_p + A_p) s + s^2.} \quad (2.43)
\end{align}
2.5 Settle time of the bulk temperature

The Padé[0,1] approximation of this transfer function has a time constant of

\[-\tilde{A}_p + A_p \over \tilde{A}_p A_p.\]  

(2.44)

As mentioned before, the settle time is defined as six times the inverse of the time constant. This is the time after which the temperature is more or less at its steady state. Figure 2.5 shows the time simulation of the cooling down of $T_p(L,t)$ and $T_p(0,t)$. According to the system equations (2.21)–(2.24), with $T_c = 275$, $v = 0.2$, and the rest of the parameters are listed in Table 2.3. The settle time of $T_p(L,t)$ that is predicted by (2.44) is $10^4$ minutes, which is in agreement with the simulation results. Other parameter choices gave similar results. The settle times of $T_p(L,t)$ and $T_p(0,t)$ are practically equal. The first order dynamics are dominant on a large time scale, which ensures an accurate prediction of the time constant and thus of the settle time of the system, but right after the start the effects of the higher order dynamics come into play. The decay of $T_p'(L,t)$ is somewhat slower in the beginning than that of $T_p'(0,t)$, and is caused by the uniform initial bulk temperature. Consequently, the air at the top is heated up by the rest of the bulk, and the top products are cooled down very little. After a while, the bottom and middle of the bulk are cooled down and the air at the top gets less heated up than in the beginning. The air now starts cooling down the products at

![Figure 2.5: The settle time for $T_p(L,t)$ and $T_p(0,t)$ is about $10^4$ minutes. The uniform initial value is $T_p(x,0) = 285$ K.](image-url)
2 Analytic control law for a food storage room

the top. Similar observations were made in [110]. The transfer function in equation (2.41) represents an unstable system if the signs in the denominator are unequal, i.e., if

$$A_p^2 - A_p^2(1 - \alpha) \exp\left(M_5\left(\frac{B_p + A_p}{-A_p}\right)\right) \leq 0.$$  (2.45)

Technically, this will give a chain reaction in which $T_p$ starts rising, which induces more heat production, which causes $T_p$ to rise further. Note that $A_p < 0$, $\alpha \in [0, 1]$, and $B_p + A_p \geq 0$. So mathematically, this occurs for small values of $\alpha$ in combination with a large argument in the exponential. The first indicates poor cooling, the second can be caused by poor ventilation, a high bulk, and a high heat production. In reality, heat production is finite, and the temperature will stop rising after some or all products have rotted away. For our parameter choice, the system is stable.

2.6 Calculation of the switching time

Because $T_p$ has its highest values at the top of the bulk, we want to control $T_p(L, t)$. We note that an desired value of the top products might result in a too low value for the bottom products. In chapter 3 we go into this. Our starting point is that $T_p(L, t)$ is at its optimal equilibrium value $T_{p,\text{opt}}$. In Subsections 2.4.1 and 2.4.2 we assumed piecewise constant inputs, which makes the analysis easier. The control problem consists of finding the optimal time to switch between these inputs. The inputs are switched between two discrete vectors, $(T_{c,1}, v_1)$ and $(T_{c,2}, v_2)$, once in an intermediate time interval of about ten minutes. The choice of this time interval has two reasons. First, the air temperatures settle within a minute, so we can approximate them by constant values on this time interval (equations (2.35) and (2.36)). Secondly, $T_p$ will move very slowly, so on a ten minute time scale its dynamics will be linear (first order) in time. The question is when to switch $T_c$ or $v$ (or both) such that $T_p(L, t)$ returns at its optimal value after each time interval. To determine the first order dynamics of $T_p(L, t)$, the transfer functions in equations (2.40) and (2.42) are combined and Padé[0,1] approximated to

$$\hat{T}_p(L, s) = \frac{\hat{B}_p B_p}{A_p A_p - (A_p + A_p)s} \hat{T}_c(s)$$

$$= \frac{B^*_p}{-A^*_p + s} \hat{T}_c(s),$$  (2.46)

with

$$A^*_p = \frac{\hat{A}_p A_p}{A_p + A_p} \quad \text{and} \quad B^*_p = -\frac{\hat{B}_p B_p}{A_p + A_p}.$$
2.6 Calculation of the switching time

The (approximated) first order dynamics of $T_p$ is thus given by

$$\frac{dT_p(L, t)}{dt} = A_p T_p(L, t) + B_p T_c(t). \quad (2.47)$$

The solution to equation (2.47) at time $\tau$ is

$$T_p(L, \tau) = T_p(L, 0) \exp(A_{p,1}^* \tau) + \int_0^\tau \exp(A_{p,1}^* (t - \tau)) B_{p,1}^* T_c,1 dt$$

$$= T_p(L, 0) \exp(A_{p,1}^* \tau) + \frac{B_{p,1}^*}{A_{p,1}^*} T_c,1 (1 - \exp(-A_{p,1}^* \tau)). \quad (2.48)$$

The second subscript refers to the discrete input $v_1$. Because $|A_{p,1}^* | \ll 1$, we can accurately linearize this to

$$T_p(L, \tau) \approx T_p(L, 0) (1 + A_{p,1}^* \tau) + B_{p,1}^* T_c,1.$$  \hspace{1cm} (2.49)

which is equivalent to a forward Euler discretization in time. Similarly, but now with a backward Euler discretization, we get

$$\frac{T_p(L, \tau_f) - T_p(L, \tau)}{\tau_f - \tau} \approx A_{p,2}^* T_p(L, \tau_f) + B_{p,2}^* T_c,2.$$ \hspace{1cm} (2.50)

Together with the condition $T_p(L, 0) = T_p(L, \tau_f) = T_{p,opt}$, where $T_{p,opt}$ is the optimal equilibrium value, this leads to

$$0 = \tau(A_{p,1}^* T_{p,opt} + B_{p,1}^* T_c,1) + (\tau_f - \tau)(A_{p,2}^* T_{p,opt} + B_{p,2}^* T_c,2).$$ \hspace{1cm} (2.51)

Simple calculus gives the optimal switching time $\tau_{opt}$

$$\tau_{opt} = -\frac{A_{p,2}^* T_{p,opt} + B_{p,2}^* T_c,2}{(A_{p,1}^* - A_{p,2}^*) T_{p,opt} + B_{p,1}^* T_c,1 - B_{p,2}^* T_c,2}. \quad (2.52)$$

The existence of solution (2.52) and the stability of the nominal system is investigated in chapter 4. As an example, Figure 2.6 shows the simulated dynamics of $T_p(L, t)$ of the nominal model. The switching time is derived from the approximated model. Each time interval $[0, \tau_f]$ with $\tau_f = 10$ minutes, the inputs are switched once, such that in each interval input 1 is applied for $\tau$ seconds, and input 2 is applied for $\tau_f - \tau$ seconds. We have $T_c,1 = 275 K$, $T_c,2 = 285 K$, $v_1 = 0.2 m/s$, and $v_2 = 0.02 m/s$. The rest of the physical coefficients is listed in Table 2.3. For $\tau = \tau_{opt}$, $T_p(L, t)$ moves slowly around its equilibrium value of 279.97 K. Since $T_{p,opt} = 280 K$, there is an offset of 0.03 degrees. The small offset indicates that errors introduced by the approximation steps from (2.21)–(2.24) to (2.47), and the approximation of (2.47) by (2.52), are small.
Figure 2.6: $T_p(L,t)$ moves slowly around its equilibrium value of 279.97 K. The difference with the optimal product temperature is 0.03 K. Here, $\tau_f = 10$ minutes, and the order of on/off switching is changed in each interval to minimize the number of switches.

Cooling element as control mechanism

A strong assumption we made, was the starting value $T_p(L,t) = T_{p,opt}$. However, Figure 2.7 shows for a similar parameter choice that if $T_p(L,t)$ is far away from $T_{p,opt}$, it will move towards it using $\tau = \tau_{opt}$. This results in the same small error between $T_p(L,t)$ and $T_{p,opt}$ as in the previous section. In chapter 4 the stability of a switching system is analyzed mathematically, but there is also an intuitive explanation to this phenomenon: the cooling element acts as a stabilizing control mechanism. Equation (2.3) implies that a higher $T_a(L,t)$ is cooled down more, and a lower $T_a(L,t)$ is cooled down less.

2.6.1 Switching time for a lumped system

The difference between a delicately modelled heat transfer mechanism and a crude model can be vital. In [74] chapter 13, and [103] chapter 6.6.1, models for a bulk storage room without spatial variabilities were used. The ventilation strategy was designed on (amongst others) the total heat production of the bulk. Using these assumptions in our model, the temperature dynamics
2.6 Calculation of the switching time

Figure 2.7: From an arbitrary initial value of 285 K, the product temperature at the top of the bulk converges close to its optimal value of 280 K. The offset is 0.2 K.

is described by

\[ V_a \rho_a c_a \frac{dT_a(t)}{dt} = \rho_p V_p(T_p(t)) \alpha - \rho_a c_a \Phi(t)(T_a(t) - T_c), \tag{2.53} \]

with \( V_a \) the total air volume in the shaft and bulk, \( V_b = (1 - \gamma)A_f L \) the total product volume, and \( \alpha \) the heat production of the products. The first r.h.s. term denotes the heat production of the products, and the second r.h.s. term denotes the heat absorption of the heat exchanger. A solution strategy similar to the one used in equations (2.49)–(2.52) results in

\[ \tau_{\text{opt,lump}} = \tau_f \frac{c_1 - c_{22}(T_{a,\text{opt}} - T_{c,2})}{c_{21}(T_{a,\text{opt}} - T_{c,1}) - c_{22}(T_{a,\text{opt}} - T_{c,2})}, \tag{2.54} \]

with \( c_1 = \frac{a \rho_p V_p T_{p,\text{opt}}}{V_a \rho_a c_a} \), \( c_{21} = \frac{\rho_a \Phi_1 \alpha}{V_a \rho_a} \), and \( c_{22} = \frac{\rho_a \Phi_2 \alpha}{V_a \rho_a} \). Without the spatial variabilities, \( T_{a,\text{opt}} = T_{p,\text{opt}} \). We choose \( T_{c,1} = 275 K \), \( T_{c,2} = 285 K \), \( v_1 = 0.2 \text{ m/s} \), and \( v_2 = 0.02 \text{ m/s} \), and compute \( \tau_{\text{opt}} = 86 \text{ s} \). Using the lumped model, we get \( \tau_{\text{opt,lump}} = 97 \text{ s} \), which results in \( T_p(L,t) = 279.5 K \) when repeating the simulation experiment in Figure 2.7 with \( \tau_{\text{opt,lump}} \). This is only a small offset, which is typical also for other choices of \( \Phi \). Therefore, the lumped model approach seems adequate for feedback controller design (chapter 4). For systems design purposes however, we note that information
2 Analytic control law for a food storage room

on the uniformity inside the bulk is important and worth modelling, see also chapter 3.

2.7 Conclusions

We derived a basic model of a storage room, containing the most essential physical properties. The resulting system equations are first validated experimentally, and then mathematically simplified using timescale decomposition, discrete switching input, and Padé approximations of transfer functions. The transfer functions of the simplified system give a good indication of the timescales, and also show how the fast states are coupled to the slow ones. These properties are supported by analysis and simulation of the nominal system.

An open loop control law that consists of the optimal switching time, is derived. The expression of the switching time contains all the physical parameters of the system explicitly. The parametric expressions of the dynamics give direct information about the sensitivities to different parameters. In chapter 3 this property is used for integrated control and design.

The mathematical techniques reduce the complexity of the model, but the first order dynamics are maintained. The result is that the optimal switching time accurately drives the product temperature at the top of the bulk to its optimal value. Also, the settle time of the bulk temperature is predicted accurately.

2.8 Appendix

2.8.1 Construction of an irrational transfer function and its Padé approximation

A transfer function of a linear pde in the variables $x$ and $t$ is constructed by substituting $\partial / \partial t = s$ in the Laplace domain, and solving the ode for the variable $x$. The solution can be written as $\hat{y}(s, x) = G(s, x)\hat{u}(s, x)$, with $\hat{y}(s, x), \hat{u}(s, x)$ and $G(s, x)$ the output, input and the transfer function, respectively.

If $G(s, x)$ is of the form

$$\frac{a_0 s^0 + \ldots + a_n s^n}{b_0 s^0 + \ldots + b_m s^m},$$

i.e. rational, then for a fixed $x$ the transfer function can be transformed back into a linear ode in the time domain. If not, the nonrational transfer function can be approximated by a rational one, for example by a Padé
approximation. A Padé[0,1] approximation of \( G(s, x) \) in \( s = 0 \) is of the form

\[
\tilde{G}(s, x) = \frac{1}{a_1(x) + a_2(x)s}, \tag{2.56}
\]

where the coefficients \( a_1(x) \) and \( a_2(x) \) are determined by setting

\[
G(0, x) = \tilde{G}(0, x) \quad \text{and} \quad \frac{\partial G}{\partial s}(0, x) = \frac{\partial \tilde{G}}{\partial s}(0, x).
\]

A clever choice of the orders \( n \) and \( m \) in a Padé\([n, m]\) approximation is made by observation of the Bode plot of the original transfer function \( G(s) \).

### 2.8.2 Notation

- \( \Phi \) air flow through shaft \((m^3/s)\)
- \( \alpha \) cooling effectiveness \((K)\)
- \( \alpha_{th} \) thermal diffusivity of air \((1.87 \times 10^{-5} \text{ m}^2/\text{s})\)
- \( \gamma \) porosity \((m^3/m^3)\)
- \( \lambda_a \) conduction of air \((2.43 \times 10^{-2} \text{ W/m K})\)
- \( \lambda_p \) conduction of product \((\text{W/m K})\)
- \( \nu \) kinematic viscosity of air \((1.35 \times 10^{-5} \text{ m}^2/\text{s})\)
- \( \rho_a \) air density \((1.27 \text{ kg/m}^3)\)
- \( \rho_p \) produce density \((\text{kg/m}^3)\)
- \( \tau \) switching time \((\text{s})\)
- \( \tau_f \) length of switching interval \((\text{s})\)
- \( A_f \) floor area of the bulk \((\text{m}^2)\)
- \( A_{ps} \) produce surface per bulk volume \((\text{m}^2/\text{m}^3)\)
- \( \text{Bi} \) Biot number \(\frac{2hR}{\lambda_p}\)
- \( L \) bulk height \((\text{m})\)
- \( L_2 \) \( R\gamma/(1 - \gamma) \), char. length \((\text{m})\)
- \( M_1 \) \( \frac{\lambda_p}{\rho_p c_p} \) \((\text{m}^2/\text{s})\)
- \( M_2 \) \( \frac{\alpha}{c_p} \) \((1/\text{s})\)
- \( M_3 \) \( \sqrt{M_2/M_1 R} \) \((\text{dimensionless})\)
- \( M_4 \) \( hA_{ps} R \) \((1/\text{s})\)
- \( M_5 \) \( \frac{\sqrt{L_2}}{\nu} \) \((\text{dimensionless})\)
- \( \text{Nu} \) Nusselt number \(\frac{2hR}{\lambda_a}\)
- \( \text{Pr} \) Prandtl number \(\frac{\nu}{\alpha_{th}}\)
- \( R \) product radius \((\text{m})\)
- \( \text{Re} \) Reynolds number \(\frac{vL_2}{\nu} \), see [110]
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$T_a$  air temperature in the bulk ($K$)
$T_c$  cooling element temperature ($K$)
$T_{ini}$  initial temperature ($K$)
$T_p$  produce temperature ($K$)
$V$  volume of shaft ($m^3$)
$a$  product heat production ($J/kg s K$)
$b$  product heat production ($J/kg s$)
$c_a$  heat capacity of air ($2 \times 10^3 J/kg K$)
$c_p$  heat capacity of produce ($J/kg K$)
$h$  heat transfer coefficient ($W/m^2 K$)
$v$  air velocity inside the bulk ($m/s$)
3

Integrated control and design of a bulk storage room

3.1 Introduction

As in chapter 2, we consider a bulk storage room for harvested foods. In order to maintain product quality, the product temperature is controlled. In general, temperature control is done in two ways; ventilation with outside air, or by means of a heat exchanger. In both situations, a fan enforces the air circulation. Cold air flows (usually upwards) through the bulk, and inside the bulk, the air warms up. Consequently, the products at the top will be warmer than those at the bottom [74]. A larger airflow will decrease these spatial variations, but will be costly, and hence there is a clear tradeoff between temperature uniformity and energy consumption. This makes the design of a storage room a nontrivial task. The design of a model-based feedback controller is also a nontrivial task, since a standard model that describes the time- and spatially dependent temperatures will consist of a set of nonlinear partial differential equations. A further complication is that systems design is strongly correlated to controller design. The controller adds dynamics to the system, causing it to behave differently than the uncontrolled system. It is therefore desirable to design and analyze the system and controller simultaneously, instead of separately.

There is extensive literature available on the modelling aspects of bulk storage rooms, see for example [58, 66, 102, 103, 109, 110]. The main goal in these papers is to gain insight in the dynamics of temperature, humidity, and product quality. There is a considerable amount of literature concerning control of nonlinear distributed parameter systems with applications to chemical and process engineering, for example [2, 27] and the review article of [17]. However, the literature on model-based control design for bulk stored food products is limited. In [44] and [103], model predictive control (MPC) algorithms were used for the temperature and humidity control of a bulk storage room with outside air ventilation, using real weather conditions.
Both proposed algorithms are model based and were tested by simulation studies. Additionally, in [59] the resulting controller was applied to a real life plant. The aim of the algorithms is to drive temperature and humidity to a desired set point at low economic costs while obeying the input and state constraints. Due to the relatively high complexity of the models, model-based control design requires computer simulations that are (very) time consuming. Furthermore, the controller has to be tested by numerical simulations for each different system configuration. Integrated system and controller design would therefore be very time consuming. To our best knowledge, the integrated modelling and control design with respect to food storage is reported only in [57], where an optimal control strategy is developed using three different kinds of input. However, in mechanical engineering integrated modelling and design is a common field, for example in [40, 69, 71, 84]. The performance of the controlled system is either measured indirectly by inspection of poles/zeros of the closed loop system, or directly by simulation studies.

This chapter is organized as follows. In section 3.2 a pde model of a bulk storage room of [95] is extended by heat loss through the walls, to make it more realistic. The control input for this model is the air flux that is generated by the fan, and the temperature of the cooling device. The control input switches regularly, which corresponds to a cooling system that is switched on and off on a regular basis. The pde model is approximated by a bilinear state space model. For this model, an open loop controller, that consists of the optimal switching times, is derived. In section 3.3 the cost criteria for the controller are defined, and the corresponding expressions are derived. In section 3.4 the relations that could not be found in the literature, are identified experimentally. These relations describe the energy usage of the fan, and the effectiveness of the heat exchanger, both as functions of the airflow. In section 3.5 a design analysis is conducted for an undisturbed test case model, and the tradeoffs of different design choices are discussed.

In section 3.6 the model is linearized and brought into linear state space form, with the switching time as an input, and the outside temperature as a disturbance. This form is suitable for linear controller design, where the switching time is adjusted by feedback, a method that was already proposed in [99] and [98] for a more basic model. For our model, it is shown that the cost criteria can be represented by a cost function that is suitable for linear optimal control. The cost function is a parametric function of all the physical system properties. For the design procedure of a controlled storage room, also known as integrated control and design, this can reduce computational costs considerably. In section 3.7 conclusions are drawn.
3.2 The model

3.2.1 Bulk storage room model

Our starting point is the (approximated) model of a temperature controlled bulk storage room, that was proposed in chapter 2. To make the model more realistic, an extra term for heat transport through the walls is added here. The storage room model is divided into two parts, namely the shaft and the bulk, see Figure 3.1. Similar to the basic model in chapter 2, the temperature model is

\[
\frac{\partial T_a(x,t)}{\partial t} = -v(t) \frac{\partial T_a(x,t)}{\partial x} + M_4(T_p(x,t) - T_a(x,t)) \quad (3.2)
\]

\[
T_a(0,t) = T_0(t) \quad (3.3)
\]

\[
A_p T_p(x,t) + B_p T_a(x,t). \quad (3.4)
\]

Equations (3.1)–(3.4) are referred to as the nominal model. We will only discuss the last r.h.s. term in equation (3.1). This describes the heat transport through the outside walls, with \( A_w \) the wall surface at the inside, \( \lambda_w \) the heat conduction coefficient, \( d_w \) the wall thickness, and \( T_{out}(t) \) the temperature outside. Notice that this term assumes a steady temperature profile.
inside the walls. This is accurate only if $T_{\text{out}}$ has very slow dynamics, which we assume is the case. According to Fourier’s law, heat conduction equals $\lambda \frac{dT}{dx} (J/sm^2)$, which is discretized as $\lambda \frac{T_{\text{out}} - T_a}{d_w}$. It is implicitly assumed that the surface temperatures of the walls equal the adjacent air temperatures. Moreover, there is only heat transport through the walls of the shaft and the floor. There is no heat transfer between the bulk and the shaft.

### 3.2.2 Open loop control

In this section, a state space model is derived for the temperature dynamics of the products in the top of the bulk. This is done by approximating model (3.1)–(3.4). In chapter 2 an open loop controller was derived, which consists of the optimal switching time that drives $T_p(L, t)$ to its optimal value $T_{p,\text{opt}}$. The same procedure is followed here, with the difference that there are now three inputs in the system instead of two, namely $T_c(t)$, $\Phi(t)$, and $T_{\text{out}}(t)$. This makes the computations somewhat more involved. Three model simplifications are made. First, the inputs $T_c(t)$ and $\Phi(t)$ take discrete values $(T_{c,1}, T_{c,2})$ and $(\Phi_1, \Phi_2)$. The input is switched between the discrete input pairs $(T_{c,1}, \Phi_1)$ and $(T_{c,2}, \Phi_2)$. Secondly, the input is switched once in every (fixed) time interval of the order of ten minutes. Third, a timescale separation is made by neglecting the fast temperature dynamics of the air. The resulting model on a fixed time interval becomes

$$0 = -\rho_a c_a \Phi(\Phi) \left( T_a(L, t) - T_c(t) \right) + \rho_a c_a T_a(L, t) +$$

$$-\rho_a c_a T_0(t) + \frac{A_w}{\rho_a} \left( T_{\text{out}}(t) - T_0(t) \right)$$

(3.5)

$$0 = -v \frac{\partial T_a(x, t)}{\partial x} + M_4(T_p(x, t) - T_a(x, t))$$

(3.6)

$$T_a(0, t) = T_0(t)$$

(3.7)

$$\frac{\partial T_p(x, t)}{\partial t} = A_p T_p(x, t) + B_p T_a(x, t).$$

(3.8)

Rearranging (3.5) gives

$$T_0(t) = c_1 T_a(L, t) + c_2 T_c(t) + c_3 T_{\text{out}}(t),$$

(3.9)

with $c_1 = \frac{1-\alpha(\Phi)}{\alpha_{\text{out}}+1}$, $c_2 = \frac{\alpha(\Phi)}{\alpha_{\text{out}}+1}$, and $c_3 = \frac{\alpha_{\text{out}}}{\alpha_{\text{out}}+1}$. Here, $\alpha_{\text{out}} = \frac{\lambda_{\text{out}} A_w}{\rho_a c_a \Phi}$. After a transformation into the Laplace domain, the solution of (3.7) equals

$$\hat{T}_p(L, s) = \frac{B_p}{-A_p + s} \hat{T}_a(L, s).$$

(3.10)
3.2 The model

Using this and solving (3.6) with boundary condition (3.7) and (3.8) gives the transfer functions from $T_c$ and $T_{out}$ to $T_a(L)$. So

$$
\hat{T}_a(L, s) = \hat{T}_0(s) \exp \left( M_5 \left( \frac{B_p}{-A_p + s} - 1 \right) \right)
= \left( c_1 \hat{T}_a(L, s) + c_2 \hat{T}_c(s) + c_3 \hat{T}_{out}(s) \right) \cdot \exp \left( M_5 \left( \frac{B_p}{-A_p + s} - 1 \right) \right),
$$

(3.11)

and thus

$$
\hat{T}_a(L, s) = \frac{\exp \left( M_5 \left( \frac{B_p}{-A_p + s} - 1 \right) \right)}{1 - c_1 \exp \left( M_5 \left( \frac{B_p}{-A_p + s} - 1 \right) \right)} \cdot (c_2 \hat{T}_c(s) + c_3 \hat{T}_{out}(s)),
$$

(3.12)

with $M_5 = \frac{M_L}{v}$. A Laplace transformation of equation (3.8) gives

$$
\hat{T}_p(L, s) = \frac{B_p}{-A_p + s} \hat{T}_a(L, s)
= \frac{B_p}{-A_p + s} \left( \frac{\exp \left( M_5 \left( \frac{B_p}{-A_p + s} - 1 \right) \right)}{1 - c_1 \exp \left( M_5 \left( \frac{B_p}{-A_p + s} - 1 \right) \right)} \right) \cdot (c_2 \hat{T}_c(s) + c_3 \hat{T}_{out}(s)).
$$

(3.13)

A suitable transfer function approximation method is the Padé method, which approximates the first order dynamics of the transfer functions very well. The resulting transfer function is rational, and therefore allows for a transformation back into the time domain. A first order Padé approximation of (3.13) gives

$$
\hat{T}_p(L, s) \approx \frac{B^*}{-A^* + s} \left( c_2 \hat{T}_c(s) + c_3 \hat{T}_{out}(s) \right),
$$

(3.14)

where

$$
A^* = \frac{A_p^2 \left( 1 - \exp \left( M_5 \left( \frac{B_p}{-A_p} - 1 \right) \right) \right)}{-M_5 B_p + A_p - A_p c_1 \exp \left( M_5 \left( \frac{B_p}{-A_p} - 1 \right) \right) - M_5 B_p + A_p - A_p c_1 \exp \left( M_5 \left( \frac{B_p}{-A_p} - 1 \right) \right)},
$$

$$
B^* = \frac{A_p B_p \exp \left( M_5 \left( \frac{B_p}{-A_p} - 1 \right) \right)}{-M_5 B_p + A_p - A_p c_1 \exp \left( M_5 \left( \frac{B_p}{-A_p} - 1 \right) \right) - M_5 B_p + A_p - A_p c_1 \exp \left( M_5 \left( \frac{B_p}{-A_p} - 1 \right) \right)}.
$$
3 Integrated control and design of a bulk storage room

Transformation of the first order approximation into the time domain leads to the state space model

$$\frac{dT_p(L, t)}{dt} = A^*T_p(L, t) + B^*(c_2T_c(t) + c_3T_{out}(t)). \quad (3.15)$$

We assumed that on a time interval \((0, \tau_f)\) there is only one switching time \(\tau\) at which the discrete inputs are switched. On the time interval \((0, \tau)\), (3.15) is approximated with a forward Euler step, which gives

$$T_p(L, \tau) = T_p(L, 0) + \tau \left( A_1^*T_p(L, 0) + B_1^*(c_{2,1}T_{c,1} + c_{3,1}T_{out}) \right). \quad (3.16)$$

The subscripts in \(A^*, B^*, c_2, c_3\) denote the relations with the inputs \(\Phi_1\) and \(\Phi_2\). Since we assumed that the dynamics of \(T_{out}\) is slow, \(T_{out}\) is assumed to be at a constant average value during \((0, \tau_f)\). A backward Euler step of magnitude of \(\tau_f - \tau\) gives

$$T_p(L, \tau_f) = T_p(L, \tau) + (\tau_f - \tau) \left( A_2^*T_p(L, \tau_f) + B_2^*(c_{2,2}T_{c,2} + c_{3,2}T_{out}) \right). \quad (3.17)$$

This is a valid approximation, provided that \(A^*\tau_f \ll 1\) [95]. Suppose that \(T_p(L, 0) = T_{p,opt}\). We look for the switching time \(\tau\) that steers \(T_p(L, t)\) back to the optimal value \(T_{p,opt}\) during the time interval \((0, \tau_f)\). This gives the following conditions on (3.16)–(3.17)

$$T_p(L, 0) = T_p(L, \tau_f) = T_{p,opt}. \quad (3.18)$$

From (3.16)–(3.18), the algebraic solution \(\tau_{opt}\) is found by simple calculus

$$\tau_{opt} = \frac{-\tau_f \left( A_2^*T_{p,opt} + B_2^*(c_{1,2}T_{c,2} + c_{3,2}T_{out}) \right)}{(A_1^* - A_2^*)T_{p,opt} + B_1^*(c_{1,1}T_{c,1} + c_{3,1}T_{out}) - B_2^*(c_{1,2}T_{c,2} + c_{3,2}T_{out})}. \quad (3.19)$$

This is an open loop controller based on the steady state of the system. Later on, the values \(T_{p,opt}\) and \(\tau_{opt}\) will be used as linearization points for the development of a linear feedback controller. To test the open loop controller that consists of the input \(\tau_{opt}\), we simulated the model (3.5)–(3.8). In this simulation, \(\Phi\) is switched between \(\Phi_1\) and \(\Phi_2\), and \(T_{c,1} = T_{c,2}\). Equation (3.2) was discretised with an upwind discretisation in the \(x\)-direction, with \(n = 25\) discretisation points, which is equivalent to 25 bulk layers. The trajectories \(T_p, T_a,\) and \(T_0\) in (3.1)–(3.4) were simulated within the Matlab Simulink environment, using the ode45 Prince-Dormand algorithm. We considered a bulk that has an initial uniform temperature of 285 K, which represents the bulk temperature right after storage. The rest of the parameters is chosen as in section 3.5 and is listed in Table 3.1. At \(t = 0\) the switching control is
applied with $T_{p,\text{opt}} = 280 \, K$. Figure 3.2 shows that the product temperature $T_p(L, t)$ converges closely to $T_{p,\text{opt}}$. The offset is smaller than $0.1 \, K$. Not only does the switching control keep $T_p(L, t)$ near its optimal value, it also drives it to $T_{p,\text{opt}}$ from an *arbitrary* initial value. Initially, the air is warmed up by the bulk, and as a consequence $T_p(L, t)$ starts to rise. The same results were reported in [95]. In [98] it was proven that for realistic parameter values this system is stable. There is also an intuitive explanation to this phenomenon: the cooling element acts as a stabilizing control mechanism, since equation (3.5) implies that a higher $T_a(L, t)$ is cooled down more, and a lower $T_a(L, t)$ is cooled down less.

![Figure 3.2: Typical trajectory of $T_p(L, t)$.](image)

### 3.2.3 The dynamics of $T_p(0, t)$

The relation between $\tau$ and $T_p(L, t)$ in steady state is given implicitly by (3.16)–(3.18). The dynamics of $T_p(L, t)$ is given by (3.15). In addition to this, we look for the dynamics of $T_p(0, t)$ and the relation between $\tau$ and $T_p(0, t)$ in steady state. These properties will be used later on for the development of an optimal feedback controller and a cost function. We start with the dynamics of $T_p(0, t)$. The derivation is similar to the one in section 3.2.2.

From the Laplace transformed equations (3.9) and (3.12) it follows that

$$
\hat{T}_0(s) = \frac{c_2 \hat{T}_c(s) + c_3 \hat{T}_\text{out}(s)}{1 - c_1 \exp \left( M_5 \left( \frac{B_p + A_p - s}{-A_p + s} \right) \right)},
$$

(3.20)
3 Integrated control and design of a bulk storage room

and after a Laplace transformation of equation (3.8) this gives

\[ \hat{T}_p(0, s) = \left( \frac{B_p}{-A_p + s} \right) \frac{c_2 \hat{T}_c(s) + c_3 \hat{T}_{out}(s)}{1 - c_1 \exp \left( M_5 \left( \frac{B_p + A_p - s}{-A_p + s} \right) \right)}. \]  

(3.21)

The dynamics of the product temperatures are mostly of first order in a switching time interval. Therefore, this transfer function is approximated with a first order Padé approximation to

\[ \hat{T}_p(0, s) \approx \frac{B^*}{-A^* + s} (c_2 \hat{T}_c(s) + c_3 \hat{T}_{out}(s)), \]  

(3.22)

with

\[ A^*_0 = \frac{A^2(1 - c_1 \exp \left( M_5 \left( \frac{B_p}{-A_p} - 1 \right) \right))}{-M_5 B_p c_1 \exp \left( M_5 \left( \frac{B_p}{-A_p} - 1 \right) \right) + A_p - A_p c_1 \exp \left( M_5 \left( \frac{B_p}{-A_p} - 1 \right) \right)}, \]

\[ B^*_0 = \frac{A_p B_p}{-M_5 B_p c_1 \exp \left( M_5 \left( \frac{B_p}{-A_p} - 1 \right) \right) + A_p - A_p c_1 \exp \left( M_5 \left( \frac{B_p}{-A_p} - 1 \right) \right)}. \]

Analogous to (3.15), a transformation from the Laplace domain to the time domain leads to

\[ \frac{dT_p(0, t)}{dt} = A^*_0 T_p(0, t) + B^*_0 (c_2 T_c(t) + c_3 T_{out}(t)). \]  

(3.23)

Again, we used that \( T_{out} \) is assumed to be constant during \((0, \tau_f)\). Following the approximation from (3.16)–(3.17), this leads to the steady state value

\[ T_p(0, t) \approx \frac{\tau B^*_{0,1}(c_{2,1} T_{c,1} + c_{3,1} T_{out}) + (\tau_f - \tau) B^*_{0,2}(c_{2,2} T_{c,2} + c_{3,2} T_{out})}{\tau A^*_{0,1} + (\tau_f - \tau) A^*_{0,2}}. \]  

(3.24)

The prediction of (3.24), with \( \tau = \tau_{opt} \), is tested by simulations of the nominal system (3.1)–(3.4), and the offset is of the same order as for \( T_p(L, t) \).

3.3 Cost criteria for feedback control

We define three cost criteria, (i) the deviation of \( T_p(L, t) \) from the set point \( T_{p, opt} \), (ii) the temperature differences between the products inside the bulk,

\[ \Delta T_p = T_p(L, t) - T_p(0, t), \]  

(3.25)

and (iii) the energy use of the fan and the heat exchanger. The expressions for the energy use are explained in the next subsection.
3.3 Cost criteria for feedback control

3.3.1 Energy costs of the fan and heat exchanger

The total energy use (in W or J/s) is defined as

$$E = E_f + E_h,$$  \hspace{1cm} (3.26)

where $E_f$ and $E_h$ are the average energy use of the fan and the heat exchanger respectively. The average is taken on the time interval $(0, \tau_f)$. The average energy demand of the fan is given by

$$E_f = \frac{\tau E_f(\Phi_1) + (\tau_f - \tau)E_f(\Phi_2)}{\tau_f}. \hspace{1cm} (3.27)$$

The relation between $E_f$ and $\Phi$ is determined experimentally in section 3.4. The average energy demand of the heat exchanger, $E_h$, was not measurable, and since a straightforward model was not found in the literature, the heat exchanger is assumed to have a constant yield of $y = 50\%$. To derive an expression for $E_h$, we need an expression for the average air temperature near the heat exchanger, $T_a(L, t)$. We assumed that $T_a(L, t)$ is approximately constant during a time interval. The average value of $T_a(L, t)$ on $(0, \tau_f)$ is

$$T_a(L) = -\frac{\tilde{B}_1(c_2,1T_{c,1} + c_3,1T_{out}) + (\tau_f - \tau)\tilde{B}_2(c_2,2T_{c,2} + c_3,2T_{out})}{\tau A_1 + (\tau_f - \tau)\tilde{A}_2}, \hspace{1cm} (3.28)$$

where

$$\tilde{A} = -\frac{c_1 A_p^2 \exp\left(\frac{B_p}{-A_p} - 1\right)}{M_5 B_p} - A_p^2, \quad \tilde{B} = -\frac{A_p^2 \exp\left(\frac{B_p}{-A_p} - 1\right)}{M_5 B_p},$$

and this is derived in a similar way as (3.24). The approximation criterion $\tilde{A} \ll 1$ is satisfied. The energy absorbed by the heat exchanger is $\rho_a c_a \Phi_1(\alpha(T_a(L) - T_c))$, see (3.1). Using a fixed yield $y$, the average energy usage in the time interval $(0, \tau_f)$ is

$$E_h = \frac{1}{y} \frac{\tau E_h(T_{c,1}) + (\tau_f - \tau)E_h(T_{c,2})}{\tau_f} = \frac{1}{y\tau_f} \left(\tau \rho_a c_a \Phi_1\alpha_1(T_a(L) - T_{c,1}) + (\tau_f - \tau)\rho_a c_a \Phi_2\alpha_2(T_a(L) - T_{c,2})\right). \hspace{1cm} (3.29)$$

The relations between $\alpha$ and $\Phi$, and between $E_f$ and $\Phi$, are still unknown. These will be identified experimentally in the next section.
3 Integrated control and design of a bulk storage room

3.4 Model identification by experiment

The model and performance criteria are still incomplete at three points. First, no model or product specifications were found that straightforwardly describe the pressure differences over a fan as a function of its variable energy usage. Second, the pressure drop over the fan equals the sum of the pressure drops over the different flow channels, such as the fan shaft and the bulk. Although the Bernoulli equations relate the pressure drop to the airflow rate, they are practically unfeasible with irregular obstacles in a standard storage room, like for example T-bar floors. Third, a model for $\alpha(\Phi)$ has not been found in the literature. The above relations were therefore determined by experiment. The experimental setup consisted of a closed storage container with inner dimensions $3.6 \times 2.9 \times 2.1 \, m$, containing a fan and a heat exchanger, see Figure 3.3. The fan in the experimental storage room was placed after the

![Figure 3.3: Schematic view of the storage room with 1 the heat exchanger, 2 the ventilator, 3 the shaft, 4 the space under the floor, and 5 the bulk space.](image)

heat exchanger, instead of before like in Figure 3.1, but this does not alter the nominal model. There was no bulk placed in the storage room. The fan was of type ITHO K315LTW, and the heat exchanger was of type Helpman LEX 22E. The data were measured with Campbell CR10 data loggers, the airflows were measured with a Lambrecht 642 anemometer, the pressures with a Micaflex MF-DP meter, and the temperatures with a Rotronic thermometer. All sensors were calibrated. The relations were determined by measuring for five fan stages, each one creating a different flux $\Phi$. For each stage, the online measurements were carried out with an interval time of one minute, for about half an hour.
3.4 Model identification by experiment

3.4.1 Identification of fan energy demands

In this subsection, we identify the function $E(\Phi)$, as needed in equation (3.29), in two steps. Using the least squares method, a good fit between fan energy demand and pressure drop over the fan is

$$E_f(\Phi) = 61.80 + 23.36\Delta P(\Phi) - 1.05\Delta P(\Phi)^2 + 0.02\Delta P(\Phi)^3,$$

(3.30)

for $\Delta P \in [5 \text{ Pa}, 25 \text{ Pa}]$, which is the pressure range of the fan. Figure 3.4 shows that equation (3.30) shows a good fit with the experimental data. The pressure drop in the empty storage room was fitted to the air flow rate as

$$\Delta P_{\text{empty}} = -11.66 + 94.86\Phi.$$  

(3.31)

Figure 3.5 shows that this curve has a good fit with the experimental data for $\Phi \in [0.1 \text{ m}^3/\text{s}, 0.34 \text{ m}^3/\text{s}]$, the flux range of the fan. The relation between the pressure drop $\Delta P$ over a storage room with a bulk, and the air flux is

$$\Delta P = \Delta P_{\text{bulk}} + \Delta P_{\text{empty}},$$

(3.32)

with $\Delta P_{\text{bulk}}$ and $\Delta P_{\text{empty}}$ the pressure drops over the bulk and over the empty storage room (without a bulk), respectively. The pressure drop over a bulk of spherical products is described by the Ergun formula (see [30])

$$\Delta P_{\text{bulk}} = L \left( 150 \frac{(1-\gamma)^2 \mu A_f}{\gamma^4 (2R)^2} \Phi + 1.75 \frac{(1-\gamma) \rho_a A_f}{\gamma^4 2R} \Phi^2 \right),$$

(3.33)
where we have used that \( v = \frac{\Phi}{A_f} \gamma \). Here, \( R \) is the product radius, and \( \mu \) the dynamic viscosity of the air. Altogether, equations (3.30)–(3.33) describe the relation between \( \Phi \) and \( E_f \). In the next subsection we identify the function \( \alpha(\Phi) \) that is needed for the prediction of the air and product temperatures.

### 3.4.2 Relation between \( \alpha \) and \( \Phi \)

For six different stages of the fan, the air temperatures were measured right before and right after the heat exchanger, together with the corresponding air fluxes and \( T_c \). \( T_c \) is chosen as the mean of the measured surface temperatures of the cooling element at the locations of the incoming and outgoing cooling liquid. In steady state, without the influence of \( T_{out} \), (3.5) becomes

\[
0 = (1 - \alpha(\Phi))T_a(L) - T_0 + \alpha(\Phi)T_c.
\]  

(3.34)

For each fan stage, \( \alpha(\Phi) \) is derived by inserting the average temperatures and flux. A least squares fit resulted in

\[
\alpha(\Phi) = -0.95\Phi + 1.04
\]  

(3.35)

for \( \Phi \in [0.1 \ m^3/s, \ 0.34 \ m^3/s] \). \( \alpha \) decreases strongly with \( \Phi \), indicating that when \( \Phi \) is increased, the energy absorption of the heat exchanger, given by \( \rho_a c_a \Phi \alpha(\Phi) \left(T_a(L,t) - T_c(t)\right) \), increases only little. Figure 3.6 shows that the fit in (3.35) is accurate.
3.5 Integrated control and design for an undisturbed system

In this section we analyze the sensitivities of the costs to different design parameters. For simplicity, we assume that the system is in steady state, and that there are no disturbances. This means that \( \tau = \tau_{\text{opt}} \) at all time intervals. This in turns implies that the temperature dynamics are equal within each time interval \((0, \tau_f)\), and hence the costs have to be evaluated on only one time interval.

The two costs are the energy costs and the bulk uniformity. For better understanding of the tradeoffs, they are analyzed separately. The design parameters are chosen as \( L, \Phi, T_{\text{out}} \) and \( T_c \). The sensitivities of the costs to these parameters is determined by simple parameter variation of a nominal system. The parameter values are listed in Table 3.1. In this table, specific data for potato are taken from [58,110]. The value of \( \lambda_w \) corresponds to brick.

The fan is switched on and off, and for the off-stage we choose \( \Phi_2 = 0 \), and \( E_f = 0 \), which corresponds to natural convection. This does not cost any fan energy. Further, \( T_c \) is assumed to be constant throughout, so \( T_{c,1} = T_{c,2} \), that is, no switching of the temperature of the cooling device. The bulk volume, \( A_f \times L \), is constant, so a higher bulk implies a smaller bulk floor area. The conditions on the design parameters are that \( \Phi_1 \) and \( \Delta P \) must lie inside or very close to the experimental ranges \( 0.1 \ m^3/s \leq \Phi_1 \leq 0.34 \ m^3/s \),

Figure 3.6: Experimental data and fitted curve of \( \alpha \) as a function of \( \Phi \).
3 Integrated control and design of a bulk storage room

\begin{table}[h]
\centering
\begin{tabular}{|l|l|}
\hline
$T_{p,\text{opt}}$ & 280 K  \\
$\lambda_w$ & 0.15 W/m K  \\
$\Phi_1$ & 0.25 m$^3$/s  \\
$L$ & 4 m  \\
$R$ & 2.5 $10^{-2}$ m  \\
$\lambda_p$ & 0.55 W/m K  \\
$a$ & 3.1 $10^{-5}$ W/kg K  \\
$\gamma$ & 0.3 m$^3$/m$^3$  \\
$T_c$ & 274 K  \\
$A_w$ & 10 m$^2$  \\
\hline
$T_{\text{out}}$ & 288 K  \\
$d_w$ & 0.1 m  \\
$\Phi_2$ & 0.02 m$^3$/s  \\
$A_f \times L$ & 400 m$^3$  \\
$V$ & 10 m$^3$  \\
$\rho_p$ & 1014 kg/m$^3$  \\
$A_{ps}$ & 40 m$^2$/m$^3$  \\
$c_p$ & 3.6 $10^3$ J/kg K  \\
$c_l$ & 2 $10^3$ J/kg K  \\
$\tau_f$ & 1.2 $10^3$ s  \\
\hline
\end{tabular}
\caption{Nominal parameter values for a bulk with potatoes.}
\end{table}

and $6 \text{ Pa} \leq \Delta P \leq 25 \text{ Pa}$.

Next, we analyze the sensitivities of the energy costs and the uniformity.

### 3.5.1 Energy costs

The total energy costs by the fan and the heat exchanger are defined here as $E$. The slopes of the graphs in Figure 3.7 show that $E$ depends very strongly on $T_{\text{out}}$, strongly on $\Phi_1$ and $T_c$, and weakly on $L$. The energy costs decrease with a more powerful fan. The high values of $L$ are not very realistic, but are used here for analysis.

### 3.5.2 Temperature uniformity

The slopes of the graphs in Figure 3.8 show that the uniformity depends strongly on all design parameters, except on $L$. This is explained by the fact that the controller keeps $T_p(L)$ at a fixed value, independent of the parameter choices, and that $T_p(0)$ depends completely on $T_0$, which in turns depends largely on $T_c$ and $T_{\text{out}}$, and not on $L$.

### 3.5.3 Discussion

The sensitivity of the system performance to parameter variations was evaluated numerically for a nominal system. The performance analysis showed the following tradeoffs. The total energy costs are minimized by a low temperature of the cooling device, but the temperature difference over the bulk is minimized by a high temperature of the cooling device. Further, the temperature difference and energy costs are decreased by a higher $\Phi_1$, caused by a more powerful fan. However, such a fan will be more expensive in purchase. In practice a higher air velocity could influence the condensation rate inside
3.5 Integrated control and design for an undisturbed system

Figure 3.7: The sensitivity of $E$ to the design parameters $L$, $\Phi_1$, $T_{out}$, and $T_c$.

the heat exchanger, and therefore negatively influence the relative humidity. Also, a higher flux could reduce poorly ventilated regions inside the bulk. These features are both not incorporated in the model. The energy usage is decreased by a lower bulk. The tradeoff is that for a fixed bulk volume, a lower bulk means a larger floor area, which is usually more expensive than a higher roof. Variation of $T_{out}$ shows that a low value of $\lambda_w \frac{T_{out} - T_0}{d_w}$, or equivalently, a thick or good isolating wall, can be costly but saves the most energy.

The modelling of the system and the design criteria was done analytically. However, since not all expressions were found in literature, the missing relations were experimentally identified. Despite that no mathematical expression could be found for the energy costs of the heat exchanger, the analysis results in realistic tradeoffs. For optimization purposes one should keep in mind that the conclusions that are drawn here, are only valid on the exper-
3 Integrated control and design of a bulk storage room

Figure 3.8: The sensitivity of $\Delta T_p$ to the design parameters $L$, $\Phi_1$, $T_{out}$, and $T_c$.

3.6 Feedback control

In this section, a system is defined that describes the dynamics of $T_p(L, t)$ and $T_p(0, t)$. This system is approximated with a discrete system that has the switching time (per time interval) as input. We show that this system allows linear optimal control, and define a suitable cost criterion. This strategy was already proposed in [98], and we follow the same algorithm.

We start by assuming that $T_c(t)$ is constant, and we regard $T_{out}(t)$ as a disturbance. Equations (3.15) and (3.23) are written as the following system

$$\frac{dx(t)}{dt} = A(\Phi)x(t) + B(\Phi)T_c + F(\Phi)T_{out}(t)$$

$$y(t) = Cx(t),$$

(3.36)
3.6 Feedback control

with

\[ x(t) = \begin{pmatrix} T_p(L, t) \\ T_p(0, t) \end{pmatrix}, \quad y(t) = \begin{pmatrix} T_p(L, t) \\ T_p(0, t) \end{pmatrix}, \]
\[ A(\Phi) = \begin{pmatrix} A^* & 0 \\ 0 & A^*_0 \end{pmatrix}, \quad B(\Phi) = \begin{pmatrix} B^*c_2 \\ B^*_0c_2 \end{pmatrix}, \]
\[ C = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad F(\Phi) = \begin{pmatrix} B^*c_3 \\ B^*_0c_3 \end{pmatrix}. \]  

(3.37)

Here, \( T_{out} \) is a disturbance. The goal is to approximate (3.36) with a form that is suitable for controller design. This controller adjusts the value of \( \tau \) on each time interval. We start by approximating (3.36) with a forward Euler step (see [99] for details)

\[
\frac{x(\tau_f) - x(0)}{\tau_f} = A(\tau)x(0) + B(\tau)T_c + F(\tau)T_{out}(0)
\]
\[ y(\tau_f) = x(\tau_f), \]  

(3.38)

with

\[
A(\tau) = \frac{\tau}{\tau_f}A_1 + \frac{\tau_f - \tau}{\tau_f}(A_2 + \tau A_1 A_2)
\]
\[ B(\tau) = \frac{\tau}{\tau_f}B_1 + \frac{\tau_f - \tau}{\tau_f}(A_2 B_1 \tau + B_2)
\]
\[ F(\tau) = \frac{\tau}{\tau_f}F_1 + \frac{\tau_f - \tau}{\tau_f}(\tau A_2 F_1 + F_2). \]  

(3.39)

The subscripts denote the implicit relations of \( A^*, B^*, \) etc., with the inputs \( \Phi_1 \) and \( \Phi_2 \). Now we split the time axis in segments \([n\tau_f, (n+1)\tau_f]\), and apply on every segment the approximation (3.38). This leads to a discrete nonlinear system of the form

\[ x^{n+1} = f(x^n, \tau^n, T^{out}_n), \]  

(3.40)

where \( x^n = x(n\tau_f) \). To make linear controller design possible, the system (3.40) is linearized. Therefore, we introduce the following variables

\[
\tau^n = \tau_{opt} + \tau_{var}^n
\]
\[ x^n = x_{opt} + x_{var}^n
\]
\[ T^{out}_n = T^{opt}_n + T^{var,n}_{out}
\]
\[ y^n = y_{opt} + y_{var}^n. \]  

(3.41)

Hence, we linearize (3.40) in the point \((\tau_{opt}, x_{opt}, T^{opt}_{out})\). The value \( \tau_{opt} \) satisfies (3.16)–(3.17), such that \( T_p(L, n\tau_f) = T_{p, opt} \). The variable \( x_{2, opt} \) is chosen
3 Integrated control and design of a bulk storage room

as the steady state value corresponding with $\tau_{opt}$ and $T_{out}^{opt}$. Further, $T_{out}^{opt}$ has no constraints, and we note that $y_{opt} = x_{opt}$. This gives

$$0 = A(\tau_{opt})x_{opt} + B(\tau_{opt}) + F(\tau_{opt})T_{out}^{opt}$$

$$y_{opt} = x_{opt}.$$  \hspace{1cm} (3.42)

The linearization of (3.40) is then

$$x_{var}^{n+1} = A_dx_{var}^n + B_d\tau_{var}^n + D_dT_{out}^{var,n}$$

$$y_{var}^n = x_{var}^n,$$  \hspace{1cm} (3.43)

with the following matrices

$$A_d = (\tau_fA(\tau_{opt}) + 1)$$

$$B_d = \tau_f\left(\frac{dA(\tau_{opt})}{d\tau}x_{opt} + \frac{dB(\tau_{opt})}{d\tau}T_{c} + \frac{dF(\tau_{opt})}{d\tau}T_{out}^{opt}\right)$$

$$D_d = \tau_fF(\tau_{opt}).$$  \hspace{1cm} (3.44)

System (3.43) has a form that is suitable for linear optimal control. Now we are ready to define a quadratic cost criterion for discrete systems

$$J = \sum_{n=1}^{N} \beta_1(T_p(L,n\tau_f) - T_{p,opt})^2 + \beta_2(T_p(L,n\tau_f) - T_{p}(0,n\tau_f))^2 + \bar{E}(n\tau_f)dt.$$  \hspace{1cm} (3.45)

Since $\bar{E}$ corresponds to a polynomial in $\Phi$ of order six, the criterion is quadratic. This cost criterion represents a standard LQ problem, and can be written as $J = J_{var}(t) + J^*$, where $J^*$ are the static costs at steady state and $J_{var}(t)$ is dynamic. The expressions that describe $J$ contain all the physical properties of the system. When searching for optimal design of the controlled system by parameter variation, the cost function can be evaluated quickly. The numerical advantage especially comes forward when disturbances are small and the system dynamics is mostly at steady state, so one can assume that $J = J^*$. This case was examined in section 3.5.

3.7 Conclusions

In chapter 2 it is shown that for the storage room model an accurate open loop switching input controller can be designed via model approximation. In this chapter, it is shown for an extended version of this model, how a realistic cost function is derived, from analysis and experimental results. This cost function makes (i) linear optimal control possible, and (ii) gives insight into the controlled process. The great numerical advantages of a parametric form
of the cost function is shown by the sensitivity analysis in section 3.5 for a realistic nominal model. Despite that no mathematical expression could be found for the energy costs of the heat exchanger, the analysis resulted in realistic tradeoffs.

### 3.8 Appendix

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Phi$</td>
<td>air flow through shaft</td>
<td>$m^3/s$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>cooling effectiveness</td>
<td>$K$</td>
</tr>
<tr>
<td>$\alpha_{th}$</td>
<td>thermal diffusivity of air</td>
<td>$1.87 \times 10^{-5} \text{ m}^2/\text{s}$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>porosity</td>
<td>$m^3 \text{ air/m}^3 \text{ bulk}$</td>
</tr>
<tr>
<td>$\lambda_{a}$</td>
<td>conduction of air</td>
<td>$2.43 \times 10^{-2} \text{ W/m K}$</td>
</tr>
<tr>
<td>$\lambda_{p}$</td>
<td>conduction of product</td>
<td>$\text{W/m K}$</td>
</tr>
<tr>
<td>$\lambda_{w}$</td>
<td>wall conduction</td>
<td>$\text{W/m K}$</td>
</tr>
<tr>
<td>$\nu$</td>
<td>kinematic viscosity of air</td>
<td>$1.3465 \times 10^{-5} \text{ m}^2/\text{s}$</td>
</tr>
<tr>
<td>$\rho_{a}$</td>
<td>air density</td>
<td>$1.27 \text{ kg/m}^3$</td>
</tr>
<tr>
<td>$\rho_{p}$</td>
<td>produce density</td>
<td>$\text{kg/m}^3$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>switching time</td>
<td>$s$</td>
</tr>
<tr>
<td>$\tau_f$</td>
<td>length of switching interval</td>
<td>$s$</td>
</tr>
<tr>
<td>$A_f$</td>
<td>floor area of the bulk</td>
<td>$m^2$</td>
</tr>
<tr>
<td>$A_p$</td>
<td>produce area of the bulk</td>
<td>$\frac{a_2}{a_3} (1/s)$</td>
</tr>
<tr>
<td>$A_{ps}$</td>
<td>produce surface per bulk</td>
<td>$m^2/m^3$</td>
</tr>
<tr>
<td>$A_w$</td>
<td>wall area</td>
<td>$m^2$</td>
</tr>
<tr>
<td>$Bi$</td>
<td>Biot number</td>
<td>$\frac{2hR}{\lambda_{p}}$ (dimensionless)</td>
</tr>
<tr>
<td>$B_p$</td>
<td>$\frac{a_1}{a_3} (1/s)$</td>
<td></td>
</tr>
<tr>
<td>$E$</td>
<td>energy</td>
<td>$W$</td>
</tr>
<tr>
<td>$L$</td>
<td>bulk height</td>
<td>$m$</td>
</tr>
<tr>
<td>$L_2$</td>
<td>$R\gamma/(1 - \gamma)$, char. length</td>
<td>$m$</td>
</tr>
<tr>
<td>$M_1$</td>
<td>$\frac{\lambda_{p}}{\rho_{p}c_{p}} (m^2/s)$</td>
<td></td>
</tr>
<tr>
<td>$M_2$</td>
<td>$\frac{a}{c_{p}} (1/s)$</td>
<td></td>
</tr>
</tbody>
</table>
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\[ M_3 = \sqrt{M_2/M_1} R \] (dimensionless)
\[ M_4 = \frac{hA_p}{\gamma a c a} \] (1/s)
\[ M_5 = \frac{M_4 L}{\nu} \] (dimensionless)
\[ M_6 = (1 - \alpha) \exp \left( -M_5 (B_p + A_p)/A_p \right) \] (dimensionless)
\[ \text{Nu} \] Nusselt number \( \frac{2hR}{\lambda a} \) (dimensionless)
\[ \text{Pr} \] Prandtl number \( \frac{\nu}{\alpha a} \) (dimensionless)
\[ R \] product radius (m)
\[ \text{Re} \] Reynolds number \( \frac{\nu L}{\nu} \), see [110] (dimensionless)
\[ T_a \] air temperature in the bulk (K)
\[ T_c \] cooling element temperature (K)
\[ T_{ini} \] initial temperature (K)
\[ T_{out} \] outside air temperature (K)
\[ T_p \] produce temperature (K)
\[ V \] volume of shaft (m³)
\[ a \] product heat production (W/kg K)
\[ a_1 \] Bi (dimensionless)
\[ a_2 \] \( 2M_3 \cot(M_3) - 2 + \text{Bi} \) (dimensionless)
\[ a_3 \] \( \frac{R^2}{M_t} \cot^2(M_3) + \frac{R^2}{M_t} - \frac{M_3}{M_t} \cot(M_3) \) (s)
\[ c_a \] heat capacity of air \( (2 \times 10^3) \text{ J/kg K} \)
\[ c_p \] heat capacity of produce \( (J/kg K) \)
\[ d_w \] wall thickness (m)
\[ h \] heat transfer coefficient (W/m²K)
\[ v \] air velocity inside the bulk (m/s)
4

Feedback controller design for discrete input

4.1 Introduction

In this chapter we design and analyze a feedback controller that switches the input between discrete values. The methodology is inspired by the application of a bulk storage room. For such a system, some control inputs are of a discrete nature. Forced air ventilation, for example, is usually realized by a fan that is switched on or off. Generally, standard model-based control design is preferred, since it is a mathematically well-understood and practically implementable method. But given the nonlinearities due to the switching input, this is not directly feasible.

Control strategies that have been developed for storage purposes, are model predictive control (MPC) and fuzzy control. In [44] and [103], MPC algorithms were used for the temperature and humidity control of a bulk storage room with outside air ventilation. Both proposed algorithms are model based and were tested by simulation studies. In [35] a fuzzy controller was tested on a mathematical model. In [37] a sensor based control law for a bulk storage room that is ventilated with outside air was proposed, and in [36] a fuzzy controller was constructed and tested experimentally. In [65] a fuzzy controller was developed for fruit storage, using neural networks, and in [64] a fuzzy controller was tested experimentally. Further, in [61] a PI controller was designed for CO₂ and O₂ concentrations, and was tested experimentally. In general, the advantages of MPC are that the control algorithm is based on a mathematical model, and that the applicability extends to extremely complicated models. A major drawback is that controller dynamics have to be solved by demanding online numerical computations. Fuzzy controllers are in practice not easily implementable due to the many tuning parameters, and have no mathematical background. Hence, controller performance is hard to guarantee.

More general, control design for systems where the switching input is the
Feedback controller design for discrete input

control parameter, is done by MPC and fuzzy control, as shown above, and switching adaptive control. Stabilizing adaptive controllers are designed in [41,111] for a large class of nonlinear MIMO systems and for a larger class of MISO systems in [48], with less restrictive assumptions. Here, the control input, that depends continuously on the system states, is switched between two functions.

In this chapter, a standard robust controller is designed via linear design theory for a class of piecewise linear systems with switching control inputs. The inputs have fixed values, and are switched at most once in each discrete time interval, in contrast to for example [41,48,111]. The organization is as follows. In section 4.2 the model is linearized to a system with the switching moment as input. A controller that dynamically adjusts this input, is designed using standard design theory for linear systems. This type of controller differs essentially from the usual switching algorithms, where the temperatures are allowed to deviate inside a certain bandwidth. In section 4.3 conditions for stability are derived. The stability region is a parametric function of all the system properties, which makes analysis easier. In section 4.4 the theoretical results are applied to a model of a bulk storage room for harvested products. The control input of this model is the air flow induced by the fan, which is switched on and off on a regular basis. It is shown that the errors that are induced by the linearization cannot destabilize the system. The performance loss due to the linearization is visualized by numerical simulation of the original pde model, and the approximated piecewise linear system. Both systems are connected to the controller, and simulated under a heavy input disturbance. Since the dynamics of both systems are essentially the same, it is concluded that no essential dynamics is lost, and hence that for this model a controller with excellent properties can be designed.

4.2 System approximation and controller design

Our class of systems consists of nonlinear, scalar SISO systems of the form

\[
\frac{dx}{dt} = A(u)x + B(u). \tag{4.1}
\]

Here, \( x \) is the system state, \( u = (u_1, u_2) \) the input that attains two discrete values, and \( A \) and \( B \) scalar functions. The continuous time is divided into discrete time intervals with length \( \tau_f \). The control problem is to determine the duration of both inputs. We assume, without loss of generality, that at the start of each time interval \( u = u_1 \). The input is switched from \( u_1 \) to \( u_2 \).
at time $\tau$, with $0 \leq \tau \leq \tau_f$. This gives the following piecewise linear system

$$\frac{dx}{dt}(t) = A(u_1)x(t) + B(u_1) \quad t \in [0, \tau), \quad (4.2)$$

$$\frac{dx}{dt}(t) = A(u_2)x(t) + B(u_2) \quad t \in [\tau, \tau_f), \quad (4.3)$$

with $x(\tau^-) = x(\tau^+)$. From now on, the notation $A(u_1) = A_1$ is used, and the subscript denotes the relation with the input. Now $\tau/\tau_f$ is the fraction of the time that $u = u_1$. The goal is to design a controller that steers $x$ to the desired state $x_{opt}$ by adjusting $\tau$ each time interval. Although we want to steer $x(t)$ to $x_{opt}$, the control action is only based on the state at the beginning of the interval. Thus in the following sections we design a sequence of switching times $\tau \in [n\tau_f, (n + 1)\tau_f]$, based on $x(n\tau_f)$ and previous samples, such that $x(n\tau_f) \to x_{opt}$ for $n \to \infty$. If the sample time $\tau_f$ is small, then this implies that $x(t) - x_{opt}$ will become small. In practice this means that the state is stabilized around $x_{opt}$.

Throughout this chapter we assume that $A_1$ and $A_2$ are negative, and that $A_1^{-1}B_1 > A_2^{-1}B_2$. Since the choice of $A_1, A_2$ in (4.2)–(4.3) was arbitrary, this imposes no real restrictions.

### 4.2.1 System approximation

In this section, the system is approximated, and a controller is designed using standard design theory for linear continuous systems. At the interval $(0, \tau_f)$, the solution to equation (4.2) at time $\tau$ is

$$x(\tau) = x(0) \exp(A_1\tau) + \int_0^\tau \exp(A_1(t - \tau))B_1 dt$$

$$= x(0) \exp(A_1\tau) - (1 - \exp(A_1\tau))A_1^{-1}B_1. \quad (4.4)$$

Similarly, the solution to equation (4.3) becomes

$$x(\tau_f) = x(\tau) \exp(A_2(\tau_f - \tau)) - (1 - \exp(A_2(\tau_f - \tau)))A_2^{-1}B_2. \quad (4.5)$$

In the interval $[n\tau_f, (n + 1)\tau_f]$ we choose the switching time $\tau^n$. We denote the state at time $n\tau_f + \tau_n$ by $\xi^n$ and the state at the time $n\tau_f$ by $x^n$. So we have

$$\xi^n = \exp(A_1\tau^n)x^n + (\exp(A_1\tau^n) - 1)A_1^{-1}B_1$$

$$x^{n+1} = \exp(A_2(\tau_f - \tau^n))\xi^n + (\exp(A_2(\tau_f - \tau^n)) - 1)A_2^{-1}B_2. \quad (4.6)$$

Combining the equations in (4.6), we find that

$$x^{n+1} = f(x^n, \tau^n). \quad (4.7)$$
4 Feedback controller design for discrete input

The switching time is chosen such that the system is in the desired state $x_{\text{opt}}$ at all time instances $n\tau_f$. The following lemma shows that for any $x_{\text{opt}} \in [-A_1^{-1}B_1, -A_2^{-1}B_2]$ there exists a unique switching time $\tau_{\text{opt}} \in (0, \tau_f)$ such that

$$x_{\text{opt}} = f(x_{\text{opt}}, \tau_{\text{opt}}).$$

(4.8)

**Lemma 4.2.1.** Consider the system (4.6). Then equation (4.8) has a solution $\tau_{\text{opt}} \in [0, \tau_f]$ if and only if

$$-A_1^{-1}B_1 \leq x_{\text{opt}} \leq -A_2^{-1}B_2.$$  

(4.9)

Furthermore, when (4.9) holds, then the solution $\tau_{\text{opt}}$ is unique.

**Proof.** See the appendix.

Since we will not start at $x_{\text{opt}}$ and since disturbances may drive $x$ away from the desired state $x_{\text{opt}}$, we want to design a feedback control law for $\tau^n$, such that $x^n \to x_{\text{opt}}$. For this we linearize system (4.7) around $x_{\text{opt}}, \tau_{\text{opt}}$, i.e., we set

$$\begin{align*}
\tau^n &= \tau_{\text{opt}} + \tau^n_{\text{var}} \\
x^n &= x_{\text{opt}} + x^n_{\text{var}}.
\end{align*}$$  

(4.10)

The linearized system equals

$$\begin{align*}
x^{n+1}_{\text{var}} &= \frac{\partial f}{\partial x^n}(x_{\text{opt}}, \tau_{\text{opt}})x^n_{\text{var}} + \frac{\partial f}{\partial \tau^n}(x_{\text{opt}}, \tau_{\text{opt}})\tau^n_{\text{var}} \\
 &= A_d x^n_{\text{var}} + B_d \tau^n_{\text{var}},
\end{align*}$$

(4.11)

where

$$\begin{align*}
A_d &= \exp(A_2(\tau_f - \tau_{\text{opt}}) + A_1\tau_{\text{opt}}) \\
B_d &= -A_2 \exp(A_2(\tau_f - \tau_{\text{opt}}))\xi(\tau_{\text{opt}}) + \exp(A_2(\tau_f - \tau_{\text{opt}})) \cdot \\
&\quad \left( A_1 \exp(A_1\tau_{\text{opt}})x_{\text{opt}} + B_1 \exp(A_1\tau_{\text{opt}}) \right) - B_2 \exp(A_2(\tau_f - \tau_{\text{opt}})) \\
&= -A_2 x_{\text{opt}} - B_2 + \exp(A_2(\tau_f - \tau_{\text{opt}})) \cdot \\
&\quad \left( A_1 \exp(A_1\tau_{\text{opt}})x_{\text{opt}} + B_1 \exp(A_1\tau_{\text{opt}}) \right).
\end{align*}$$

(4.12)

Since $A_1$ and $A_2$ are negative, and since $0 \leq \tau_{\text{opt}} \leq \tau_f$, we see that $A_d \in (0, 1)$. We note that any nonlinear MIMO system with switching input can be brought to the form of (4.11), i.e. linearized and with $\tau^n_{\text{var}}$ as input, and hence enable linear (discrete) control design. The reason that we consider scalar systems of the form (4.1), is that they allow a rigorous stability analysis. This results in a stability area that contains all the physical knowledge of the system.
4.3 Stability analysis

4.2.2 Controller design

In the appendix we show how a standard PI controller can be designed for our discrete time system. The resulting controller in discrete time is

\[ \zeta^{n+1} = -\frac{(A_d - 1)^2}{B_d} x_{\text{var}}^n + \zeta^n \]

\[ \tau_{\text{var}}^n = \frac{A_d - 1}{B_d} x_{\text{var}}^n + \zeta^n. \] (4.13)

From the second equation in (4.13) it is clear that \((x_{\text{var}}^n, \tau_{\text{var}}^n)\) converges to zero if and only if \((x_{\text{var}}^n, \tau_{\text{var}}^n)\) converges to zero. Using (4.13) and (4.11) we have

\[ \tau_{\text{var}}^{n+1} = \frac{A_d - 1}{B_d} (A_d x_{\text{var}}^n + B_d \tau_{\text{var}}^n) - \frac{(A_d - 1)^2}{B_d} x_{\text{var}}^n + \zeta^n \]

\[ = \frac{A_d - 1}{B_d} (A_d x_{\text{var}}^n - (A_d - 1)x_{\text{var}}^n + B_d \tau_{\text{var}}^n) + \tau_{\text{var}}^n - \frac{A_d - 1}{B_d} x_{\text{var}}^n \]

\[ = (A_d - 1) \tau_{\text{var}}^n + \tau_{\text{var}}^n \]

\[ = A_d \tau_{\text{var}}^n. \] (4.14)

Thus the closed loop system for \(x_{\text{var}}^n\) and \(\tau_{\text{var}}^n\) is

\[ x_{\text{var}}^{n+1} = A_d x_{\text{var}}^n + B_d \tau_{\text{var}}^n \]

\[ \tau_{\text{var}}^{n+1} = A_d \tau_{\text{var}}^n. \] (4.15)

Since \(A_d \in (0, 1)\), this is stable. We underline that this holds for the system without any external noise or modelling perturbations. Also, it is interesting to see that the initial condition plays an important role for this particular case. For example, choosing \(\tau_{\text{var}}^0 = 0\) results in \(\tau_{\text{var}}^n = 0\) for all \(n\), and this is the open loop controller from chapter 2, which obviously results in a globally stable system. This explains the stability that was observed in Figure 2.7. In the following section we investigate the stability of the controller (4.13) on the original system 4.1.

4.3 Stability analysis

In this section we prove that the controller (4.13) stabilizes the original system (4.1). The control action on the original system is modified such that realistic time switches are applied to the original system. The rules are

If \(\tau_{\text{var}}^n + \tau_{\text{opt}} > \tau_f\), then \(\tau^n = \tau_f\)

If \(\tau_{\text{var}}^n + \tau_{\text{opt}} < 0\), then \(\tau^n = 0\)

If \(0 \leq \tau_{\text{var}}^n + \tau_{\text{opt}} \leq \tau_f\), then \(\tau^n = \tau_{\text{var}}^n + \tau_{\text{opt}}\). \(\) (4.16)
Next we show that if \( \tau^n \) is chosen according to these rules, then \( x^n \) stays bounded. Later we show that \( x^n \to x_{opt} \).

**Lemma 4.3.1.** Let \( \tau^n \) be a sequence in the interval \([0, \tau_f]\) and let \( x^0 \) be given. For any \( \delta > 0 \) there exists a \( N \) such that \( x^N \in (-A_1^{-1}B_1 - \delta, -A_2^{-1}B_2 + \delta) \) for \( n \geq N \). Here \( x^n \) is the solution of (4.7).

**Proof.** See the appendix. \( \square \)

Using the linearized model (4.11) we can write (4.7) as

\[
x^{n+1}_{var} = A_dx^n_{var} + (B_d + \varepsilon(x^n_{var}, \tau^n_{var}))\tau^n_{var}.
\]

(4.17)

Here we have used that (4.7) is linear in \( x \). Similar as in (4.14), we obtain the following difference equation for \( \tau_{var} \)

\[
\tau^{n+1}_{var} = (A_d + \frac{A_d - 1}{B_d} \varepsilon(x^n_{var}, \tau^n_{var}))\tau^n_{var}.
\]

(4.18)

Here \( \varepsilon \) is the error induced by linearization. Our closed loop system becomes

\[
\begin{align*}
x^{n+1}_{var} &= A_dx^n_{var} + (B_d + \varepsilon(x^n_{var}, \tau^n_{var}))\tau^n_{var} \\
\tau^{n+1}_{var} &= (A_d + \frac{A_d - 1}{B_d} \varepsilon(x^n_{var}, \tau^n_{var}))\tau^n_{var}.
\end{align*}
\]

(4.19)

We know from Lemma 4.3.1 that \((x^n_{var}, \tau^n_{var})\) will lie in a bounded set. Using the second equation of (4.19), we conclude that if \( \varepsilon \) is sufficiently small, then \( \tau^n_{var} \to 0 \). Since \( \varepsilon \) contains higher order terms, and since \( \varepsilon(0,0) = 0 \), the condition that \( \varepsilon \) is small in a neighbourhood of (0,0) is not a strong assumption. Concluding, we have

**Theorem 4.3.2.** Consider equation (4.19). Let \( \Omega = \{(x_{var}, \tau_{var}) \mid x_{var} + x_{opt} \in [-A_1^{-1}B_1, -A_2^{-1}B_2] \text{ and } \tau_{var} + \tau_{opt} \in [0, \tau_f]\} \). If

\[
\sup_{(x_{var}, \tau_{var}) \in \Omega} \left| A_d + \frac{A_d - 1}{B_d} \varepsilon(x_{var}, \tau_{var}) \right| < 1,
\]

(4.20)

then (4.19) is asymptotically stable.

### 4.4 Application to food storage

In this section, the controller design and the stability analysis from the previous section are applied to a model of a bulk storage room for harvested food products.
4.4 Application to food storage

4.4.1 The model

In this subsection the (approximated) model that was derived and validated in chapter 2, is described. The storage room model is schematically drawn in Figure 4.1. Air is circulated by a fan, and the air is cooled down by a heat exchanger right below the fan. The air enters the bulk at the bottom, and consequently the products at the top will be the warmest. The nominal model describing the product temperature in the top of the bulk is

\[
\frac{dT_p(t)}{dt} = A(\Phi(t))T_p(t) + B(\Phi(t), T_c(t)),
\]

with \(T_p(t)\) the product temperature at the top of the bulk. For the expressions for \(A\) and \(B\) we refer to chapter 2. We assume that the temperature of the cooling device is constant. The most important physical parameters that correspond to a storage room with a bulk of potatoes, are the temperature of the cooling device \(T_c = 275\ K\), the fluxes generated by the fan in on and off position \(\Phi_1 = 1\ m^3/s\), and \(\Phi_2 = 0.001\ m^3/s\), the height 4 \(m\), the floor area 5 \(m^2\), and the shaft volume 10 \(m^3\). The rest of the parameter values is listed in chapter 2. In Table 4.1 the numerical values of the key parameters in this chapter are given.

![Figure 4.1: Schematic representation of a bulk storage room.](image)

4.4.2 Controller

The controller measures the product temperature at the top of the bulk, \(T_p(t)\). The optimal switching time corresponds to \(T_p(t) = T_{p,\text{opt}}\). Realistic
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<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>$-2 \times 10^{-5}$ 1/s</td>
<td>$A_2$</td>
<td>$-2 \times 10^{-8}$ 1/s</td>
</tr>
<tr>
<td>$B_2$</td>
<td>$8.1 \times 10^{-6}$ K/s</td>
<td>$A_d$</td>
<td>$1.0 - 3 \times 10^{-4}$</td>
</tr>
<tr>
<td>$\tau_f$</td>
<td>600 s</td>
<td>$B_d$</td>
<td>$-1.2 \times 10^{-4}$ K/s</td>
</tr>
<tr>
<td>$B_1$</td>
<td>$6.6 \times 10^{-3}$ K/s</td>
<td>$T_{p,opt}$</td>
<td>280 K</td>
</tr>
</tbody>
</table>

Table 4.1: Numerical key parameter values.

Disturbances in the air temperature are caused by open doors, heat leakage through the walls, etcetera. For mathematical simplicity we assume that the disturbances in air temperature occur in the vicinity of the heat exchanger, and that they therefore act on the system as the temperature of the cooling element $T_c$. We assume that disturbances in $T_c$ have the same qualitative influence on $T_p$ as disturbances in $\tau$. Therefore, we can assume that realistic disturbances act on the system as depicted in Figure 4.3. Hence, the controller design as proposed in section 4.2.2 is appropriate for the bulk storage room model.

### 4.4.3 Stability

In this section the influence of the linearization error on the stability is investigated. We define the linearization as in (4.11), with the variable $x$ replaced with $T_p(L)$. Also, $T_p(L)$ is denoted by $T_p$ for convenience. We recall the system (4.19) with $x_{var}$ replaced with $T_{p,var}$

\[
T_{p,var}^{n+1} = A_d T_{p,var}^n + (A_d + \varepsilon) \tau_{var}^n
\]

\[
\tau_{var}^{n+1} = (A_d + \varepsilon) \tau_{var}^n.
\]

(4.22)

We have that

\[
\varepsilon = \frac{1}{2} \frac{\partial^2 f}{\partial T_p \partial \tau} (T_{p,opt}, \tau_{opt}) T_{p,var}^n + \frac{1}{2} \frac{\partial^2 f}{\partial \tau \partial T_p} (T_{p,opt}, \tau_{opt}) T_{p,var}^n
\]

\[
+ \frac{1}{2} \frac{\partial^2 f}{\partial (\tau)^2} (T_{p,opt}, \tau_{opt}) \tau_{var}^n + \text{h.o.t.}
\]

(4.23)

We neglect the higher order terms of $\varepsilon$, which gives

\[
\varepsilon = \frac{1}{2} \left( (A_1 - A_2) \alpha_2 - A_2 + A_1 \alpha_2 \right) T_{p,var}^n + \left( \frac{1}{2} (A_1 - A_2) \left( (A_1 - A_2) \alpha_2 T_{p,opt} - \frac{A_2 B_1}{A_1} \alpha_2 + B_1 \alpha_2 \right) - \frac{1}{2} A_2 (\frac{A_2 B_1}{A_1} - B_2) \exp(A_2(\tau_f - \tau_{opt})) \right) \tau_{var}^n,
\]

(4.24)

with $\alpha_2 = \exp(A_2(\tau_f - \tau_{opt}) + A_1 \tau_{opt})$. Numerical evaluation gives

\[
\varepsilon = -2.4 \times 10^{-5} T_{p,var} + 7.9 \times 10^{-8} \tau_{var}.
\]

(4.25)
We have that $A_d = 1 - 3 \times 10^{-4}$, and $B_d = 6.5 \times 10^{-3}$, so the stability criterion of Theorem 4.3.2 $|A_d + \frac{A_d - 1}{B_d} \epsilon| < 1$ becomes $|1 - 3 \times 10^{-4} + 4.6 \times 10^{-2} \epsilon| < 1$, which is fulfilled if $|\epsilon| < 64.6$. We have that $0 < \tau_{var} < 600$, according to (4.16), and that $T_p$ will converge to the range $(-A_1^{-1}B_1 - \delta, -A_2^{-1}B_2 + \delta)$ for any $\delta$, by Lemma 4.3.1. Since for our case $(-A_1^{-1}B_1, -A_2^{-1}B_2) = (275.1, 398.2)$, we have that $|T_{p,var}| < 123.1$ for any choice of $T_{p,opt}$. We conclude that $T_{p,var}$ and $\tau_{var}$ cannot grow large enough to destabilize the system, and hence the system is asymptotically stable according to Theorem 4.3.2.

### 4.4.4 Simulation study

In the previous section the stability robustness was analyzed, and in this section we analyze the loss of performance due to the linearization. This is done by connecting controller (4.13) to the linearized system (4.26) and to the nominal system (4.21). The differences in $T_p(t)$ and $\tau(t)$ should give an indication whether any essential dynamics is discarded. Further, a heavy input disturbance $d$ is added, such that the system dynamics are clearly visible. The initial product temperature is set uniform at 285 K, while the optimal product temperature is 280 K. The input disturbance is $d = a \sin(\omega t)$, with $a = 10$ s, and $\omega = 3 \times 10^{-6}$ Hz.

The dynamics of $\tau(t)$ and $T_p(t)$ are shown in Figure 4.2(a). For both controlled systems the dynamics of $T_p(L, t)$ and $\tau(t)$ are more or less the same, indicating that the linearization error between (4.21) and (4.26) is small. Even when initially the product temperature differs considerably from the linearization point of 280 K, the differences are small. Furthermore, the controller seems to perform quite well under these large input disturbances. For various frequencies of $d$ similar results were obtained. Figure 4.2(b) shows the results for the large interval of $\tau_f = 10$ hours. The amplitude of the disturbance is scaled with $\tau_f$. The results are very similar, which indicates that the size of $\tau_f$ has no considerable influence on the performance robustness of the controller. For different amplitudes and frequencies, the linearization was also found to be very accurate.

### 4.5 Conclusions

We showed that for a large class of nonlinear scalar systems with discrete input, it is possible to make an approximation that allows the design of a linear controller that controls the switching time of the input. This is done by a linearization around the optimal switching time, and the optimal state. Lemmas 4.2.1 and 4.3.1 give conditions for the existence of such a linearization point, state its uniqueness, and guarantee that the state is bounded.
Theorem 4.3.2 gives the condition for asymptotic stability of the controlled system. The conditions are in analytical form, which gives a more structural insight into the influence of errors and perturbations on the stability.

As an example, a controller was designed and connected to a temperature model of a bulk storage room. For controller design, the original (or nominal) model was linearized. It was shown that the stability cannot be jeopardized by the linearization error. Numerical simulations show that under large input disturbances the nominal and the approximated system have similar dynamics in $T_p$ and $\tau$. This also holds for different disturbance amplitudes and frequencies, indicating that the linearization does not discard any essen-
tial dynamics. Hence a controller with excellent properties can be designed for this bulk storage room model. The linearization and the controller design can be applied to any system. However, for more complex systems, such as higher order systems, the controller design and the stability analysis do generally not result in parametric expressions, and will therefore be more numerically involved. Nevertheless, a next step would be the design of a switching input controller for higher order systems, together with a numerical stability analysis.

4.6 Appendix

4.6.1 Controller design

The following strategy is used. Equation (4.11) is approximated by a continuous system, by taking $\tau_f \rightarrow 0$. The idea is that the dynamics of $x$ are slow on $(0, \tau_f)$, and that therefore $\frac{\partial x}{\partial t} \approx \frac{x^{n+1} - x^n}{\tau_f}$. We start by rewriting (4.11) as

$$\frac{x^{n+1} - x^n}{\tau_f} = A_{lin}x_{var} + B_{lin}\tau_{var}, \quad (4.26)$$

with $A_{lin} = \frac{A_d - 1}{\tau_f}$, and $B_{lin} = \frac{B_d}{\tau_f}$. We approximate it by a continuous system, by taking $\tau_f \rightarrow 0$, so (4.26) becomes

$$\frac{dx_{var}(t)}{dt} = A_{lin}x_{var}(t) + B_{lin}\tau_{var}(t). \quad (4.27)$$

For (4.27) it is now possible to design a controller by standard linear theory. For the formulation of design specifications, system (4.27) is transformed into the Laplace frequency domain to

$$\hat{x}_{var}(s) = \frac{B_{lin}}{-A_{lin} + s}\hat{\tau}_{var}(s)$$

$$= G(s)\hat{\tau}_{var}(s). \quad (4.28)$$

In this section it is assumed for simplicity, that there is only one disturbance, $d$, which acts on the input $\tau_{var}$. Figure 4.3 shows the interconnection of $G(s)$ and the controller $K(s)$, together with the input disturbance. Various designs are possible, e.g. LQG or optimal control design. We propose the following design specifications that are standard for linear SISO systems (see for example [51] for more details).

- The sensitivity function $S = \frac{1}{1 + K(s)G(s)}$ from $d$ to $x_{var}$ should be small for low frequencies, and close to 1 for high frequencies for good performance.
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- A very high crossover frequency of $S$ will result in a very fast controller, with the tradeoff that under disturbances the performance and stability will be poor.

Input disturbances with a higher frequency than the crossover frequency of $G$ are already attenuated by $G$. Therefore, a good choice would be that $S$ is small up to the crossover frequency of $G$: $-A_{lin}$. In other words, we have to find $K$ such that

$$
\frac{1}{1 + G(s)K(s)} = \frac{\bar{s}}{1 + \bar{s}},
$$

(4.29)

with $\bar{s} = \frac{s}{-A_{lin}}$. In this way, $S_2$ is small for all frequencies, $S$ is small for frequencies up to $s = -A_{lin}$, and $S$ tends to 1 for high frequencies. Straightforward calculation gives the PI controller

$$
K(s) = \frac{A_{lin} - s}{B_{lin} s},
$$

$$
\Leftrightarrow \hat{\tau}(s) = (-\frac{A_{lin}^2}{B_{lin} s} + A_{lin})\hat{x}_{var}(s).
$$

(4.30)

With the substitution

$$
\hat{\zeta}(s) = -\frac{A_{lin}^2}{B_{lin} s} \hat{x}_{var}(s)
$$

(4.31)

and forward Euler, our controller in discrete time becomes

$$
\frac{\zeta^{n+1} - \zeta^n}{\tau_f} = -\frac{A_{lin}^2}{B_{lin}}x_{var}^n
$$

$$
\tau_{var}^n = \zeta^n + \frac{A_{lin}}{B_{lin}}x_{var}^n.
$$

(4.32)

Note that the controller is an explicit parametric function of all the system characteristics. Figure 4.3 shows the controlled system with input disturbance $d$ schematically.

Figure 4.3: Schematic overview of the controlled system with input disturbance $d$. 
4.6.2 Proofs

Proof of lemma 4.2.1

Using (4.6) we can write \( f(x, \tau) \) as

\[
f(x, \tau) = f_1(\tau)x + f_2(\tau).
\]

(4.33)

Further, it is not hard to see that for \( \tau \in [0, \tau_f] \) \( f_1(\tau) \in (0, 1) \), and

\[
\begin{align*}
f_2(0) &= \exp(A_2 \tau_f) - 1)A_2^{-1}B_2 \\
f_1(\tau_f) &= \exp(A_1 \tau_f) - 1)A_1^{-1}B_1.
\end{align*}
\]

(4.34)

Solving

\[
x_{opt} = f_1(\tau)x_{opt} + f_2(\tau)
\]

for \( \tau \in [0, \tau_f] \) is possible if and only if

\[
x_{opt} = \frac{f_2(\tau)}{1 - f_1(\tau)}
\]

(4.35)

is solvable for \( \tau \in [0, \tau_f] \). Since the right hand side is a continuous function of \( \tau \), we see that solving (4.36) is possible if and only if \( x_{opt} \) lies in the range of \( f_2/(1 - f_1) \). We have that

\[
\begin{align*}
\frac{f_2(0)}{(1 - f_1(0))} &= -A_2^{-1}B_2 \\
\frac{f_2(\tau_f)}{(1 - f_1(\tau_f))} &= -A_1^{-1}B_1.
\end{align*}
\]

(4.37)

Thus if \( x_{opt} \) lies between these values, then (4.36) is solvable. If the range of \( f_2/(1 - f_1) \) for \( \tau \in [0, \tau_f] \) would be larger, then

\[
\begin{align*}
\frac{f_2(\tau)}{(1 - f_1(\tau))} &= -A_2^{-1}B_2 & \text{or} \\
\frac{f_2(\tau)}{(1 - f_1(\tau))} &= -A_1^{-1}B_1
\end{align*}
\]

(4.38)

must be solvable for at least two \( \tau \in [0, \tau_f] \). We show that this is not possible. We do this for the second equation, the first one goes similarly. Using (4.38) in (4.6) gives

\[
\begin{align*}
\xi &= -A_1^{-1}B_1 \\
-A_1^{-1}B_1 &= \exp(A_2(\tau_f - \tau))\xi^n + (\exp(A_2(\tau_f - \tau)) - 1)A_2^{-1}B_2 \\
\Leftrightarrow A_1^{-1}B_1 &= \exp(A_2(\tau_f - \tau))A_1^{-1}B_1 - (\exp(A_2(\tau_f - \tau)) - 1)A_2^{-1}B_2 \\
\Leftrightarrow A_1^{-1}B_1 &= (\exp(A_2(\tau_f - \tau)) - 1)(A_1^{-1}B_1 - A_2^{-1}B_2).
\end{align*}
\]

(4.39)
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Since $A_1^{-1}B_1 \neq A_2^{-1}B_2$, we must have $\exp(A_2(\tau_f - \tau)) - 1 = 0$, which gives $\tau = \tau_f$. Now we will prove the uniqueness of $\tau_{opt}$. Assume that $\tau_1$ and $\tau_2$ are times such that

$$x_{opt} = f(x_{opt}, \tau_i) \quad i = 1, 2. \quad (4.40)$$

Assume that $\tau_1 < \tau_2 \leq \tau_f$, and let

$$\xi_i = \exp(A_1\tau_i)x_{opt} + A_1^{-1}B_1(\exp(A_1\tau_i) - 1) \quad i = 1, 2. \quad (4.41)$$

We observe from (4.6) that

$$\xi_i + A_1^{-1}B_1 = \exp(A_1\tau_i)(x_{opt} + A_1^{-1}B_1) \quad (4.42)$$
$$x_{opt} + A_2^{-1}B_2 = \exp(A_2(\tau_f - \tau_i))(\xi_i + A_2^{-1}B_2) \quad (4.43)$$

Since $A_1 < 0$, and since $\tau_1 < \tau_2$, we have by (4.42) and $x_{opt} > -A_1^{-1}B_1$ that

$$\xi_1 + A_1^{-1}B_1 > \xi_2 + A_1^{-1}B_1. \quad (4.44)$$

This implies that

$$\xi_2 + A_2^{-1}B_2 > \xi_1 + A_2^{-1}B_2. \quad (4.45)$$

Now using (4.43) and the fact that $\tau_f - \tau_1 > \tau_f - \tau_2$ we find

$$\exp(A_2(\tau_f - \tau_2))(\xi_2 + A_2^{-1}B_2) > \exp(A_2(\tau_f - \tau_1))(\xi_1 + A_2^{-1}B_2). \quad (4.46)$$

However, both expressions must be equal to $x_{opt}$. Hence $\tau_1$ cannot be unequal to $\tau_2$. □

**Proof of lemma 4.3.1**

We want to show that for some $N$ $x^N \in [-A_1^{-1}B_1 - \delta, -A_2^{-1}B_2 + \delta]$ for any $\delta > 0$. Suppose that this does not hold, then $x^n \notin [-A_1^{-1}B_1 - \delta, -A_2^{-1}B_2 + \delta]$ for all $n$. Suppose

$$x^0 < -A_1^{-1}B_1 - \delta \Rightarrow (4.42) \quad \xi^0 < -A_1^{-1}B_1 - \delta, \quad (4.47)$$

which implies that $x_1 < -A_2^{-1}B_2$. Since $x^1 \notin [-A_1^{-1}B_1 - \delta, -A_2^{-1}B_2 + \delta]$ we have

$$x^1 < -A_1^{-1}B_1 - \delta. \quad (4.48)$$

Furthermore, $x^0 < x^1$. Repeating the above argument gives

$$x^0 < x^1 < x^2 \ldots x^n \leq -A_1^{-1}B_1 - \delta. \quad (4.49)$$

Hence $x^n \to x^\infty \leq -A_1^{-1}B_1 - \delta$. Similarly, $\xi^n \to \xi^\infty \leq -A_1^{-1}B_1 - \delta$. From (4.42) we conclude that if $x$ and $\xi$ both converge, then so does $\tau$. So $\tau^n \to \tau^\infty$. Thus we have that $(x^\infty, \tau^\infty)$ is a fixed point that satisfies $x^\infty = f(x^\infty, \tau^\infty)$ and $x^\infty < -A_1^{-1}B_1$. Lemma 4.2.1 implies that $x^\infty \geq -A_1^{-1}B_1$. □
4.6 Appendix

4.6.3 Notation

\( \Phi \) air flow through shaft \((m^3/s)\)
\( \alpha \) cooling effectiveness \((K)\)
\( \alpha_{th} \) thermal diffusivity of air \((1.87 \times 10^{-5} \, m^2/s)\)
\( \gamma \) porosity \((m^3/m^3)\)
\( \lambda_a \) conduction of air \((2.43 \times 10^{-2} \, W/m \, K)\)
\( \lambda_p \) conduction of product \((W/m \, K)\)
\( \nu \) kinematic viscosity of air \((1.35 \times 10^{-5} \, m^2/s)\)
\( \rho_a \) air density \((1.27 \, kg/m^3)\)
\( \rho_p \) produce density \((kg/m^3)\)
\( \tau \) switching time \((s)\)
\( \tau_f \) length of switching interval \((s)\)
\( A_f \) floor area of the bulk \((m^2)\)
\( A_{ps} \) produce surface per bulk volume \((m^2/m^3)\)
\( Bi \) Biot number \( \frac{2hR}{\lambda_p} \)
\( L \) bulk height \((m)\)
\( L_2 \) \( R \times \gamma/(1 - \gamma) \), char. length \((m)\)
\( Nu \) Nusselt number \( \frac{2hR}{\lambda_p} \)
\( Pr \) Prandtl number \( \frac{\nu_a}{\alpha_{th}} \)
\( R \) product radius \((m)\)
\( Re \) Reynolds number \( \frac{\nu L_2}{\rho} \), [110]
\( T_a \) air temperature in the bulk \((K)\)
\( T_c \) cooling element temperature \((K)\)
\( T_{ini} \) initial temperature \((K)\)
\( T_p \) produce temperature \((K)\)
\( V \) volume of shaft \((m^3)\)
\( a \) product heat production \((J/kg \, s \, K)\)
\( b \) product heat production \((J/kg \, s)\)
\( c_a \) heat capacity of air \((2 \times 10^3 \, J/kg \, K)\)
\( c_p \) heat capacity of produce \((J/kg \, K)\)
\( h \) heat transfer coefficient \((W/m^2 K)\)
\( v \) air velocity inside the bulk \((m/s)\)
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4.6.4 Expressions

\[ A = A^* \]
\[ B = B^* T_c \]
\[ A^* = \frac{\tilde{A}_p A_p}{A_p + \tilde{A}_p} \]
\[ B^* = -\frac{A_p + \tilde{A}_p}{B_p} T_c \]
\[ \tilde{A}_p = -\frac{A_p^2}{M_5 B_p} + \frac{A_p^2 (1 - \alpha)}{M_5 B_p} \exp\left(M_5 \left( \frac{B_p + A_p}{-A_p} \right) \right) \]
\[ \tilde{B}_p = \frac{\alpha A_p^2}{M_5 B_p} \exp\left(M_5 \left( \frac{B_p + A_p}{-A_p} \right) \right) \]
\[ A_p = \frac{\frac{R^2}{\alpha} \cot^2(M_3) + \frac{R^2}{\alpha_1} - \frac{M_2 M_5}{M_2} \cot(M_3)}{2 M_3 \cot(M_3) - 2 + Bi} \]
\[ B_p = \frac{\frac{R^2}{\alpha} \cot^2(M_3) + \frac{R^2}{\alpha_1} - \frac{M_2 M_5}{M_2} \cot(M_3)}{\frac{\lambda_p}{Bi}} \]
\[ M_1 = \frac{\rho_p c_p}{\lambda_p} \]
\[ M_2 = \frac{\lambda}{c_p} \]
\[ M_3 = \sqrt{\frac{M_2}{M_1} R} \]
\[ M_4 = \frac{h A_p}{\gamma \lambda_0 C_0} \]
\[ M_5 = \frac{\gamma^3}{A_f} \]
\[ v = \frac{f'}{A_f} \]
5

Modelling and controller design for a UV disinfection plant

5.1 Introduction

For the disinfection of fluids, UV-treatment becomes increasingly popular, and replaces more conventional options. For example, in drinking water treatment, chlorination is still the most used disinfection method. However, since the residuals are toxic to aquatic life [105], and since some by-products of chlorination have been proven to be mutagenic, the environmentally friendly UV disinfection method has been stimulated as an alternative [3, 101]. In [10, 13, 79, 85, 107, 108] design and modelling of a photoreactor for disinfecting drinking water is proposed. In [54, 55, 76, 77, 112] experimental results are presented that link the UV dosage to the inactivation of microorganisms in wastewater treatment in agriculture and horticulture. Air disinfection by UV is investigated in [31] and [75]. In [23] the required UV dosage for apple cider pasteurization is examined, and in [14] the possibility of UV treatment of process water in the food and beverage industries is discussed.

In order to improve the operation of UV disinfection, proper dynamic control needs to be incorporated. However, surprisingly little literature is available on control design for UV disinfection. Only one approach is known to the authors. In [44] a basic model was developed, and a controller was designed. The fluid mechanics was modelled by a plug flow, and the attenuation of the microorganisms due to the UV irradiation was modelled as a first order reaction. For effective control design, it is desirable to have a low order linear model that contains the essential system dynamics. This allows for standard linear control, which is very well documented and gives generally good performance, see for example [9, 51]. Unfortunately, models describing a UV reactor are generally not of this type. Disinfection is a complex process with often higher order reaction kinetics [13], and sometimes reactivation. Moreover, a detailed description of the fluid flow through the reactor is in general
described by the full Navier-Stokes equations [42]. Altogether, a detailed reactor model leads to a nonlinear model, and hence it is not possible to design a controller in the traditional sense, employing standard techniques. In this chapter, a model-based controller is designed for a linear and low order approximation of the original model. In section 5.2, we describe the full transport model for the fluid and microorganism concentration, under the influence of UV irradiation. The reaction kinetics are described in a basic way, following [44]. Subsequently, we adopt the assumption of laminar flow and invoke basic symmetry properties of the solution. This yields a simplified model of which the fluid-mechanics part can be solved analytically. By specifying the model to conditions relevant to disinfection of cider [23], we obtain the reference ‘nominal’ model.

In section 5.3 the nominal model is linearized and approximated by a linear state space form by a Padé transfer function approximation. Next, the model is input/output balanced and truncated down to a first order model. In section 5.4 a feedback controller is designed for the resulting linear, first order model. The controller is tested by a simulation study and found to perform equally well on the reduced model as on the nominal model. This indicates that under the reduction steps, the essential dynamics of the nominal model is maintained. Hence, for the original model a classical controller with excellent properties can be designed.

5.2 Basic modelling of the disinfection plant

In this section we first sketch the general physical model. Then we proceed to analyze laminar flow in the reactor and subsequently consider the dynamics of the active microorganism concentration in case diffusive transport can be neglected. Finally, the parameter values for the nominal model are chosen, corresponding to the particular case of UV-disinfection in a cider plant.

5.2.1 Physical model

We propose a general model that describes the fluid flow and the reaction kinetics in a cylindrical reactor. This model is composed of the Navier-Stokes equations for an incompressible fluid, to describe conservation of mass and momentum, and a convection-diffusion-reaction equation which quantifies the concentration of active microorganisms in the reactor. Figure 5.1 shows the reactor with a UV lamp in the center along the longitudinal direction. Here, $R_2$ is the outer radius, $R_1$ the radius of the lamp, and $L^*$ the length of the reactor. The variables with an asterisk are the variables with physical dimensions that are scaled later on. The general equations, describing continuous fluid flow and mass transport in the reactor are (gravitational
5.2 Basic modelling of the disinfection plant

Effects are neglected and the fluid is assumed to be incompressible)

Continuity:

$$\nabla^* \cdot \mathbf{v}^* = 0$$  \hspace{1cm} (5.1)

Momentum:

$$\frac{\partial \mathbf{v}^*}{\partial t^*} = -(\mathbf{v}^* \cdot \nabla^*)\mathbf{v}^* - \frac{\nabla^* p^*}{\rho} + \nu \nabla^2 \mathbf{v}^*.$$  \hspace{1cm} (5.2)

Transport:

$$\frac{\partial C^*}{\partial t^*} = -(\mathbf{v}^* \cdot \nabla^*)C^* + D \nabla^2 C^* - K^* C^*,$$  \hspace{1cm} (5.3)

where $\mathbf{v}^*$ is the velocity vector. Furthermore,

$$D = \frac{\nu}{Pr}$$  \hspace{1cm} (5.4)

denotes the microorganisms diffusion coefficient. $C^* > 0$ is the concentration of microorganisms in cells per $m^3$, $\nu$ the kinematic viscosity, and $Pr = \frac{\eta C_p}{\lambda}$ the Prandtl number, with $C_p$ the heat capacity, $\eta$ the dynamic viscosity, and $\lambda$ the thermal conductivity.

Finally, we consider the first order reaction mechanism as proposed in [44], using results from [10], [79], and [85]. This is described by the disinfection reaction rate

$$K^*(r^*) = \epsilon I^* \frac{R_1}{r^*} \exp\left( - E^*(r^* - R_1) \right),$$  \hspace{1cm} (5.5)

which is governed by the intensity $I^*$ ($W/m^2$), taken as the UV intensity at the surface of the lamp at $R_1$, the microorganism susceptibility factor $\epsilon$ ($m^2/J$), and the monochromatic absorbance $E^*$ ($1/m$). It is assumed that $\epsilon$ and $E^*$ are constant. The most important assumptions that have been made in the derivation of (5.5) are

- The reaction constant only depends on $r$ and not on $z$.  

Figure 5.1: Schematic overview of a UV reactor.
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- The reaction kinetics are of first order, and there is no reactivation.
- Solids are completely suspended in the medium, i.e. there are no lumps of material.
- Reflection and refraction effects are negligible.

The boundary conditions are discussed later on.

5.2.2 Scaling

It is convenient to work with dimensionless quantities and for that purpose all variables are scaled. As reference length-scale we adopt $R_2$. The reference velocity is denoted by $u_0$ for which we select the mass-average velocity, i.e., $u_0 A = Q$ where the reference area $A = \pi (R_2^2 - R_1^2)$, and $Q$ is the volume flow of fluid through the reactor in $m^3 s^{-1}$. These reference scales also define a time-scale $R_2 / u_0$. To finalize the scaling of the fluid flow part, we adopt the kinematic viscosity $\nu$ to quantify the so-called viscous fluxes in the Navier-Stokes equations. In order to scale the convection-diffusion-reaction equation for the active microorganism concentration $C$ we adopt the same time- and length-scales, and use the average concentration at the inlet of the reactor, $\overline{C}_0$, as reference scale for the concentration. The intensity is scaled using the average value of the lamp intensity, $\overline{I}$, as reference scale. Altogether, the parameters with asterisks are scaled as follows

\[
\begin{align*}
(z^*, r^*) &= R_2(z, r) \\
v^* &= u_0 v \\
t^* &= \frac{R_2 t}{u_0} \\
C^* &= \overline{C}_0 C \\
I^* &= \overline{I} I \\
E^* &= \frac{E}{R_2} \\
p^* &= \rho u_0^2 \rho \\
\alpha &= R_1 / R_2
\end{align*}
\]

(5.6)

This yields a normalized disinfection reaction rate $K$ that is given by

\[
K(r) = \frac{I}{r} \exp \left( - E(r - \alpha) \right) = IF(r).
\]

(5.7)

Here $r$ denotes the scaled radial coordinate, $I$ is the scaled intensity, and the scaled absorbance is denoted by $E$. The susceptibility factor and $R_1$ are
5.2 Basic modelling of the disinfection plant

Included in the overall amplitude of the total disinfection rate that results from the combined scaling factors, as we shall see later on.

Written out in full, the scaled model equations for the fluid flow in cylindrical coordinates are given by (see [43] pp. 59-60) the continuity equation:

\[
\frac{1}{r} \frac{\partial}{\partial r} (rv_r) + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0
\]

(5.8)

and by the conservation of momentum:

\[
\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + v_\theta \frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial z} = -\frac{\partial p}{\partial r} + \frac{1}{Re} \left[ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} (rv_r) \right) + \frac{1}{r^2} \frac{\partial^2 v_r}{\partial \theta^2} - 2 \frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2} \right]
\]

\[
\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta v_\theta}{r} + v_z \frac{\partial v_\theta}{\partial z} = -\frac{1}{r} \frac{\partial p}{\partial \theta} + \frac{1}{Re} \left[ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} (rv_\theta) \right) + \frac{1}{r^2} \frac{\partial^2 v_\theta}{\partial \theta^2} + 2 \frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2} \right]
\]

\[
\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + v_\theta \frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z} = -\frac{\partial p}{\partial z} + \frac{1}{Re} \left[ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} (rv_z) \right) + \frac{1}{r^2} \frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2} \right].
\]

(5.9)

Here \( Re = \frac{u_0 R_2}{\nu} \) is the Reynolds number, which quantifies the ratio of the convective forces over the viscous forces in the flow. We use cylindrical coordinates \((r, \theta, z)\) (cf. Figure 5.1) with velocity components \((v_r, v_\theta, v_z)\) and dimensionless pressure \(p\). The convection-diffusion-reaction equation for the active microorganism concentration can be written as:

\[
\frac{\partial C}{\partial t} = -(v_r \frac{\partial C}{\partial r} + v_\theta \frac{1}{r} \frac{\partial C}{\partial \theta} + v_z \frac{\partial C}{\partial z}) + \frac{1}{Pe} \left[ \frac{1}{r} \frac{\partial}{\partial r} (r \frac{\partial C}{\partial r}) + 1 \frac{\partial^2 C}{\partial \theta^2} + \frac{\partial^2 C}{\partial z^2} \right] - DaKC,
\]

(5.10)

Here, \( Pe = \frac{u_0 R_2}{D} \) is the Peclet number, which, analogous to \( Re \), measures the relative importance of the convective mass transfer over the diffusive mass transfer. \( Da = \varepsilon \alpha R_2 / u_0 \) is the Damköhler number, which indicates the chemical reaction rate relative to the convective mass transfer rate. The description of the fluid flow and concentration dynamics given in (5.8)–(5.10) does not allow further analytical treatment in its general form. Therefore, we proceed with analyzing the velocity and concentration profiles by introducing a number of simplifying assumptions. Most importantly, these limit the applicability of the model to laminar flow conditions, as apply, e.g., to UV treatment in cider plants [23]. Further, suitable boundary conditions are chosen, together with the controlled and measured variables.
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5.2.3 Laminar velocity field model

In addition to the laminar flow assumption, we assume that the reactor is very long. This implies that the fluid motion is in the $z$ direction only, i.e., $v_\theta = v_r = 0$. This requires that the Reynolds number is sufficiently small and that the flow has become fully developed before the reactor inlet. In other words, there are no entry effects, and as a consequence the velocity profile depends on $r$ and $t$, and not on $z$ nor on $\theta$. As a result we have $\mathbf{v} = (0, 0, v_z(r, t))$, which satisfies (5.8) and also that $\frac{\partial^2 v_z}{\partial \theta^2} = \frac{\partial^2 v_z}{\partial z^2} = 0$. Therefore, the Navier-Stokes system (5.9) contains only one non-trivial equation. Only the longitudinal momentum equation needs to be retained, yielding

$$\frac{\partial v_z}{\partial t} = -\frac{\partial p}{\partial z} + \frac{1}{\text{Re}} \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial v_z}{\partial r} \right) \right].$$

(5.11)

Taking the derivative of this equation with respect to $z$, and using the fact that $v_z$ does not depend on $z$, yields that $\frac{\partial p}{\partial z}$ is independent of $z$. Since the pressure does not depend on $r$ or $\theta$, the pressure derivative depends on $t$ only. We set

$$-\frac{\partial p}{\partial z}(t) = \beta(t).$$

(5.12)

Further we assume no-slip boundary conditions at the walls, which is the most common choice for incompressible fluid flow,

$$v_z(\alpha) = v_z(1) = 0.$$  

(5.13)

This completes the laminar fluid-mechanics description of the reactor flow.

5.2.4 Convection-diffusion-reaction equation

Since the velocity field, the UV-radiation field, the initial condition, and the geometry are assumed to be independent of $\theta$, the concentration of active microorganisms will also not depend on $\theta$. Hence, (5.10) becomes

$$\frac{\partial C}{\partial t} = -v_z \frac{\partial C}{\partial z} + \frac{1}{\text{Pe}} \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial C}{\partial r} \right) + \frac{\partial^2 C}{\partial z^2} \right] - \text{Da}KC.$$

(5.14)

To complete the formulation for $C$ we introduce boundary conditions in the $z$ and $r$-direction. First, we assume that the concentration at the inlet is well-mixed, making it a function of $t$, but not of $r$. Moreover, we assume that there is no concentration gradient in $z$ at the inlet. This corresponds with the absence of UV radiation for $z \leq 0$. Thus

$$C(0, r, t) = C_0(t), \quad \frac{\partial C}{\partial z}(0, r, t) = 0.$$  

(5.15)
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At the walls the velocity is zero and the only mass transport comes from diffusion. The diffusion is small compared to the chemical reaction rate if \( \frac{Da}{Pe} \ll 1 \). We restrict ourselves to such cases and assume that the concentration at the walls is zero, as the radiation will have sufficient time to eliminate the active microorganisms:

\[
C(z, \alpha, t) = C(z, 1, t) = 0.
\]

This completes the formulation for the dynamics of the microorganism concentration. Recapitulating, the basic model describing the flow and concentration inside the reactor is

\[
\frac{\partial v_z(r, t)}{\partial t} = \beta(t) + \frac{1}{Re} \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial v_z(r, t)}{\partial r} \right) \right]
\]

\[
v_z(\alpha, t) = v_z(1, t) = 0
\]

\[
\frac{\partial C(z, r, t)}{\partial t} = -v_z(r, t) \frac{\partial C(z, r, t)}{\partial z} + \frac{1}{Pe} \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial C(z, r, t)}{\partial r} \right) + \frac{\partial^2 C(z, r, t)}{\partial z^2} \right] - DaK(r, t)C(z, r, t)
\]

\[
C(0, r, t) = C_0(t), \quad \frac{\partial C}{\partial z}(0, r, t) = C(z, \alpha, t) = C(z, 1, t) = 0.
\]

where \( v_z \) is a function of \( r \) and \( t \), and \( C \) a function of \( z \), \( r \) and \( t \). In practical UV-treatment, control is exerted with \( I(t) \), the intensity of the lamp, see (5.7). The average concentration at the outflow is a natural monitoring parameter that needs to be controlled. Here, we define the average concentration in terms of the total outflow of microorganisms relative to the average velocity of the liquid. This is defined by

\[
\bar{C}(L, t) = \frac{\int_0^1 C(L, r, t)v_z(r)rdr}{\int_0^1 v_z(r)rdr}.
\]

Here, \( L \) is the scaled length of the reactor. The control will be aimed at reducing \( \bar{C}(L, t) \) below a pre-set acceptance level, by adjusting the input \( I \). We turn to this in the next section.

5.2.5 Analytical solutions for special cases

To get some insight into the disinfection process, the laminar flow and corresponding concentration profiles are analyzed next. For a reactor with a constant flow rate, i.e., \( \frac{\partial v_z}{\partial t} = 0 \) and \( \beta \) constant, the solution of the differential equation for the velocity in (5.17) is given by

\[
v_z(r) = -\frac{\beta \text{Re}}{4} \left[ r^2 + \frac{1 - \alpha^2}{\ln(\alpha)} \ln(r) - 1 \right].
\]
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This shows a characteristic quadratic profile, reminiscent of the Poiseuille profile, with logarithmic corrections arising from the UV lamp along the center of the cylinder. After choosing the parameters from Table 5.1 for the cider plant in section 5.2.7, we get \( \beta \text{Re} = 41.5 \). The parameter \( \alpha \) can be seen as a design parameter, which is for example small for a wide reactor with a thin lamp. Figure 5.2 shows the velocity profile for different values of \( \alpha \). The velocity profile decreases strongly with \( \alpha \). The shape of the profile is more or less parabolic as in an ordinary tube flow. The peak shifts a little bit from the center of the radius for lower \( \alpha \). Using this velocity profile,

![Velocity profile](image)

Figure 5.2: The velocity profile \( v_z(r) \) for different values of \( \alpha \), with \( \beta \text{Re} = 1.16 \times 10^3 \).

we analyze the concentration of active microorganisms next. As mentioned above, we restrict to cases in which diffusive transport is negligible compared to convection and UV irradiation, or equivalently, \( \text{Pe} \gg 1 \) and \( \text{Pe} \gg \text{Da} \). For the particular case of UV disinfection in a cider plant \([23]\), this is the case, and in section 5.2.6 the influence of mass diffusion is shown to be negligible by numerical analysis. This motivates neglecting mass diffusion. By doing so, the third equation in (5.17) becomes

\[
\frac{\partial C}{\partial t} = -v_z \frac{\partial C}{\partial z} - \text{Da}K C
\]

\[
C(0, r, t) = C_0(t),
\]

which is a standard convection-reaction process. Note that with \( v_z(r) \) from (5.19) the boundary conditions in (5.17) are satisfied. The general solution
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to equation (5.20) is,

\[ C(z, r, t) = g(r, z - v_z(r)t) \exp \left( - Da \int_0^t K(r, \tau) d\tau \right). \tag{5.21} \]

The function \( g \) is determined by the initial condition \( C(0, r, t) = C_0(t) \), which gives the equation

\[ g(r, -v_z(r)t) = C_0(t) \exp \left( Da \int_0^t K(r, \tau) d\tau \right) \tag{5.22} \]

\[ g(r, \tilde{t}) = C_0(-\frac{\tilde{t}}{v_z(r)}) \exp \left( Da \int_0^{-\frac{\tilde{t}}{v_z(r)}} K(r, \tau) d\tau \right), \]

with \( \tilde{t} = -v_z(r)t \). If we know \( C_0(t) \) for \( t > 0 \), we know \( g(r, \tilde{t}) \) for \( \tilde{t} < 0 \). A suitable initial condition would be \( C(z, r, 0) = 0 \). Substituting this in (5.21) gives \( g(r, \tilde{t}) \) for \( t > 0 \). So if we define \( C_0(t) = 0 \) for \( t < 0 \), then \( g(r, z - v_z(r)t) \) is given by

\[ g(r, z + \tilde{t}) = C_0\left( -\frac{z + \tilde{t}}{v_z(r)} \right) \exp \left( Da \int_0^{-\frac{z + \tilde{t}}{v_z(r)}} K(r, \tau) d\tau \right). \tag{5.23} \]

Inserting this in (5.21), gives the solution. We are interested in the concentration in \( L \), and the solution in \( z = L \) is

\[ C(L, r, t) = C_0(t - \frac{L}{v_z(r)}) \exp \left( - Da \int_{t - \frac{L}{v_z(r)}}^t K(r, \tau) d\tau \right). \tag{5.24} \]

This expresses the instantaneous solution \( C(L, r, t) \) in terms of the inflow-value at a previous time \( t - t_r(r) \) where the residence time \( t_r = L/v_z(r) \). In addition, the accumulated effect of the UV irradiation at distance \( r \), acquired during a time-interval \( t_r \) is expressed by the exponential. To simplify the analysis, assume that \( C_0 \) and \( K \) do not depend on \( t \). Expression (5.24) then becomes

\[ C(L, r) = C_0 \exp \left( - \frac{DAK(r)L}{v_z(r)} \right). \tag{5.25} \]

This is plotted on a logarithmic scale for different values of \( \alpha \) in Figure 5.3. The physical parameters are the same as in the nominal model that will be introduced in section 5.2.7. We observe that close to the boundaries \( r = \alpha \) and \( r = 1 \) the concentration drops strongly - this is associated with the very long residence time of microorganisms that enter the reactor close to a wall. Conversely, only the 'inner' region of the reactor contributes significantly to the outflow of still active microorganisms. Hence, especially for quite large values (\( \alpha > 1/3 \)), the contribution of \( C(L, r) \) to \( \overline{C}(L) \) is significant only
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in a small part of the reactor. Expressions (5.18) and (5.25) illustrate that \( \overline{C}(L) \) is influenced strongly by the fluid particles for which the residence time \( L/v_z(r) \) is the smallest. A realistic value is \( \alpha > 1/3 \), see for example [79] where \( \alpha = 0.4 \) is used.

![Figure 5.3: Concentration profile of \( C(L,r) \) for different values of \( \alpha \). The relevant parameter values are listed in Table 5.1.](image)

5.2.6 Numerical analysis of the basic model

In this section the influence of mass diffusion is investigated numerically for the steady state form of the basic model (5.17). For that, we define a discrete update scheme and consequently we determine an appropriate grid. The parameter values that are used are the same as in the nominal model of section 5.2.7, and they are listed in Table 5.1. The discrete update scheme
for the concentration plus boundary conditions is

\[
\frac{v_j C_{j,k+1} - C_{j,k}}{\Delta z} = \frac{1}{\text{Pe} \Delta r^2 r_j} \left( (r_j + \frac{1}{2} \Delta r) C_{j+1,k} - 2 r_j C_{j,k} + (r_j - \frac{1}{2} \Delta r) C_{j-1,k} \right) \\
+ \frac{1}{\text{Pe} \Delta z^2} \left( C_{j,k+1} - 2 C_{j,k} + C_{j,k-1} \right) - \text{Da} K_j C_{j,k}
\]

\[\begin{align*}
C_{n_r,k} &= C_{0,k} = 0 \\
C_{2..n_r-1,0} &= C_{0} \\
C_{n_r} &= \frac{\sum_{j=1}^{n_r-1} \left( C_{n_z,j} v_j r_j + C_{n_z,j+1} v_{j+1} r_{j+1} \right) \Delta r}{\sum_{j=1}^{n_r-1} \left( v_j r_j + v_{j+1} r_{j+1} \right) \Delta r},
\end{align*}\]

with \( r_j = j \Delta r \). Here, \( C_{j,k} = C(j \Delta r, k \Delta z) \). The subscript \( j = 1 \ldots n_r \) denotes the discrete radial space with mesh-size \( \Delta r \), and \( k = 1 \ldots n_z \) denotes the discrete longitudinal space with mesh-size \( \Delta z \). A central discretisation in \( r \) and \( z \) is used, except for the convective term, which has an upwind discretisation. For the initial condition an Euler step is used. Further, \( C = C_0 \) at the inlet, except for the walls, where \( C \equiv 0 \). \( \bar{C}(L) \) is computed with the trapezoidal integration rule. The velocity profile is taken from the model in (5.17). Plot (a) in Figure 5.4 shows \( \bar{C}(L) \) as a function of the number of grid points in \( z \)-direction, \( n_z \), with a fixed value of \( n_r = 15 \). A sufficient convergence is attained for \( n_z = 10^4 \), since for higher values of \( n_z \) the average concentration is more or less the same. Plot (b) in Figure 5.4 shows \( \bar{C}(L) \) as a function of the number of grid points in \( r \)-direction, \( n_r \), with a fixed value of \( n_z = 10^4 \). A sufficient convergence is attained for \( n_r = 15 \). Altogether, a sufficient fine grid consists of \( n_z \times n_r = 10^4 \times 15 \) grid points, and this is the grid that will be used from now on. The resolution of the grid in \( z \)-direction is three orders higher than in \( r \)-direction. This is partially explained as follows. The convective term is discretised as

\[
v_j \frac{\partial C_{j,k}}{\partial z} = v_j \frac{C_{j,k+1} - C_{j,k}}{\Delta z} + v_j \frac{\Delta z}{2} \frac{\partial^2 C_{j,k}}{\partial z^2} + O(\Delta z^2),
\]

where the second r.h.s. term is the discretisation error of order \( \Delta z \). The solution of \( C \) without mass diffusion is given by equation (5.25). Inserting its second derivative gives an error of

\[
\frac{\Delta z}{2v_j} (\text{Da} K_j)^2 C_0 \exp \left( - \frac{\text{Da} K_j z}{v_j} \right),
\]

which grows infinitely large for \((z, r) \to (0, \alpha)\) and \((z, r) \to (0, 1)\), since \( v = 0 \) at the walls. However, it was shown in Figure 5.3 that the concentration near the wall has a very small influence on the average concentration. For the bulk of the flow, the error is maximal in \( z = 0 \), and in the order of
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Figure 5.4: The numerical solution of $\overline{C}(L)$ as a function of $n_z$ (a) with $n_r = 15$, and as a function of $n_r$ (b) with $n_z = 10^4$.

$\Delta z \approx 10^2$, which means that the grid is sufficient fine if $\Delta z \ll 10^{-2}$, and since $n_z \Delta z = L$, this means that $n_z \gg 10^2$. The discretisation error in radial direction is caused by the discretisation of the diffusive term, which has an error of

$$\frac{1}{Pe} \frac{\Delta r^2}{6} \frac{\partial^3 C_{j,k}}{\partial r^3}.$$  \hspace{1cm} (5.29)

Since this decreases quadratically with $\Delta r$, and because $\frac{1}{Pe}$ is very small, this gives an indication on why a coarse grid in $r$-direction is sufficient.

Figure 5.5 shows a contour plot of $C$ for a stationary flow in the domain
5.2 Basic modelling of the disinfection plant

$r \in [\alpha, 1]$, and $z \in [0, L]$. The lamp is placed on the bottom horizontal axis. The relatively large distances between the contour lines indicate a small concentration gradient in the center, caused by the high convection. As a contrast, the concentration gradient is very steep near the walls at the inlet.

We now investigate the influence of mass diffusion. The significance of mass diffusion depends on the relative contributions of reaction and mass diffusion, indicated by $Da$ and $Pe$. We define $\overline{C}_d(L)$ and $\overline{C}(L)$ as the average concentrations at $L$ with and without mass diffusion, respectively. Figure 5.6 shows $\frac{\overline{C}(L)_d-\overline{C}(L)}{\overline{C}(L)}$ as a function of $Pe$ and $Da$. In our case, $Pe = 3.6 \times 10^5$ and $Da = 5.6$, and the difference is smaller than 1%. In addition to the arguments in section 5.2.5, this motivates the choice of discarding the term $\frac{1}{Pe} \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial C(z,r,t)}{\partial r} \right) + \frac{\partial^2 C(z,r,t)}{\partial z^2} \right]$ from (5.17) in the following.

5.2.7 The nominal model

Using the results of the previous sections, we obtain a model for an apple cider plant. The dimensionless constants for this plant are

$$Pe = 3.5 \times 10^5, \; Da = 5.8, \; Re = 1.3 \times 10^3, \; \text{and} \; \alpha = 0.5,$$  

(5.30)
5 Modelling and controller design for a UV disinfection plant

![Isoines of the percentage difference](image)

Figure 5.6: Isolines of the percentage difference $\frac{C(L)_{d} - C(L)}{C(L)}$ for different values of Pe and Da.

and they are determined by using parameter values specific for apple cider, [23]. In Table 5.1 all the physical constants are listed \(^1\). From section 5.2.1

<table>
<thead>
<tr>
<th>$R_1$</th>
<th>0.2 m</th>
<th>$R_2$</th>
<th>0.4 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E^*$</td>
<td>1.1 m(^{-1})</td>
<td>$\epsilon$</td>
<td>0.2624 m(^2)J(^{-1})</td>
</tr>
<tr>
<td>$\frac{dp^*}{dz}$</td>
<td>-0.9 kgm(^{-2})s(^{-2})</td>
<td>$\eta$</td>
<td>3 (10^{-2}) kgm(^{-1})s(^{-1})</td>
</tr>
<tr>
<td>$I^*$</td>
<td>10.5 Wm(^{-2})</td>
<td>$C_0^*$</td>
<td>(10^7) m(^{-3})</td>
</tr>
<tr>
<td>$C_p$</td>
<td>5.11 (10^3) Jkg(^{-1})K(^{-1})</td>
<td>$\lambda$</td>
<td>0.55 Wm(^{-1})K(^{-1})</td>
</tr>
<tr>
<td>$L^*$</td>
<td>1 m</td>
<td>$u_0$</td>
<td>9.4 (10^{-2}) ms(^{-1})</td>
</tr>
</tbody>
</table>

Table 5.1: Physical parameters of the apple cider plant.

we have that $Da = \epsilon \bar{T} \alpha R_2 / u_0$, where $\bar{T}$ is the average lamp intensity. The light intensity is determined such that it gives a '5 log reduction' of the inlet concentration of the Escherichia coli 0157:H7 bacteria, i.e., $C(L) = 10^{-5}C_0$. Furthermore, we assume that the flow rate through the reactor is constant. By section 5.2.5 this implies that $v_z$ is given by (5.19). Combining (5.7),

\(^1\)The viscosity $\eta$ is a factor 10 higher than that of water. However, this is the value that is mentioned several times in [23], so we assume this to be a realistic value.
5.3 Model reduction for the nominal model

(5.19) and (5.20), we obtain the following nominal model

\[ v_z(r) = -\frac{\beta \text{Re}}{4} \left[ r^2 + \frac{1 - \alpha^2}{\ln(\alpha)} \ln(r) - 1 \right] \]

\[ \frac{\partial C(z, r, t)}{\partial t} = -v_z(r) \frac{\partial C(z, r, t)}{\partial z} - Da I(t)f(r)C(z, r, t) \]

\[ C(0, r, t) = C_0(t). \]  

(5.31)

In this model, we regard \( I(t) \) as the control input, and \( C_0(t) \) as the disturbance. The measurement is given by (5.18).

5.3 Model reduction for the nominal model

In the nominal model we see that our control input \( I(t) \) gets multiplied with the state \( C(z, r, t) \), and so it is a non-linear model. Furthermore, it is a distributed parameter model. Using linearization and reduction via balancing [113], we obtain a first order linear model for which the control design is easy.

Since the nominal model is given in the scaled variables, we have as reference values \( C_0^{\text{ref}} = 1, I^{\text{ref}} = 1 \). For these reference values, the steady state solution of (5.31) gives the reference concentration profile

\[ C^{\text{ref}}(z, r) = C_0^{\text{ref}} \exp \left( -\frac{Da f(r)z}{v_z(r)} \right). \]  

(5.32)

Defining \( I(t) = I^{\text{ref}} + I^{\text{var}}(t) \), \( C_0(t) = C_0^{\text{ref}} + C_0^{\text{var}}(t) \), and \( C(z, r, t) = C^{\text{ref}}(z, r) + C^{\text{var}}(z, r, t) \), and linearizing (5.31) around the reference concentration, gives the following p.d.e. for \( C^{\text{var}} \)

\[ \frac{\partial C^{\text{var}}(z, r, t)}{\partial t} = -v_z(r) \frac{\partial C^{\text{var}}(z, r, t)}{\partial z} - Da I^{\text{var}}(t)f(r)C^{\text{ref}}(z, r) - Da f(r)C^{\text{var}}(z, r, t) \]

\[ C^{\text{var}}(0, r, t) = C_0^{\text{var}}(t). \]  

(5.33)

We use this to obtain the transfer functions from input and disturbances to the state. Let \( \hat{x} \) denote the Laplace transform of the variable \( x \). Then after Laplace transformation with respect to time, equation (5.33) becomes

\[ s\hat{C}^{\text{var}}(z, r, s) = -v_z(r) \frac{\partial \hat{C}^{\text{var}}(z, r, s)}{\partial z} - Da \hat{I}^{\text{var}}(s)f(r)C^{\text{ref}}(z, r) - Da f(r)\hat{C}^{\text{var}}(z, r, s) \]

\[ \hat{C}^{\text{var}}(0, r, s) = \hat{C}_0^{\text{var}}(s). \]  

(5.34)
5 Modelling and controller design for a UV disinfection plant

The output depends on the concentration at \( z = L \), see (5.18), and we find

\[
\hat{C}_{\text{var}}(L, r, s) = \frac{Da \, f(r)}{s} \left[ -\exp \left( -\frac{Da \, f(r) L}{v_z(r)} \right) + \exp \left( -\frac{s + Da \, f(r)}{v_z(r)} L \right) \right] \hat{v}_{\text{var}}(s) \\
+ \exp \left( -\frac{s + Da \, f(r)}{v_z(r)} L \right) \hat{C}_{\text{var}}(s) \\
= \tilde{G}_1(r, s) \hat{v}_{\text{var}}(s) + \tilde{G}_2(r, s) \hat{C}_{\text{var}}(s).
\] (5.35)

The transfer function \( \tilde{G}_1(r, s) \) is a non-rational function in \( s \). Hence it is not straightforward to find a balanced realization of it. Therefore, we approximate \( \tilde{G}_1(r, s) \) by a rational function. For fixed \( r \) we use the first order Padé approximation, i.e.,

\[
\tilde{G}_1(r, s) \approx \frac{\tilde{G}_1(r, 0)^2}{-\frac{\partial \tilde{G}_1(r, 0)}{\partial s} s + \tilde{G}_1(r, 0)} = \frac{Da \, f(r) C_{\text{ref}}(L, r)}{s + \frac{2v_z(r)}{L}}.
\] (5.36)

For the transfer function from \( I_{\text{var}}(t) \) to \( C_{\text{var}}(L, t) \) we obtain the following approximation, see (5.18)

\[
\tilde{G}_1(s) = \left( \int_\alpha^1 \frac{Da \, f(r) C_{\text{ref}}(L, r)}{s + \frac{2v_z(r)}{L}} v_z(r) rdr \right) / \left( \int_\alpha^1 v_z(r) rdr \right).
\] (5.37)

A state space realization of this is given by the parameterized ode

\[
\frac{dx(r, t)}{dt} = a(r) x(r, t) + b(r) I_{\text{var}}(t) \\
C_{\text{var}}(L, t) = \frac{1}{\bar{v}_r} \int_\alpha^1 x(r, t) v_z(r) rdr,
\] (5.38)

where

\[
a(r) = -2 \frac{v_z(r)}{L}, \quad b(r) = Da \, f(r) C_{\text{ref}}(L, r) \quad \text{and} \quad \bar{v}_r = \int_\alpha^1 v_z(r) rdr.
\] (5.39)

Since the state \( x \) at time \( t \) is a function of \( r \), this is an infinite-dimensional system. Next, we approximate this by a finite-dimensional one. Therefore we discretize \( x(r, t) \) with respect to \( r \). We introduce the uniform grid for \( r \), \( r_j = \alpha + j\Delta r \). The new state vector is \( x(t) = [x(r_1, t) \ldots x(r_n, t)]^T \), and the model becomes

\[
\frac{dx}{dt}(t) = Ax(t) + BI_{\text{var}}(t) \\
C_{\text{var}}(L, t) = Cx(t)
\] (5.40)
5.3 Model reduction for the nominal model

with

\[ A = \text{diag}[a(r_1), \ldots, a(r_n)] \]
\[ B = [b(r_1), \ldots, b(r_n)]^T \]
\[ C = \frac{1}{\nu_T} \left[ \frac{v(r_1)r_1}{2}, \frac{v(r_2)r_2}{2}, \ldots, \frac{v(r_{n-1})r_{n-1}}{2}, \frac{v(r_n)r_n}{2} \right]. \]

Matrix \( C \) is obtained by trapezoidal integration. For this model we calculate the Hankel singular values, see Figure 5.7. When the first singular value dominates the others, the model may be truncated down to one state [113]. Since the first state is much larger than the second by a factor 10^3, we may truncate (5.40) down to the first order differential equation

\[
\frac{d\hat{C}_{\text{var}}(L, t)}{dt} = A_{\text{red}}\hat{C}_{\text{var}}(L, t) + B_{\text{red}}I_{\text{var}}(t). \tag{5.41}
\]

Using the constants of our nominal model, steps (5.31)–(5.41) lead to \( A_{\text{red}} = -1.2, B_{\text{red}} = -7.3 \times 10^{-3} \).

To see whether all approximations have not discarded any essential dynamics, the Bode plot of (5.41) is compared to that of system \( \tilde{G}_1(s) \) in (5.35) with output (5.18), see Figure 5.8. It is clear that (5.41) is a crude approximation of (5.35) with output (5.18) for large \( \omega \). More importantly, it captures the first order dynamics of the model, characterized by the static gain and the time constant. Hence, we use (5.41) as our model for controller design. Using (5.35), (5.18), and (5.41) we have the following model in the s-domain

\[
\hat{C}_{\text{var}}(L, s) = \frac{B_{\text{red}}}{s - A_{\text{red}}} \hat{I}_{\text{var}}(s) + \frac{1}{\nu_T} \int_{\alpha}^{1} \tilde{G}_2(r, s)v_z(r)rdr\hat{C}_{0\text{var}}(s) \tag{5.42}
\]
\[ := G_1(s)\hat{I}_{\text{var}}(s) + \tilde{G}_2(s)\hat{C}_{0\text{var}}(s). \]

Figure 5.7: The Hankel singular values of the laminar flow model without diffusion.
Figure 5.8: Gain $|\tilde{G}_1(s)|$ and phase arg($\tilde{G}_1(s)$) (in radians) plot of $G_1(s)$ in model (5.35) (dashed line), and of model (5.41) (solid line). Here, $s = i\omega$.

The approximation of $\tilde{G}_2(s)$ is made as follows. The amplitude Bode plot of $\tilde{G}_2(s)$ is flat. Hence we approximate it by a pure time delay,

$$
\tilde{G}_2(s) \approx G_2(s) = c \exp(-\tau s).
$$

with $c = \tilde{G}_2(0)$, and $\tau = -\frac{1}{\omega} \frac{d\tilde{G}_2}{ds}(0)$. Figure 5.9 shows the phase and gain plots of $\tilde{G}_2(s)$ and $G_2(s)$. The gain as well as the phase of $G_2(s)$ matches that of $\tilde{G}_2(s)$ indistinguishably.
5.4 Controller design

For the approximate model in the $s$-domain (see (5.42) and (5.43))

$$\hat{C}_{var}(L, s) = G_1(s)\hat{I}_{var}(s) + G_2(s)\hat{C}_{0\, var}(s)$$  \hspace{1cm} (5.44)

we design a controller. This controller will be tested on our nominal model (5.31) with output equation (5.18).

Since no dynamic properties of $C_{0\, var}(t)$ are known, and since $G_2(s)$ is a
pure time delay, we regard the term \( G_2(s)C_0^{\text{var}}(s) \) as delayed white noise. Since the dynamics of \( I^{\text{var}} \) to \( C^{\text{var}} \) is of first order, we design a simple PI controller. The controller should meet the following (standard) design specifications (see also [51]):

- Up to some (later specified) crossover frequency, the sensitivity function \( S(s) \), should be small, in order to attenuate disturbances, get good command response, and have robustness at low frequencies.

- As a consequence, \( T(s) \) should be close to one for low frequencies, and \( S(s) \) should be close to one for high frequencies. Here, \( S \) is defined as \( \frac{1}{1+L} \), with \( L = G_1K \), and \( T = \frac{L}{1+L} \).

Input disturbances with a higher frequency than the crossover frequency are already attenuated by \( G_1 \). Therefore, the ideal crossover frequency of \( S \) and \( T \) equals the crossover frequency of \( G_1 : -A_{\text{red}} \). In other words, we have to find a controller \( K \) such that \( T = \frac{1}{1+s} \), with \( s = \frac{s}{A_{\text{red}}} \). Solving

\[
T = \frac{L}{1+L} = \frac{1}{1+s}, \tag{5.45}
\]

with \( L = KG_1 \), gives the PI controller

\[
K(s) = \frac{k_1 + s}{k_2s}, \tag{5.46}
\]

with \( k_1 = -A_{\text{red}} \) and \( k_2 = \frac{-B_{\text{red}}}{A_{\text{red}}} \). Figure 5.10 shows the control loop schematically.

### 5.4.1 Savings

We now give an idea of the possible economic savings by means of a short illustration. We assume a fixed \( r \) for convenience, and rewrite equation (5.25) to

\[
C(L,t) = C_0(t) \exp(-K(t)g(r)), \tag{5.47}
\]

with \( g(r) = \frac{DaL}{v_s(r)} \). Further, we impose a sinusoidal disturbance in the input: \( C_0(t) = C_0^*(1 + A \sin(t)) \). We assume that the controller is fast enough such that \( C(L,t) \) is kept constant in time. This leads to the controlled input \( K(t) = -1/g \ln(C(L)/C_0(t)) \). An uncontrolled input has to be set to the worst case value that corresponds to the highest input concentration: \( K_2 = -1/g \ln(C(L)/C_0^*(1 + A)) \). This gives the percentage difference between the average of the two inputs over time

\[
S = \frac{1}{2\pi} \int_0^{2\pi} \frac{K_2 - K(t)}{K_2} dt, \tag{5.48}
\]
5.4 Controller design

Figure 5.10: Schematic representation of the controlled model.

which is independent of $g$. In figure 5.11 $S$ is plotted as a function of $A$ for two cases, namely $C(L) = 10^{-3}C_0^*$, and $C(L) = 10^{-8}C_0^*$, i.e. a 3 log and a 8 log reduction. We restrict ourselves to the case that $A < 1$, such that the input concentration cannot become negative. Not only does the input difference increase with $A$, as could be expected, but it also increases as $C(L)$ increases. The differences can be translated into economic savings, since $K$ is related linearly to the energy usage in $J/s$.

5.4.2 Simulations

Simulation studies are conducted to see whether the reduced model (5.41) is a sufficiently accurate approximation of the nominal model (5.31). This is done by connecting the controller to both models. The nominal model is solved numerically by a forward Euler method in time, and an upwind scheme in space. This gives the update scheme

$$\frac{C_{k,j}^{n+1} - C_{k,j}^n}{\Delta t} = -v_j \frac{C_{k-1,j}^n - C_{k+1,j}^n}{\Delta z} - C_{k,j}^n D_\alpha I^n f_j.$$

(5.49)

Here $v$ is solved analytically in equation (5.19). Similarly, the update scheme for the controller (5.46) is, with the reference mean concentration $\bar{C}(L)^{ref}$ defined as in (5.18), with $C = C^{ref}$,

$$\tilde{I}_0^{n+1} = -\frac{d^{n+1}}{k_2} - \frac{1}{k_2} (\bar{C}(L)^n - \bar{C}(L)^{ref})$$

$$\frac{d^{n+1} - d^n}{\Delta t} = k_1^n (\bar{C}(L)^n - \bar{C}(L)^{ref}).$$

(5.50)
5 Modelling and controller design for a UV disinfection plant

Figure 5.11: The percentage in input saving \( S \), as a function of \( A \). Plot (a): \( C(L) = 10^{-3}C_0^* \), and plot (b): \( C(L) = 10^{-8}C_0^* \).
5.5 Conclusions

Here, $n$, $j$ and $k$ denote the discrete time, radial space, and longitudinal space respectively. The reduced model is a first order ode, and this is simulated within the Matlab Simulink environment. To visualize the difference in output dynamics clearly, $C_0(t)$ is disturbed by $A \sin(\omega t)$, with an amplitude of $A = 0.5$ and a frequency of $\omega = 0.8$. For this frequency, the error in gain in Figure 5.8 is relatively large. The first plot in Figure 5.12(a) shows the concentration $C(L,t)$ for the reduced model (5.41) and the nominal model (5.31). Initially, the difference between the two models is large. This is caused by the transient behavior of the nominal model. The small initial concentration near the inlet is decreased further by the lamp, resulting in a smaller and smaller $C(L)$. After the transient dynamics has gone, the outputs of the nominal and the reduced model match very well. Further increase of $A$ and $\omega$ leads to larger errors. To illustrate this, the amplitude and the frequency of the input disturbance are increased to 0.9 and 2 respectively. The dynamics is shown in Figure 5.12(b). The higher amplitude causes a larger approximation error in (5.33), and the high frequency causes a larger phase error in Figure 5.8.

5.5 Conclusions

A basic model was developed, describing the fluid and concentrations dynamics inside an annular disinfection plant. It was shown by the analysis of the Hankel singular values, and the Bode plots, that the essential dynamics of an apple cider plant can be approximated accurately by a first order model. Consequently, a simple classical model-based controller with excellent properties can be designed. This was further confirmed by a simulation study with significant disturbances.

Up to the balancing, the formulas are analytical and contain all the physical properties of the nominal model. This allows a clear analysis, like in section 5.2.5. For more complex models, for example with an irregular geometry, or without discarding mass diffusion as was done here, the reduction steps as well as the model simulations can become numerically more involved, since an analytical linearization may not be possible. These practical drawbacks leave the door open for alternative design methods, such as nonlinear control or the method that is developed in the next chapter. The next fundamental steps would be to validate the basic model experimentally, and to check the controller performance in a real-life situation.
Figure 5.12: The dynamics of the average concentration $\overline{C}(L, t)$ and the reduced order linearized model (5.41), and the nonlinear nominal model (5.31). Left: $A = 0.5$ and $F = 0.8$. Right: $A = 0.9$ and $F = 2$. 
6

Modelling via residence time distribution

6.1 Introduction

The control of outlet concentration in chemical plants is an important topic in process engineering. For classical PI types of controllers that are not model based, the adjustment is made manually. For a reactor with slow system dynamics this requires a long tuning process. Moreover, without a model, a controller structure has to be chosen ad hoc, which obviously does not guarantee optimal or even good performance. It is therefore desirable to have model based control, since this allows a direct transfer of physical system properties into the controller design. However, the design of a model based controller for a reactor can be difficult due to model complexities that are caused by a flow field, and thermal and reaction processes.

Often, these processes lead to CFD models that require millions of states to accurately describe the process behavior. This number can be reduced, and there are some promising results on model reduction for various applications, see for example [7,8,38] and [6,73,100] for applications to thin film deposition processes. Nevertheless, simulation and control optimization still require a lot of computational time and modelling effort. In addition to these methods, there are mathematically more standard techniques, such as linearization [70], singular perturbation theory [47], transfer function approximation [5], and input output balancing [113]. These are well documented methods, but have a limited applicability. For example, the latter two only apply to linear systems.

A method to avoid the complexities of physical modelling, is to construct a model that is based on a residence time distribution. Such a distribution is obtained by measuring the output concentration of the fluid after a pulse in the input concentration. The resulting distribution gives information on how long particles reside inside the reactor. For systems without a chemical reaction or production, the residence time distribution can be seen as an
6 Modelling via residence time distribution

impulse response from input concentration to output concentration. The development of a model from an impulse response is standard in systems theory, see for example [70]. In [49] an alternative theory is developed, and in [1,80] numerical algorithms are developed for the realization of finite order linear systems. In chemical engineering, modelling via the residence time distribution is a standard procedure, see e.g. [34]. A drawback is that the obtained model holds no physical information of the system, which makes it hard to make predictions for configurations other than the experimental setup. A benefit of this method is that, besides avoiding the modelling issues, the realized system is in state space form. This makes it suitable for standard linear control design, which generally results in high performance control and relatively easy design. Surprisingly, it was only since recently that this feature was employed. In [53] the cumulative residence time distribution was used to develop various controllers for reactors with first order reaction kinetics and linear dynamics.

In this chapter, a linear realization method is proposed for systems with first order reaction kinetics. The inputs of the model are the inflow concentration and the reaction rate coefficient, and the output is the concentration in the outflow. In contrast to [53], there is a nonlinear relation between an in- and output. Between this input and output, a residence time distribution cannot be measured. Further, the realization may contain a dead time, which is shown to increase the accuracy of the realized model considerably.

This type of system is motivated by a UV disinfection reactor, in which the concentration of microorganisms in a fluid is controlled by UV light. Disinfection by UV irradiation is a common method for purification of drinking water [3,101]. A lot of experimental research is conducted on the effect of UV light on the attenuation of microorganisms [54,55,76,77,112], and on the mathematical modelling of UV reactors [10,13,79,85,107,108]. The disinfection rate can be dynamically adjusted by the intensity of the UV radiation. The reaction rate coefficient as input variable represents also reactors in which the reaction rate is adjusted by temperature or a catalyst.

The organization is as follows. First, a nonlinear convolution model is derived from the residence time distribution between inflow and outflow. Thereafter, the model is linearized and brought into state space form. Then, dead time is incorporated into the modelling procedure. This method is tested on two types of systems. First, for a series of ideally mixed tanks, the residence time distribution is of an analytical form, which enables an exact state space form of a linearized version of the original model. Simulations indicate that the realization method is accurate. Second, for a UV reactor with a dead time, the residence time distribution of the original physical model is approximated with a good fit in the region of low residence times, but a less accurate fit for the higher regions. This results in a model that
6.2 Derivation of the convolution model

We start with a chemical reactor for fluids, in which some component reacts. Our goal is to design a controller for the component that steers the reaction rate coefficient or the inlet concentration. Therefore, we first derive a model, which is in convolution form. We make the following assumptions on the reaction rate.

- The reaction rate is of first order;
- The reaction rate is uniform over the reactor.

This leads to the following first order reaction,

\[
\frac{dC(t)}{dt} = -C(t)K(t),
\]

with \( C \) the concentration of the component and \( K \) the reaction rate coefficient. For simplicity, we assume that \( K \geq 0 \). The solution is

\[
C(t_1) = C(t_0) \exp\left(-\int_{t_0}^{t_1} K(\tau) d\tau\right),
\]

and implies that fluid with residence time \( t_r \) leaves the reactor with the concentration

\[
C_{out}^{t_r}(t_r - t_0) = C_{in}(t_0) \exp\left(-\int_{t_0}^{t_0 + t_r} K(\tau) d\tau\right),
\]

where \( C_{in} \) and \( C_{out} \) denote the ingoing and outgoing concentration, respectively. This is equivalent with

\[
C_{out}^{t_r}(t) = C_{in}(t - t_r) \exp\left(-\int_{t - t_r}^{t} K(\tau) d\tau\right).
\]

The average concentration of the outgoing fluid is \( C_{out}(t) \), and equals the integrand of the concentrations of all the fluid particles with residence time \( t_r \), weighed with the residence time distribution \( \rho(t_r) \)

\[
C_{out}(t) = \frac{\int_0^\infty C_{out}^{t_r}(t) \rho(t_r) dt_r}{\int_0^\infty \rho(t_r) dt_r}.
\]
6 Modelling via residence time distribution

The distribution $\rho$ can be obtained by creating a pulse in the ingoing concentration, and measuring the outgoing concentration as a function of time. Using (6.4), this gives our convolution model

$$C_{out}(t) = \frac{1}{\alpha} \int_0^\infty C_{in}(t - t_r) \exp(-\int_{t-t_r}^t K(\tau)d\tau) \rho(t_r) dt_r,$$

with $\alpha = \int_0^\infty \rho(t)dt$.

6.2.1 Calibration of $K$

Our goal of modelling via retention time distributions is to design a controller. For example, a controller that steers the reaction rate coefficient $K$. It is not always easy to relate the value of $K$ to the physical output of the controller. For example, for control of the UV disinfection reactor in section 6.6, the relation between $K$ and the power of the UV lamp has to be determined. This is not straightforward, since it depends on many factors, like the clearness of the liquid, the type of microorganisms that have to be attenuated, and the yield of the lamp. Therefore, $K$ has to be calibrated, and this can be done by measuring the residence time distribution for a constant lamp power, and a constant input concentration. As a result, during calibration, the output concentration will also be constant. Equation (6.6) then becomes

$$C_{out}(t) = \frac{1}{\alpha} \int_0^\infty C_{in} \exp(t_r K) \rho(t_r) dt_r,$$

and from this the value of $K$ can be solved (numerically).

6.3 State space realization

In this section, the nonlinear convolution model (6.6) is approximated by means of linearization, in order to enable a linear state space realization. We define

$$C_{in}(t) = C_{in}^* + \tilde{C}_{in}(t)$$
$$C_{out}(t) = C_{out}^* + \tilde{C}_{out}(t)$$
$$K(t) = K^*_0 + \tilde{K}(t),$$

and after inserting this in (6.6) and ignoring the cross-terms with $\tilde{C}_{in}(t)\tilde{K}(t)$, this gives

$$C_{out}(t) \approx \int_0^\infty \frac{1}{\alpha} \tilde{C}_{in}(t - t_r) \exp(-K^*t_r) \rho(t_r) dt_r + \int_0^\infty \frac{1}{\alpha} C_{in}^* \exp(-K^*t_r) \cdot \exp\left(\int_{t-t_r}^t -\tilde{K}(\tau)d\tau\right) \rho(t_r) dt_r.$$
6.3 State space realization

Linearization of the exponential term in the second r.h.s. term gives

\[
\tilde{C}_{out}(t) = \int_{0}^{\infty} \frac{1}{\alpha} \tilde{C}_{in}(t - t_r) \exp(-K^* t_r) \rho(t_r) dt_r + \int_{0}^{\infty} \frac{1}{\alpha} C^*_{in} \exp(-K^* t_r) \left( \int_{t - t_r}^{t} \tilde{K}(\tau) d\tau \right) \rho(t_r) dt_r. \tag{6.10}
\]

The state space realization from input \(\tilde{C}_{in}\) to output \(\tilde{C}_{out}\) is obtained as follows. By change of variables \(z = t - t_r\), the first r.h.s. term in (6.10) is written as

\[
\tilde{C}_{out,1}(t) = -\int_{-\infty}^{-t} \frac{1}{\alpha} \tilde{C}_{in}(z) \exp(-K^*(t - z)) \rho(t - z) dz
\]

\[
= \int_{-\infty}^{t} \frac{1}{\alpha} \tilde{C}_{in}(z) \exp(-K^*(t - z)) \rho(t - z) dz. \tag{6.11}
\]

The second term becomes, after the same change of variables,

\[
\tilde{C}_{out,2}(t) = \int_{t}^{-\infty} \frac{1}{\alpha} C^*_{in} \exp(-K^*(t - z)) \int_{t}^{t} \tilde{K}(\tau) d\tau \rho(t - z) dz
\]

\[
= -\int_{-\infty}^{t} \int_{z}^{t} \frac{C^*_{in}}{\alpha} \exp(-K^*(t - z)) \cdot \tilde{K}(\tau) \rho(t - z) d\tau dz. \tag{6.12}
\]

Changing the order of integration gives

\[
\tilde{C}_{out,2}(t) = -\int_{-\infty}^{t} \int_{-\infty}^{\tau} \frac{C^*_{in}}{\alpha} \exp(-K^*(t - z)) \cdot \rho(t - z) dz \tilde{K}(\tau) d\tau. \tag{6.13}
\]

Now we look for a state space that is equivalent to these convolution forms. Suppose that the argument of the convolution integral in (6.11) is

\[
\frac{1}{\alpha} \exp(-K^*(t - z)) \rho(t - z) = C \exp(A(t - z)B). \tag{6.14}
\]

We assume \(D = 0\) since there is no direct feed-through in a well-designed chemical reactor. The state space realization from input \(\tilde{C}_{in}\) to output \(\tilde{C}_{out,1}\) has the form

\[
\frac{dx(t)}{dt} = Ax(t) + B\tilde{C}_{in}(t)
\]

\[
C_{out,1}(t) = C x(t). \tag{6.15}
\]
6 Modelling via residence time distribution

The integral (6.13) can then be written as

\[
\tilde{C}_{\text{out,2}}(t) = \int_{-\infty}^{t} \int_{-\infty}^{\tau} -C_{\text{in}}^{*} C \exp(A(t-z)) B dz \tilde{K}(\tau) d\tau \quad \text{(by (6.14))}
\]

\[
= \int_{-\infty}^{t} C_{\text{in}}^{*} C A^{-1} \exp(A(t - \tau)) B \tilde{K}(\tau) d\tau,
\]

(6.16)

where we assumed that \( A \) is exponentially stable, which is not a strong assumption. The last equation in (6.13) has a state space realization \((A, B_2, C)\), with \( B_2 = C_{\text{in}}^{*} B A^{-1} \). Written out in full, this is

\[
\frac{dx}{dt}(t) = \begin{pmatrix} A & 0 \\ 0 & A \end{pmatrix} x(t) + \begin{pmatrix} C_{\text{in}}^{*} B A^{-1} & 0 \\ 0 & B \end{pmatrix} \begin{pmatrix} \tilde{C}_{\text{in}}(t) \\ \tilde{K}(t) \end{pmatrix}
\]

\[
\tilde{C}_{\text{out}}(t) = \begin{pmatrix} C \\ C \end{pmatrix} x(t).
\]

(6.17)

To find the matrices \( A, B, C \), that provide a good fit in (6.14), various numerical routines are available, for example from [1,80]. In the next example we shall see that the matrices can also be obtained via an analytical way.

6.4 Example: mixed tanks in series

To test our realization method, we take a model of which the residence time distribution is known analytically. From this distribution and the first order reaction model, a linear state space model is derived. The output dynamics of the realized model and of the original model are compared by simulation. We consider a model of a UV disinfection reactor that consists of \( n \) ideally mixed tanks in series, with total volume \( V \). In each tank, there is a first order reaction as described by equation (6.1). This leads to the following model

\[
\frac{dx(t)}{dt} = F(K(t)) x(t) + G C_{\text{in}}(t)
\]

\[
C_{\text{out}}(t) = H x(t),
\]

(6.18)
6.4 Example: mixed tanks in series

\[
F(K(t)) = \begin{pmatrix}
-K(t) - n/\tau & 0 & \ldots & 0 \\
n/\tau & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & 0 \\
0 & \ldots & n/\tau & -K(t) - n/\tau
\end{pmatrix}
\]

\[
G = \begin{pmatrix}
n/\tau \\
0 \\
\vdots \\
0
\end{pmatrix}
\]

\[
H = \begin{pmatrix}
0 & \ldots & 0 & 1
\end{pmatrix}
\]

\[
x(t) = \begin{pmatrix}
c_1(t) \\
c_2(t) \\
\vdots \\
c_n(t)
\end{pmatrix}^T.
\]

Here \(K(t)\) and \(C_{in}(t)\) are the inputs, \(C_{out}(t)\) is the output, and \(c_i(t)\) are the concentrations in each tank. Further, the subscript \(i\) denotes the tank number, \(\tau = \Phi/V\) is the average residence time, and \(\Phi\) is the volume throughput. Note that the model is nonlinear, since the input \(K\) does not enter the system as in (6.15). The residence time distribution of this system is given by

\[
\rho(t) = \frac{n^n \theta^{n-1}}{(n-1)!} \exp(-n\theta),
\]

(6.19)

with \(\theta = t/\tau\), see also [43]. We continue with this distribution to see if our realization procedure results in an accurate approximation of the original model (6.18). Like in (6.14) we make the realization

\[
\frac{1}{\alpha} \exp(-K^*(t - z)) \rho(t - z) = C \exp(A(t - z))B,
\]

(6.20)

with \(\alpha = \int_0^\infty \rho(t)dt = \tau\) and \(K^*\) a fixed nominal value of \(K\), using the following theory. If, according to [70] p.96,

\[
C \exp(A(t))B = \sum_i \sum_j \beta_{ij} \frac{t^{j-1}}{(j-1)!} \exp(\lambda_i t),
\]

(6.21)

then the transfer function from input to output equals

\[
C(sI - A)^{-1}B = \sum_i \sum_j \frac{\beta_{ij}}{(s - \lambda_i)^j}.
\]

(6.22)

After inserting (6.19) into (6.20), the solution of (6.21) is \(i = j = n\), \(\beta_{nn} = n^n/(\alpha \tau^{n-1})\), and \(\lambda_n = -n/\tau - K^*\), so that

\[
C(sI - A)^{-1}B = \frac{\beta_{nn}}{(s - \lambda_n)^n},
\]

(6.23)
and via standard realization techniques the system \((A, B, C)\) can be obtained, see for example [70] p.215. We note that if we know the model (6.18), we can use this to make the following linear realization, \((A, B, C) = (F(K^*), G, H)\).

### 6.4.1 Simulations

The original system \((F, G, H)\) from (6.18) and the realized system \((A, B, C)\) are simulated numerically with a forward Euler discretisation, while the inputs are disturbed

\[
K(t) = K^* + a_1 \sin(\omega t) \\
C_{in}(t) = C_{in}^* + a_2 \sin(\omega_2 t).
\] (6.24)

The physical parameters are listed in Table 6.1, and are chosen randomly. The disturbances are significantly large. Figure 6.1 shows the outgoing concentration \(C_{out}\) as a function of time. Both systems have more or less the same dynamics under the input disturbances. We conclude that the realization procedure results in an accurate linear approximation of the physical model. The advantage of the system with perfect mixed tanks is that the residence time distribution has an expression that could be incorporated into the realization without approximation errors.

### 6.5 Time delay

There is often a minimal residence time for all the fluid particles. This is denoted by dead time, or \(\tau_d\). Consequently, the residence time distribution is exactly zero on \((0, \tau_d)\). Pure dead time cannot be modelled by a finite dimensional linear state space realization. Therefore, the numerical algorithms of [1,80] that result in such a realization give no guarantee on success. One way to model a residence time distribution with dead time is by choosing a model of a high number of mixed tanks in series. This gives a distribution that is almost zero near \(t_r = 0\), but never exactly. To illustrate that this
6.5 Time delay

Figure 6.1: Dynamics of $C_{out}$ of the original system (–), and of the realized system (− −).

can lead to considerable model errors, we take $K$ constant in (6.3) to get

$$C_{out}^{tr}(t_r) = C_{in}(0) \exp(-K t_r)$$

(6.25)

for fluid particles with residence time $t_r$. Since $K \geq 0$, the particles with the smallest residence time have the largest contribution to $C_{out}$, and can therefore cause large errors if their dynamics is not properly modelled.

This illustration motivates us to derive a model with the property that $\rho = 0$ on $[0, \tau_d]$. First, we model the realization from the input $\tilde{C}_{in}$ to the output $C_{out}$. When $\rho = 0$ on $[0, \tau_d]$, (6.11) becomes

$$C_{out,1}(t) = -\int_{-\infty}^{t-\tau_d} \frac{1}{\alpha} \tilde{C}_{in}(z) \exp(-K^*(t-z)) \rho(t-z) dz.$$  \hspace{1cm} (6.26)

We transform $\tilde{z} = z + \tau_d$ and define $\rho^*(t) = \rho(t + \tau_d)$, so (6.26) becomes

$$C_{out,1}(t) = -\int_{-\infty}^{t} \frac{1}{\alpha} \tilde{C}_{in}(\tilde{z} - \tau_d) \exp(-K^* \tau_d) \exp(-K^*(t - \tilde{z})) \rho^*(t - \tilde{z}) d\tilde{z}.$$  \hspace{1cm} (6.27)

The realization $(A, B, C)$ is found as in (6.14), with (discarding the tilde)

$$\frac{1}{\alpha} \exp(-K^* \tau_d) \exp(-K^*(t-z)) \rho^*(t-z) = C \exp(A(t-z)) B,$$  \hspace{1cm} (6.28)
6 Modelling via residence time distribution

which gives the following state space realization

\[
\frac{dx(t)}{dt} = Ax(t) + B\tilde{C}_{in}(t - \tau_d) \\
C_{out,1}(t) = Cx(t).
\]

(6.29)

Note that \(C_{in}\) has a delay of \(\tau_d\). Next, we look at the state space realization from \(\tilde{K}\) to \(C_{out}\). This is found by starting at (6.12). By using the transformation \(\tilde{z} = z + \tau_d\) and \(\rho^*(t) = \rho(t + \tau_d)\), (6.12) becomes

\[
C_{out,2}(t) = -\int_{-\infty}^{t} \frac{C_{in}^*}{\alpha} \exp(-K^*(t - \tilde{z} + \tau_d))\rho(t - \tilde{z}) \int_{\tilde{z} - \tau_d}^{t} \tilde{K}(\tau)d\tau d\tilde{z}
\]

\[
\approx -\int_{-\infty}^{t} \frac{C_{in}^*}{\alpha} \exp(-K^*(t - \tilde{z} + \tau_d))\rho(t - \tilde{z}) \cdot \left(\int_{\tilde{z}}^{t} \tilde{K}(\tau)d\tau + \tau_d \tilde{K}(\tilde{z})\right) d\tilde{z},
\]

(6.30)

where we approximated \(\tilde{K}\) with a constant on the interval \([0, \tau_d]\). We justify the approximation by assuming that \(\tilde{K}\) moves at a lower frequency than \(1/\tau_d\). Following section 6.3, equation (6.30) becomes

\[
C_{out} = -\int_{-\infty}^{t} C_{in}^* \tau_d C \exp(A(t - z))B\tilde{K}(z)dz + \int_{-\infty}^{t} C_{in}^* CA^{-1} \exp(A(t - z))B\tilde{K}(z)dz,
\]

(6.31)

with \((A, B, C)\) found from (6.27). So (6.31) corresponds to the realization \((A, B, C_2)\) with \(C_2 = C_{in}^* CA^{-1} - C_{in}^* \tau_d C\). Written out, this is

\[
\frac{dx}{dt}(t) = \begin{pmatrix} A & 0 \\ 0 & A \end{pmatrix} x(t) + \begin{pmatrix} B & 0 \\ 0 & B \end{pmatrix} \left(\begin{pmatrix} \tilde{K}(t) \\ \tilde{C}_{in}(t - \tau_d) \end{pmatrix}\right)
\]

\[
\tilde{C}_{out}(t) = \begin{pmatrix} C_{in}^* CA^{-1} - C_{in}^* \tau_d C \\ C \end{pmatrix} x(t).
\]

(6.32)

6.6 Example: UV disinfection reactor

We test the realization method from section 6.5 on a model with a considerable time delay. An example of such a model is an annular UV reactor with a laminar flow. For this model, the residence time distribution is known analytically, and hence we can make a model realization from this distribution. The accuracy of the realization is again tested by simulation of the output dynamics under input disturbances.
We make the following assumptions. The flow field is parabolic. Equivalently, there are no lamps or other obstacles placed inside the reactor. Further, the light intensity, and therefore the reaction rate, is uniform over the fluid domain. In chapter 5 it is shown that this is a reasonable assumption. In the same chapter it is also shown for a reactor with a laminar flow, that the dynamics is mainly linear and of first order. Further, the fluid velocity is constant in time. The corresponding model is given by (see chapter 5 for details)

\[
v(r) = -\frac{\beta}{4}(r^2 - R^2)
\]  

\[
\frac{\partial C(z, r, t)}{\partial t} = -v(r) \frac{\partial C(z, r, t)}{\partial z} - K(t)C(z, r, t)
\]

\[
C_{out}(t) = \frac{\int_0^R C(L, r, t)v(r)rdr}{\int_0^R v(r)rdr}.
\]

Here, \(z \in [0, L]\) and \(r \in [0, R]\) are the longitudinal and radial direction, \(K\) is the reaction rate coefficient caused by the light intensity, \(\beta\) is the pressure gradient, and \(C_{out}\) the total outflow of the microorganisms relative to the average velocity of the fluid. The boundary condition is \(C(0, r, t) = C_{in}(t)\), assuming a radially uniform inflow concentration. Equation (6.34) consists of a first order reaction with a convection term for the fluid flow. Model (6.33)–(6.35) is referred to as the physical model.

We now look at the model realization. According to [43] the laminar tube flow has the following residence time distribution

\[
\rho(t_r) = \begin{cases} 
0 & \text{if } t_r < \tau_d \\
\frac{1}{2\theta^3} & \text{if } t_r \geq \tau_d
\end{cases}
\]

\[
\tau_d = \frac{4L}{\beta R^2}
\]

where \(\tau_d\) is the dead time, and \(\theta = t_r/2\tau_d\). We have two difficulties; first, there is a delay, and second, even if \(\tau_d = 0\) the residence time distribution cannot be written in the form \(C \exp(At_r)B\). Therefore, we have to find an approximate \((A, B, C)\) from (6.28). First we have to approximate \(\rho\) by \(\tilde{\rho}\), and define \(\tilde{\rho}^*(t) = \tilde{\rho}(t + \tau_d)\). We try a first order model, and we do this by using the approximation \(\tilde{\rho}^*(t_r) = b_1 \exp(a_1 t_r)\), and choose \(a_1\) and \(b_1\) such that at \(t_r = \tau_d\) there is an exact match, and that both distributions have the same scale, so

\[
\tilde{\rho}^*(0) = \rho^*(0)
\]

\[
\int_0^\infty \tilde{\rho}^*(t_r)dt_r = \int_0^\infty \rho^*(t_r)dt_r.
\]
This results in $b_1 = 4$ and $a_1 = -4/2\tau_d$. Figure 6.2 shows $\tilde{\rho}^*(t_r)$ and $\rho^*(t_r)$. Near $\tau_d$ the approximation is accurate, but for large $t_r$ the error is considerable. For some $\tilde{K}^*$ that corresponds to $C^*_{\text{out}}$ and $C^*_{\text{in}}$, the realization procedure (6.26)–(6.31) results in the following state space

$$
\frac{dx(t)}{dt} = \begin{pmatrix} A & 0 \\ 0 & A \end{pmatrix} x(t) + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \tilde{K}(t) \\ \tilde{C}_{\text{in}}(t - \tau_d) \end{pmatrix}
$$

$$
\tilde{C}_{\text{out}}(t) = \left( C^*_\text{in} C \frac{1}{A - \frac{4L}{\beta R^2}} \right) x(t),
$$

with

$$
A = -K^* - \frac{\beta R^2}{2L},
$$

$$
C = \frac{\beta R^2}{2L} \exp\left( \frac{-4K^*L}{\beta R^2} \right).
$$

### 6.6.1 Controller design

In this section we give an outline of how a controller can be designed for model (6.38) by classic design theory. Since no dynamic properties of $C_{\text{in}}$ are known, we consider it as a (white noise) disturbance. For controller design, we regard $\tilde{K}$ as control input. Since the dynamics of $\tilde{K}$ to $C_{\text{out}}$ is of
first order, we can design a simple PI controller. In the frequency domain we have

\[
\tilde{C}_{out}(s) = G_1(s) \tilde{K} + G_2(s) \tilde{C}_0(s),
\]

(6.40)

with \( G_1(s) = \frac{BC}{s - A} \) and \( G_2(s) = \exp(-s\tau_d)BC_2/(s - A). \) We choose the following control structure

\[
K(s) = \frac{A}{BC} \left( \frac{A - s}{s} \right).
\]

(6.41)

This is a standard controller that has the property that the crossover frequency of the complementary sensitivity function equals that of the transfer function \( G_1(s). \) We note that this standard controller is of sufficient quality for the simulation study in the next section, i.e. no fine tuning is necessary. Figure 6.3 shows the control loop schematically. For more details on the design, we refer to chapter 5.

**6.6.2 Simulations**

To see whether all essential dynamics of the physical model (6.33)–(6.35) are captured by the realization (6.38), both models are connected to the controller (6.41) and simulated. As parameter values we choose \( \beta = 30, \) \( K^* = 2, \) \( C_{in}^* = 1, \) \( C_{out}^* = 2.7 \times 10^{-4}, \) \( L = 1, \) and \( R = 0.2. \) We used a grid of \( 15 \times 1000 \) points in radial and longitudinal direction respectively, and this gave a sufficient convergence. The large number of grid points needed in
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Figure 6.4: Simulation results for the physical (--) and the realized model (---) for \((a, f) = (0.5, 0.4)\) (a), and for \((a, f) = (0.5, 0.8)\) (b)

longitudinal direction is probably due to the high attenuation, which gives a steep concentration gradient in this direction. The spatial derivatives are discretized by upwind discretisation, and the time derivatives by forward Euler. For more details, see chapter 5. The time step was determined by CFL analysis to guarantee stability.

To visualize the differences clearly, the input \(C_{in}\) is disturbed like

\[ C_{in} = C_{in} (1 + a \sin(ft)), \]  

(6.42)

with a significant amplitude of \(a = 0.5\) and a frequency of \(f = 0.4\). Figure 6.4 shows \(C_{out}(t)\) for both models. The transient dynamics are not shown, since they are not modelled. In Figure 6.4 (a) the outputs of the physical and the realized model match very well. Further increase of \(f\) leads to larger
6.7 Example: Experimental residence time distribution

It was predicted after equation (6.30) that dynamics in $K$ with a frequency higher than $1/\tau_d$ can lead to considerable model errors. Since $1/\tau_d = 0.3$ in our case, and since $K$ moves with the same frequency as $C_{in}$, it is not surprising that the input frequency of 0.8 gives a large error. It was observed that for frequencies lower than 0.4 the differences become smaller.

6.7 Example: Experimental residence time distribution

Up to now we looked at models for which the residence time distribution is given by an analytical expression. Now we consider two residence time distributions that are obtained via experiment\(^1\). The experimental setup consisted of an annular reactor with one UV lamp, placed in the longitudinal direction. The volume of the reactor is $7 \text{ dm}^3$, and $R = 5.5 \text{ cm}$. The residence time distributions of two flows of $10 \text{ m}^3/h$ and $20 \text{ m}^3/h$ were measured at discrete times and are marked with ‘•’. This is plotted in Figure 6.5. The experimental distributions are fitted by a model of 4 mixed reactors in series with dead time. This model type is suitable for standard linear control design, and thus desirable. Figure 6.5 (a) and (b) shows that the fitted distributions match the experimental ones very well for low values of $t_r$. For the higher values there are considerable differences. This was also the case for the example in section 6.6. This resulted in an accurate realization, at least for control purposes. Therefore, we conclude that the good fit for low $t_r$ indicates that the realization (6.38) will result in a good approximation of the experimental plant, and that consequently a high quality controller can be designed.

6.8 Conclusions

In this chapter, we proposed a model realization using a residence time distribution and a first order reaction model. The inputs are the ingoing concentration and the reaction rate coefficient, and the output is the outgoing concentration. Also, dead time is incorporated into the realization algorithm.

For two nonlinear models of mixed tanks in series, and a UV disinfection reactor, it is shown by simulation that the proposed realization method results in a linear model that accurately captures the dynamics of the nonlinear

\(^1\)Data obtained from tests done in 1999 by Priva Hortimation B.V., De Lier, The Netherlands
Figure 6.5: Experimental residence time distribution (*) and fitted distribution (–) for 10 m³/h (a), and for 20 m³/h (b).
model, at least for control purposes. For the UV reactor example, it was illustrated that when the residence time distribution is modelled accurately for small, but inaccurately for large residence times, the realization still results in an accurate model.

For an experimental UV disinfection reactor, the residence time distribution was measured and approximated with the residence time distribution of a model of four mixed tanks in series with time delay. Like in the example of the UV reactor model, the fit was accurate for small, but inaccurate for large residence times. This indicates that for this real life plant a high quality controller design is possible.
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7

Conclusions

In this thesis we focused on two applications, namely food storage and UV disinfection. We first present the conclusions for each case separately, and then some general conclusions are drawn.

7.1 Food storage

In chapter 2 a bulk storage room is modelled in a basic, but practically relevant way, containing the most essential physical properties. The complicating factors that stand in the way of a simple reduction or control design are the flow and the switching input. The resulting system equations are first validated experimentally, and are then mathematically simplified using timescale decomposition, discrete switching input, and Padé approximations. An open loop control law, describing the optimal switching time between two flow rates, is derived. This switching time accurately drives the product temperature at the top of the bulk to its desired value. The expression of the switching time contains all the physical parameters of the system explicitly. Hence, the parametric expression gives information about the sensitivities to different parameters.

It is shown in chapter 3 that for an extended version of the above mentioned model, a realistic cost function can be derived. This is done by analysis and through experiments. This cost function (i) makes linear optimal control possible, and (ii) gives insight into the controlled process. The great numerical advantage of the parametric forms is illustrated by a realistic example. A more general result is obtained in chapter 4. We showed that for a class of nonlinear scalar systems with discrete input, it is possible to make an approximation that allows linear controller design. The input then consists of the switching times. This type of controller differs essentially from the usual switching algorithms, where the temperatures are allowed to deviate inside a certain bandwidth. Also, a stability criterion is derived in terms of a parametric stability region. The controller is realistic due to the switching,
and easily implementable, and therefore practically relevant. As an example, a switching controller is designed and connected to a temperature model of the bulk storage room. Via numerical simulations it is shown that a controller with excellent stability and robustness properties can be designed for that model.

### 7.2 UV disinfection

In chapter 5 a basic model for a UV disinfection reactor is developed, describing the fluid and concentrations dynamics inside an annular disinfection plant. The complicating factors that stand in the way of a simple reduction or control design are the flow, the nonlinear input and the extremely high number of states that is needed for an accurate finite dimensional discretisation. The model is simplified by physical arguments and mathematical reduction techniques, such as linearization, transfer function approximation, and input/output balanced reduction. For the application of an apple cider plant it is shown that the essential dynamics are mainly of first order. Consequently, a practical, classical model-based controller with excellent properties can be designed.

In chapter 6 we proposed a model realization using a residence time distribution and a first order reaction model. Dead time is incorporated into the realization algorithm. It is shown that the proposed realization method results in an accurate linear model. As an example, the theory is successfully applied to a model of a UV reactor with dead time. The results are used to show that it is possible to design a high quality controller with practical relevance for a real life plant.

### 7.3 General conclusions

In this section, we summarize the conclusions that hold for both applications. The dynamics of the original models is mainly linear and of first order. Therefore, the essential properties that have to be obtained for good control are the static gain and the time constant. These distributed parameter systems are too complicated for straightforward application of one standard technique. However, combinations of standard techniques lead to the desired simplification, while retaining physical information as much as possible. As a consequence, model based PI control can be designed. This results in practical high quality control.

The combinations of techniques have been proven to be a powerful tool towards control design. Moreover, physical model information is retained, which gives great advantages for the design of a real life plant and controller.
7.4 Future work

However, for very complex systems (for example a complex flow structure) success is not guaranteed. Therefore, an alternative modelling method is developed that uses a measured residence time distribution. This results in linear models that are suitable for standard control, without the need for complex modelling and model reduction. The drawback is that no physical model information is retained, so that changes in system design have an unpredictable influence on the dynamics.

7.4 Future work

The distributed parameter systems that we used, are simple in the sense that they have a one dimensional flow and a regular flow domain. For systems with two or three dimensional flows and irregular domains, for example in refrigerated transport, their form and dynamics may become considerably more complex. For those types of systems, the question arises whether they can be approximated with low order linear models, and whether this is possible using the techniques used in chapters 2–5.

Incorporating different dynamics into the existing models is very well possible. One can think for example of adding higher order reactions, and multiple species of microorganisms into the UV disinfection reactor model. Or incorporating moisture transport and frost formation on the cooling element into the bulk storage room model. This makes the models more realistic, and therefore it will lead to better control design. It would be worthwhile to investigate the possibility of standard control design for such extended models.
7 Conclusions
Samenvatting

In praktijk bestaat het regelaarontwerp voor systemen uit drie stappen: modellering, modelreductie en regelaarontwerp voor het gereduceerde model. Systemen met stroming zijn vaak gecompliceerd, en er is dan ook geen standaard algoritme dat deze drie stappen integreert. In dit proefschrift doen we hiertoe een bescheiden poging aan de hand van twee praktijkgevallen: klimaatregeling van een opslagplaats voor voedsel, en UV desinfectie van vloeistof. Het doel is tweeledig. Ten eerste proberen we tot een regelaarontwerp te komen dat praktisch relevant is. De regelaar moet makkelijk te implementeren zijn, en van hoge kwaliteit. Ten tweede proberen we zoveel mogelijk de fysische informatie uit het systeem te behouden. Het voordeel van een regelaar die alle fysische informatie bevat van het te regelen systeem, is dat bij het ontwerpen van zo’n systeem en regelaar veel tijd kan worden bespaard.

Van de voedselopslagplaats is een realistisch maar relatief eenvoudig basismodel gemaakt. Dit model is gevalideerd en gekalibreerd door middel van experimenten. De factoren die een standaard modelreductie en regelaarontwerp in de weg staan, zijn de luchtstroming en de input die niet continu is maar schakelt tussen twee waarden. Via een combinatie van standaard modelreductie technieken (Padé approximatie, linearisatie, tijdschaaldecompositie) is het systeem gereduceerd tot een eerste orde lineair systeem met de schakeltijd als input. Hiervoor is een hoogwaardige en eenvoudige regelaar ontworpen. Via simulaties is de regelaar succesvol getest op het basismodel. De dynamica van het geregelde systeem is een parametrische uitdrukking, hetgeen grote numerieke voordelen geeft bij het systeemontwerp.

Voor de UV desinfectie reactor is een realistisch basismodel afgeleid. De factoren die een standaard modelreductie en regelaarontwerp in de weg staan, zijn de stroming, de niet-lineaire input en de extreem grote toestandsruimte die nodig is voor een nauwkeurige discretisatie. Via een combinatie van standaard reductietechnieken (Padé approximatie, input/output balancing, linearisatie) is het systeem gereduceerd tot een eerste orde lineair systeem. Hiervoor is met succes een hoogwaardige en eenvoudige regelaar ontworpen, die geschikt is voor het basismodel. Een nadeel is dat bij input/output balancing de fysische informatie van het model niet behouden blijft. Verder beschrijft het model een zeer specifiek geval.
7 Conclusions

Als alternatief is daarom een modelleringstechniek ontworpen die gebruik maakt van een gemeten verblijftijdsspreiding. Het voordeel hiervan is dat het model lineair is en geschikt voor praktisch regelaarontwerp. Bovendien is het model automatisch gekalibreerd aan de experimentele resultaten waarop zij is gebaseerd. Het nadeel is dat het model geen fysische informatie over het systeem bevat.
Summary

In practice, feedback control design consists of three steps: modelling, model reduction and controller design for the reduced model. Systems with flow are often complicated, and there is yet no standard algorithm that integrates these steps. In this thesis we make a modest effort by considering two applications: climate control for food storage, and UV disinfection of fluids. The goal is twofold. First, the aim is to come to a controller design that is practically relevant. The controller has to easy implementable and of high quality. Second, we try to retain as much physical information from the system as possible. The advantage of a controller that contains physical information lies in the time saving when designing the system and the controller simultaneously.

For the food storage room a realistic but relatively simple model is derived. This model is validated and calibrated by experimental results. The factors that stand in the way of standard model reduction and controller design, are air flow and the input that is not continuous, but switches between two values. Via a combination of standard model reduction techniques (Padé approximation, linearization, timescale decomposition) the system is reduced to a first order linear system with the switching time as input. For this, a simple and high quality controller is designed. The controller is tested successfully on the basic model. The system dynamics is parameterized by the physical properties of the system, which can give great numerical advantages for the system design.

For a UV disinfection reactor a realistic basic model is derived. The factors that stand in the way of standard model reduction and controller design, are flow, nonlinear input, and the extremely large state space that is needed for an accurate discretisation. Via a combination of standard model reduction techniques (Padé approximation, linearization, input/output balancing) the system is reduced to a first order linear system. For this, a simple and high quality controller is designed. The controller is tested successfully on the basic model. A drawback is that the input/output balancing does not retain any physical system information. Further, the model describes a very specific case.

As an alternative, a modelling technique is designed that uses the measured residence time distribution. The advantage here is that the resulting model
7 Conclusions

is linear and suited for practical controller design. Moreover, the model is automatically calibrated to the experimental data that it is based on. The drawback is that the model does not hold any physical system information.
Een proefschrift van een dergelijke kwaliteit schrijf je nooit alleen. Dat kan ook niet. Daarom wil ik een aantal mensen bedanken.


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