On the acoustics of turbulent non-premixed flames

S.A. Klein
The research project was supported by NOVEM, Demkolec and Thomassen International and was carried out in the Laboratory of Thermal Engineering of the University of Twente.
ON THE ACOUSTICS OF TURBULENT
NON-PREMIXED FLAMES

PROEFSCHRIFT

ter verkrijging van
de graad van doctor aan de Universiteit Twente,
op gezag van de rector magnificus,
prof.dr. F.A. van Vught,
volgens besluit van het College voor Promoties
in het openbaar te verdedigen
op vrijdag 18 februari 2000 om 13.15 uur.

door

Sikke Ate Klein

geboren op 17 september 1971
te Hengelo (Ov.)
Dit proefschrift is goedgekeurd door de promotor:

Prof. dr. ir. J.J.H. Brouwers

en de assistent promotork:

Dr. ir. J.B.W. Kok
Summary

Gas turbines are clean, compact and efficient engines for electric power generation. They are used at a large scale to this end and are usually fired with natural gas. A novel development is to fire them with coal gas produced by a coal gasifier. This opens ways to use the huge coal resources in a clean and efficient operation. As the combustion properties and conditions of coal gas are very different from those of natural gas, it is necessary to redesign the burners and combustors of the gas turbine. Because of the complexity of the burners and combustors in a gas turbine, many difficulties have to be taken care of.

In 1993 the (at that moment) largest coal gasification combined cycle power plant (ICGCC) became in operation, the Denholc power plant in Buggenum (NL). One of the problems during the start up of this power plant were acoustic oscillations in the combustion chamber of the gas turbine ('humming'). These oscillations can be very dangerous for the operation of the gas turbine. By now the humming problem is solved. The acoustic oscillations in the Buggenum gas turbine are the cause for the research, described in this thesis. The results of this research describe the phenomena, which caused these oscillations, and they explain the design solutions implemented in the Buggenum gas turbine to avoid the humming. The results of this research have been used in the design and operation of the recently installed coal-gas-fired gas turbine in Puertollano (Spain).

The acoustic oscillations in the Buggenum gas turbine were caused by the coupling between the acoustics of the combustion chamber and the heat release by the flame. This flame is large and intense, thermal power can be 30 MW per burner at a diameter of about 30 cm. The velocities are high, in the order of 50 to 100 m/s. Temperatures are 1000-1800°C and the pressure is 10-40 bar. The flame can be characterized as a turbulent combustion process with fluctuations in all variables. In this thesis the interaction between acoustic pressure fluctuations, turbulence and combustion is studied. The research focuses on turbulent non-premixed flames with coal gas or synthesized gas as a fuel.

To describe the acoustic role of a flame an integral expression is derived for the pressure fluctuations generated by the flame. In this expression the sound spectrum of the flame is written in terms of the turbulence spectrum at the flame front. To validate it, measurements were performed on a bluff body burner. Furthermore, a new measurement technique is developed to asses the flame transfer function, which gives the amplification of the acoustic pressure by the flame. This measurement technique for the flame transfer function was applied to a bluff body burner and to two swirl burners, akin to the burners used in the Buggenum gas turbine.
The propagation of small pressure fluctuations in a compressible medium (like a gas) can be described by the wave equation. Unsteady combustion induces in the acoustic wave equation a monopole (volume) source term, via the fluctuating heat release by the flame. Because the acoustic wave equation is a linear equation, this heat release source term can be written as the sum of two source terms:

1. A source term which is independent of the acoustic pressure field (the flame as autonomous source of sound).

2. A source term that is a function of the acoustic pressure field (the flame as amplifier of sound).

In process 1 the flame generates on its own account a source term for the acoustic wave equation. If a flame with this source term is placed in a cavity, resonance will occur around the acoustic eigenfrequency of the cavity. In process 2 the flame is as a positive or negative damping term in the acoustic wave equation, driven by the acoustic pressure field. If a flame with this behavior is placed in a cavity, instability might occur. The occurrence of the instability is given by Rayleigh’s criterion. The pressure amplitudes in the case of instability (process 2) are in general much higher than in the case of resonance (process 1). These pressure amplitudes can be 1-5% of the mean pressure in the gas turbine. The flame as amplifier of sound can therefore be very dangerous for the operation of the gas turbine. Both the flame as autonomous source of sound and the flame as amplifier of sound are studied in this thesis.

For the turbulent non-premixed flame as autonomous source of sound (process 1) an integral expression has been derived that gives the (acoustically one-dimensional) noise spectrum from the flame in terms of the turbulence spectrum at the flame front. The main assumption for this expression is that the chemistry may be treated as infinitely fast and that therefore the combustion is determined by the mixing of fuel and air. An assumed shape is used for the turbulence spectrum. The input, necessary to specify the turbulent spectrum and the flame geometry, is derived from a steady state CFD calculation. The noise spectrum calculations are compared with experiments in an acoustically one-dimensional atmospheric combustion chamber with a non-premixed bluff body burner. The calculations compare very well with the experiments, both for the sound level and the shape of the sound spectrum, as long as the chemistry in the flame is infinitely fast. It is shown that this condition is not fulfilled if methane is present in the cool gas. A fuel containing methane gives a different noise spectrum. The experiments have been repeated under an elevated mean pressure (up to 4 bar). The shape of the sound spectrum does not change with increasing pressure with/without methane. The sound level scales quadratically with the mean pressure, as predicted by the model.

To study the flame as amplifier of sound (process 2), the flame transfer function is defined. The flame transfer function gives the (normalized) fluctuating heat release by the flame as a function of the fluctuating fuel or air inlet velocity. To determine the flame transfer function a new experimental method has been developed. This method is applicable to acoustically one-dimensional combustion systems in which the acoustics can be described by upstream and downstream traveling waves. In this method an acoustic fluctuation is imposed on the fuel (or air) flow, the acoustic response of the system on this fluctuation is measured using multiple microphones. An acoustic model is derived for the system with the flame transfer
function as an unknown. The model is solved numerically, using the measurement data as input. This yields the flame transfer function.

The flame transfer function has been measured for a number of series of flames using a non-premixed bluff body burner. It shows that all studied non-premixed flames are insensitive to air flow fluctuations. Further, the phase of all measured flame transfer functions shows a time delay behavior. For one series of flames, it appears to be possible to scale the frequency with the Strouhal number, for another series of flames this is not possible. The validity of Strouhal number scaling is determined by the mechanism of flame stabilization. The measured flame transfer functions are used in a system analysis of the combustion rig. In this system analysis the acoustic system of the combustion rig is described with a number of transfer matrices, one of them is the flame transfer matrix that contains the flame transfer function. This system analysis is compared with the measured system response and the agreement is very satisfactory. It appears however that the acoustic modeling of the fuel supply has a very strong influence on the outcome of the system analysis.

The method to measure the flame transfer function has been applied to two scaled gas turbine burners. From the measurements it is found that, in contrast to the bluff body burner, the influence of the pressure drop over the burner mouth has to be taken into account in the acoustic system description for these burners. To be able to do this a burner transfer function is introduced, which is determined from experiments. The measured burner transfer function shows behavior consistent with acoustic theory. The phase of the flame transfer functions of these burners shows the familiar linear time delay behavior. The time delay depends on a number of variables, like the burner design and the thermal power of the flame. The frequency of the flame transfer function can be scaled reasonably well with the Strouhal number.

An acoustic system study is performed for a typical gas turbine situation, very similar to the Demkolder gas turbine. For the axial modes the system is described with transfer matrices, for the cylindrical modes (radial and azimuthal) an eigenmode analysis is performed. It appears that it is possible to perform an acoustic system study of a real gas turbine using these two methods. Again the outcome of the system analysis is very sensitive to the acoustic description of the supply system.
Samenvatting

Gasturbines zijn schone, compacte en efficiënte machines om elektriciteit op te wekken. Ze worden op een grote schaal gebruikt voor dit doel. De brandstof is in de meeste gevallen aardgas. Een nieuwe ontwikkeling is om kolengas, geproduceerd door een kolenvergasser, als brandstof te gebruiken. Hierdoor is het mogelijk om de enorme kolenreserves op een schone en efficiënte manier te benutten. Omdat de verbrandings eigenschappen van kolengas sterk verschillen van die van aardgas, moeten de branders en verbrandingskamers van de gasturbine opnieuw ontworpen worden. Gezien de complexiteit van de branders en verbrandingskamer is dit geen eenvoudige opgave.

In 1993 is er in Nederland de (op dat moment) grootste kolenvergassings-elektriciteitscentrale ter wereld in gebruik genomen, de Devoelecentrale in Buggenum. Eén van de problemen bij de ingebruikname van deze elektriciteitscentrale waren akoestische trillingen ('brommen'), die optraden in de verbrandingskamer van de gasturbine. Deze trillingen zijn zeer vlijtige verhogend voor de gasturbine. Het trillingsprobleem is inmiddels opgelost. De akoestische trillingen in de Buggenum gasturbine vormen de aanleiding van het onderzoek, dat beschreven staat in dit proefschrift. De resultaten van dit onderzoek beschrijven de verschijnselen, die deze trillingen veroorzaakten, en verklaren de werking van de oplossingen, die toegepast zijn in de Buggenum gasturbine om de akoestische trillingen te verhelpen. De resultaten van het onderzoek zijn gebruikt bij het ontwerp en het bedrijven van de recent geïnstalleerde kolengas-gasturbine in Puertoellano (Spanje).

De akoestische trillingen in de Buggenum gasturbine werden veroorzaakt door de koppeling van de akoestiek van de verbrandingskamer met de warmte-afgifte door de vlam. De vlam in zo'n industriële gasturbine is groot en intens, het thermisch vermogen kan 30 MW per brander bedragen op een diameter van ongeveer 30 cm. De snelheden in de vlam zijn hoog, in de orde van 50 tot 100 m/s. De temperatuur in de verbrandingskamer bedraagt 1000-1800°C en de druk is 10-40 bar. De vlam kan gekarakteriseerd worden als een turbulent verbrandingsproces met fluctuaties in alle variabelen. In dit proefschrift wordt de interactie tussen akoestische drukfluctuaties, turbulentie en verbranding onderzocht. Het onderzoek richt zich met name op turbulentie niet-voorgemengde vlammen met kolengas of synthetisch gas als brandstof.

Om het akoestische gedrag van een vlam te beschrijven is er een integraal-vergelijking voor de geluidsgeneratie door de vlam geformuleerd. In deze vergelijking wordt het geluidspectrum van een vlam uitgedrukt als functie van het turbulentiespectrum ter plaatse van het vlamfront. Om de vergelijking te valideren zijn er metingen uitgevoerd met een bluff-bodybrander. Verder is er een nieuwe meetmethode ontwikkeld om de vlamoverdrachtsfunctie te bepalen, die de versterking van de akoestische druk door een vlam geeft. Deze meetme-
thode is toegepast op een bluff-body-brander en op twee swirl-branders, vergelijkbaar met de branders die gebruikt worden in de Buggenum gasturbine.

De voortplanting van een kleine drukvariatie in een medium kan beschreven worden met behulp van de akoestische golfvergelijking. Verbranding genereert een monopool (volume) bronterm in deze golfvergelijking via de fluctuerende warmte-afgifte door de vlam. Omdat de golfvergelijking een lineaire vergelijking is, kan deze warmte-afgifte-bronterm geschreven worden als de som van twee brontermen:

1. Een bronterm die onafhankelijk is van de akoestische druk (de vlam als autonome gehuidsbron).

2. Een bronterm die afhangt van de akoestische druk (de vlam als versterker van gehuid).

In situatie 1 genereert de vlam autonoom een bronterm voor de akoestische golfvergelijking. Als een vlam met deze bronterm geplaatst wordt in een omhulling, zoals een verbandingskamer van een gasturbine, zal er resonantie optreden rond de akoestische eigenfrequentie van die ruimte. In situatie 2 genereert de vlam een (positieve of negatieve) dempingterm in de akoestische golfvergelijking. Als een vlam met dit gedrag geplaatst wordt in een omhulling kan er instabiliteit optreden. Het al dan niet optreden van instabiliteit wordt gegeven door Rayleigh’s criterium. In het algemeen zijn de drukamplitudes bij instabiliteit (situatie 2) veel groter dan in het geval van resonantie (situatie 1), deze drukamplitudes kunnen 1-5% van de gemiddelde druk in de gasturbine bedragen. De vlam als versterker kan daarom zeer gevaarlijk zijn bij het bedrijven van een gasturbine. Zowel de vlam als autonome gehuidsbron als de vlam als versterker is bestudeerd in dit proefschrift.

Voor een turbulent niet-voorgemengde vlam als gehuidsbron (situatie 1) is er een integraal-vergelijking afgeleid. Deze vergelijking geeft het (akoestisch ééndimensionale) gehuisspectrum van de vlam als functie van het turbulentiespectrum ter plaatse van het vlamfront. De belangrijkste aanname voor deze vergelijking is dat verondersteld mag worden dat de chemie oneindig snel is, zodat de verbranding bepaald wordt door de menging van brandstof en lucht. Er wordt een aangenomen vorm van het turbulentiespectrum gebruikt. De input, die nodig is om het turbulentiespectrum en de vlamvorm te karakteriseren, wordt afgeleid uit een steady-state CFD berekening. De berekeningen van het gehuisspectrum zijn vergeleken met experimenten in een akoestisch ééndimensionale atmosferische verbrandingskamer met een niet-voorgemengde bluff-body-brander. De berekeningen komen zeer goed overeen met de metingen, zolang de chemie veel sneller is dan de menging. Het is aangetoond dat aan deze conditie niet voldaan wordt als er methaan aanwezig is in de brandstof. Een brandstof met methaan levert een ander gehuisspectrum op. De experimenten zijn herhaald onder verhoogde druk (tot 4 bar). De vorm van het gehuiisspectrum verandert niet met toenemende druk, zowel met als zonder methaan. Het gehuissniveau schaalt kwadratisch met toenemende druk, zoals ook voorspeld wordt door het model.

Om de vlam als versterker van gehuid (situatie 2) te bestuderen, wordt de vlamoverdrachtsfunctie gedefinieerd. De vlamoverdrachtsfunctie geeft de (genormaliseerde) warmte-afgifte door de vlam als functie van de fluctuerende brandstof- of luchtinhoudsnelheid. Om de vlamoverdrachtsfunctie te bepalen is er een nieuwe experimentele methode ontwikkeld. Deze methode is toepasbaar op akoestisch ééndimensionale verbrandingsystemen, waarin
de akoestiek beschreven kan worden met twee lopende golven. In deze methode wordt een akoestische fluctuatie opgelegd in de brandstof- of luchtvoerder, de akoestische reactie van het systeem op deze fluctuatie wordt gemeten met een aantal microfoons. Een akoestisch model is opgesteld als onbekende de vlamoverdrachtsfunctie. Het model wordt numeriek opgeloost met de meetdata als input. Dit levert de vlamoverdrachtsfunctie.

De vlamoverdrachtsfunctie is gemeten voor een aantal series vlammen van een niet-
voorgemengde bluff-body-brander. Het blijkt dat de voorgemengde vlammen gemiddeld gevoelig zijn voor fluctuaties in de luchtstroom. De fase van vlam- overdrachtsfuncties vertoont een gedrag duidend op een tijdsvertraging. Voor een aantal vlammen blijkt het mogelijk te zijn om de frequentie te schalen met het Strouhalgetal, voor andere vlammen is dit niet mogelijk. De geldigheid van de schaling met het Strouhalgetal wordt bepaald door het mechanisme van vlamstabilisatie. De gemeten vlamoverdrachtsfuncties zijn gebruikt in een akoestische systeemanalyse van de verbrandingsinstallatie. In deze systeemanalyse wordt het akoestische systeem van de verbrandingsinstallatie beschreven met een aantal overdrachtsmatrices, één daarvan is de vlamoverdrachtsfunctie. Deze systeemanalyse is vergeleken met de meetresultaten van de overeenstemmende frequentie is zeer goed. Het blijkt echter dat de akoestische modellering van het brandstofvoersysteem een grote invloed heeft op het resultaat van de systeemanalyse.

De methode om de vlamoverdrachtsfunctie te meten is toegepast op twee geschalde gasturbine-branders. Uit de metingen blijkt dat, in tegenstelling tot de bluff-body-brander, de invloed van de drukverlies en de branderontwerp in rekening gebracht moet worden. Om dit te kunnen doen wordt de branderoverdrachtsfunctie geïntroduceerd, die bepaald wordt uit metingen. De gemeten branderoverdrachtsfunctie vertoont een gedrag dat consistent is met de akoestische theorie. De fase van de gemeten vlamoverdrachtsfuncties van deze branders vertoont het lineaire gedrag van een tijdsvertraging. De tijdsvertraging hangt van een aantal variabelen af, zoals het branderontwerp en het thermische vermogen van de vlam. De frequentie van de vlamoverdrachtsfunctie kan geschaald worden met het Strouhalgetal.

Voor een typische gasturbine situatie, gelijksoortig aan de Demoelc gasturbine, is er een akoestische systeemanalyse uitgevoerd. Voor de axiale modes wordt het systeem beschreven met overdrachtsmatrices. Een analyse van de eigenmodes is uitgevoerd voor de cilindrische modes (radiaal en azimuthaal). Het blijkt dat het mogelijk is om een akoestische systeemanalyse van een gasturbine uit te voeren, gebruikmakende van deze twee methoden. Ook in dit geval, is het resultaat van de systeemanalyse zeer gevoelig voor de akoestische beschrijving van het toevloersysteem.
Preface

This thesis is the result of the research I performed during the last four years at the Laboratory of Thermal Engineering of the University of Twente. During this project I have had support from a large number of people, for which I would like to thank them.

First of all I would like to thank my supervisor dr. J.B.W. Kok for the good co-operation and for the possibilities he created to perform research. I wish to thank my promotor, prof. J.J.H. Brouwers, for his useful suggestions and support.

The research project has been made possible by the financial support from Denkolec, NOVEM and Thomassen International, for which I would like to thank them. During the research we have had several meetings to discuss the progress and the direction of the research with W. Willeboer and G. Zon from Denkolec, with P. Stollwerk from NOVEM and with P. Kamminga from Thomassen International. I appreciated these meetings very much and I would like to thank them for their efforts to make this project successful.

In the development of the theory on combustion noise of non-premixed flames (chapter 3) I have had a number of discussions with prof. L. van Wijngaarden. I would like to thank him for his critical comments and useful suggestions.

At the end stage of the project we have been in contact with Siemens KWU about combustion driven oscillations. Their practical questions and feedback helped me to improve the insight in this subject. I especially would like to thank M. Huth and A. Helios. Also the interest in this project from Elcogas (Puertollano, Spain) is appreciated. I would like to thank especially J. Pisa and F.G. Peia for their interest and support.

During this research project we have been in contact with ECN (Netherlands Energy Research Foundation) in Petten, what resulted in the noise experiments under elevated pressure. I would like to thank the following persons at ECN: J. Kiel, A.J. Dijkhuizen, G. Beckers, L.P.M. Rabou and J. Haas.

A large part of the work has been experimental work. For the construction of the laboratory facility and of the combustion rig I needed a lot of support from the mechanics of the laboratory of Thermal Engineering, for which I wish to thank them. Special thanks go to Chris Bakker, who constructed the swirl burner. I also would like to thank Hans Kamminga, who contributed a lot to the construction of the laboratory facility.

A number of students has contributed to this project by performing the project for their graduation in the framework of my research project. I would like to thank them for their contributions: Peter Swart, Arjan Dijkhuizen, Luke Vrielink, René Hekkens, Martijn Pronk (Hogeschool Alkmaar) and Bas van de Rest.

Marco Beltman and Marco Derksen put a lot of effort in the correction of the text of my thesis, for which I wish to thank them.
I wish to thank prof. T. Sattelmayer from the TU München for taking place in the committee for my PhD graduation.

All the students, colleagues and former colleagues have contributed to the nice working atmosphere at the laboratory, for which I would like to thank them.

My girl fried Margo Meijer has helped me a lot during this research project. I wish to thank her for her encouragement during the whole project and particularly for the support in the last months, when I had to finish this thesis.

Sikke Klein

Deventer, December 1999
Contents

1 Introduction .................................................. 5
   1.1 The role of combustion in acoustics ....................... 6
       1.1.1 Combustion roar ................................... 6
       1.1.2 Combustion driven oscillations .................. 6
   1.2 Denko  .................................................. 7
   1.3 Scope of the thesis .................................... 9
   1.4 Turbulence and combustion ............................ 10
       1.4.1 Turbulence ....................................... 10
       1.4.2 Turbulent combustion ........................... 14

2 Acoustics .................................................... 21
   2.1 Introduction ........................................... 21
   2.2 The acoustic wave equation ............................ 21
   2.3 The role of the flame ................................ 22
   2.4 Sound propagation ..................................... 23
       2.4.1 Harmonic waves .................................. 23
       2.4.2 Description of an acoustic system ............... 24
   2.5 The mode analysis method ............................. 25
       2.5.1 The flame in the acoustic system, Rayleigh's criterion ... 26
       2.5.2 The flame as source versus the flame as amplifier .... 28

3 Noise generation by turbulent non-premixed flames .......... 31
   3.1 Introduction ........................................... 31
   3.2 Literature on turbulent combustion noise ............... 31
   3.3 Theory ................................................ 32
       3.3.1 The mixed-is-burnt approach .................... 34
       3.3.2 The sound from a non-premixed turbulent flame .... 35
       3.3.3 An overview of the assumptions .................. 40
   3.4 Evaluation of the noise source ........................ 41
   3.5 Coupling to a CFD calculation ........................ 43
       3.5.1 The turbulence variables ....................... 43
       3.5.2 The discretized equation ....................... 44
   3.6 The experimental method .............................. 44
   3.7 Results ............................................... 45
CONTENTS

3.7.1 The studied cases ........................................ 45
3.7.2 Description of the calculations ......................... 46
3.7.3 The results for Flame 3.2 ............................... 47
3.7.4 The results for Flame 0 ................................. 49
3.7.5 The results for Flame 3.4 ............................... 50
3.7.6 The results for Flame 3.5: a fuel containing methane 52
3.7.7 The overall sound level .................................. 54
3.8 Conclusions for the noise models .......................... 55
3.9 Pressure effect on noise generation ...................... 56
  3.9.1 The experimental method .............................. 56
  3.9.2 The experimental setup ............................... 57
  3.9.3 Results .............................................. 58
  3.9.4 Conclusions ......................................... 62
3.10 Experiments with a swirl burner ........................ 62
  3.10.1 The influence of the power .......................... 62
  3.10.2 The influence of the air factor ...................... 63
  3.10.3 The fall off of the sound spectrum ................. 65
  3.10.4 Conclusions ....................................... 65
3.11 Summary and conclusions .................................. 65

4 Combustion acoustic system interaction .................... 67
  4.1 Introduction ............................................ 67
  4.2 Possible energy transfer mechanisms .................... 67
  4.3 Acoustic system description ............................ 69
  4.4 The flame transfer function ............................ 70
    4.4.1 The propagation of a fluctuation ................ 70
    4.4.2 Examples of a flame transfer function .......... 71
  4.5 Measurement of the flame transfer function ............. 73
    4.5.1 The measurement of OH-radicals ................. 74
    4.5.2 The acoustic method .............................. 74
    4.5.3 The burner transfer function .................... 77

5 Experimental results for a bluff body burner ............... 79
  5.1 Introduction ............................................ 79
  5.2 The burner transfer function ........................... 80
  5.3 The flame transfer function for Flame 0 ............... 81
    5.3.1 Fuel side excitation .............................. 81
    5.3.2 Air side excitation ................................ 83
    5.3.3 The influence of the air factor ................. 83
    5.3.4 The influence of the thermal power .............. 85
  5.4 The flame transfer functions for Flame 3.2, 3.4 and 3.5 85
    5.4.1 The influence of the chemical composition ....... 86
    5.4.2 The influence of the thermal power .............. 87
  5.5 Conclusions for the flame and burner transfer function 88
  5.6 The acoustic system behavior ........................... 89
    5.6.1 The flame reflection factor ....................... 90
## CONTENTS

5.6.2 Instability and instability frequency ........................................ 94
5.6.3 Conclusions for the system analysis ........................................ 99

6 Experimental results for swirl burners ........................................ 101
6.1 Introduction ............................................................................ 101
6.2 The experimental set up .......................................................... 101
6.3 The burner transfer function ..................................................... 102
   6.3.1 The influence of the flow velocity ...................................... 104
   6.3.2 The influence of the swirl ................................................ 105
   6.3.3 The influence of a swirl perturbator in the fuel swirler .......... 106
6.4 The flame transfer function ...................................................... 106
   6.4.1 The influence of the power .............................................. 107
   6.4.2 The influence of the swirl .............................................. 108
   6.4.3 The influence of a swirl perturbator in the fuel swirler ....... 109
   6.4.4 The influence of the chemical composition ....................... 110
6.5 System behavior, the flame reflection factor .............................. 111
6.6 Conclusions ......................................................................... 113

7 System study ........................................................................ 115
7.1 Introduction ......................................................................... 115
   7.1.1 The dimensions of the gas turbine .................................. 115
7.2 Methodology ......................................................................... 117
   7.2.1 Scaling rules .................................................................. 117
   7.2.2 Acoustic system description .......................................... 119
7.3 Results ................................................................................. 122
   7.3.1 The axial modes ............................................................ 122
   7.3.2 The cylindrical modes ................................................... 125
7.4 Conclusions ......................................................................... 127

8 Conclusions .......................................................................... 129
8.1 General .............................................................................. 129
8.2 The flame as source of sound ................................................ 129
8.3 The flame as amplifier of sound .............................................. 130

Bibliography ........................................................................... 131

A Nomenclature ........................................................................ 137

B The experimental setup .......................................................... 141
   B.1 The fuel and air supply ..................................................... 141
   B.2 The combustion rig ........................................................ 143
   B.3 The acoustical experimental setup .................................... 143

C The thermo-acoustic source term ............................................. 147
D  The transfer matrix method .................................. 153
   D.1 Element for the propagation of sound .................. 154
   D.2 Connection element ..................................... 154
   D.3 Special elements ........................................... 156
      D.3.1 Burner element ...................................... 156
      D.3.2 Flame element ...................................... 157
   D.4 Elements for acoustic boundaries ....................... 158
      D.4.1 An open end ........................................ 158
      D.4.2 A closed end ....................................... 159
      D.4.3 A choked end ....................................... 159
   D.5 The matrix description of the system ................... 159
   D.6 The solution of the matrix equation ..................... 160

E  Comparison of the transfer matrix method with the mode analysis method 163

F  Analysis of the experimental method ......................... 167
   F.1 Error in the speed of sound .............................. 168
   F.2 Error in the Mach Number ................................ 169
   F.3 Errors in the measured transfer function ............... 169
   F.4 Conclusions ............................................. 171

G  The two-microphone method ................................ 173
Chapter 1

Introduction

The study of the interaction between acoustics and combustion has received considerable interest in the last decades. In almost all situations the acoustic effects of combustion are unfavourable for the operation of the apparatus.

The noise from a combustion process can generally be characterized as one of two types (see also figure 1.1):

1. Combustion roar

2. Combustion driven oscillations.

Combustion roar, which shows a broad spectrum, is caused by the (autonomous) noise generation by a flame due to the turbulent fluctuations in the flame, possibly in combination with acoustic resonance. Combustion roar is especially interesting to study in view of acoustical pollution of the environment. Combustion roar is further interesting from an academic point of view since the noise from a flame can be thought of as a 'finger print' of the turbulence in the flame; noise measurements can be used to qualify flames.

Combustion driven oscillations involve a feedback cycle that converts chemical energy into oscillatory energy. Combustion driven oscillations always involve one (or more) specific frequencies. The pressure amplitude at this frequency can be very high (in certain cases up

![Flame → Combustor](image1.png)

(a) Combustion roar

![Flame → Combustor](image2.png)

(b) Combustion driven oscillations

Figure 1.1: A simplified scheme to show the difference between combustion roar and combustion driven oscillations: in the case of combustion driven oscillations there is feedback from the acoustics to the flame.
to 50% of the mean pressure in the combustion chamber). These large pressure amplitudes may cause serious damage to the combustion installation.

In this thesis both combustion roar and combustion driven oscillations are studied. The focus in this thesis is on non-premixed syngas (coal gas) flames. These flames showed acoustic problems in several coal gasification power plants, like the Denkolec power plant in Buggenum (NL) (Vörtmeyer et al. (1995)) and in a Dow Chemical coal gasification power plant in the USA (Scalzo et al. (1990)).

In the present research is Denkolec the main participant. The other participants are NOVEM, Thomassen International Gas Turbines and the University of Twente.

1.1 The role of combustion in acoustics

In an unsteady flame the fluctuating density acts as a volume source of sound (the volume of the flame fluctuates with the reciprocal of the density). The main cause for the fluctuating density are fluctuations in the heat release. The distinction between combustion roar and combustion driven oscillations is based on the origin for these heat release fluctuations: In combustion roar the density fluctuations are caused by autonomous (e.g. turbulent) fluctuations in the heat release. In the case of combustion driven oscillations the heat release of the flame depends on the acoustic phenomena in the combustion chamber, due to a feedback loop between acoustics and combustion.

1.1.1 Combustion roar

The study of combustion roar has become important with the increasing interest in environmental pollution. Especially the noise generation by jet engines from aircrafts has received a lot of attention. From that research it appeared to be very hard to make the flame more silent. The way to minimize the combustion roar in air crafts jet engines has been in many cases the use of liners designed to provide acoustic damping. Other examples which are interesting for combustion roar research are flares and hot air balloon burners.

The phenomenon of combustion roar can be split up into the noise generation by the flame and the resonance in the combustion chamber. In this thesis the noise generation by turbulent non-premixed flames is discussed extensively. The resonance in the combustion chamber is only treated briefly.

1.1.2 Combustion driven oscillations

The study of combustion driven oscillations goes back to 1777, when Higgins (1802) observed the singing flame. This is a laminar (diffusion) flame in a vertical tube. If this flame is inserted sufficiently far into the tube, the flame may produce a 'singing'. The frequency of the singing is near the acoustic natural frequency of the tube. The occurrence of the singing, the mode and the amplitude depend on the diffusivity and heating value of the fuel, the fuel supply rate, the configuration of the fuel supply line and the dimensions of the combustion tube (Putnain (1971)). Lord Rayleigh was the first to explain the phenomenon of the singing flame and formulated a criterion, what is now known as Rayleigh's criterion, stating that the occurrence of the singing is determined by the phase between the heat release of the flame and the pressure (Rayleigh (1945)). The occurrence of singing is also determined by
the coupling between the acoustics and the flame. Rayleigh’s criterion will be discussed in more detail in section 2.5.1 of this thesis.

In combustion driven oscillations the flame is always placed in an acoustic resonator (the tube in the experiment of Higgins), which accumulates acoustic energy. If the flame adds more energy to the resonator than is dissipated, the acoustic energy in the resonator grows in time and there is an instability. In the study of combustion driven oscillations the active role of a flame as part of an acoustic system is studied. One of the most important factors is the feedback (coupling) from the acoustic pressure on the combustion process. To be able to study the occurrence of combustion driven oscillations it is necessary to take the acoustic system of the complete combustion installation into account.

In the period of the first rocket launches (about 1950-1970) a lot of research was performed on the very severe combustion driven oscillations which were observed in rocket engines. One way to reduce these instabilities was the introduction of baffles in the flame. Due to these baffles the acoustic behavior of the combustion system changed strongly. This remedy appeared to be very successful.

At about 1970 there was a renewed interest for combustion driven oscillations. One of the reasons was the switch over of many combustion installations from oil to natural gas. Under certain circumstances these installations with gas burners showed instabilities where the same installation with oil burners had been stable. One of the remedies in power plants was to redistribute the burners in the boiler ‘wall’, by this redistribution the coupling between the acoustics and combustion was changed.

Since about 1990 the interest in the occurrence of combustion instabilities is growing, especially in gas turbine combustion research. Due to stricter environmental legislation with respect to the emission of pollutants, especially $NO_x$, it has become necessary to use lean premixed combustion. It appears that this type of combustion is accompanied in many cases with instability problems. Many modern gas turbines (of all manufacturers) have a limited window of parameters in which they can operate stably. This window of stable operation is determined by trial and error during operation in the field. It is not (yet) possible to predict under which circumstances an installation will be stable or unstable.

From history it can be found that for (almost) all new types of burners there have been problems with combustion driven oscillations. Also the burners used in the Demkolec gas turbine showed these combustion driven oscillations in first instance. The novelty of these burners was that it was the first time that syngas was combusted in a gas turbine at such a high firing rate. The manufacturer of the Demkolec gas turbine, Siemens K.W.U, solved the instability problems by changing the burner design. The optimum design was found after a number of tests.

1.2 Demkolec

Since 1993 the Demkolec power plant is in operation. This plant is a medium size power plant (250 MW). The Demkolec plant was built to demonstrate the possibility to generate electricity on a large scale on the basis of coal gasification technology integrated with a combined cycle. After the demonstration period was finished at the end of 1997, the Demkolec power plant became in regular operation.

The coal gas (syngas) in the Demkolec power plant is produced by the gasification of
coal in an entrained flow reactor. The syngas from the gasifier is cleaned and diluted with nitrogen and steam. The syngas is burnt in a gas turbine, the exhaust gases of the gas turbine are led to a boiler to generate steam for the steam cycle (combined cycle). To make the process as efficient as possible, the heat flows and mass flows of the gasification process and of the combined cycle (Integrated Coal Gasification Combined Cycle, IGCC) are strongly integrated.

The advantages of an IGCC power plant over a conventional coal power plant are its higher coal-to-power efficiency (43% for the Demokolc power plant instead of 40% for a state of the art conventional coal power plant) and the much lower emissions of pollutants. The latter is achieved by removing all polluting components from the coal gas before it is combusted. Byproducts are slag, fly ash, sulphur and salt.

The oxidizer in the coal gasifier is pure oxygen. This pure oxygen is produced by an air separation plant on the same location. The syngas from the gasifier is diluted with nitrogen from the air separation plant and steam. The dilution of the syngas is necessary to get a sufficiently low flame temperature, necessary for low NOx emissions and to get the correct mass flow for the turbine. The main components in the syngas are carbon monoxide (CO), hydrogen (H2), nitrogen (N2) and steam (H2O). The calorific value of the syngas is low, typically 4.3 M.J/kg.

The syngas is combusted in a Siemens V94.2 gas turbine, a picture of this gas turbine is
shown in figure 1.2. This gas turbine has two vertically placed silo combustion chambers, each of them equipped with 8 burners. The full load thermal power is 400 MW, the pressure ratio of the compressor under full load is 1:11. The burners for syngas operation in this gas turbine are of the non-premixed type, in order to prevent flashback due to the high content of hydrogen in the syngas.

In 1999 a second large ICGCC power plant has become operational in Puertollano (Spain). This power plant is owned by Elcogas. This plant uses a different gasifier design and a different gas turbine from the same manufacturer (a Siemens V94.3 gas turbine, full load thermal power: 512 MW, pressure ratio 1:16). The models and methods developed and applied for the Demkolec power plant, described in this thesis and in the thesis by Louis (1997), have also been used successfully for this Elcogas power plant.

1.3 Scope of the thesis

In this thesis the acoustic role of a turbulent (non-premixed coal gas) flame is studied. The role of the flame in combustion roar and combustion driven oscillations is discussed separately.

Section 1.4 and chapter 2 form the introduction of the thesis. In section 1.4 the general principles of combustion and turbulence are summarized. In chapter 2 the acoustic role of a flame is described: the flame is a source term in the acoustic wave equation. The Rayleigh’s criterion, very important in the study of combustion driven oscillations, is also discussed in this chapter.

In chapter 3 the turbulent non-premixed flame as autonomous source of sound is discussed (combustion roar). It appears that the noise spectrum of such a flame can be described in terms of the turbulence spectrum at the flame front. The analysis of the combustion noise generation results in a CFD model. The results of this CFD model are compared with experiments.

In the chapters 4 - 7 combustion driven oscillations are studied and especially the role of the flame in it. In chapter 4 the theory of combustion driven oscillations is discussed. In this chapter a new method is presented to measure the role of the flame in combustion driven oscillations.

In chapter 5 experimental results for a bluff body burner are presented. The influence of several parameters on the amplification behavior of the flame are studied.

In chapter 6 acoustic measurements performed with two model gas turbine burners are presented. The design of these burners is based on the burners in the Demkolec gas turbine. For these burners the influence of several parameters, like the chemical composition of the fuel, on the acoustic flame behavior is studied.

In chapter 7 an acoustic system analysis for a realistic gas turbine situation, very similar to the Demkolec gas turbine, is performed. The results from chapter 6 are used in this analysis.

In the last chapter, chapter 8, conclusions are drawn.

For the experiments described in this thesis, a new experimental facility has been built. The set up of this laboratory facility is described briefly in appendix B.

In appendix D the transfer matrix method is discussed, this method is used to perform studies of acoustically one-dimensional systems.
1.4 Turbulence and combustion

In combustion there is a strong interaction between fluid mechanics, chemical reactions and heat and mass transfer. These processes are all described by conservation equations. The influence of the chemical and thermal processes on the flow field is due to the change of density. The density changes because of changes in temperature and mean molecular weight. The effect of the flow field on the chemical and thermal processes is felt through the convection of the scalar variables governing combustion. The scalar equations for the transport of the different species and enthalpy form a stiff set, due to the strong dependency of the chemical reaction rates on the temperature. Even when using numerical methods, the complete set of equations is hard to solve.

When the flow is turbulent the situation becomes even more complex since all variables fluctuate in time. These turbulent fluctuations influence both the transport terms and the source terms in the (scalar) transport equations. In principle it is possible to solve this set of equations numerically, this is called Direct Numerical Simulation (DNS). However, the large number of species that play a role in combustion (20-100 depending on the used chemical scheme) and the wide range of time and length scales that has to be solved for makes that this is only possible if large restrictions are applied: low Reynolds number (about 1000 at maximum at the moment), simple geometries and reduced reaction schemes.

To be able to compute combustion in practical situations the number of transport equations has to be reduced and the turbulence has to be described with model equations. The reduction of the number of transport equations for species together with the influence of the turbulence on the source terms of these equations are described by (turbulent) combustion models. For the description of turbulent transport there are turbulence models available, which are (almost) identical to the turbulence models for isothermal non-reactive flow.

In section 1.4.1 some background theory on turbulence is given. Turbulent combustion is briefly discussed in section 1.4.2. This theory is used in chapter 3 to describe the interaction between turbulence and combustion.

1.4.1 Turbulence

A turbulent flow exhibits an irregular behavior both in space and time. This irregular behavior is caused by fluctuations in all variables, which seem to behave rather randomly. The fluctuations in a turbulent flow are caused by the so-called eddies. These eddies 'stir' the flow and increase that way the mean transport of flow quantities. The larger the eddies the better the flow is stirred.

One fundamental result of turbulence theory is that these eddies are not of one particular size, but that a (broad) continuous range of large to small eddies exists in a turbulent flow. In general the size of the largest eddies is determined by the geometry of the flow configuration. The energy for the generation of these eddies is extracted from the mean flow. The smaller eddies do not extract their energy directly from the mean flow but are fed by a continuous decay of large eddies which break up into smaller ones. These smaller ones in turn decay to even smaller eddies, until this cascade reaches the smallest scale of turbulent motion (in turbulence theory, this process is known as 'energy cascade'). This smallest length scale is called the Kolmogorov length scale. The Kolmogorov scale is determined by the amount of energy transferred along the energy cascade towards the small eddies and by the kinematic
viscosity of the fluid.

**Turbulent scales and spectra**

Associated with the motions in the turbulent flow are several time, velocity and length scales. The length scale of the largest eddies is the integral scale \( l \), the (fluctuating) velocity scale of these eddies is \( u' \). The kinetic energy of the large scale eddies is \( u'^2 \).

The loss of kinetic energy of the large-scale eddies to the smaller eddies is called the dissipation rate \( \varepsilon \). The dissipation rate is independent of the small scale eddies and the fluid properties since it is fully determined by the large scale eddies. This is expressed by the following relation, which is a fundamental result in turbulence theory:

\[
\varepsilon \propto \frac{u'^3}{l} \propto \frac{u'^2 u'}{l} \propto \frac{\text{Kin. Energy}}{\text{Eddy life time}} \tag{1.1}
\]

The quantity \( l/u' \) is the eddy life-time. Within their life time, the large eddies lose their energy due to a break-up into smaller eddies.

A relation for the Kolmogorov length scale can be found by using dimensional analysis:

\[
\eta = \left( \frac{\nu^3}{\varepsilon} \right) \tag{1.2}
\]

A commonly used method to describe turbulence is to use the Fourier transform of a turbulent quantity in space (and/or time). With the Fourier transform a quantity in the space/time domain is translated to a quantity in the wavenumber/frequency domain. The technique of Fourier transformation is very useful in homogeneous turbulence to show the correlations in space and time of the turbulence. The Fourier transform in space of a quantity \( \phi \) is defined as:

\[
F^k(\phi) = \int_{-\infty}^{\infty} \phi(x) e^{ikx} dx \tag{1.3}
\]

where \( k \) is the three dimensional wave number vector.

In the case of homogeneous isotropic turbulence it is possible to work in one dimension. The energy spectrum \( (E(k)) \) of the turbulent velocity fluctuations is defined as:

\[
E(k) = \int \overline{u' \cdot u'}(\Delta x) e^{ik\Delta x} d\Delta x \tag{1.4}
\]

where \( \overline{u' \cdot u'}(\Delta x) \) is the spatial correlation function of \( u' \).

From dimensional analysis it can be shown that for \( k_{\text{integral}} < k < k_{\text{Kol}} \) \( (k_{\text{integral}} \propto 1/l, \quad k_{\text{Kol}} \propto 1/\eta) \) the shape of the energy spectrum is:

\[
E(k) = \text{const} \cdot \varepsilon^{2/3} \cdot k^{-5/3} \tag{1.5}
\]

This shape of the turbulent spectrum has been confirmed by a large amount of experimental data (Hinze (1975)). Many experiments have shown that the energy spectrum of non-isotropic, non-homogeneous turbulence at a high Reynolds number also shows this shape.
Numerical simulation of turbulence

In the numerical simulation of turbulence there are two main different ways, besides DNS, to describe the turbulence. The first way, which is called Large Eddy Simulation (LES), is based on spatial filtering. The Navier Stokes equations are spatially filtered, the length scales larger than the filter width (normally the grid spacing) are solved explicitly, for the length scales smaller than the filter width a subgrid-scale model is used. In fact this is a cheap version of DNS, only the largest scales are simulated directly, their interaction with the smallest scales is modeled. The other way to describe the turbulence is to average the Navier Stokes equations in time (Reynolds averaged Navier Stokes (RANS)). By the time averaging process, equations for the mean quantities are derived. In these equations there are terms due to turbulence, which have to be modeled. Two well known RANS models are the k-ε-model and the Reynolds stress model. In this thesis only turbulence models based on RANS are used.

In the RANS method transport equations for the time-averaged variables are obtained by decomposition of the instantaneous variables in a mean part and a time fluctuating part:

\[ \phi (\mathbf{x}, t) = \overline{\phi} (\mathbf{x}) + \phi' (\mathbf{x}, t) \]  

(1.6)

This is called Reynolds decomposition. The time averaged part is indicated with an overline bar, the fluctuating part with a prime (\( ' \)). Reynolds decomposition is mainly used for constant density flows. For flows with a fluctuating density, like a combusting flow, the time average is weighted by the density (Favre averaging). In Favre averaging the time (and density) averaged part is indicated with a tilde (\( \tilde{\phantom{\phi}} \)), the fluctuating part with a double prime(\( '' \)):

\[ \phi = \tilde{\phi} + \phi'' \]  

(1.7)

The Favre averaged value of \( \phi \) is defined by:

\[ \tilde{\phi} = \frac{1}{\overline{\rho} . T} \int_0^T \rho(t) \phi(t) dt \]  

(1.8)

, where \( T \) is the time interval over which is averaged. Because combustion is typical non-isothermal, Favre averaging is used throughout this thesis. The Favre averaged Navier Stokes equations are:

\[ \frac{\partial \overline{\rho \mathbf{u}}}{\partial t} + \nabla \cdot \overline{(\rho \mathbf{u} \mathbf{u})} = -\nabla \overline{p} + \nabla \cdot \left( \overline{\mathbf{u} \mathbf{u}} - \overline{\rho \mathbf{u}'' \mathbf{u}''} \right) \]  

(1.9)

, where \( \mathbf{u} \) is the Favre averaged viscous stress tensor.

The first term at the right hand side of equation (1.9) gives the influence of the turbulent fluctuations on the mean flow. Because the shape of it is similar to a stress term, this term is called the Reynolds stress term. To close the Navier Stokes equations the Reynolds stresses should be known. It is possible to derive extra transport equations for the Reynolds stresses, in which new unknown terms appear, which have to be modeled. This model is called the Reynolds Stress Model (RSM). This model will not be discussed further in this thesis.

Another way to close the Favre averaged Navier Stokes equation is to model the Reynolds stresses. The most popular model to do this is the k-ε model (Launher and Spalding (1972), which is discussed here. In the k-ε model extra (modeled) transport equations are solved for the turbulent kinetic energy (\( k \), in this thesis denoted by \( tke \)) and the turbulent dissipation
(\epsilon). In the k-\epsilon model the Reynolds stresses are treated similar to the viscous stresses by introducing a turbulent viscosity (\mu_t). This is the Boussinesq hypothesis:

\[ \overline{\rho u'_{ij} u'_{ij}} = \frac{2}{3} \delta_{ij} \left( \overline{\rho \dot{k} e} + \mu_t \frac{\partial \overline{u_i u_j}}{\partial x_k} - \mu_t \left( \frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i} \right) \right) \]  

(1.10)

, for reasons of convenient notation, index notation is used here. In equation (1.10) is k\epsilon the turbulent kinetic energy, defined by:

\[ k\epsilon = \frac{1}{2} \overline{u_i' u_i'} \]  

(1.11)

In the k-\epsilon model the following expression for the turbulent viscosity is used:

\[ \mu_t = C_{\mu} \frac{k\epsilon^2}{\epsilon} \]  

(1.12)

The constant \( C_{\mu} \) is a modeling constant (usually 0.09).

For the turbulent kinetic energy (k\epsilon) and the turbulent dissipation (\epsilon) transport equations are used, which contain several modeled terms.

Also a Favre averaged transport equation for a scalar \( \phi \) can be derived. In this equation the term \( \overline{\mu_i^p \phi_i^p} \) arises. This term is called the Reynolds flux and it is also modeled with the Boussinesq hypothesis. The transport equation for \( \phi \) is:

\[ \frac{\partial \overline{\phi \rho}}{\partial t} + \nabla \cdot \left( \overline{\rho \phi u} - \left( \frac{\mu_t}{\sigma_\phi} + \overline{D_L} \right) \nabla \overline{\phi} \right) = \overline{\nabla \phi} \]  

(1.13)

, where \( \omega_\phi \) is the source term in the \( \phi \) transport equation, \( \sigma_\phi \) is the empirical turbulent Schmidt/Prandtl number, \( D_L \) is the laminar diffusion coefficient.

If \( \phi \) represents the mass fraction \( Y_i \) of a species, \( \omega_\phi \) is the chemical source term, which depends non-linearly on the instantaneous temperature and mass fractions by the Arrhenius equation. In a turbulent flow the turbulent fluctuations in the instantaneous temperature and mass fractions can be large, which makes it very difficult to get a good description of \( \overline{\nabla \phi} \). The goal of most combustion models is to get a good expression of \( \overline{\nabla \phi} \) while keeping the computational efforts as low as possible.

**PDF averaging**

The unclosed terms in the transport equation of \( \phi \) and the Naveir-Stokes equations are the mean density \( \overline{\rho} \) and the mean reaction rate \( \overline{\omega} \) (if \( \phi \) represents the mass fraction of a species). Both terms depend non-linearly on the combustion scalars, which are the mass fractions of the species and the temperature. In a turbulent flame these combustion scalars show large fluctuations in time. To be able to account for the influence of these fluctuations the Probability Density Function (PDF) approach is used. For example if the reaction rate \( \omega \) is a function of a combustion scalar \( \phi \), the mean value of \( \omega \) can be calculated from:

\[ \overline{\omega} = \int_{-\infty}^{\infty} \omega(\phi) \cdot P(\phi) d\phi \]  

(1.14)
where the PDF $P(\phi)$ gives the probability that the instantaneous value of $\phi$ is between $\phi$ and $\phi + d\phi$. If $\omega$ would be a function of several combustion scalars a multidimensional PDF should be used.

The problem now has shifted to the calculation of the PDF. It is possible to derive a transport equation for the PDF in chemical reacting flows (Pope (1985)). The solution of this equation is approximated by the solution for the trajectory of a large number of stochastic fluid particles, which represent different realisations of the flow. A Monte Carlo method is used to solve the transport equations. The computational costs of this method are very high. The great advantage of this method is that no extra modeling is necessary for the mean chemical source. Modeling is still required for the turbulent mixing of the fluid particles.

Another, more common, way to determine the PDF is to use an assumed shape PDF. The parameters that describe the shape of this PDF are the mean of $\phi$ and the variance of $\phi \bar{\phi}$. The mean value of $\phi$ can be derived from its transport equation. For $\bar{\phi} \bar{\phi}$, a new transport equation is derived, which contains several terms that have to be modeled. The most common assumed shape PDF's are: the $\beta$-function, the single $\delta$ function and the double $\delta$ function (Warnatz et al. (1990)).

1.4.2 Turbulent combustion

The occurrence of a wide range of time and length scales in a turbulent combusting flow complicates the description of turbulent flames. Further there is strong interaction between chemical reactions, heat release and turbulence. Because of this complexity the combustion models for turbulent flows are much less developed than the corresponding models for laminar flows. Most turbulent combustion models are based on the simplification of the real physical process.

The interaction between chemistry and turbulence may be classified by two criteria: premixed or non-premixed, slow or fast chemistry (Peters (1992)). The first criterion is relevant with respect to applications: combustion in large furnaces and many (older type natural gas or syngas) gas turbines is essentially non-premixed while in modern natural gas gas turbines the combustion occurs in the premixed regime. Slow chemistry is not very often of practical interest: there are a few situations like formation and destruction of pollutants in post-flame regions where chemistry is slow compared to convection and diffusion. Reacting flows with fast chemistry occur in nearly all the applications mentioned above, the burners in these applications have been designed to give a complete and stable combustion, which requires short chemical time scales. In this thesis only the fast chemical time scales are considered, because they appear to be the most important for the description of the acoustic behavior of a turbulent flame. The flames studied in this thesis are all of the non-premixed type, but for completeness premixed flames are discussed briefly.

Premixed combustion

In the case of premixed combustion the fuel and the oxidizer (air) are well mixed before the mixture is ignited. The advantage of premixed combustion over non-premixed combustion is that it is possible to get a low flame temperature, which is necessary to reduce the $NO_x$ formation. Until 15 years ago premixed flames were avoided for their disadvantages:
1.4 Turbulence and combustion

Turbulence may cause the flame front to be unstable and imprecise, as it may lead to uncontrolled heat release. To stabilize a premixed flame, a non-premixed flame, small non-premixed flames are used as pilot flames for a large premixed flame in many industrial applications.

The combustion in a premixed flame takes place in a small layer, the combustion zone. The combustion rate is a function of the area of this layer and the transport of heat and radicals from downstream of the layer to the fresh mixture upstream of the layer, where they ignite the fresh mixture. Due to the turbulence the area of the layer is increased and the transport is enhanced. To characterize this layer the following three dimensionless parameters are used: the turbulent Reynolds number ($Re_t$), the Damköhler number ($Da$) and the Karlovitz number ($Ka$).

The turbulent Reynolds number is defined by:
\[ Re_t = \frac{\rho u' l}{\mu} \]  
(1.15)

where $l$ is the integral length scale and $u'$ the scale of the turbulent velocity fluctuation.

The Damköhler number gives the ratio of the turbulent integral time scale over the chemical time scale:
\[ Da = \frac{t_{turb}}{t_{chem}} = \frac{l \cdot s_{fr}}{\delta_L \cdot u'} \]  
(1.16)

where $s_{fr}$ is the laminar burning velocity and $\delta_L$ is the laminar flame thickness. A large Damköhler number ($Da >> 1$) indicates that the chemistry is very fast compared to the turbulent time scales.

The Karlovitz number denotes the ratio of the time scale of the laminar flame ($t_L$) and the Kolmogorov time scale $t_{Kd}$:
\[ Ka = \frac{t_L}{t_{Kd}} = \frac{\delta_L^2}{\eta^2} \]  
(1.17)

It is now possible to construct the Borgli diagram, see figure 1.3. In this diagram the departure from a plane flame front to an increasingly three-dimensional structure is shown. In the Borgli diagram the ratio of the turbulent velocity over the laminar flame speed is plotted double logarithmic against the ratio of the turbulent integral length scale over the laminar flame thickness. Several lines divide the diagram into different domains with different flame behavior. When the turbulent Reynolds number ($Re_t$) is smaller than one, laminar combustion is observed. The domain of turbulent combustion is separated into three zones. The separation lines of these zones are given by $Ka = 1$ and $Da = 1$.

When the Karlovitz number is smaller than one, the flame thickness is smaller than the Kolmogorov length scale and the system is described as a locally laminar premixed flame embedded into a turbulent flow. This regime below the $Ka = 1$ line is called the flamelet regime ($Ka < 1$). The influence of the turbulence on the laminar flame (flamelet), is that the flamelet is wrinkled and stretched.

When $Da < 1$, the time needed for chemical change is larger than the time needed for turbulent mixing. This means that in this regime nearly all turbulent eddies are embedded in the reaction zone, which becomes so broad that the term 'flame front' is not useful anymore. In the Borgli diagram this area is located above the line $Da = 1$ and is called the ideally stirred reactor regime.
Figure 1.3: The Borghi diagram. On the x-axis the ratio of the turbulent integral length over the laminar flame thickness is plotted. On the y-axis the ratio of the scale of the turbulent velocity fluctuations over the laminar flame speed is plotted. Both axis are plotted logarithmic.

Between the stirred reactor zone and the flamelet zone is the distributed reaction zone, where a fraction of the eddies is embedded in the flame front. This will thicken the flame front.

In many turbulent premixed flames several flame regimes appear in one flame, because the turbulence is not identical at all locations in the flame. A premixed gas turbine flame is normally for the largest part in the 'distributed reaction zone' regime, but also parts of the flame maybe in the flamelet regime or the perfectly stirred reactor regime.

Combustion models for premixed combustion will not be discussed here.

Non-premixed combustion

In the case of non-premixed combustion the fuel and the oxidizer are introduced into the combustion chamber separately. The fuel and the oxidizer should first mix before combustion can take place. In general the flame zone is located at the position where the fuel and the oxidizer are mixed in the stoichiometric ratio, the mixing also determines the flame position. At the flame front the temperature is near the stoichiometric adiabatic flame temperature. The flame zone separates the fuel flow from the oxidizer flow.

It is not possible to use the Borghi diagram to characterize non-premixed turbulent flames since a laminar flame speed does not exist for these flames. There is a combustion diagram for turbulent non-premixed combustion which is characterized by two parameters: the mixing fluctuations at the flame front and the ratio of the Kolmogorov time to the chemical time. For a discussion of this diagram the reader is referred to Peers (1992).

In a turbulent non-premixed flame the mixing of fuel and oxidizer is described by the
mixture fraction ($\xi$), a conserved scalar, that equals one in the fuel stream and zero in the oxidizer stream. The reaction rates are large at the location where the mixture fraction has its stoichiometric value ($\xi = \xi_{st}$).

An important parameter that characterizes turbulent non-premixed flames is the ratio of the Kolmogorov time to the chemical time (Peters (1992)). This ratio is the inverse Karlovitz number. If combustion is fast compared to the Kolmogorov time, it may be considered as quasi steady. If however, the Kolmogorov time is of the same order or shorter than the chemical time, regions of intense mixing and reaction occur. As in premixed combustion, this regime is called the distributed reaction zones regime.

The models used for non-premixed turbulent combustion in this thesis are: the mixed-is-burnt model and the reaction progress variable model. Both models are only valid for large Damköhler numbers, the chemical reactions are much faster than the turbulent mixing. For this situation it is possible to separate the (turbulent) mixing, described with $\xi$, from the chemical reactions. Because the deviation from chemical equilibrium is small in this case, it possible to reduce the reaction mechanism (or even assume chemical equilibrium). These models are described briefly in the next two sections.

The mixed-is-burnt model The mixed-is-burnt model is based on infinitely fast chemistry (very large Damköhler number): the chemical reactions can be considered completed as soon as the reactants are mixed. All chemical species are in chemical equilibrium. This implies that all species can be expressed as a function of the mixture fraction only. The precondition for this is that the diffusion coefficients for all species are equal. For turbulent flows in which the turbulent transport is described with a turbulent diffusion coefficient, which is much larger than the molecular diffusion and equal for all species, the mixture fraction approach can be used (see e.g. Bilger (1980)). The mixture fraction is defined by the mass fractions of the (arbitrary) element $i$ ($Z_i$):

$$\xi = \frac{Z_i - Z_{i,\text{air}}}{Z_{i,\text{fuel}} - Z_{i,\text{air}}}$$  \hspace{1cm} (1.18)

($\xi = 0$ in the air flow and $\xi = 1$ in the fuel flow)

The (laminar) transport equation for the mixture fraction is:

$$\frac{\partial \rho \xi}{\partial t} + \nabla \cdot (\rho \vec{U} \xi - \rho D_L \nabla \xi) = 0$$  \hspace{1cm} (1.19)

The mixture fraction is a conserved scalar as can be noticed from equation (1.19) (the transport equation for it does not contain a source term). If there is no heat loss to the environment (adiabatic situation) and the energy diffuses at the same rate as the species ($Le = 1$), the enthalpy field is also an unique function of the mixture fraction. Thus, assuming (1) equilibrium chemistry, (2) equal diffusivity and $Le = 1$ and (3) no heat loss, all scalar variables (temperature, mass fractions and density) are known functions of the mixture fraction only.

The influence of the turbulent fluctuations in the mixture fraction on the scalar variables has to be taken into account. The way to do this, is to use a Probability Density Function as described in section 1.4.1. A PDF with a specified (assumed) shape will be used. This PDF is determined by the mean of the mixture fraction ($\xi$) and the variance of the mixture...
fraction \( (\xi^T \xi'' = g) \). The (mean) transport equation for \( g \) is:

\[
\frac{\partial \overline{g}}{\partial t} + \nabla \cdot \left( \overline{p} \overline{L} g - \overline{p} (D_L + D_{turb}) \nabla g \right) = 2\overline{p} D_{turb} \nabla \overline{\xi} \cdot \nabla \overline{g} - 2 \rho D_L \nabla \xi^T \cdot \nabla \xi'^T \quad (1.20)
\]

The last term at the right hand side of this equation contains the scalar dissipation rate \( \chi \): this is the rate at which turbulent fluctuations in the mixture fraction are dissipated by molecular mixing. It is the mixing rate at molecular level. A common way to model the scalar dissipation is to relate it to the turbulent time scale of the velocity fluctuations:

\[
\chi = 2D_L \nabla \xi^T \cdot \nabla \xi'^T \simeq C_{\rho 2} \frac{g}{\nabla \xi_{turb}} = C_{\rho 2} \frac{e}{t_{ke} \rho} \quad (1.21)
\]

where \( C_{\rho 2} \) is a modeling constant (normally 2.0).

There are two approaches to relate the scalar parameters (temperature, mass fraction, density) uniquely to the mixture fraction. The first way is to describe the combustion reaction as an irreversible one-step reaction, this is the original mixed-is-burnt-approach. The other way is to assume chemical equilibrium, there is also a reverse reaction, this is called the local equilibrium PDF approach. The mass fractions of the oxidizer, the reactive components in the fuel and the product, calculated with both approaches are plotted in figure 1.4. With the one step method, the relation between the mass fractions and the mixture fraction is represented by a straight line with a discontinuity at \( \xi = \xi_{st} \). With the equilibrium approach the lines are smoothed and there is no discontinuity at \( \xi = \xi_{st} \).

The advantage of the mixed-is-burnt approach is that the transport equation for the mixture fraction does not contain a source term, what makes this equation relatively easy to solve. In certain cases one is interested in the chemical source term, for example if the heat release by the flame should be calculated. The starting point to express a chemical source term in terms of \( \xi \), is the (laminar) transport equation for the mass fraction of a species,
1.4 Turbulence and combustion

e.g. the fuel, with the source term $\omega_{fuel}$. This mass fraction is written as a function of the mixture fraction: $Y_{fuel} = Y_{fuel}(\xi)$. After some simple algebraic operations this transport equation for $Y_{fuel}$ can be written as:

$$\left[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u \xi - \rho D_L \nabla \xi) \right] \cdot \frac{dY_{fuel}}{d\xi} - \frac{d^2 Y_{fuel}}{d\xi^2} = \omega_{fuel}$$

The part between the square brackets at the left hand side in this equation is the transport equation for the mixture fraction and equals zero. This means that the chemical source term equals:

$$\omega_{fuel} = -\frac{d^2 Y_{fuel}}{d\xi^2} - \rho D_L \nabla \xi \cdot \nabla \xi$$

If this equation is time averaged (assuming that there is no correlation between $d^2 Y_{fuel} / d\xi^2$ and $\rho D_L \nabla \xi \cdot \nabla \xi$) the last part can be recognized as half the scalar dissipation ($\chi$). This gives physical the interpretation of this term: the chemical reaction rate is determined by the molecular mixing of fuel and air.

The term $\frac{d^2 Y_{fuel}}{d\xi^2}$ in equation (1.23) shows a Dirac delta behavior with only a value at $\xi = \xi_{st}$ ($\delta(\xi - \xi_{st})$). This can be seen from figure 1.4, because $\frac{d^2 Y_{fuel}}{d\xi^2}$ has a discontinuity at $\xi = \xi_{st}$. This means that reaction takes place only at the location where $\xi = \xi_{st}$ and that the instantaneous flame sheet is infinitely thin.

The mixed-is-burnt model has shown to give reasonable results for the prediction of the flame shape, the concentration of the mean species, the temperature and the heat release. The mixed-is-burnt model gives however an underprediction for the pollutant formation because the concentrations of radicals, which are responsible for the pollutant formation, are not well predicted (Bilger (1980), Warnatz et al. (1998)). Therefore extensions have been made, where chemical non-equilibrium is incorporated. One of them is discussed here: the reaction progress variable model.

The reaction progress variable model In a reaction progress variable model the complex chemical reaction scheme with many dimensions (species) is projected on a small number of reaction progress variables. Transport equations are solved for these reaction progress variables. In theory the number of reaction progress variables is unlimited but to keep the advantage of this method the number of reaction progress variables should be minimized, while keeping the important chemical properties.

In the F.I.R.S. model, which is developed at the University of Twente (Louis (1997) and Kok and Louis (1998)) and which is based on the work by Coore (1985) and Janicka and Kollman (1978), the complex chemical scheme of CO/H2-oxidation (21 reversible reactions, 10 species) is projected on two reaction progress variables. To do this partial equilibrium and steady state assumptions are used (other methods like ILDM give similar results (Egels et al. (1997))). The reaction progress variables are $r$, which gives the deviation from equilibrium for the $H_2$ reactions, and $s$, which gives the progress of the CO reaction. It is assumed that all scalars are statistically independent, which makes it possible to factorize the multi-dimensional PDF to the product of one-dimensional PDFs. To describe the mixing between fuel and air the transport equation for the mixture fraction (in this model called $f$) is solved. Heat loss can be taken into account by using the $i$ variable, which is the normalized enthalpy.
Chapter 2

Acoustics

2.1 Introduction

Acoustics describes the propagation and generation of sound. Acoustical phenomena are fluid mechanical phenomena and are governed by the following equations: the equation for the conservation of mass, the momentum equations (the Navier Stokes equations) and an energy equation. In this study the enthalpy equation is taken for the energy equation, because combustion processes are more easy to describe in terms of enthalpy. These three equations are given here for the mixture of components and not for the separate components (Bird et al. (1960)). The equation for the conservation of mass is:

\[
\frac{\partial p}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0
\]  \hspace{1cm} (2.1)

The equation for the conservation of momentum is:

\[
\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) = -\nabla p + \nabla \cdot \mathbf{\tau}
\]  \hspace{1cm} (2.2)

where \( \mathbf{\tau} \) is the viscous stress tensor. External body forces (like the gravity) have been neglected.

The equation for the enthalpy is (Bird et al. (1960)):

\[
\rho \frac{Dh}{Dt} = -\left( \nabla \cdot \mathbf{q} \right) - (\mathbf{\tau} : \nabla \mathbf{u}) + \frac{Dp}{Dt}
\]  \hspace{1cm} (2.3)

where \( \mathbf{q} \) is the heat conduction vector.

To complete this set of equations the equation of state, like the ideal gas law, should be added together with the transport equations for the different components in the mixture. These equations are given by equation (C.7) and (C.12) respectively.

2.2 The acoustic wave equation

For the derivation of the acoustic wave equation, the procedure of Lighthill is followed (Lighthill (1954)), except that the pressure is now chosen as acoustic variable instead of
the density. In this procedure the time derivative of the equation for the conservation of mass (equation (2.1)) is taken and the divergence operation is applied to the Navier Stokes equations (equation (2.2)). The results are subtracted:

$$\frac{\partial \rho}{\partial t} - \nabla^2 \rho = \nabla \cdot (\rho \mathbf{u} \mathbf{u} - \mathbf{E})$$

(2.4)

Subsequently the term $\frac{1}{v_0} \frac{\partial^2 (p - p_0)}{\partial t^2} - \frac{\partial^2 p_0}{\partial t^2} + \nabla^2 p_0$ is added both to the left hand side and the right hand side of equation (2.4), the term $\frac{\partial^2 p}{\partial t^2}$ is moved to the right hand side. The subscript $0$ denotes a constant (in space and time), mean value in the distant acoustic field. This yields:

$$\frac{1}{v_0^2} \frac{\partial^2 \rho}{\partial t^2} - \nabla^2 \rho' = \nabla \cdot (\nabla \cdot \mathbf{T}) - \frac{\partial \rho_c}{\partial t}$$

(2.5)

In equation (2.5) is $\rho'$ the acoustic pressure fluctuation defined as $\rho' = p - p_0$, $\mathbf{T}$ is the modified Lighthill stress tensor, defined as $\rho \mathbf{u} \mathbf{u} - \mathbf{E}$ and $\rho_c$ is the 'excess' density defined as:

$$\rho_c = \rho - \rho_0 - (p - p_0) / v_0^2$$

(2.6)

The left hand side of the wave equation, equation (2.5), gives the propagation of a pressure disturbance with the speed of sound $v_0$. At the right hand side there are two source terms: the flow noise source term: $\nabla \cdot (\nabla \cdot \mathbf{T})$ and the thermo-acoustic source term: $- \frac{\partial \rho_c}{\partial t}$. In a turbulent flame both noise source terms are present. The thermo-acoustic source term is evaluated in appendix C. From an order of magnitude analysis, performed in that appendix, it is found that the thermo-acoustic source term is the most important one in a (low Mach number) flame. This thermo-acoustic source term is mainly a function of the heat release ($q$) in the flame. The wave equation with only this source term is:

$$\frac{1}{v_0^2} \frac{\partial^2 \rho'}{\partial t^2} - \nabla^2 \rho' = \frac{\partial}{\partial t} \left[ \frac{\rho \gamma - 1}{\rho} \frac{c^2}{\gamma} q \right]$$

(2.7)

The subscript $0$ in this equation refers to a mean constant value in the far field, the variables $\rho$ and $\frac{\gamma c^2}{\gamma - 1}$ should be evaluated at the location of the heat release.

### 2.3 The role of the flame

The discussion in the previous section demonstrates that the fluctuating heat release from a flame appears in the acoustic wave equation as a monopole source term. Two types of source behavior will now be discerned, the nature of the source term depends on the origin of the heat release fluctuations. Because the wave equation is linear, it is allowed to write the fluctuating heat release by the flame as the sum of these two different terms ($q = q_{\text{noise}} + q_{\text{amp}}$). The two different source terms are now:

- The flame as (autonomous) source of sound ($q_{\text{noise}}$). The fluctuations in the heat release are independent of the acoustic pressure in the combustion chamber. The fluctuations in the heat release are caused by random turbulent fluctuations, vortex shedding etc.
2.4 Sound propagation

A solution of the homogeneous part of the wave equation (2.7) in a quiescent medium with a constant speed of sound is:

\[ p' = f(d \cdot \vec{z} - c_0 t) \]

(2.8)

in which \( f \) is a twice differentiable function, determined by the boundary and initial conditions, and \( |d| = 1 \). This solution describes the propagation of a disturbance at the speed of sound in the direction of \( d \).

The effect of convection on the propagation of sound has been neglected in equation (2.7). This effect is included in the Lighthill stress tensor in equation (2.5). The convective effects on the propagation of sound can be included explicitly for a flow with a constant mean velocity and a constant density by linearisation of the mass conservation equation and the Navier-Stokes equations (Dowling and Flows Williams (1983)). The convective homogeneous wave equation is:

\[ \frac{1}{c_0^2} \frac{D^2 p'}{D t^2} - \nabla^2 p' = 0 \]

(2.9)

A solution for the one-dimensional convective homogeneous wave equation is:

\[ p'(x, t) = f(x - (c_0 + \bar{u}) \cdot t) + g(x + (c_0 - \bar{u}) \cdot t) \]

(2.10)

This equation describes the propagation of two disturbances, one to the left with the velocity \( c_0 - \bar{u} \) and one to the right with the velocity \( c_0 + \bar{u} \). The propagation of sound is also influenced by the mean flow to the order \( Ma \). The initial condition prescribes \( f \) and \( g \). Equation (2.10) is valid for an infinitely long one-dimensional domain, the reflection of sound waves at boundaries is not included.

2.4.1 Harmonic waves

The description of the propagation of sound is facilitated by considering harmonic waves only. For linear acoustics any solution to the wave equation can be written as a sum of harmonic waves. The solution \( p' \) can be written as:

\[ p'(\vec{z}, t) = \sum_{j=1}^{n} \tilde{p}_j(\vec{z}) e^{i\omega_j t} \]

(2.11)

where \( \tilde{p}_j(\vec{z}) \) is a complex quantity (the Fourier transform of \( p' \) in time at \( \omega_j \)), \( i = \sqrt{-1} \) and \( \omega_j \) is the angular frequency.
The convective wave equation for a harmonic wave is obtained by introduction of equation (2.11) into equation (2.9):\[ \omega_j^2 \ddot{p} + (\alpha_0 - \mathbf{u} \cdot \mathbf{u}) \nabla^2 \dddot{p} = 0 \] (2.12)
The solution of this equation is fully determined by the acoustic boundary conditions.

The solution of the one-dimensional convective wave equation is:
\[ p'(x,t) = \sum_{j=1}^{n} \left( p_j^+ e^{i(\omega_j t - k_j^+ x)} + p_j^- e^{i(\omega_j t + k_j^- x)} \right) \] (2.13)
with \( k_j^+ = \omega_j / (c_0 + U) \) and \( k_j^- = \omega_j / (c_0 - U) \), \( k_j^+ \) and \( p_j^+ \) are in the positive x direction, \( k_j^- \) and \( p_j^- \) are in the negative x direction. The complex amplitudes \( p_j^+ \) and \( p_j^- \) are given by the boundary conditions and possible source terms in the wave equation. If the mean velocity equals zero, \( k_j^- \) will be equal to \( k_j^+ \) and the solution of the wave equation can be written with geometric functions. The complex amplitudes \( p_j^+ \) and \( p_j^- \) are used throughout this thesis.

2.4.2 Description of an acoustic system

One-dimensional acoustic wave propagation is described by the following set of variables: \( u' \), \( p' \) and \( \varphi' \). The variable \( p' \) can be written as \( p'/c^2 \). This means that the acoustics in an isentropic situation can be described by two variables. This set of variables can be: the two traveling acoustic pressure waves \( (p^+ \) and \( p^-) \) or the acoustic pressure and the acoustic velocity \( (p' \) and \( u') \).

The acoustic equations are linear equations, which means that the relation between the acoustic properties at two different locations in a one-dimensional system can be written with a linear transfer matrix. This matrix method is a well known method from literature (see e.g. Munjal (1987)) to study acoustic systems. In this thesis it is chosen to use the traveling waves \( (p^+ \) and \( p^-) \) as the acoustic variables. Deiker (1995) uses the acoustic pressure and the acoustic velocity, Paschereit and Polifke use the two Riemann invariants for the acoustic variables (Paschereit and Polifke (1998), Schuermans et al. (1999)), which are a combination of the acoustic pressure and the acoustic velocity.

The combustion system of a gas turbine can be thought of as a system of acoustic components, each of which can be characterized by a transfer matrix. It is prescribed that at the connection of two (or more) components the acoustic solution is continuous. All component matrices together yield a system matrix equation, that can be solved numerically. This matrix method is used in this thesis to describe the interaction between acoustics and combustion for acoustically one-dimensional system (chapter 4-7). This method is explained in appendix D.

In addition to this matrix method another method is used in this thesis. In this method the acoustic eigenfunctions of the combustion chamber are used. By using essentially Galerkin's approach it is then possible to eliminate the space dependency. (Culick (1994)), this method is called the mode analysis method. The result of this analysis is a normal differential equation with respect to time (second order system), which can easily be solved for the linear case. This latter method assumes that the difference between the real acoustic situations and the acoustic eigenfunctions is small. This method is used in chapter 7 to perform a system study of a real gas turbine.
For both methods computations are performed for a Rijke tube. A comparison of the results is presented in appendix E.

2.5 The mode analysis method

By using Galerkin’s method, which is based on the mode analysis of the combustion chamber, it is possible to transform the partial differential equation of the wave equation to an ordinary differential equation with respect to time. This method is referred to in this thesis as the mode analysis method. Culick (1994) used this method to study the non-linear behavior of combustion instabilities. Such a study of non-linear behavior is not possible with the transfer matrix method (appendix D). In this thesis the mode analysis method will only be used to study the linear behavior of acoustic systems.

The wave equation with only the combustion source term, equation (2.7), is taken as a starting point. A weighting function $\psi_m$ is introduced, which should satisfy equation (2.12) (the influence of the mean flow velocity is neglected):

$$k_m^2 \psi_m + \nabla^2 \psi_m = 0$$

This equation is known as the Helmholtz equation, $k_m$ is the acoustic wave number. The solutions of equation (2.14) are determined by the acoustic boundary conditions of the combustion chamber. These boundary conditions are assumed here to be identical to those for the acoustic pressure $p'$ (this is not essential for the method, but assumed here to keep the expressions simple). The function $\psi_m$ is the eigenfunction of the acoustic cavity (in our case the combustion chamber). The wave equation, equation (2.7), is multiplied with $\psi_m$. Equation (2.14) is multiplied with the acoustic pressure $p'$. The results are added and integrated over the volume of the combustion chamber. This gives:

$$\int \left[ p' \nabla^2 \psi_m - \psi_m \nabla^2 p' \right] dV + \frac{1}{c_0^2} \int \psi_m \frac{\partial^2 p'}{\partial t^2} dV + k_m^2 \int p' \psi_m dV = \int \psi_m \gamma - \frac{1}{c^2} \frac{\partial q}{\partial t} dV$$

Green’s theorem is applied to the first term at the left hand side of equation (2.15), this gives:

$$\int \mathbf{n} \cdot \left( p' \nabla \psi_m - \psi_m \nabla p' \right) dA + \frac{1}{c_0^2} \int \psi_m \frac{\partial^2 p'}{\partial t^2} dV + k_m^2 \int \psi_m p' dV = \int \psi_m \gamma - \frac{1}{c^2} \frac{\partial q}{\partial t} dV$$

where $\mathbf{n}$ is the normal vector, perpendicular to the boundary.

If the acoustic boundary conditions for $p'$ and $\psi_m$ are identical, the first integral at the left hand side of equation (2.16) disappears. It can be proved that the eigenfunctions $\psi_m$ form a complete orthogonal set\(^1\). The acoustic pressure can be expressed as a function of them by ($\mathbf{\eta}$ is an arbitrary constant):

$$p' = \mathbf{\eta} \sum_{n=1}^{\infty} \eta_n(t) \psi_n(z)$$

\(^1\)This can easily be seen if the shape of the combustion chamber is rectangular or cylindrical. The eigenfunctions $\psi_m$ (the solution of equation (2.14)) can then be written in terms of geometric functions, which form a complete orthogonal set.
Equation (2.16) reduces then to a normal differential equation \( \dot{\psi}_n(t) = \omega_n / c_0 \), \( \omega_n \) is the \( n \)th
eigenfrequency of the combustion chamber belonging to mode \( \psi_n \):
\[
\frac{d^2 \psi_n(t)}{dt^2} + 2 \zeta \omega_n \frac{d\psi_n}{dt} + \omega_n^2 \psi_n = S \tag{2.18}
\]

The normal (modal) linear damping term \( 2 \zeta \omega_n \), not present in equation (2.16), for a second order system has been included here. This damping can for example be caused by viscosity or non-ideal reflection at the boundaries. In this equation \( S \) is the source term, defined by:
\[
S = \frac{c_0^2}{p} \int \psi_n \frac{\gamma - 1}{c^2} \frac{\partial q}{\partial t} dV \tag{2.19}
\]

If the acoustic eigenfunctions of the combustion chamber and the heat release as function of time and space are specified, equation (2.19) can be solved to obtain a value.

In this research only linear acoustics are studied. The eigenmode analysis can however also be applied to non-linear acoustics, the modes are coupled in that situation [Culick (1994)]. The linear approach is applicable as long as the non-linear terms can be neglected with respect to the linear terms. It is possible to study the occurrence and onset of an instability using the linear approach, to determine the amplitude of the instability non-linear (damping) effects should be taken into account.

### 2.5.1 The flame in the acoustic system, Rayleigh’s criterion

The mode analysis method is a good way to analyze theoretically the interaction between the flame and the acoustics in the cavity. The fluctuating heat release by the flame can have two origins, see also section 2.3:

1. The flame as an autonomous source of sound. This role of the flame is indicated with \( q_{\text{noise}} \).

2. The flame as an amplifier of sound. The total heat release by the flame fluctuates (with a constant phase) with the pressure in the combustion chamber. Just as the pressure, the heat release can be split in a time dependent part and a spatial part. Because of the coupling, the time dependent part of the heat release is identical to that of the pressure \( (\dot{\psi}_n) \). The spatial part is written with the partial derivative of the heat release with respect to the pressure:
\[
\frac{\partial q (x, t)}{\partial t} = \frac{\partial q}{\partial p} \left( \frac{\partial p}{\partial x} \right) + \frac{\partial q}{\partial p} \sum \psi_n (x) \left[ \frac{\partial \dot{\psi}_n}{\partial t} \right] (t) \tag{2.20}
\]

With the two roles of the flame the equation for the cavity (equation (2.18)) becomes:
\[
\frac{d^2 \psi_n}{dt^2} + 2 \zeta \omega_n \frac{d\psi_n}{dt} + \omega_n^2 \psi_n = \frac{c_0^2}{p} \int \psi_n \frac{\gamma - 1}{c^2} \frac{\partial q_{\text{noise}}}{\partial t} dV + \frac{\partial q}{\partial p} \psi_n \frac{d\psi_n}{dt} \tag{2.21}
\]

Because equation (2.21) is a linear equation, the solution of it is the sum of the solutions related to either of both source terms. The flame as an autonomous source of sound gives
an ordinary second order differential equation with a forcing term. This role of the flame is not discussed here further, the solution of the spectrum of $p$ belonging to this source term is given in equation (2.30). The focus is here on the case where a coupling between the pressure and the heat release exists.

The term with $\frac{\partial q}{\partial p}$ in equation (2.21) can be moved to the left hand side, where it appears as a damping term (containing the term $\frac{dp}{dt}$):

$$\frac{d^2 \eta_n}{dt^2} + \left(2\zeta \omega_n - \frac{c_0^2}{\int \psi^2_n dV} \int \psi^2_n \frac{\partial q}{\partial p} dV \right) \frac{d\eta_n}{dt} + \omega_n^2 \eta_n = 0 \tag{2.22}$$

As long as the total damping term, the term between brackets in equation (2.22), remains positive, the system will be damped and stable. If the total damping term becomes negative, the amplitude of the oscillation will grow exponentially in time and the system is unstable.

The relationship between pressure and heat release can generally be written in the following form, where $T_{flame}$ is a complex quantity and $\phi$ the phase between the pressure fluctuation and the heat release fluctuation:

$$\frac{c_0^2}{\int \psi^2_n dV} \int \psi^2_n \frac{\partial q}{\partial p} dV = T_{flame} = R \cdot e^{i\phi} = R(\cos(\phi) + i\sin(\phi)) \tag{2.23}$$

From this equation it can be derived that the total (real) damping term in equation (2.22) ($= $ damping - amplification) is negative if:

$$R \cos(\phi) > 2\zeta \omega_n \tag{2.24}$$

Equation (2.24) can only be valid if the left hand side of it is larger than zero:

$$\cos(\phi) > 0 \Rightarrow -90^\circ < \phi < 90^\circ \tag{2.25}$$

This equation is known as Rayleigh’s criterion. It gives the possibility for combustion driven acoustic oscillations in a system, it is however not sufficient: If the phase $\phi$ between the pressure and the heat release is between $-90^\circ$ and $90^\circ$ the flame ‘amplifies’ the acoustic pressure in the combustion chamber and an instability might exist, if the amplification by the flame is larger than the damping in the system, see equation (2.24). This criterion was first formulated by Lord Rayleigh (Rayleigh (1878)):

If heat be periodically communicated to, and abstracted from, a mass of air vibrating (for example) in a cylinder bounded by a piston, the effect produced will depend upon the phase of the vibration at which the transfer of heat takes place. If heat be given to the air at the moment of greatest condensation, or be taken from it at the moment of greatest rarefaction, the vibration is encouraged. On the other hand, if heat be given at the moment of greatest rarefaction, or abstracted at the moment of greatest condensation, the vibration is discouraged.

Putnam has formulated Rayleigh’s criterion mathematically for harmonic waves (Putnam (1971)). Lang extended the criterion of Putnam to non-harmonic waves by formulating it in terms of the cross spectrum between the heat release and the pressure (Lang and Vormeyer (1987)). The cross spectrum formulation of Lang has the same condition as equation (2.25) in order to have combustion driven oscillations in a combustion chamber.
The fulfillment of Rayleigh’s criterion is essential to have combustion instabilities. To determine if these oscillations really occur, equation (2.24) can be used. In most combustion installations, especially complex ones like gas turbines, it is difficult to determine the modal damping (\( \zeta \)) of the system.

To calculate the eigenfrequency of the system and the linear growth rate, the time dependent function \( \eta_n(t) \) is written as a harmonic function:

\[
\eta_n(t) = \tilde{\eta}_n e^{i\omega t}
\]  
(2.26)

In this equation is \( \omega \) a complex number \( \omega = \omega_{re} + i\alpha \), the real part \( \omega_{re} \) indicates the frequency in rad/s, the imaginary part \( \alpha \) gives the negative value of the growth rate.

Equation (2.22) can be rewritten to:

\[
\left( - (\omega_{re} + i\alpha)^2 + i(\omega_{re} + i\alpha) \left( 2\zeta \omega_n - R \cdot e^{i\phi} \right) + \omega_n^2 \right) \tilde{\eta}_n = 0
\]  
(2.27)

Equation (2.27) can be solved to obtain \( \omega_{re} \) and \( \alpha \). This equation indicates that there exists only a discrete number of non-trivial solutions for \( \omega_{re} \) and \( \alpha \). This means that in the frequency domain instabilities are characterized by peaks with an infinitely narrow bandwidth at \( \omega_{re} \). With some simplifications \( (\alpha / \omega_{re}, (\omega_{re} - \omega_n) / \omega_n << 1) \) an estimate for the instability frequency and the growth rate can be found:

\[
\omega_{re} = \omega_n + R \sin (\phi)
\]  
(2.28)

\[
\alpha = \zeta \omega_n - \frac{R \cos (\phi)}{2}
\]  
(2.29)

Equation (2.28) indicates that the frequency of the instability may be shifted with respect to the eigenfrequency of the combustion chamber.

### 2.5.2 The flame as source versus the flame as amplifier

For the flame as source of sound it can be derived that the pressure amplitude spectrum is (see equations (2.18) and (2.19)):

\[
p(\omega) = \sum_n \left\{ \frac{\psi_n(\omega) / \omega_n^2}{\sqrt{1 - (\omega / \omega_n)^2}} \frac{\omega_n^2}{\sqrt{\int \psi_n^2 dV}} \int \frac{\gamma - 1}{c^2} \psi_n q_{noise} dV \right\}
\]  
(2.30)

The spectrum of \( p \), due to the combined action of the noise generation by the flame \( q_{noise} \) and the resonance in the combustion chamber, is a broad spectrum around the eigenfrequency of the combustion chamber (the eigenfrequency can be shifted a little due to damping). From the sound spectrum measured in a combustion chamber it can be seen if there is instability or (just) resonance. The differences between both are summarized in table 2.1.

An example of the difference in the sound spectrum between resonance and instability is given in figure 2.1. A similar figure has been shown by Schimmer and Vortmeyer (1977). In this figure two sound spectra are plotted, one for a combustion chamber with a length of 1.3 m (eigenfrequency about 150 Hz) and one for which the combustion chamber was elongated to 2.0 m (eigenfrequency about 100 Hz). The flame was identical in both cases.
2.5 The mode analysis method

<table>
<thead>
<tr>
<th>Resonance</th>
<th>Instability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive damping</td>
<td>Negative damping</td>
</tr>
<tr>
<td>Broad spectrum</td>
<td>Spike in spectrum</td>
</tr>
<tr>
<td>(in case of small damping)</td>
<td>(infinitely narrow bandwidth)</td>
</tr>
<tr>
<td>Peak around the acoustic eigenfrequency of the combustion chamber</td>
<td>Eigenfrequency shifted with respect to the acoustic eigenfrequency of the combustion chamber</td>
</tr>
<tr>
<td>Finite amplitude</td>
<td>Infinitely high amplitude</td>
</tr>
<tr>
<td></td>
<td>(ultimately limited by non-linear effects)</td>
</tr>
</tbody>
</table>

Table 2.1: The differences between resonance and instability in a combustion chamber.

![Graph](image)

Figure 2.1: The difference between the flame as amplifier of sound and the flame as source of sound. The pressure amplitude, measured by a microphone in the combustion chamber, is plotted linear ($\Delta f_{FFT} = 0.78125$ Hz).
The experimental set up is discussed in section 5.6.1. The figure shows that with the longer combustion chamber the sound spectrum is a broad spectrum: the flame acts only as a source of sound, Rayleigh’s criterion is not fulfilled here. For the short combustion chamber the spectrum shows a sharp peak with a very narrow bandwidth, the flame acts as an amplifier of sound. Rayleigh’s criterion is fulfilled and the instability frequency is shifted with respect to the acoustic eigenfrequency. The amplitude of the instability peak is about 100 times the amplitude of the resonance peak.
Chapter 3

Noise generation by turbulent non-premixed flames

3.1 Introduction

In this chapter the autonomous sound generation by low Mach number non-premixed turbulent flames with infinitely fast chemistry is studied. The study presented in this chapter is very similar to the paper by Klein and Kok (1999b), some changes in the description of the theory and small corrections are incorporated in this chapter.

The contents of this chapter are: First a short summary of the literature on the subject of sound generation by turbulent flames is given in section 3.2. In section 3.3 a model for combustion noise from non-premixed turbulent flames is derived. In section 3.4 some simplifications to the noise generation model are applied and the governing variables for the noise generation are determined. In the next section, section 3.5, the coupling of the acoustic model to an existing steady state turbulent non-premixed flame code is discussed. In section 3.6 the experimental method is discussed. In section 3.7 the calculations made with the model are compared with experiments, performed with a bluff body burner. In section 3.8 the influence of the mean pressure on the noise generation is studied. In section 3.10 results of experiments with a swirl stabilized burner are presented. Conclusions are drawn in section 3.11.

3.2 Literature on turbulent combustion noise

Compared to the amount of research performed on turbulent jet noise very little research has been performed on combustion noise. Strahle is the one with probably most publications on this subject. Strahle studied both premixed and non-premixed turbulent flames. In the models for premixed flames of Strahle (see e.g. Strahle (1983), Strahle (1971)) fluctuations in the flame surface area are the source of sound (the flame front is wrinkled by the turbulence). The total sound emission from a flame can be calculated by writing these fluctuations of the flame surface as a function of the mean flow quantities. Strahle's models do not give a sound spectrum.
Clavin and Siggia (1991) did research on the sound spectrum of turbulent premixed flames. Like Strahle he took the fluctuations in the flame surface area as the source of sound. These fluctuations are described in terms of the turbulent spectrum. For a turbulent spectrum with Kolmogorov shape (a fall off of $k^{-5/3}$ in the inertial subrange) he comes to a fall off of the sound spectrum with $\omega^{-5/2}$. In his paper no predictions are made for the sound level.

Ohiwa et al. (1993) did an experimental study on noise characteristics of turbulent diffusion flames. Both the directional and spectral characteristics of combustion noise were experimentally examined for three turbulent non-premixed flames with different turbulent structure. The influence of the turbulent structure was clearly visible in the acoustic measurements. The source of sound of the flame further showed a weak quadrupole behavior.

The sound generation from a turbulent premixed flame was studied by Boinéau et al. (1995). In their paper they derive an expression for the noise from a turbulent premixed flame using spatial coherence functions. They come to a three-dimensional sound spectrum that behaves as $f^4$ for low frequencies and falls off following $f^{-2}$. Their model predictions agree well with measurements for a test case.

In this study an integral equation is derived that describes the sound spectrum as a function of the local turbulent mixture fraction spectrum at the flame front. The input, necessary for the calculation of the sound spectrum, can be derived from a steady state combustion calculation. This follows the approach of Béchara et al. (1994) who calculated the sound output from a turbulent jet with information about the turbulence from a k-ε calculation.

The basic idea of the present model is that sound is generated by turbulent fluctuations in the heat release rate at the flame front. In the non-premixed flame studied here, the mixture fraction determines the heat release. The most important part of the model is then the description of the turbulent heat release spectrum as a function of the turbulent mixture fraction spectrum at the flame front. This is achieved by the use of the mixed-is-burnt approach to describe the combustion. For the spectrum of the turbulent mixture fraction fluctuations a model spectrum is used. The use of turbulent model spectra has also been done by Rubinstein and Zhou (1997) to calculate the sound from isotropic turbulence. Introduction of the model spectra into the combustion noise model yields a local heat release spectrum and a local sound spectrum. The sound spectrum from the total flame is calculated by the integration of all local contributions. It should be pointed out that only the sound generation by the flame is studied in this chapter, the rest of the acoustic system (like the combustion chamber and the supply tubes) are supposed to be anechoic, so they don’t influence the resulting sound spectrum. Further this study presumes that there is no feedback from the acoustics to the flame. This method can therefore not be used to predict (thermo-)acoustic system instability.

### 3.3 Theory

The propagation and generation of sound are described by the acoustic wave equation. As discussed in chapter 2 and appendix C, this equation is derived by combining the equation for the conservation of mass, the equation for the conservation of momentum (Navier-Stokes equations) and the enthalpy equation. The resulting wave equation contains several source
3.3 Theory

In the theory of acoustics, the most relevant source term in low Mach number combustion is the source term caused by the fluctuating heat release due to chemical reactions. The wave equation (2.7), for low Mach number flows, with only this source term, is:

\[
\frac{1}{c_0^2} \frac{\partial^2 p}{\partial t^2} - \nabla^2 p = \frac{\gamma - 1}{c^2} \frac{\partial q}{\partial t} \quad (3.1)
\]

The ratio \( \frac{q}{p} \) from equation (2.7) is assumed to be 1. The subscript 0 in equation (3.1) refers to a mean value in the far field.

The heat release source term is given by:

\[
q = \sum_i \omega_i h_i^0 \quad (3.2)
\]

With \( \omega_i \) the chemical source term of species \( i \) and \( h_i^0 \) the enthalpy of formation of species \( i \).

In most cases one is interested in the sound in the acoustic far field, where the source term in equation (3.1) equals zero. If the flame is compact compared to the acoustic wavelength, the sound in the far field from the flame can be calculated with a Green’s function.

In this study the one-dimensional acoustic field from flames is studied, the one-dimensional Green’s function is:

\[
G(x_0, t|x, \tau) = \frac{c_0}{S} \left( t - \tau - \frac{|x_0 - x|}{c_0} \right) \quad (3.3)
\]

For non-1D acoustic situations the appropriate Green’s function (e.g., equation (C.15) for the free space Green’s function) should be used instead of equation (3.3). Multiplying equation (3.1) with the Green’s function (equation (3.3)) and taking the partial integration over time gives for the sound pressure at location \( x_0 \) in the far field \((c_0/S)\) is assumed to be constant in space and time):

\[
p(x_0, t) = \frac{c_0}{S} \int \int \int \gamma - 1 \frac{q}{c^2} \left( t - \tau - \frac{|x_0 - x|}{c_0} \right) dx \quad (3.4)
\]

In general one is interested in the sound spectrum from a flame. This can be calculated by taking the Fourier transform in time of the correlation function of equation (3.4), this is the same approach as has been followed by Crighton et al. (1992) on page 327 for the calculation of the sound from a turbulent jet. The spectrum of pressure fluctuations due to combustion is given by:

\[
pp(x_0, \omega) = \int_{\Delta} \int_{\Delta} \frac{1}{S} p(x_0, 0)p(x_0, \Delta \omega)e^{i\omega \Delta t} d\Delta t = \quad (3.5)
\]

\[
\left[ \omega \right]^2 \int_{\Delta} \int_{\Delta} \frac{1}{S} q \left( x_1, \frac{x_0 - x_1}{c_0} \right) dx_1 \cdot \int_{\Delta} \int_{\Delta} \frac{1}{S} q \left( x_2, \Delta t - \frac{x_0 - x_2}{c_0} \right) dx_2 e^{i\omega \Delta t} d\Delta t
\]

The co-ordinate \( x_2 \) is written as \( x_2 = x_1 + \Delta x \). In the acoustic far field \((x_0 >> |x_1|, x_0 >> |x_0|)\) only the axial component of the acoustic time delay is important. The axial components of \( x_1 \) and \( \Delta x \) are \( x_1 \) and \( \Delta x \) respectively. The new time variable \( \Delta t^* \) is introduced as \( \Delta t^* = \Delta t + \Delta x/c_0 \). The factor \( \frac{1}{\sigma^2} \) is taken constant over \( \Delta x \). In the acoustic far field
the sound spectrum is not a function of the distance between the source and the listener’s location. The following expression for the sound spectrum is found:

$$pp(\omega) = \left[ \frac{c_0}{S} \right]^2 \int_{D_0} [\frac{\gamma - 1}{\gamma}]^{\frac{3}{2}} \int_{D_0}^{D_1} \frac{1}{\Delta s} \frac{1}{\Delta t^*} \Phi(x; \Delta x, \Delta t^*) \text{e}^{i \omega \Delta t^*} \text{e}^{i \frac{2\pi}{\lambda_0} \Delta x} \text{d}x \text{d}t^* \text{d}x\text{d}t$$

(3.6)

Because the time variable over which is integrated has been changed in equation (3.6) the spatial Fourier transform over $\Delta x$ is introduced. So equation (3.6) reads in terms of the Fourier transform of $\Phi$ $(F(x, k))$ as:

$$pp(\omega) = \left[ \frac{c_0}{S} \right]^2 \int_{D_0} \left[ \frac{\gamma - 1}{\gamma} \right]^{\frac{3}{2}} \int_{D_0}^{D_1} \frac{1}{\Delta s} \frac{1}{\Delta t^*} F(\omega, k) (\Phi(x; \omega, k) = (-\omega/c_0, 0, 0)) \text{d}x\text{d}t^* \text{d}x\text{d}t$$

(3.7)

Equation (3.7) indicates that only the energy at the acoustic wave number ($\omega/c_0$) in the heat release spectrum contributes to the sound spectrum at a specific frequency.

The main problem in the theory of combustion noise is to get a good description of the fluctuating heat release. In this study the mixed-is-burnt approach is used to write the heat release as a function of the mixture fraction ($\xi$).

### 3.3.1 The mixed-is-burnt approach

For many non-premixed turbulent flames the chemistry appears to be much faster than the mixing process, so for this case the chemistry can be assumed to be infinitely fast and the combustion is controlled by the mixing. This is called the mixed-is-burnt approach, which has been described in section 1.4.2. The mixing is described with a dimensionless conserved scalar, the mixture fraction ($\xi$). It can be shown that all species can be described as a function of the mixture fraction only ($Y_i = Y_i(\xi)$). The chemical source term ($\omega_i$) of a species $Y_i$ as function of the mixture fraction is given by equation (1.23), which is repeated here:

$$\omega_i = -\frac{\partial Y_i}{\partial s} \rho D_L \nabla \xi \cdot \nabla \xi$$

(3.8)

The second derivative of $Y_i$ with respect to $\xi$ is a Dirac delta function, with only a contribution at $\xi = \xi_{st}$ as can be seen from figure 1.4. In the mixed-is-burnt approach three different components of the gas mixture can be distinguished: fuel, air, and products. The heat release source term in the mixed-is-burnt approach is (combining equation (1.23) and (3.2)):

$$q = -\rho D_L \nabla \xi \cdot \nabla \xi \left( h_0 \frac{\partial Y_u}{\partial x} + h_f \frac{\partial Y_f}{\partial x} + h_p \frac{\partial Y_p}{\partial x} \right) =$$

$$-\rho D_L \nabla \xi \cdot \nabla \xi \left( \frac{h_0}{\xi_{st} - \xi_{st}} \left( (1-\xi_{st}) + \epsilon p \xi_{st} - \epsilon p \rho_{prod} \right) \delta (\xi - \xi_{st}) \right)$$

(3.9)

If the specific heat, $c_p$, is constant and the same for all three components and if the inlet temperature of the fuel and the air are equal, the denominator in the last term of equation (3.9) can be written as $c_p (T_{inlet} - T_0)$. With these assumptions the source term of equations (3.1) and (3.4) can be rewritten to ($c_p$ is constant):

$$\frac{\gamma - 1}{\gamma c^2} q = -\rho D_L \nabla \xi \cdot \nabla \xi A* \delta (\xi - \xi_{st})$$

(3.10)
3.3 Theory

, with \( A^* = (T_{flame} - T_0) / (T_{flame} \xi_{st} (1 - \xi_{st})) \).

Because the chemical kinetics are assumed to be infinitely fast, the heat release is instantaneous and at the stoichiometric interface. Taking finite chemical kinetics into account would lead to additional and different terms in equation (3.10) (Kok and Klein (1999)).

3.3.2 The sound from a non-premixed turbulent flame

To calculate the one-dimensional sound generated by the fluctuating heat release from a turbulent non-premixed flame a similar approach as in the previous sections is followed. Combining equation (3.4) and equation (3.10) gives for the sound from a turbulent non-premixed flame in an acoustically one-dimensional situation:

\[
p(x_0, t) = -\frac{c_0}{S} \int \int \rho D_L A^* \delta (\xi - \xi_{st}) \nabla \xi \cdot \nabla \xi \bigg|_{x_{st} = \frac{x_0}{a}} dV \quad (3.11)
\]

The Dirac delta function behavior in the mixture fraction domain makes that the flame is an infinitely thin sheet in physical space. The local value of the mixture fraction determines the instantaneous location of this flame sheet. At the lean side of this flame sheet there is a mixture of air and combustion products and at the rich side a mixture of fuel and combustion products. The rate of combustion is given by the mixing of fuel and air across this sheet by molecular diffusion (\( \rho D_L \nabla \xi \cdot \nabla \xi \)).

To be able to evaluate equation (3.11) the factor \( \rho D_L A^* \delta (\xi - \xi_{st}) \nabla \xi \cdot \nabla \xi \) is factorized into two parts:

1. \( A^* \delta (\xi - \xi_{st}) \) This term gives the position of the flame front. It is everywhere zero except at the flame sheet, where the local mixture fraction has the stoichiometric value. The position of the flame front is mainly determined by the gross scale mixing of fuel and air. This position may fluctuate in time due to (large scale) turbulent fluctuations.

2. \( \rho D_L \nabla \xi \cdot \nabla \xi \) This term describes the scalar dissipation. The scalar dissipation is the irreversible fine scale mixing by molecular diffusion. This fine scale mixing is a local turbulence property.

Because the term \( A^* \delta (\xi - \xi_{st}) \) is determined by gross scale mixing of fuel and air, its time scale is coupled to the ‘long’ time scale of the large scale motion of the flame sheet. In this thesis the term \( A^* \delta (\xi - \xi_{st}) \) will be treated as being constant in time, that is a reasonable assumption for short time scales (and therefore high frequencies).

A new co-ordinate system is introduced to describe the flame sheet: \( y_1 \) and \( y_2 \) are the co-ordinates in the flame sheet, \( y_3 \) is the co-ordinate perpendicular to the flame sheet. This co-ordinate system can be thought of as a flamelet co-ordinate system (Warnatz et al. (1998)). Because the mixture fraction is constant on the flame sheet, it becomes an unique function of \( y_3 \) and time in the new co-ordinate system (e.g., \( \nabla \xi = \partial \xi / \partial y_3 \)). To complete this co-ordinate transformation the Jacobian \( J \) has to be introduced. Equation (3.11) becomes in the new co-ordinate system:
\[ p(x_0, t) = \frac{\partial}{\partial \xi} \int \int \rho D_L A^* \delta(\xi - \xi_a) \frac{\partial \xi_a}{\partial y_a} \cdot \frac{\partial \xi_a}{\partial y_a} \bigg|_{y_a = x_0, t} Jdy_1 dy_2 dy_3 \]

\[ = \frac{\alpha}{\beta} \int \int \rho D_L A^* \delta(\xi - \xi_a) \frac{\partial \xi_a}{\partial y_a} \cdot \frac{\partial \xi_a}{\partial y_a} \bigg|_{y_a = x_0, t} J \frac{\delta y_1}{\delta y_a} dy_1 dy_2 \]

(3.12)

\[ \approx \frac{\alpha}{\beta} \int \int \rho D_L A^* \delta(\xi - \xi_a) \frac{\partial \xi_a}{\partial y_a} \cdot \frac{\partial \xi_a}{\partial y_a} \bigg|_{y_a = x_0, t} J \frac{\delta y_1}{\delta y_a} dy_1 dy_2 \]

The variable \( x_0 \) is not changed, because it does not have any influence on the one dimensional sound spectrum.

In the third step of equation (3.12) the partial derivative \( \partial y_2 / \partial \xi \) is replaced by \( \Delta y_2 / \Delta \xi \) where \( \Delta y_2 \) gives the thickness of the boundary layer between the fuel and air flow. In the fourth step \( A^* \Delta y_2 \) is written as \( A \). Because the assumption is made that the flame sheet location is constant in time, \( A \) and \( J \) will also be constant in time and only a function of \( y_1 \) and \( y_2 \). \( A \) and \( J \) are treated here as constants, in the sections 3.4 and 3.5 it will be explained how their numerical value is calculated.

Applying a Fourier transform in time to the correlation function of the last term of equation (3.12) in the same way as has been done with equation (3.6) gives:

\[ p(\omega) = \left[ \frac{\alpha}{\beta} \right]^2 \int_{A} \left[ \int_{y_1, y_2} \rho D_L A \frac{\partial \xi}{\partial y_a} \cdot \frac{\partial \xi}{\partial y_a} dy_1 dy_2 \right] e^{i \omega \Delta t} d\Delta t \]

(3.13)

Because the term \( AJ \) in equation (3.13) is mainly determined by the gross scale mixing of fuel and air, it is assumed that this term is constant over the turbulent correlation length, this means that it can be taken outside of the integral over \( \Delta y_1 \) and \( \Delta y_2 \). Also the term \( \rho D \) is taken constant in time and constant over the correlation length.

As has been done in equation (3.6) a new variable for the time is introduced. The results from this equation can be used directly to give the sound spectrum from a non-premixed flame:

\[ p(\omega) = \left[ \frac{\alpha}{\beta} \right]^2 \int_{y_1, y_2} \left[ \rho D_L A \right]^2 F(\omega, k) \left( \frac{\partial \xi}{\partial y_a} \cdot \frac{\partial \xi}{\partial y_a} \right) \left( \Delta t, \Delta \xi \right) (y_1, y_2; \omega, k) dy_1 dy_2 \]

(3.14)

In equation (3.14) the wave number vector \( k \) is the same as in equation (3.7).

To evaluate the turbulent mixing term in equation (3.14) the mixture fraction is split in a time averaged part, indicated with \( \bar{\xi} \) and a time fluctuating part \( \xi' \) (Reynolds decomposition). After the Reynolds decomposition a quadruple product of \( \partial \xi' / \partial y_a \) arises. This term is replaced by the square of the twofold correlation function of \( \partial \xi' / \partial y_a \). This is allowed if
normal joint probability of $\partial \xi / \partial y_3$ is assumed (Ribner (1969)). So with these operations the quadruple product of $\partial \xi / \partial y_3$ in equation (3.14) can be written as:

\[
\frac{\partial \xi}{\partial y_3} \frac{\partial \xi}{\partial y_3} \frac{\partial \xi}{\partial y_3} \frac{\partial \xi}{\partial y_3} \bigg|_{y_1 + \Delta y_1, y_2 + \Delta y_2, \Delta t} = \\
\frac{\partial \xi}{\partial y_3} \frac{\partial \xi}{\partial y_3} \frac{\partial \xi}{\partial y_3} \frac{\partial \xi}{\partial y_3} \bigg|_{y_1 + \Delta y_1, y_2 + \Delta y_2, \Delta t}^2 + \\
2 \frac{\partial \xi}{\partial y_3} \frac{\partial \xi}{\partial y_3} \frac{\partial \xi}{\partial y_3} \frac{\partial \xi}{\partial y_3} \bigg|_{y_1 + \Delta y_1, y_2 + \Delta y_2, \Delta t} \frac{\partial \xi}{\partial y_3} \frac{\partial \xi}{\partial y_3} \bigg|_{y_1 + \Delta y_1, y_2 + \Delta y_2, \Delta t} + \\
\frac{\partial \xi}{\partial y_3} \frac{\partial \xi}{\partial y_3} \frac{\partial \xi}{\partial y_3} \frac{\partial \xi}{\partial y_3} \bigg|_{y_1 + \Delta y_1, y_2 + \Delta y_2, \Delta t} \frac{\partial \xi}{\partial y_3} \frac{\partial \xi}{\partial y_3} \bigg|_{y_1 + \Delta y_1, y_2 + \Delta y_2, \Delta t} + \\
\frac{\partial \xi}{\partial y_3} \frac{\partial \xi}{\partial y_3} \frac{\partial \xi}{\partial y_3} \frac{\partial \xi}{\partial y_3} \bigg|_{y_1 + \Delta y_1, y_2 + \Delta y_2, \Delta t} \frac{\partial \xi}{\partial y_3} \frac{\partial \xi}{\partial y_3} \bigg|_{y_1 + \Delta y_1, y_2 + \Delta y_2, \Delta t}
\]  

(3.15)

Only the first two parts at the right hand side of equation (3.15) will contribute to the sound spectrum, the other terms only contribute to the sound spectrum at $\omega = 0$.

As in the theory for low Mach number turbulent jets (Mankbadi (1990)) the quadratic term (the first term at the right hand side) of equation (3.15) will give the main contribution to the generated sound. For the linear term, wave numbers in the spectrum of $\partial \xi / \partial y_3$ that equal the acoustic wave number will give contribution to the sound spectrum. The energy in the turbulence spectrum at the acoustic wave number is however very small and therefore the linear term can be neglected. The sound spectrum from a turbulent non-premixed flame becomes with these considerations (the $'s$ are dropped from here on):

\[
pp(\omega) = \left[ \frac{co}{S} \right]^2 \int_{y_1} \int_{y_2} [\rho DL A L]^2 F^{\omega, k} \left( \frac{\partial \xi}{\partial y_3} \frac{\partial \xi}{\partial y_3} \bigg|_{y_1 + \Delta y_1, y_2 + \Delta y_2, \Delta t} \right)^2 (y_1, y_2; \omega, k) dy_1 dy_2
\]  

(3.16)

The co-ordinates on the flame sheet can be chosen in such a way that $y_1$ coincides with the mean flow direction, the mean velocity in $y_2$ direction then equals zero. This is done because it appears, see further on, that a turbulent quantity (e.g. the mixture fraction) at two points is correlated in time due to the transport of that quantity by the mean flow. This means that if there is no flow present in the $y_2$ direction the correlation in this direction does not have a time component and will also not influence the shape of the sound spectrum. The correlation in the $y_2$ direction only has influence on the energy in the sound spectrum: in the case of a strong correlation in the $y_2$ direction the sound output will be higher than in the case of a weak correlation. This influence of the correlation in the $y_2$ direction on the sound level is given by the introduction of the correlation length in the $y_2$ direction:

\[
F^{\omega, k} \left( \frac{\partial \xi}{\partial y_3} \frac{\partial \xi}{\partial y_3} \bigg|_{\Delta t, \Delta y_1, \Delta y_2} \right)^2 (\omega, k) = I_{corr, y_2} F^{\omega, k} \left( \frac{\partial \xi}{\partial y_3} \frac{\partial \xi}{\partial y_3} \bigg|_{\Delta t, \Delta y_1, \Delta y_2} \right)^2 (\omega, k)
\]  

(3.17)

In this thesis it will be assumed that the correlation length in the $y_2$ direction is identical to the correlation length in the mean flow direction.

The Fourier transform from equation (3.17) is calculated with the convolution theorem (Crighton et al. (1992) pp. 86). In this calculation the fact is used that the acoustic wave number $\omega/c$ is much smaller than the typical turbulent wave numbers for low Mach number flows. The acoustic wave number can then be set to zero. The turbulence is assumed to be
symmetric in the wave number space in the \( y_1 \) direction, so \( F^{\omega,k}(\omega, k) = F^{\omega,k}(\omega, -k) \).

\[
F^{\omega,k} \begin{bmatrix} \frac{\partial \delta}{\partial y_3}, \frac{\partial \delta}{\partial y_3} \\ \Delta_t, \Delta y_1 \end{bmatrix}^2 (y_1, y_2; \omega, k = 0) = 
\]
\[
\frac{1}{2\pi} \int_{\lambda} \int_{\Delta t} F^k \begin{bmatrix} \frac{\partial \delta}{\partial y_3}, \frac{\partial \delta}{\partial y_3} \\ \Delta_t, \Delta y_1 \end{bmatrix} (\lambda, \Delta t) e^{i\omega\Delta t} d\lambda d\Delta t = 
\]
\[
\frac{1}{2\pi} \int_{\lambda} \int_{\Delta t} F^k \begin{bmatrix} \frac{\partial \delta}{\partial y_3}, \frac{\partial \delta}{\partial y_3} \\ \Delta_t, \Delta y_1 \end{bmatrix} (\lambda) r(\lambda, \Delta t) r(\lambda, \Delta t) e^{i\omega\Delta t} d\lambda d\Delta t = 
\]
\[
\frac{1}{2\pi} \int_{\lambda} \int_{\beta} F^k \begin{bmatrix} \frac{\partial \delta}{\partial y_3}, \frac{\partial \delta}{\partial y_3} \\ \Delta_t, \Delta y_1 \end{bmatrix} (\lambda) r(\lambda, \beta) r(\lambda, \omega - \beta) d\lambda d\beta = 
\]

In equation (3.18) the correlation function has been factorized in a wave number part \( (F^k \begin{bmatrix} \frac{\partial \delta}{\partial y_3}, \frac{\partial \delta}{\partial y_3} \\ \Delta_t, \Delta y_1 \end{bmatrix} (\lambda)) \) and a wave number-time part \( (r) \). If the relevant length and time scales are smaller than the turbulent eddy scales, the function \( r \) is mainly determined by the transport of turbulent quantities by the mean flow \( (U) \). This due to the fact that a turbulent quantity at two different locations is correlated in time by the transport of that quantity by the mean flow if the separation of these locations is smaller than the eddy length scale. For larger separation distances the correlation is influenced by the turbulent fluctuations (turbulent diffusion). This is called the sweeping hypothesis by Rubinstein and Zhou (1997), it is also known as the Taylor hypothesis (Hinze (1975)). A widely used form for \( r(k, \Delta t) \) is:

\[
r(k, \Delta t) = e^{-\frac{k^2U^2}{2\Delta t}} 
\]

(3.19)

, with \( U \) the mean flow velocity in the \( y_1 \) direction.

The introduction of the Fourier transform of equation (3.19) into equation (3.18) gives integrals over \( \lambda \) and \( \beta \), which are not easy to evaluate. This problem is solved by taking a simpler shape for the space time correlation function: If a turbulent quantity at two locations is only correlated in time by the transport of that quantity by the mean flow, the space time correlation function of that quantity can be represented with a Dirac delta function. The correlation function in the frequency/wavenumber domain will then also be a Dirac delta function:

\[
r(k, \omega) = \frac{2\pi \delta}{U} \delta \left( k - \frac{\omega}{U} \right) 
\]

(3.20)

Equation (3.20) is introduced into equation (3.18) and the integration over \( \lambda \) and \( \beta \) are performed:

\[
F^{\omega,k} \begin{bmatrix} \frac{\partial \delta}{\partial y_3}, \frac{\partial \delta}{\partial y_3} \\ \Delta_t, \Delta y_1 \end{bmatrix}^2 (y_1, y_2; \omega, k = 0) = 
\]
\[
\frac{1}{U} \int_{\beta} \int_{\Delta t} F^k \begin{bmatrix} \frac{\partial \delta}{\partial y_3}, \frac{\partial \delta}{\partial y_3} \\ \Delta_t, \Delta y_1 \end{bmatrix} (\lambda) \delta(\lambda - \frac{\beta}{U}) \delta(\lambda - \frac{\omega - k}{U}) d\lambda d\beta = 
\]
\[
\frac{1}{U} \int_{\beta} F^k \begin{bmatrix} \frac{\partial \delta}{\partial y_3}, \frac{\partial \delta}{\partial y_3} \\ \Delta_t, \Delta y_1 \end{bmatrix} (\lambda) \delta(2\lambda - \frac{\omega - k}{U}) d\lambda = 
\]
\[
\frac{1}{2U} F^k \begin{bmatrix} \frac{\partial \delta}{\partial y_3}, \frac{\partial \delta}{\partial y_3} \\ \Delta_t, \Delta y_1 \end{bmatrix} \frac{1}{2\pi} 
\]
3.3 Theory

Very typical about the last term in equation (3.21) is the appearance of $\omega/2U$ instead of $\omega/U$. This is explained by the square of the correlation function of $\xi$.

The last term at the right hand side of equation (3.21) is the square of the Fourier transform of the correlation in the $y_1$ direction of the scalar dissipation in the $y_3$ direction of $\xi$. This term is modeled by taking this correlation function of $\xi$ in the $y_1$ direction. This assumption is based on the idea that the scalar dissipation will be larger if the fluctuations in $\xi$ are larger, the modeling of the time averaged scalar dissipation, $\chi$, is based on the same assumption (see also equation (1.21)): $\chi \propto \xi^2$. The Fourier transform in space of the correlation function of $\xi$ is the one-dimensional turbulent energy spectrum of $\xi$ ($E_{\xi\xi}^{1D}$):

$$F^k \left[ \frac{\partial \xi}{\partial y_3}, \frac{\partial \xi}{\partial y_5} \right]_{\Delta y_1} = B \int \xi(\Delta y_1) e^{ik\Delta y_1} \Delta y_1 = B E_{\xi\xi}^{1D}(k) \quad (3.22)$$

The factor $B$ from equation (3.22) can be calculated with Parseval's theorem (Crighton et al. (1992) pp. 87) from the scalar dissipation ($\chi$):

$$\frac{1}{\pi} \rho_D B \int_0^\infty E_{\xi\xi}^{1D}(k) dk = \frac{1}{2} \rho \chi \approx \frac{1}{2} \rho \frac{\xi}{\tau_{turb}} \quad (3.23)$$

The equations (3.21) and (3.22) together with equation (3.17) are introduced into equation (3.16). The sound spectrum from a turbulent non-premixed flame is then:

$$pp(\omega) = \left[ \frac{c_0}{S} \right]^2 \int_{y_3} \int_{y_2} \rho_D B_2^2 A \frac{I_{cor,y_2}}{2U} E_{\xi\xi}^{1D} \left( y_1, y_2; \frac{\omega}{2U} \right) A J_{dy_1} J_{dy_2} \quad (3.24)$$

This equation will be used in the remainder of this thesis.

A remark should be made about the term $J_{cor,y_2}$. This term cannot be calculated explicitly and needs to be modeled. A reasonable assumption is that the correlation length in the $y_2$ direction is equal to the correlation length in the $y_1$ direction. This assumption is based on homogeneous turbulence.

As can be seen from equation (3.24), the shape of the sound spectrum of a turbulent non-premixed flame is determined by the shape of the one-dimensional turbulent energy spectrum of $\xi$. The one-dimensional turbulent energy spectrum can be calculated from the three-dimensional turbulent energy spectrum with (see e.g Hinze (1975)):

$$E_{\xi\xi}^{1D}(k_1, t) = \int_{k_1}^\infty E_{\xi\xi}^{3D} \frac{dk}{k} \quad (3.25)$$

For high Reynolds numbers the three-dimensional turbulent energy spectrum for a passive scalar is the same as that for the velocity (Hinze (1975)) and can be described with the modified von Karman spectrum (Bêchera et al. (1994)):

$$E_{\xi\xi}^{3D}(k) \simeq \frac{(k/k_c)^4}{\left[ 1 + (k/k_c)^2 \right]^{17/6}} \exp \left[ -2 \left( k/k_{Kol} \right)^2 \right] \quad (3.26)$$

$k_c$ is the integral wave number and $k_{Kol}$ the Kolmogorov wave number.
Because the non-premixed turbulent flames studied in this thesis all have a high Reynolds number \((10^4 - 10^6)\), it is allowed to take this shape for the turbulent spectrum.

The application of equation \((3.25)\) to equation \((3.26)\) gives that for wave numbers below the integral wave number the turbulent spectrum is (more or less) constant, for wave numbers above the integral wave number the spectrum has a fall off of \(k_1^{-5/3}\) up to the Kolmogorov wave number:

\[
E_{1D}(k_1) \simeq \begin{cases} 
1 & k_1 < k_e \\
\left( k_1 / k_e \right)^{-5/3} & k_e < k_1 < k_{Kd} \\
0 & k_1 > k_{Kd}
\end{cases}
\]  

(3.27)

Equation \((3.27)\) is introduced into equation \((3.24)\) with \(k_1 = \omega / 2U\) and \(k_e = k_e(y_1, y_2)\). It is found that the sound spectrum will fall off with \(\omega^{-10/3}\) for \(\omega > 2U k_e\) (\(k_e\) is assumed to be constant in the flame).

Next to the shape of the sound spectrum another interesting aspect is the total sound emission from the flame. This can be calculated by applying Parseval’s theorem to equation \((3.24)\):

\[
\bar{p}^2 = \frac{1}{\pi} \int_0^\infty pp(\omega) d\omega
\]  

(3.28)

The equation that has been derived for the sound generation by a turbulent non-premixed flame is valid for a one-dimensional acoustical situation (for example a flame in a tube with a small diameter). For a flame radiating to the free field, like flares and hot air balloon burners, the appropriate three-dimensional Green’s function should be used instead of the one-dimensional Green’s function (equation \((3.3)\)). The resulting equation for the three-dimensional sound spectrum is:

\[
pp(\omega) = \left[ \frac{\omega}{4\pi} \right]^2 \int_{y_1}^{y_2} [\rho D_I B]^2 \frac{J}{2U} E \int \left( \frac{\omega}{2U} \right) A J dy_1 dy_2
\]  

(3.29)

where \(r\) is the distance between the observer and the flame. According to this equation the three-dimensional sound spectrum will have a fall off of \(\omega^{-4/3}\).

### 3.3.3 An overview of the assumptions

The assumptions made for the derivation of equation \((3.24)\) are:

1. One is interested in the sound spectrum in the acoustic far field of the flame. The flame is a compact source of sound (small compared to the acoustic wave length).
2. The mixed-is-burnt approach is used, this means that the chemistry is assumed to be infinitely fast and the flame front can be represented by an infinitely thin sheet. The diffusion across this sheet determines the combustion rate and also the heat release.
3. The frequency of the fluctuations in the position of the flame front is much lower than the typical frequency of the turbulent fine scale mixing. In this case the flame front position can be treated as being quasi-steady for the higher frequencies in the sound spectrum. The change in flame front position within a turbulent correlation length is assumed to be small.
4. The Taylor hypothesis is valid for the space-time correlation. This is only true for
length scales smaller than the eddy length and therefore for frequencies higher than
the turnover frequency of the eddy.

5. The one-dimensional spectrum of \( \frac{\partial \xi}{\partial t} \) can be written as a constant \( (B) \) times
the one-dimensional spectrum of \( \xi \). This constant is calculated from the scalar
dissipation.

### 3.4 Evaluation of the noise source

To get results from the noise generation equation (3.24) the numerical values for the differ-
ent parameters in this equation should be known. These parameters can be derived from
a steady state CFD calculation, this method will be described in the next section. In the
present section simplifications are made to equation (3.24) to analyze the effect of the dif-
ferent parameters and to give a model from which rough estimates can be made about the
sound generation by a turbulent non-premixed flame. The main simplifications are that the
turbulence and the flow are constant across the entire flame. To be able to use equation
(3.24) the next parameters should be known: \( A, AJ, B, l_{cor}, \Delta \beta, c \) and \( \xi \).

The factor \( A \) in equation (3.24) can be determined from the third term of equation (3.12):

\[
A = \overline{F} \Delta \beta
\]  
(3.30)

, in this equation is \( \Delta \beta \) the thickness of the boundary layer between fuel and air, for which
a value should be estimated . To calculate the factor \( AJ \) in equation (3.24) the co-ordinate
transform applied to equation (3.11) is considered:

\[
\overline{F} J \Delta \beta \rho_1 \rho_2 = \overline{F} dV
\]  
(3.31)

Further it can be derived from equation (3.9) that \( \overline{F} \) in this equation can be written as a
function of the time averaged local heat release and the scalar dissipation:

\[
\overline{F} = \frac{\gamma - 1}{c^2} \frac{\overline{Q}}{\pi \rho_x}
\]  
(3.32)

The factor \( B \) in equation (3.24) can be expressed as function of the scalar dissipation \( (\chi) \)
and the 1D energy spectrum of \( \xi \), using equation (3.22):

\[
B = \frac{1}{\pi D} \int_0^\infty E^{1D} (k) dk
\]  
(3.33)

It is now assumed that all parameters are constant over the flame volume. This gives the
following simplified shape of equation (3.24):

\[
pp (\omega) = \left[ \frac{c_0}{S} \right]^2 \left[ \frac{\gamma - 1}{c^2} \frac{Q_{tot}}{\pi} \int_0^\infty E^{1D} (k) dk \right]^{\frac{\gamma}{\gamma - 1}} \frac{H_{cor} \Delta \beta}{2UV_{flame}} E^2 \left( \frac{\omega}{2U} \right)
\]  
(3.34)
\[ Q_{\text{total}} \] is the total heat release by the flame, defined by \( Q_{\text{total}} = \bar{q}_s V_{\text{flame}} \) and \( V_{\text{flame}} \) is the volume of the flame. The integral in the denominator of this equation is calculated using the 1D energy spectrum of equation (3.27) and is equal to \( \frac{5}{2} k_e \). Further \( V_{\text{flame}} \) is written as: \( V_{\text{flame}} = J_l c_{\text{flame}} \Delta y_{\text{flame}}. \) This gives the following equation for the sound spectrum:

\[
pp(\omega) = \left[ \frac{c_p}{S} \right]^2 \left[ \frac{\gamma - 1}{c^2} - \frac{\gamma - 1}{c^2} Q_{\text{total}} \right]^2 \frac{1}{2U V_{\text{flame}}} E^2 \left( \frac{\omega}{2U} \right)
\]  

(3.35)

The total sound emission can be calculated from this equation by integrating over \( \omega \), using Parseval's theorem. Again the shape of the 1D spectrum of equation (3.27) for \( E \) is used. The integral wave number \( k_e \) is written as \( \pi/l_e \), where \( l_e \) is the integral length scale. This gives the following expression for the total sound emission:

\[
pp = K \cdot \left[ \frac{c_p}{S} \right]^2 \left[ \frac{\gamma - 1}{c^2} - \frac{\gamma - 1}{c^2} Q_{\text{total}} \right]^2 \frac{l_e}{l_{\text{flame}}}
\]  

(3.36)

, where \( K \) is a factor following from the integration over the spectrum. If the spectrum of equation (3.27) is used, \( K \) has the value \( 8/35 \). The total sound emission from the flame is, in this simplified equation, only a function of the thermal power of the flame, \( c_p \) times the flame temperature (written as \( (\gamma - 1)/c^2 \) ), the ratio of the turbulent integral length scale over the flame length and the ratio \( c/\Delta y \) which is due to 1D acoustics. This is a surprisingly simple result after the algebraic operations that have been performed in the previous section. The scaling factor \( K \) in this equation is mainly due to the choice of the spectrum. Equation (3.36) is referred to as the simplified model in the remainder of this chapter.

Equation (3.36) resembles very much the general equation that has been derived by Strahle to predict the sound from a flame (Strahle (1983)). Strahle derived the sound power from combustion noise for a flame radiating to the free field. If his model is applied to an acoustical one-dimensional situation, the following equation is found:

\[
pp = \frac{1}{\eta} \cdot \left[ \frac{c_p}{S} \right]^2 \left[ \frac{\gamma - 1}{c^2} - \frac{\gamma - 1}{c^2} Q_{\text{total}} \right]^2 \frac{l_e}{l_{\text{flame}}}
\]  

(3.37)

The equation (3.37) differs from equation (3.36) in the factor \( 1/\eta \) instead of \( K \). The variable \( \eta \) gives the part of the time there is actual combustion, this factor is always smaller than one. \( 1/\eta \) is therefore always larger than one. The model of Strahle gives a sound level that is a factor \( 1/(\eta \cdot K) \) higher than the sound level calculated by the present model.

Several important properties can be seen from equation (3.36). First, the sound emission is a linear function of the correlation length, the shorter the correlation length the less sound is generated. If the correlation length is zero, the correlation function is a delta function, no sound is produced by the flame. Second, the sound power scales quadratically with the thermal power of the flame. And third, a longer flame, with the same thermal power and the same correlation length, produces less sound than a short flame.

The properties of the sound from a turbulent non-premixed flame (for which the mixed-is-burnt approach is valid) can also be summarized to be:

1. The fall off of the sound spectrum shows a \( f^{-10.5} \) behavior.

2. The frequency at which the fall off starts, is \( f_e = U/l_e \).
3. The total sound emission can be described by equation (3.36) and is only a function of the thermal power, the flame temperature and the ratio of the integral length scale over the flame length.

### 3.5 Coupling to a CFD calculation

In the foregoing section drastic simplifications have been made to equation (3.24) to be able to analyze the behavior of this equation. In this section it is shown how the different parameters in equation (3.24) are derived from a steady computational fluid dynamics (CFD) combustion calculation.

For the calculation of the sound spectrum the following turbulence/flow variables are necessary: the mean velocity $U$, the integral wave number $k_e$, the Kolmogorov wave number $k_{K, ed}$, the correlation length in the $y_2$ direction $l_{cor,y_2}$ and the scalar dissipation $\chi$. Further the thickness of the mixing boundary layer $\Delta y_3$, the location of the flame front $J$ and the heat release factor $A^*$ should be known. The turbulence/flow variables can be derived from the results of any CFD (combustion) calculation using standard modeling approaches. The other variables ($A^*$ and $J$) have to be incorporated into the combustion model, these variables have already been discussed in section 3.4.

In this research a combustion model is used as described in Kok and Louis (1998). The combustion model and the acoustic noise model are implemented in CFX 4.2, a CFD code from AEA Technologies. The noise model is implemented as a post processor.

#### 3.5.1 The turbulence variables

The integral wave number $k_e$ is computed with:

$$
k_e = \frac{\pi}{l_e}
$$

(3.38)

where $l_e$ is a typical length scale of the used geometry.

The turbulent time scale $\tau_{turb}$ in equation (3.38) should be derived from the used turbulent model. If the $k$-$\epsilon$ model is used, the turbulent time scale is calculated from:

$$
\tau_{turb} = (1/C_{g2}) \frac{tke}{\epsilon}
$$

(3.39)

In equation (3.39) is $tke$ the turbulent kinetic energy, $\epsilon$ is the dissipation of the turbulent kinetic energy. The values for $tke$ and $\epsilon$ are available in a $k$-$\epsilon$ calculation. $C_{g2}$ is a modeling constant that links the time scale of the turbulent scalar fluctuations of $\xi$ with the time scale of the turbulent velocity fluctuations. A standard value for $C_{g2}$ is 2.0 (Bijler (1980)).

The Kolmogorov wave number is calculated from (Dèchara et al. (1994)):

$$
k_{K,d} = \left[ \frac{\epsilon}{\nu^3} \right]^{1/4}
$$

(3.40)

The correlation length in the $y_2$ direction, $J l_{cor,y_2}$, should be estimated. See also the remark about this quantity at equation (3.24). In the present calculations it is assumed that $l_{cor,y_2}$ equals the correlation length in the $y_1$ direction ($\equiv l_e$).
The scalar dissipation $\chi$, should be known to calculate the factor $B$ in equation (3.24) with equation (3.23). The scalar dissipation is modeled with:

$$\chi = \frac{g}{\tau_{urb}} = C_{y} \frac{\epsilon}{tke} g$$  \hspace{1cm} (3.41)

The variable $g$ is the variance of $\xi$, it is available from the results of the combustion calculation.

The thickness of the boundary layer between fuel and air, $\Delta y_{B}$, is calculated from:

$$\Delta y_{B} = \frac{1}{\sqrt{\nabla \bar{\xi} \cdot \nabla \bar{\xi}}}$$  \hspace{1cm} (3.42)

where $\nabla \bar{\xi}$ can be calculated from the mixture fraction field.

### 3.5.2 The discretized equation

The combustion calculations are performed with a finite volume method. This means that the variables are only known at the centers of all grid cells. Equation (3.24) should also be discretized to be able to calculate it numerically. It is chosen to express the sound spectrum as function of the frequency ($f$) in Hz instead of as function of $\omega$, this gives a factor $2\pi$ ahead of the equation to keep the same total emission by integration over $f$ instead of $\omega$.

The discretisation of equation (3.24) yields:

$$pp(f) = 2\pi \left[ \frac{c_0}{S} \right]^2 \sum_{i=1}^{n_{cell}} \left[ (\rho DB \bar{\Delta})^2 \frac{J l_{cor,y_b} \Delta y_{B} E^2}{2U} \right]_i V_i$$  \hspace{1cm} (3.43)

The terms between the square brackets in equation (3.43) should be evaluated at the center of the cell $i$. $V_i$ is the volume of cell $i$.

To use equation (3.43) only the variable $J l_{cor,y_b}$ should be estimated, all other variables can be calculated from the results of the combustion calculation. For the spectrum, the spectrum of equation (3.27) is used.

In the results of the calculations, the sound spectrum is written in dB sound pressure level defined as ($\Delta f_{ref} = 1\ Hz, p_{ref} = 20\ \mu Pa$):

$$SPL(f) = 20 \log \left( \frac{\sqrt{pp(f)}}{20 \cdot 10^{-6}} \right)$$  \hspace{1cm} (3.44)

The total sound pressure level is calculated using a similar definition for SPL in dB.

### 3.6 The experimental method

An experimental setup is designed to measure the noise induced by a flame. To perform these noise measurements, it is necessary to make the combustion chamber as anechoic as possible. This has been described in appendix B.3. It is not possible to make the combustion installation complete anechoic. The fuel and air tube of the burner may show
3.7 Results

The output from the auto spectrum measurement by the FFT analyzer is the amplitude \( |p_i| \) at a number of discrete frequencies \( f_i \). The required power density spectrum of the acoustic pressure is calculated from this with:

\[
p_p(f_i) = \frac{\pi p_i^2}{2\Delta f_{FFT}}
\]

in which \( \Delta f_{FFT} \) is the resolution of the frequency domain, determined by the settings of the FFT analyzer. In the plots the sound spectrum is plotted as sound pressure level defined in equation (3.44). The overall sound level can be calculated with Parseval’s theorem applied to equation (3.45). The overall sound pressure level is then calculated from the RMS value of the overall sound level:

\[
SPL_{overall} = 20 \cdot \log \left( \frac{\sqrt{\sum p_i^2}}{2 \cdot 10^{-5}} \right) = 20 \cdot \log \left( \frac{\sqrt{1/2 \sum p_i^2}}{2 \cdot 10^{-5}} \right)
\]

3.7 Results

In this section a comparison is drawn between the CFD calculations performed with the model presented in equation (3.43) and measurements in the acoustically 1D combustion chamber. The used burner is the bluff body stabilized burner, the tests have been performed at atmospheric pressure. Further, the overall sound level, both from the measurements and the CFD calculations, is compared with the simplified model from equation (3.56).

3.7.1 The studied cases

One experiment is performed with the same flame as has been used by Louis (1997). The combustion chamber, the burner, the fuel, the air factor and the power are all identical to the flame described by Louis (1997). Louis performed concentration measurements of the main species with a suction probe in this flame. The agreement between the experiments and the CFD calculation for the main species is very good. It is therefore believed that also the heat release from this flame will be predicted correctly. This flame is referred to as flame 0.

The other experiments are performed with fuels that all have the same stoichiometric mixture fraction: \( \xi_{st} = 0.48 \). The volume flows of air and fuel (in Nm³/s) are identical for all compositions, the mass flow of the fuel varies for the different compositions (different density) just as the air factor and the thermal power. In Table 3.1 a summary of the used fuels is given. The numbering of the fuels is a continuation of the numbers presented in a previous study (Klein and Kok (1997)). Table 3.2 contains the experimental conditions at which the different flames are studied. Flame 3.4 is studied at four different powers.

All flames have been studied at one air factor, only Flame 3.4 has been studied at different air factors. It shows that the air factor only has a very minor influence on the
sound spectrum of the flame. In the discussion of the results of flame 3.4 a plot of the sound spectrum from this flame at different air factors is included.

3.7.2 Description of the calculations

The calculations are performed with a self developed local equilibrium $\beta$-PDF combustion model (mixed-is-burnt) and the standard CFX 4.2 $k$-$\epsilon$ turbulence model. The modeled geometry is 2D axi-symmetric. The computational domain is 50 mm (radial) times 650 mm (axial). The used (non uniform) grid is 50 (radial) times 300 (axial) cells. All species, the temperature, the density and $\overline{\omega}$ are calculated as a function of $\xi$ and $\varphi$ before the CFD-computation and stored in a database. The density is interpolated from this database during the calculation. After the last iteration all species, the temperature and $\overline{\omega}$ are interpolated from the database for all cells. With $\overline{\omega}$ and the turbulence quantities the sound spectrum is calculated in a noise post processor as described in the previous section.

The boundary conditions for the velocity and turbulence variables used in the calculations are presented in table 3.3. The boundary conditions for $\kappa$ and $\epsilon$ are the default CFX 4.2 boundary conditions.

<table>
<thead>
<tr>
<th>Name</th>
<th>Composition</th>
<th>Ad. flame temp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flame 0</td>
<td>40% CO 40% H$_2$ 20% N$_2$</td>
<td>2333 K</td>
</tr>
<tr>
<td>Flame 3.2</td>
<td>10% CO 25% H$_2$ 65% N$_2$</td>
<td>1816 K</td>
</tr>
<tr>
<td>Flame 3.4</td>
<td>38% CO 5% H$_2$ 57% N$_2$</td>
<td>2024 K</td>
</tr>
<tr>
<td>Flame 3.5</td>
<td>4% CH$_4$ 20% CO 5% H$_2$ 71% N$_2$</td>
<td>1808 K</td>
</tr>
</tbody>
</table>

Table 3.1: The composition and adiabatic flame temperature of the used fuels.

<table>
<thead>
<tr>
<th>Name</th>
<th>Power</th>
<th>$\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flame 0</td>
<td>16.0 kW</td>
<td>1.3</td>
</tr>
<tr>
<td>Flame 3.2</td>
<td>12.3 kW</td>
<td>1.6</td>
</tr>
<tr>
<td>Flame 3.4</td>
<td>16.6 kW, 33.2 kW, 40.0kW</td>
<td>1.3</td>
</tr>
<tr>
<td>Flame 3.4</td>
<td>25 kW</td>
<td>1.0, 1.3, 1.75 2.0</td>
</tr>
<tr>
<td>Flame 3.5</td>
<td>14.4 kW and 28.8kW</td>
<td>1.35</td>
</tr>
</tbody>
</table>

Table 3.2: The thermal power and air factor for the studied flames. (All thermal powers are the lower heating value).

<table>
<thead>
<tr>
<th>FL 0 16 kW</th>
<th>FL 3.2 123 kW</th>
<th>FL 3.4 16.6 kW</th>
<th>FL 3.4 25.0 kW</th>
<th>FL 3.4 33.2 kW</th>
<th>FL 3.4 29.2 kW</th>
<th>FL 3.4 40 kW</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_{fuel}$</td>
<td>3.7 m/s</td>
<td>7.3 m/s</td>
<td>11.0 m/s</td>
<td>14.7 m/s</td>
<td>17.7 m/s</td>
<td></td>
</tr>
<tr>
<td>$\kappa_{fuel}$</td>
<td>0.15 m$^3$/s$^2$</td>
<td>0.11 m$^3$/s$^2$</td>
<td>0.24 m$^3$/s$^3$</td>
<td>0.43 m$^3$/s$^3$</td>
<td>0.63 m$^3$/s$^3$</td>
<td></td>
</tr>
<tr>
<td>$\epsilon_{fuel}$</td>
<td>4.4 m$^2$/s$^3$</td>
<td>5.2 m$^2$/s$^3$</td>
<td>17.6 m$^2$/s$^3$</td>
<td>42.1 m$^2$/s$^3$</td>
<td>73.6 m$^2$/s$^3$</td>
<td></td>
</tr>
<tr>
<td>$U_{air}$</td>
<td>2.9 m/s</td>
<td>3.1 m/s</td>
<td>4.7 m/s</td>
<td>6.3 m/s</td>
<td>7.5 m/s</td>
<td></td>
</tr>
<tr>
<td>$\kappa_{air}$</td>
<td>0.12 m$^3$/s$^2$</td>
<td>0.02 m$^3$/s$^2$</td>
<td>0.04 m$^3$/s$^3$</td>
<td>0.08 m$^3$/s$^3$</td>
<td>0.11 m$^3$/s$^3$</td>
<td></td>
</tr>
<tr>
<td>$\epsilon_{air}$</td>
<td>3.46 m$^2$/s$^3$</td>
<td>0.26 m$^2$/s$^3$</td>
<td>0.9 m$^2$/s$^3$</td>
<td>2.13 m$^2$/s$^3$</td>
<td>3.66 m$^2$/s$^3$</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.3: The boundary conditions used in the calculations.
3.7 Results

The following estimations are used in the noise post processor calculations: \( J \nu_{av}_{yz} = l_e \) (isotropic turbulence), where \( l_e \) is 0.1 m (the diameter of the combustion chamber) and \( c = 650 \text{ m/s} \) \((T \approx 1050 \text{ K})\). The estimation of \( l_e \) is based on the idea that the main vortices in the combustion chamber are not generated by the shear on the interface of fuel and air but by the sudden expansion at the burner. The value of \( l_e \) has been obtained by optimization.

For the different flames, measurements of the concentration of the main species have been performed with a suction probe. As already mentioned, the measurements of Flame 0 have been performed by Louis and they show a good agreement with the CFD predictions (Louis (1997)). It shows however from the comparison of the measurements of the Flames 3.2, 3.4 and 3.5 with the results of the steady state calculations that these flames are in reality much shorter than those predicted by the CFD calculations. In the predictions there is still fuel left downstream of the bluff body, in the measurements all fuel is burnt ahead of the bluff body. The mixing between fuel and air is apparently in reality much faster than predicted by the calculations. This phenomenon is subject of current research at the Laboratory of Thermal Engineering.

3.7.3 The results for Flame 3.2

The local noise source strength for Flame 3.2 is plotted in figure 3.1. The local source strength is the part between the square brackets of equation (3.43). From this figure it can be seen that noise is generated at the interface between fuel and air. The maximum source strength is predicted to be somewhat downstream of the bluff body. As mentioned above, measurements indicate that this region is upstream of the bluff body. This ill prediction of the location of the heat release seems to have no negative effect on the noise generation calculation.

The measured and calculated sound spectrum for flame 3.2 at 12.3 kW are plotted in figure 3.2. In the measured sound spectrum there is a small elevation of the pressure amplitude visible at about 340-360 Hz. This frequency is the eigenfrequency of the combustion chamber. The performance of the damper has been tested by measuring the transfer function between the two microphones at different axial locations. From this transfer function measurement it shows that the damper works well, except around 340-360 Hz. The peak in the sound spectrum comes also from the reflection at the exit.

The measured sound spectrum compares well with the calculated one. At lower frequencies the sound spectrum is predicted to be flat, in reality it is not completely flat. Also the sound level is a little overpredicted at low frequencies, this is due to the fact that the measured spectrum starts to fall off at a lower frequency than the calculated spectrum. The fall off of the sound spectrum at higher frequencies is calculated very well. Also the sound level is predicted very well.

In figure 3.3 the sound spectrum in \([Pa^2/Hz]\) has been plotted double logarithmic against the frequency. From this figure it can be seen that for high frequencies the fall off of the spectrum follows a curve of \( pp(f) \propto f^{-10/3} \). It also shows from this figure that the measured spectrum between 20 and 70 Hz falls off less steeply than \( f^{-10/3} \). At higher frequencies the calculated fall off coincided with the measured fall off. The less smooth transition between 20 Hz and 70 Hz in the calculations compared to the experiments is due to the discontinuity in the first derivative of the assumed turbulence spectrum, equation (3.27), at \( k_1 = k_e \). A more realistic turbulence spectrum will improve the calculations.
Figure 3.1: The local contribution to the total sound level. The dark grey zones are the zones with the largest contribution.

Figure 3.2: The measurements and the CFD calculations of the sound spectrum of Flame 3.2, plotted in SPL dB ($\Delta f=1$ Hz).
3.7 Results

Figure 3.3: The measurements and the numerical calculations of the sound spectrum of Flame 3.2 plotted double logarithmically.

It can be concluded from figures 3.2 and 3.3 that the agreement between the measured sound spectrum and the calculated one is good. The level of the sound spectrum is well predicted. The calculated spectrum shows the same fall off as the measured one.

3.7.4 The results for Flame 0

In figure 3.4 the measured and calculated sound spectra for Flame 0 and Flame 3.2 are plotted. The sound spectrum of Flame 3.2 is included as a reference. The agreement between the measurements and the predictions for Flame 0 is good. The measured spectrum of Flame 0 starts to fall off at a lower frequency (about 10 Hz) than the measured spectrum of Flame 3.2 (about 20 Hz). A similar shift can be seen in the calculations. This shift is caused by the lower velocities in Flame 0.

The thermal power of Flame 0 (16 kW) is higher than the thermal power of Flame 3.2 (12.3 kW). According to the simplified model (equation(3.36)) the sound level scales with the thermal power and it could be expected that the sound level of Flame 0 would be higher than that of Flame 3.2. The experiments show that the level of the spectrum of Flame 0 is a little lower than that of Flame 3.2. The trend is correctly predicted by the numerical calculations, the difference between the two experiments is however larger than predicted.

Both differences between the sound spectrum of Flame 3.2 and that of Flame 0 (the other start frequency of the fall off and the sound level) are predicted by the numerical calculations.
Figure 3.4: The measurements and the numerical calculations of the sound spectrum of Flame 0 and Flame 3.2, plotted in dB SPL ($\Delta f = 1$ Hz).

3.7.5 The results for Flame 3.4

For Flame 3.4 experiments have been performed at different air factors and thermal powers. The measured influence of the air factor on the sound spectrum is shown in figure 3.5. The sound spectra of Flame 3.4 at 25 kW at different air factors, ranging from 1.0 to 2.0 are plotted here. It shows from this figure that the air factor does only have a small influence on the sound spectrum. The calculations, which are not plotted here, also show that the influence of the air factor on the sound spectrum is very small.

In figure 3.6 the sound spectra of Flame 3.4 at four different powers are plotted. In the measured spectra of 25 kW, 33.2 kW and 40 kW there is a broad peak around 340-360 Hz. This peak is again because of reflection at the damper.

The influence of an increase in power is mainly twofold: 1. the sound level increases for the higher frequencies and 2. the start frequency of the fall off increases.

The increase in the sound level at higher frequencies with increasing power is predicted by the calculations, the effect is a little overpredicted: For the power of 16.6 kW the sound level at high frequencies is a bit underpredicted (3 dB), for 25 kW and 33.2 kW the agreement is excellent and for 40 kW the predictions are a little too high.

At low frequencies, below the start frequency of the fall off, the sound level increases strongly from 16.6 kW to 25 kW. Above 25 kW the increase in sound level is much less strong. This trend is predicted by the model. The start frequency of the fall off shifts to higher values with increasing power, this effect is overpredicted by the calculations.

In section 3.7.7 the measured total sound level will be compared with the numerical calculations and with the results from the simplified model (equation(3.36)).
3.7 Results

Figure 3.5: The measurements of the sound spectrum of Flame 3.4 at 25 kW and different air factors, plotted in dB SPL ($\Delta f=1$ Hz).

Figure 3.6: The measurements and the numerical calculations of the sound spectrum of Flame 3.4 at four different thermal powers, plotted in dB SPL ($\Delta f=1$ Hz).
3.7.6 The results for Flame 3.5: a fuel containing methane

The main differences between Flame 3.5 and the other flames with the same stoichiometric mixture fraction (Flame 3.2 and Flame 3.4) is that Flame 3.5 contains methane and that the adiabatic flame temperature of Flame 3.5 is the lowest of the series. In figure 3.7 the measured and calculated sound spectra of Flame 3.5 at two different powers are plotted. The agreement between the measurements and the numerical calculations is worse than that of Flame 3.2 and 3.4. At low frequencies the measured sound level for Flame 3.5 is higher than predicted by the numerical calculations, at higher frequencies the sound level is overpredicted because the measured spectrum falls off more steeply than the predicted spectrum. The effect of a steeper sound spectrum for non-premixed flames with the same stoichiometric mixture fraction but with a higher content of methane is also shown in the paper by Kok and Klein (1999).

Because the stoichiometric mixture fraction of Flame 3.5 is the same as that of Flame 3.2 and 3.4, it may be expected that the mixing process is similar for these flames. A possible explanation for the difference between Flame 3.2 and Flame 3.4 at one hand and Flame 3.5 at the other hand is that the chemical kinetics in Flame 3.5 can not be considered as being infinitely fast: the mixed-is-burnt approach is not valid for this flame and therefore the sound generation is not only a function of the turbulent mixing but also of the chemistry in the flame. To test this hypothesis the circumstances under which Flame 3.5 is studied should be altered in such a way that the mixed-is-burnt approach becomes valid for Flame 3.5. This can be achieved in two ways, the first way is to increase the mean pressure in the combustion chamber. It is known that the reaction rates increase strongly as function of the
Figure 3.8: The measurements of the sound spectrum of Flame 3.4 (33.2 kW) and Flame 3.5 (29.2 kW), preheated and non-preheated, plotted double logarithmic.

pressure \((\text{Louis (1997)})\), especially if the limiting reaction steps are of an order higher than one, the reaction rate will then increase stronger than linear with the pressure. Experiments have been performed at an elevated pressure up to 4.0 bar, these experiments are described in section 3.9. The other way to make the reactions faster is to increase the inlet temperature of fuel and air, the flame temperature will then also rise. A higher flame temperature will speed up the reaction rates according to the Arrhenius relation. Especially reactions with a large activation energy can be sensitive to a rise in temperature.

The inlet temperature of the fuel and the air flow is increased by preheating with electrical heaters. The new inlet temperature of the air flow is about 100 °C, that of the fuel flow is about 150 °C. The adiabatic flame temperature will also rise by approximately 100 K. Experiments have been performed for Flame 3.4 and Flame 3.5 at a power of 16.6 kW and 33.2 kW (Flame 3.4) and 14.6 kW and 29.2 kW for Flame 3.5, both with preheating and without preheating.

The measured sound spectra for Flame 3.4 at 33.2 kW (without preheating) and Flame 3.5 at 29.2 kW (with and without preheating) are plotted in figure 3.8. In this figure only the spectrum for Flame 3.4 without preheating is shown, the spectrum with preheating coincides with this spectrum. For Flame 3.5 the difference between the spectrum with preheating and without preheating is very big. The spectrum of Flame 3.5 without preheating has a higher sound level at lower frequencies and it shows a fall off of \(f^{-4}\). The spectrum of Flame 3.5 with preheating coincides with the spectrum of Flame 3.4, both for the sound level at low frequencies and the fall off.

The spectrum of Flame 3.5 with preheating follows the curve \(f^{-10/3}\), which is the pre-
dicted fall off if the mixed-is-burnt approach is valid. The fall off of $f^{-4}$ for Flame 3.5 without preheating is the same fall off as predicted for premixed flames by Boinneau et al. (1996), they predict a fall off of $f^{-2}$ for an open premixed flame, this equals a $f^{-4}$ fall off for a flame in an acoustically 1D cavity. Apparently the fuel and air are first well mixed in Flame 3.5 without preheating before the combustion takes place.

From these measurements it can be concluded that for Flame 3.5, without preheating, the heat release, and therefore the sound generation, is not only a function of the mixing but also of the chemistry. For this situation the mixed-is-burnt approach is not valid. By preheating the fuel and air flow of Flame 3.5 the chemical reaction rate increases, the heat release is now determined by the mixing of fuel and air, the chemical reactions are much faster and the mixed-is-burnt approach is valid. Apparently the sound spectrum of a flame in which the heat release rate is limited by the chemistry shows a steeper fall off and a higher sound level at low frequencies than a similar flame in which the mixing is the limiting step. Both observed phenomena, the higher sound level at low frequencies and the steeper fall off, indicate that flames with slow chemistry are more active at low frequencies, which can be explained by the long (chemical) time scales in these flames.

### 3.7.7 The overall sound level

In this section the measured total sound level is compared with the results from the numerical calculations and the results from the simplified model (equation (3.36)). In the calculations with the simplified model, equation (3.36), the factor $(\gamma - 1)/c^2$ is set to $4.1 \cdot 10^{-7}$, the factor $\alpha_0/S$ to $650/((\pi/4) \cdot D^2)$, where $D$ is the diameter of the combustion chamber (0.1 m). The ratio $l_e/l_{flame}$ is set to 1/2.

In table 3.4 the total sound emission from the different flames is summarized. There is a good agreement between the numerical calculations and the experiments for all flames except Flame 3.5. The fact that the prediction for Flame 3.5 is not good is because of the fact that for this flame the mixed-is-burnt approach is not valid. The sound levels predicted by the simplified model are too high for all flames.

The influence of the thermal power on the total sound level for Flame 3.4 is shown in figure 3.9. In this figure the predictions with the numerical calculations and the simplified model are compared with the experiments. The experiments go to a lower value at 40 kW,

<table>
<thead>
<tr>
<th>Fuel (Thermal power)</th>
<th>Measurements</th>
<th>Numerical Model (eq. (3.43))</th>
<th>Simplified Model (eq. (3.36))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flame 0 (16.0 kW)</td>
<td>125.2 dB</td>
<td>125.3 dB</td>
<td>139.3 dB</td>
</tr>
<tr>
<td>Flame 3.2 (12.3 kW)</td>
<td>124.3 dB</td>
<td>124.4 dB</td>
<td>137.0 dB</td>
</tr>
<tr>
<td>Flame 3.4 (16.6 kW)</td>
<td>125.9 dB</td>
<td>125.2 dB</td>
<td>139.6 dB</td>
</tr>
<tr>
<td>Flame 3.4 (25.0 kW)</td>
<td>130.5 dB</td>
<td>128.4 dB</td>
<td>143.1 dB</td>
</tr>
<tr>
<td>Flame 3.4 (33.2 kW)</td>
<td>132.0 dB</td>
<td>131.0 dB</td>
<td>145.3 dB</td>
</tr>
<tr>
<td>Flame 3.4 (40.0 kW)</td>
<td>131.4 dB</td>
<td>132.2 dB</td>
<td>145.2 dB</td>
</tr>
<tr>
<td>Flame 3.5 (14.6 kW)</td>
<td>132.6 dB</td>
<td>124.1 dB</td>
<td>138.5 dB</td>
</tr>
<tr>
<td>Flame 3.5 (29.2 kW)</td>
<td>134.4 dB</td>
<td>129.9 dB</td>
<td>144.5 dB</td>
</tr>
</tbody>
</table>

Table 3.4: The overall sound level from the different flames. Comparison of experiments, calculations with the numerical model and the simplified model.
3.8 Conclusions for the noise models

The CFD noise post-processor model predicts the total sound level and the shape of the sound spectrum well for flames in which the mixed-is-burnt approach is valid. This has been verified for different fuel compositions and at different thermal powers. For flames where the heat release is limited by the chemistry the sound spectrum shows a steeper fall off and a higher sound level at low frequencies. The simplified model overpredicts the sound level, which cannot yet be explained. The increase of the sound level with the power, predicted with the simplified model, shows the same trend as the results from the numerical calculations.

The numerical model can be used to calculate the sound emission from turbulent non-premixed flames in advance. The simplified model, equation (3.36) is a very useful tool to scale up the sound spectrum from a flame measured at a specific power to the sound level at a higher power.

Figure 3.9: The effect of the thermal power on the overall sound level. Plotted are measurements and predictions with the two different models. which is not calculated by either of the two models. The lines of the numerical predictions and the calculations with the simplified model go parallel, this indicates that the simplified model contains the same important features as the numerical model. The simplified model however, overpredicts the sound level. This overprediction cannot be explained yet and needs further investigation.

If the model of Strahle (1983), equation (3.37), would have been used for the prediction of the total sound level, the overprediction of the sound level would even be higher than with the used simplified model, equation (3.36).
3.9 Pressure effect on noise generation

In an additional series of experiments the effect of the mean pressure on the sound generation by turbulent non-premixed flames has been studied. The influence of the pressure can be twofold:

1. If the inlet velocities are kept identical to those at atmospheric conditions, the thermal power of the flame increases linearly with the pressure. According to equation (3.36), the total sound emission will increase quadratically.

2. The pressure will change the chemical behavior of the flame. Because of the higher pressure the reactions become faster, especially the higher order reactions, what will bring the chemical conditions in the flame closer to equilibrium.

The first aspect is important to study to be able to use atmospheric experiments to predict the sound generation by a flame under elevated pressure, like a gas turbine flame. The second aspect is interesting to study, because under high pressure conditions the mixed-is-burnt approach becomes valid for fuels for which it is not valid under atmospheric conditions.

The influence of pressure on the sound generation has been studied in an experimental set up at ECN ¹. The flames with the codes Flame 3.2, Flame 3.4 and Flame 3.5 have been studied. Flame 3.2 and Flame 3.4 are used as reference cases and can be used to relate the sound emission to the mean pressure. Flame 3.5 is the fuel that contains methane and it has been shown in previous experiments that this flame shows a different sound spectrum as that from the theory. The difference is contributed to the chemistry in this flame, which is not infinitely fast, what makes the mixed-is-burnt approach not valid. At higher pressure the reaction rate of Flame 3.5 should increase what could make the mixed-is-burnt approach valid for this flame.

Experiments have been performed at pressures from 1.4 bar to 4.0 bar. For all pressures the inlet velocities of the fuel and air are kept constant, they are identical to the velocities in the third column of table 3.3. The thermal power during the experiments increases also linearly with the pressure.

3.9.1 The experimental method

Because it was not clear what the acoustic behavior of the combustion chamber would be, the two microphone method is used to separate the acoustic behavior of the combustion chamber from the sound generation by the flame (see appendix G). Because of the small diameter of the combustion chamber the acoustics can be treated as being one-dimensional, what means that there are two traveling waves:

\[ p(x, t) = \text{Real} \left( \left( p^+ e^{-i k^+ x} + p^- e^{i k^- x} \right) e^{i \omega t} \right) \]  

(3.47)

where \( p^+ \) and \( p^- \) are the complex amplitudes and \( k^+ \) and \( k^- \) the acoustic wave numbers.

Two microphones are positioned at different axial locations. The transfer function between the two microphones and the autospectrum of both microphones is measured. By measuring the transfer function between the two microphones it is possible to separate the

upstream traveling acoustic wave \( (p^-) \) and the downstream traveling wave \( (p^+) \). The ratio between \( p^- \) and \( p^+ \) is fully determined by the acoustic exit conditions of the combustion chamber. It is further assumed that the following boundary condition is valid at the burner \( (x = 0) \):

\[
p^+ = p^- + \tilde{S}_{\text{noise}}
\]

(3.48)

where \( \tilde{S}_{\text{noise}} \) is the Fourier transform in time of the noise generation by the flame. The downstream going wave \( (p^+) \) is also the sum of the reflected upstream going wave \( (p^-) \) (closed end at the burner) and the noise generation by the flame. \( \tilde{S}_{\text{noise}} \) can be calculated from the measured autospectrum of one microphone and the ratio of \( p^- \) over \( p^+ \), known from the measured transfer function. This method is inaccurate at low frequencies (long wave lengths), because the difference between the signals at the two microphones is then very small. An estimate for the lowest frequency for which this method is valid, is:

\[
f_{\text{min}} > 0.05 \frac{c}{\Delta x}
\]

(3.49)

where \( \Delta x \) is the separation distance of the microphones. This equation gives for this experimental set up \( (\Delta x = 0.8 \, \text{cm}, \, c \approx 600 \text{m/s}) \) a minimum frequency of 35 Hz. Similar problems with acoustic measurements at low frequencies have also been encountered in the measurement of the flame transfer function, see appendix F.

### 3.9.2 The experimental setup

For the experiments at a higher pressure a new combustion chamber has been built by ECN. A drawing of this combustion chamber is given in figure 3.10. This combustion chamber was fitted in the already present pressurized combustion test rig of ECN. The diameter of the combustion chamber equals the diameter of the atmospheric combustion chamber at the University of Twente (100 mm).

The combustor wall in this pressurized combustion chamber is water cooled. This is necessary because the heat transfer to the wall is too high for air cooling.

At the exit of the combustion chamber there is a disk with a small hole, similar to the damper used in the atmospheric test rig. The flue gases from the combustion chamber are cooled in a heat exchanger by a cold water flow. The cross sectional areas of the hole in the disk and of the heat exchanger are much smaller than that of the combustion chamber. Downstream of the heat exchanger the combustion chamber pressure is regulated by a valve.
Figure 3.11: The noise generation of Flame 3.4 (1.4 bar) derived from the measurements. Two methods are used: the two microphone method (solid line) and the method in which it is assumed that the exit of the combustion chamber acts as a closed end (dotted line).

To get the highest resolution of the acoustic measurements the reference pressure for the microphones is set equal to the pressure in the combustion chamber. This achieved by placing the microphones in a cavity, this cavity is connected to the air inlet by small tubes with valves. Before the acoustic experiments these valves are open to equalize the mean pressure. During the acoustic experiments the valves are closed to prevent the influence of a fluctuating reference pressure.

3.9.3 Results

From the measured transfer function between the two microphones, it appeared that the exit of the combustion chamber acts as a closed end. This can also be seen in figure 3.11, which shows the noise generation from the flame calculated by two methods. The first method is with the two microphone method as described in section 3.9.1, in the second method it is assumed that the exit acts as a closed end, this gives the ratio between $p^-$ and $p^+$, this value is used instead of the ratio from the measured transfer function. It can be seen from this figure that for higher frequencies, above 30 Hz, there is a good agreement between the two methods. The deviation for the lower frequencies is because of errors in the two microphone method. The peak at 240 Hz with the closed end method is a computational artefact, it is at the eigenfrequency of the combustion chamber. It has been chosen to model the combustion chamber with a closed end boundary condition at the exit to derive the sound generation from the measurements.
3.9 Pressure effect on noise generation

![Graph showing the spectra of flame 3.4 at 1.4 and 4.0 bar in Pa²/Hz, plotted double logarithmic.](image)

Figure 3.12: The spectra of flame 3.4 at 1.4 and 4.0 bar in Pa²/Hz, plotted double logarithmic.

The fall off of the sound spectrum

In figure 3.12 the sound spectra of Flame 3.4 for 1.4 and 4.0 bar and the curve $f^{-10/3}$ are shown. It appears from this figure that both measured spectra follow the expected fall off of $f^{-10/3}$. Similar results have been found for Flame 3.2. The spectrum of Flame 3.5 will be discussed further on.

The scaling of the spectrum with the mean pressure

According to equation (3.24) the sound power spectrum should scale quadratically with the density and therefore with the mean pressure. With reference to the flame at 1.4 bar the spectrum of the flame at 3.0 bar should be $20 \cdot \log(3.0/1.4) = 6.6$ dB higher and the spectrum of the flame at 4.0 bar should be 9.1 dB higher. In figure 3.13 the spectra of flame 3.4 at 1.4, 3.0 and 4.0 bar are plotted. It shows from this figure that the spectrum from the flames at the higher pressure are shifted to a higher sound pressure level. The average difference in dB SPL between the flames at 3.0 bar and 1.4 bar is about 6.0 dB, between 4.0 and 1.4 bar is about 8.5 dB. This is in reasonable agreement with the predictions. It shows further from this figure that the peak in the spectrum does not change with the pressure. The position of this peak is a function of the turbulent integral length scale and the velocity, these parameters stay constant with increasing pressure, as expected.

For the different flames the total sound emission as function of the mean pressure have been calculated. The results are plotted in figure 3.14. In this figure also a theoretical curve has been plotted based on equation (3.30), for this theoretical curve the SPL at 1.4 bar
Figure 3.13: The spectra of flame 3.4 at 1.4, 3.0 and 4.0 bar in dB SPL ($\Delta f=1$ Hz).

Figure 3.14: The total sound emission from the flames as function of the mean pressure. Also the theoretical curve has been plotted.
3.9 Pressure effect on noise generation

Figure 3.15: The sound spectrum at 1.4 bar and 4.0 bar from Flame 3.4 and Flame 3.5 plotted double logarithmic.

has been set to 126 dB. It is seen from this figure that the increase in SPL with increasing pressure follows the theoretical curve very well. This observation makes it possible to scale the results from tests under atmospheric conditions to higher pressures.

The difference between the curve of Flame 3.2 and Flame 3.4 can be explained by the different powers of flame 3.2 (12 kW) and 3.4 (17 kW). According to equation (3.36) this would give a difference of about 3 dB, which can be seen in figure 3.14.

The influence of methane (Flame 3.5)

One of the main reasons to perform experiments under conditions with a higher pressure was to study the influence of the finite chemistry on the sound spectrum. It has been shown in section 3.7.6 that the effect of finite chemistry on the sound spectrum is present in Flame 3.5. The influence of the finite chemistry can be reduced by preheating the fuel and air flow or by increasing the mean pressure (see also Louis (1997)). In this section the effect of increasing the mean pressure is studied.

From the experiments presented in section 3.7.6 it is seen that the difference between Flame 3.5 and the other flames is that Flame 3.5 has a higher sound level at low frequencies and that the spectrum of Flame 3.5 shows a steeper fall off. In figure 3.15 the sound spectrum from Flame 3.4 and Flame 3.5 are plotted double logarithmic at two different pressures (1.4 bar and 4.0 bar). It is seen from this figure that for both pressures Flame 3.5 has a higher sound level at low frequencies (below about 70 Hz). The fall off of the spectrum of Flame 3.5 is a little steeper than the fall off of the spectrum of Flame 3.4 at the two pressures. Both aspects indicate that at 4.0 bar the influence of the chemistry on the sound spectrum
is still present. Apparently the mixed-is-burnt approach is not valid at 4.0 bar. This set of experiments shows that increasing the mean pressure to 4.0 bar does not reduce the influence of the chemistry on the sound spectrum for Flame 3.5 significantly. The chemical influence can be reduced by increasing the inlet temperatures of fuel and air as has been shown in section 3.7.6.

3.9.4 Conclusions

The measurement of noise generation by a flame under an elevated mean pressure is more complicated than under atmospheric circumstances. This is mainly because of the exit condition of the combustion chamber. Under atmospheric conditions it is well possible to make the exit of the combustion chamber nearly anechoic by the use of a damper. For the pressurized combustion chamber this is not possible, the exit of the combustion chamber acts as an acoustical closed end.

It has been shown in this section that the sound output from a flame scales well with the mean pressure in the combustion chamber. The sound level increases quadratically with the mean pressure. The shape of the sound spectrum does not change significantly with the mean pressure in the combustion chamber. Flames that show the theoretical fall off of $f^{-10/3}$ under atmospheric conditions show the same fall off under an elevated pressure. The influence of the finite chemistry for Flame 3.5 on the sound spectrum could not be reduced by increasing the mean pressure.

3.10 Experiments with a swirl burner

Experiments have been performed with a model gas turbine burner to investigate the influence of the swirl on the sound generation by the flame. In this section only the most important results and trends will be discussed. The used burner is the GT1 burner described in section 6.2. Only experiments will be presented here that have been performed with the fuel of Flame 3.4 (the composition is given in table 3.1). The experiments are performed in a similar test rig as the bluff body burner.

The sound pressure level in this section is made dimensionless with $\frac{P_{L}}{Q_{tot}}$ where $Q_{tot}$ is the thermal power of the flame. The results are plotted in dB SPL, the reference pressure for the dB calculation is arbitrary and will not be specified here.

3.10.1 The influence of the power

To study the influence of the thermal power on the sound spectrum, a series of flames is studied at the following thermal powers: $Q_{ref}$, $1.5 \times Q_{ref}$, $2.0 \times Q_{ref}$ and $2.5 \times Q_{ref}$, where $Q_{ref}$ is an arbitrary reference power.

In figure 3.16 the spectra for the four different powers at an air factor of $\lambda = 1.75$ are plotted. At the x-axis the Strouhal number is plotted, which is defined by:

$$St = \frac{f \cdot D}{U}$$

(3.50)

For $U$ the inlet fuel flow speed is taken, for $D$ a typical dimension of the burner.
3.10 Experiments with a swirl burner

Figure 3.16: The sound spectra at different powers for Flame 3.4 ($\lambda = 1.75$), plotted in dB SPL. The amplitude has been made dimensionless. At the x axis the Strouhal number is plotted.

The general shape of the spectra scales well with the Strouhal number. For low Strouhal numbers the spectra nearly coincide, especially those for the higher thermal powers. The scaling with the factor $\frac{\gamma - 1}{\gamma} Q_{\text{atm}}$ works well, the dimensionless amplitudes at the different powers are almost identical. The peaks which appear at higher Strouhal numbers scale well with the Strouhal number for the higher powers ($Q_{\text{ref}} : St = 2.6, 1.5 \times Q_{\text{ref}} : St = 2.4, 2.0 \times Q_{\text{ref}} : St = 2.2$ and $2.5 \times Q_{\text{ref}} : St = 2.2$). These peaks are probably caused by vortex shedding.

3.10.2 The influence of the air factor

Contrary to the bluff body burner, the air factor has an influence on the measured sound spectrum of the swirl burner as can be seen in figure 3.17. With increasing air factor the amplitude of the spectrum increases at low frequencies and it decreases at high frequencies (above 200 Hz). The low frequency spectrum is mainly caused by the (turbulent) fluctuations from the air flow, the high frequency spectrum is mainly a function of the (turbulent) fluctuations from the fuel flow. The explanation for the fact that the sound spectrum is influenced by the air factor for the swirl burner is that the flow field is changed by changing the air flow. With increasing air flow the importance of the air swirl grows and the recirculation zone is changed what changes the flame behavior.
Figure 3.17: The sound spectrum at $1.5 \times Q_{ref}$ at different air factors, plotted in dB SPL.

Figure 3.18: The sound spectrum of Flame 3.4 at a power of $2.5 \times Q_{ref}$ ($\lambda = 1.75$) plotted double logarithmic.
3.10.3 The fall off of the sound spectrum

In figure 3.18 the sound spectrum of Flame 3.4 at a power of $2.5 \times Q_{ref}$ and an air factor of 1.75 is plotted double logarithmic. The fall off of the sound spectrum agrees well with the predicted fall off of $f^{-10/3}$, except for the peak around 250 Hz. This peak is probably caused by vortex shedding from the fuel swirler, see also figure 3.16.

3.10.4 Conclusions

It can be concluded from the noise measurements with the swirl burner that it is well possible to scale the sound level with the total heat release by the flame. Within a certain range, the spectrum can be scaled with the Strouhal number. Contrary to the bluff body burner, the air factor has influence on the noise spectrum from the swirl burner.

3.11 Summary and conclusions

An integral expression has been derived for the description of the sound generation by non-premixed turbulent flames in the infinitely fast chemistry limit. The assumption is made that the instantaneous combustion zone is infinitely thin. The sound spectrum can be expressed as a function of the 1D turbulence spectrum of the mixture fraction at the flame front. The information about the turbulence spectrum is derived from a CFD combustion calculation. The sound spectrum is then calculated numerically. For the overall sound level an simple analytical model has been derived.

The calculations are compared with experiments performed in an acoustically one-dimensional, atmospheric combustion chamber. The burner used in these experiments is a bluff body burner. The total sound emission from the flames is well predicted with the present model and scales with the thermal power. The shapes of the calculated and measured sound spectra for flames from fuels without methane show good global comparison for frequencies above 100 Hz. The sound spectra from these flames show a fall off of $\omega^{-10/3}$. The increase of the sound spectrum with the thermal power is well predicted.

For fuels containing methane (Flame 3.5) the measured fall off of the sound spectrum is steeper than for fuels without methane. This can be ascribed to the effect that the mixed-is-burnt approach is not valid for these fuels: the chemical reaction rate is the limiting step instead of the mixing. This hypothesis has been verified by preheating the fuel and air flow. The experiments with the preheated air and fuel flow show the theoretical fall off for infinite fast chemistry, which confirms the hypothesis.

The experiments with the bluff body burner have been repeated under an elevated mean pressure (up to 4.0 bar) to study the influence of the mean pressure. In this set of experiments the inlet flow velocity has been kept constant. The measured sound level scales quadratically with the mean pressure, what confirms the theory. In this series of experiments, the elevated mean pressure seems to have little influence on the shape of the sound spectrum, what indicates that the turbulent structure and chemistry are not changed significantly by increasing the pressure (to 4.0 bar).

Experiments have been performed with a swirl stabilized burner. The spectrum from this burner shows the theoretical fall off of $\omega^{-10/3}$. The level of the sound spectrum shows the expected scaling with the thermal power. The general shape of the sound spectrum can
be scaled with the Strouhal number. In the measured spectra peaks are visible caused by flow instabilities. The frequency of these peaks could not be scaled with the thermal power.
Chapter 4

Combustion acoustic system interaction

4.1 Introduction

The flame is a positive or negative damping term in the acoustic system if there exists a feedback loop from the acoustics to the combustion process (section 2.5.1). In the case of negative damping, the flame is an amplifier of sound and the amplitude of the acoustic pressure grows exponentially in time till non-linear processes become important. The pressure amplitude in the case of such an instability is much higher than that in the case of the flame as source of sound (where it is limited by the linear damping in the system). In many combustion installations, where the flame is located in a cavity (combustion chamber, boiler), most acoustic problems are therefore caused by the flame as amplifier of sound. The role of the flame as amplifier (or damper) of sound is studied in this chapter.

In this chapter it is first discussed what mechanisms can be the cause for the coupling between acoustics and combustion. Therefore a list of the feedback mechanisms from acoustics to the flame is presented in section 4.2. In section 4.3 it is shown that the amplification behavior of the flame can be described by the flame transfer function. This flame transfer function is discussed in more detail in section 4.4. In section 4.5 a new method to measure the flame transfer function is presented.

The new experimental method to measure the flame transfer function, discussed in this chapter, is used to study flames from a bluff body burner and two swirl burners in chapter 5 and 6 respectively.

4.2 Possible energy transfer mechanisms

The instability problem can also be considered as an energy transfer problem: if the flame adds more energy to the acoustic system than is dissipated (by viscous damping or radiation at the boundaries) the amount of energy in the acoustic system will grow in time and instability occurs. If the amount of dissipation equals (or exceeds) the energy transfer from the flame to the acoustic system, the system remains stable.
For the flame as amplifier of sound, the interaction of the flame with the acoustic system is important, especially the fluctuating heat release as function of the acoustic pressure in the combustion chamber (Rayleigh's criterion). The coupling between the pressure in the combustion chamber and the heat release is a complex interaction of fluid dynamics, thermodynamics and chemistry. This coupling is called the energy transfer mechanism. In the literature many possible mechanisms are described (Putnam (1971), Hermann (1997)), the most important ones are summarized here:

- Fluctuating mass flow through the burner. The acoustic pressure fluctuations in the combustion chamber will induce velocity and density fluctuations in the burner mouth (fuel and/or air injector). The mass flow of fuel and/or air then fluctuates with the acoustic pressure. Fluctuations in the fuel mass flow give fluctuations in the total chemical enthalpy that is fed to the flame and therefore fluctuations in the heat release. Air mass flow fluctuations may influence the heat release by the flame if the reaction kinetics are changed (fluctuating composition), if flow instability occurs or if the flame front area fluctuates.

- Vortex shedding/Flow instability. This mechanism can amplify the effect of a fluctuating mass flow through the burner mouth. The flow from the burner may show a dominant frequency. This can be a consequence of vortex shedding from a flame holder, swirl or other flow instabilities. If this dominant flow frequency coincides with an acoustic eigenfrequency of the system a strong coupling between the flow and the acoustics may occur. Such a flow instability may change the flame shape strongly and will then also influence the heat release by the flame.

- Fluctuating air factor/composition. This mechanism plays especially an important role in premixed flames. If the mass flow of fuel and/or air fluctuates, the composition of the mixture at the flame front will also fluctuate. These mixture fluctuations may influence the chemical kinetics. It has been shown that this phenomenon plays an important role in instability problems in lean premixed (natural) gas flames (Lieuwen and Zinn (1998)).

- Fluctuating flame front area. Flame front fluctuations are caused by fluctuations in the mass flow of fuel and/or air, possibly in combination with flow instabilities. The flame front fluctuations will result in heat release fluctuations.

- Droplet formation/Fuel break up. In some cases the interaction between the mass flow of fuel and the acoustics is so strong that the fuel jet is broken up. In the case of a liquid fuel jet this results in the formation of droplets. The droplets will result in a strongly fluctuating heat release.

- Dependency of chemical reaction rate on pressure. The chemical reaction rate of the combustion process depends on the pressure. For gaseous combustion this correlation is rather weak but for solid fuels, like those used in rocket engines, there exists a strong correlation between the chemical reaction rate and the pressure. Installations fired on solid fuels show therefore very often combustion instabilities.
4.3 Acoustic system description

From the different energy transfer mechanisms listed in the previous section, it becomes clear that for gaseous flames the coupling between the heat release and the acoustic pressure in the combustion chamber is mainly because of a fluctuating mass flow through the burner which changes the flame behavior. This fluctuating mass flow is mainly because of the fluctuating inlet velocity.

The coupling between the pressure in the combustion chamber and the heat release by the flame can therefore be split into two parts: 1. The relationship between the pressure in the combustion chamber and the velocity fluctuation in the burner mouth (the admittance of the burner) and 2. The relation between the velocity fluctuation in the burner mouth and the fluctuating heat release. This latter is called the flame transfer function, defined by (the noise generation by the flame is not taken into account):

\[
\frac{\gamma - 1}{c_s} Q = H_{\text{flame}} \cdot \bar{u}_{\text{burner}}
\]  

(4.1)

Following this way of reasoning a combustion system can acoustically be split into three parts:

1. The combustion chamber. The combustion chamber acts as a resonator. Acoustic energy is accumulated in the combustion chamber. In acoustical sense only propagation, reflection, radiation and damping of sound takes place in the combustion chamber. This means that it is well possible to describe the one-dimensional acoustic behavior of the combustion chamber with a set of standard 4 pole transfer matrices.

2. The supply-system. The supply system consists of the burner, the piping to the burner, the premix duct and the plenum or dome (if present). Propagation, reflection and damping of sound waves are the most important acoustical phenomena in the supply system. In many cases the components of the supply system can be described with a system of standard 4 pole matrices. In some components, especially the burner mouth, the acoustics can become very complicated because of interaction with the mean flow. This cannot be described with the standard 4 pole matrices. For this the burner transfer function can be used, which is defined by equation (D.11) in appendix D.

The velocity fluctuation in the burner mouth, as function of the pressure in the combustion chamber, is in general fully determined by the acoustic behavior of the supply system.

3. The flame. To be able to describe the role of the flame, one should know the effect of a fluctuating inlet velocity on the heat release by the flame. This gives a 4 pole matrix that describes the role of the flame in the acoustic system. The most important part in the flame transfer matrix is the flame transfer function.

The acoustic behavior of the combustion chamber and the supply system will not be discussed here, they are discussed in appendix D. The acoustic behavior of the flame will be studied in detail in the following sections.
4.4 The flame transfer function

The flame transfer function is a part of the (acoustic) flame transfer matrix, see appendix D.3.2. It describes the relation between the fluctuating inlet velocity and the fluctuating heat release by the flame. To study the flame transfer function, it is also necessary to know how this velocity fluctuation is propagated to the flame front. The flame transfer function was introduced by Merk (1966) and has been used by a number of people like Becker and Günther (1970), Matsui (1981), Priesmeier (1986), Schuermans et al. (1999), Abbet (1998) and with a different definition by Blossidge et al. (1988).

4.4.1 The propagation of a fluctuation

For a one-dimension situation, e.g., a flat flame in a small tube, a fluctuation in the velocity propagates as a mass flow fluctuation with the speed of sound. An acoustically compact flame has also the same velocity fluctuation at all locations. A fluctuation in a scalar, like the enthalpy or a mass fraction, is propagated by convection (see the equations (C.10) and (C.12)).

For a two-dimensional situation the velocity can in general be written in terms of a potential ($\phi$) and a stream function ($\psi$):

$$u = \nabla \phi + \nabla \times \psi$$  \hspace{1cm} (4.2)

A typical two-dimensional situation for a burner connected to a combustion chamber is shown in figure 4.1. In this figure a flow with the velocity $\vec{u}_1$ leaves the burner and forms a jet in the combustion chamber.

The acoustic velocity fluctuation can be described with the velocity potential fluctuation $\phi'$ (Hirschberg and Nienstra (1992)), which is defined by: $\nabla \phi' = \nabla \phi$. This velocity potential fluctuation is described by the linearized Bernoulli equation (it is assumed that the density is constant):

$$\frac{\partial \phi'}{\partial t} + \frac{\nabla \cdot \vec{u}}{\rho} + \nabla \cdot \vec{u}' = C$$  \hspace{1cm} (4.3)
4.4 The flame transfer function

\[ C \text{ is the Bernoulli constant, which has a constant value along a stream line. If the jet is acoustically compact, the first term at the left hand side can be neglected (see also appendix D.2).} \]

At the edge of the burner there is a jump in the Bernoulli constant (the right hand side of equation (4.3)): The Bernoulli constant in the jet equals \( C_{2b} = \rho_2 + \frac{g{\mathbf{\pi}}}{{\mathbf{\pi}}} u_2 \), the Bernoulli constant outside the jet equals \( C_{2a} = \rho_2 \). It is assumed here that the cross sectional area of the combustion chamber is much larger than that of the burner, what means that the velocity fluctuation outside the jet is small (about \( \left( S_{\text{burner}} / S_{\text{comb. chamber}} \right) u_2 \)). It is further assumed that the pressure is continuous across the jet.

According to the Kutta-condition, the velocity should remain finite at the edge, what implies that a thin shear layer is shed at the edge (Hirschberg and Rienstra (1992) ). The distribution of vorticity in this shear layer compensates the singularity in the Bernoulli constant at the edge. These vortices are propagated by the mean flow. The velocity in these vortices can be described by the rotation of a stream function \( \psi \), the second term in the right hand side of equation (4.2). It can be concluded that in a two-dimensional situation a velocity fluctuation in the burner mouth is propagated along two different characteristics: information about the mass flow is propagated with the speed of sound, the vortices, shed at the edge, are propagated by convection with the mean flow velocity.

The vorticity generated at the edge can be amplified by the mean flow, resulting in flow instabilities. In this situation energy is fed from the mean flow to the vortices. The most unstable types of flow are jets and wakes (Hirschberg and Rienstra (1992) , Lucas et al. (1992) ).

Important to notice is that fluctuations in the radial velocity are induced by the axial velocity fluctuation. This effect has been studied experimentally for a turbulent jet by Priesmeier (1986).

Three-dimensional situations are not discussed here but the same effects occur as in the two-dimensional situation.

The interaction between acoustics, vortex generation and combustion driven oscillations has for example been studied experimentally by Poinset et al. (1987). In their paper it is described how coherent vortices are generated in a turbulent premixed flame because of a fluctuating inlet condition. These vortices are convected and combustion takes place in the wakes of the vortices and at the edge of the main vortex where smaller vortices are generated and the mixing is intense.

The flame transfer function is in general a function of the generation of vorticity at the edge of the burner, the propagation to the flame front and the effect of the vortex on the combustion process.

4.4.2 Examples of a flame transfer function

In this section some very simple examples of theoretical flame transfer functions are analysed. These flame transfer functions describe the most important properties of practical flames. The following flame transfer functions will be discussed:

- A concentrated (premixed) flame front.
- A distributed (premixed) flame.
A concentrated (premixed) flame front

This situation could be a flat flame front located at a certain distance downstream of the burner.

In the case of the concentrated flame front located at $x = x_0$, the heat release can be written as:

$$Q(x, t) = \delta(x - x_0) \cdot Q(t)$$  \hspace{1cm} (4.4)

Let us assume that there is a fluctuation in the mass flow of the mixture of fuel and air in the burner mouth (the heat release depends then linearly on the local velocity). The velocity fluctuation propagates to the flame front with the mean velocity ($U_{mean}$). The flame transfer function is then:

$$H_{flame} = \frac{\gamma - 1}{c^2} \frac{Q_{total}}{U_{burner}} e^{-i \frac{U_{mean}}{c} x_0}$$  \hspace{1cm} (4.5)

, where $Q_{total}$ is the total heat release by the flame, $U_{burner}$ the mean velocity in the burner mouth and $U_{mean}$ the mean velocity by which the velocity fluctuation is convected to the flame front.

A distributed (premixed) flame

Most analytical flame transfer functions from literature are based on a distributed flame front. The most simple shape is presented here, but more complicated flame forms could be used, like a cone for a laminar premixed flame (Matsui (1981)).

It is assumed here that the flame is located between $x_0$ and $x_0 + l_{flame}$, that the heat release over the flame is uniform, that the mean velocity ($U_{mean}$) is constant and that the heat release depends linearly on the local velocity. The flame transfer function is then:

$$H_{flame} = \frac{\gamma - 1}{c^2} \int_{x_0}^{x_0 + l_{flame}} \frac{q \cdot S_{flame}}{U_{mean}} e^{-i \frac{U_{mean}}{c} (x_0)} dx =$$

$$\frac{\gamma - 1}{c^2} \frac{Q_{total}}{U_{burner}} \frac{U_{mean}}{S_{flame}} e^{-i \frac{U_{mean}}{c} x_0} \left( e^{-i \frac{U_{mean}}{c} l_{flame}} - 1 \right)$$  \hspace{1cm} (4.6)

, where $q \cdot S_{flame} \cdot l_{flame}$ is the total heat release by the flame ($Q_{total}$).

The flame transfer function of equation (4.6) is plotted in figure 4.2 for two different flame lengths. The amplification factor has been made dimensionless with $\frac{\gamma - 1}{c^2} \frac{Q_{total}}{U_{burner}}$. The values of the different variables are: $x_0 = 0.01 \text{ m}$, $U_{mean} = U_{burner} = 10 \text{ m/s}$, $l_{flame}$ has been set to 0.01 m and 0.04 m respectively.

The dimensionless amplification factor of the flame transfer function in figure 4.2 is for both flame lengths always smaller than one. The longer the flame the stronger the decrease of the amplification factor with the frequency. The phase plot for the short flame shows the typical time delay behavior. In the phase plot for the longer flame some jumps are visible. These jumps are caused by the distributed heat release.

This decrease of the burner transfer function with flame length has been observed in many experiments. It is also known from practice that in general combustion installations with short fat flames are much more sensitive to combustion driven oscillations than similar installations with long flames (Putnam (1971)).

For many real situations it is not possible to write down an analytical expression for the flame transfer function. The flame transfer function is namely the result of the complex
interaction of unsteady fluid dynamics, chemistry and heat release. In those situations the flame transfer function has to be measured or calculated from numerical (CFD) calculations. In this thesis an experimental method will be discussed. For CFD methods the reader is referred to Klein and Kok (1999a) and Krüger (1998).

4.5 Measurement of the flame transfer function

To determine the flame transfer function from measurements, the heat release from the flame should be correlated to the fluctuating inlet velocity. The most common method to measure the flame transfer function is to impose a fluctuation on the inlet velocity and to measure the response in the heat release on this fluctuation. The shape of the imposed fluctuation can either be a sweep, white noise or a pure tone. The fluctuating inlet velocity in the burner mouth can be measured with hot wire anemometry or laser Doppler anemometry. There exists however no experimental method to measure the heat release by the flame. The heat release should therefore be derived from quantities that can be measured. All methods to determine the flame transfer function from measurements are therefore indirect methods.

There are two main methods to determine the flame transfer function:

1. The heat release from the flame is determined by the measurement of the chemiluminescence of OH or CH radicals in the flame. This method is applicable to hydrocarbon flames because the heat release in these flames shows a strong correlation with the OH/CH-concentration. The velocity fluctuation in the burner mouth can be measured with the above mentioned techniques.

2. A fully acoustic method. An acoustic fluctuation is imposed on the system. By using several microphones the response of the system to this fluctuation is measured. These
measured signals are the input for an acoustic model that yields the flame transfer function.

4.5.1 The measurement of OH-radicals

The chemiluminescence of the OH-radicals is measured with optical techniques. One method is to excite these radicals with a laser sheet and to measure the fluorescence at the specific wavelength of that radical with a ICCD camera (this method is called planar laser induced fluorescence (PLIF)). With this method it is possible to get a two dimensional picture of the local distribution of the heat release within the flame. The total heat release can be calculated by integration. This method has been used by Paschereit (Paschereit et al. (1998)) to measure the heat release from a natural gas gas turbine flame. Due to the strong turbulent fluctuations in the flame and the low maximum frequency of the laser shots (typically 10 Hz) it is necessary to apply phase averaging.

Another optical method is to measure the light emission at the wavelength of the OH/CH radical from the whole flame with a photo multiplier equipped with a filter. This signal is also correlated to the total heat release by the flame. This method has been used by many people like Hermann (1997), Langborne (1988), Priesmeier (1986), Joos and Vortmeyer (1986), Lang and Vortmeyer (1987) and Becker and Günther (1970).

It is possible to get qualitative results with these optical methods. To get quantitative results the relation between the heat release and the light emission should be known in advance. This relation can be estimated from steady state measurements.

These optical methods are not well applicable to syngas flames since there does not exist a clear relationship between the concentration of OH/CH radicals and the heat release by syngas flames. Further this method should be calibrated for all different gas compositions what makes it difficult to study the influence of the gas composition on the acoustic behavior of the flame.

4.5.2 The acoustic method

The method used in this thesis and which has been developed by the author is a fully acoustic method. A similar method has been developed by Paschereit and Polifke at the same time, see Paschereit and Polifke (1998) and Schiermanns et al. (1999).

In the method used in this thesis an oscillation is imposed on the flow, the response of the acoustic system to this fluctuation is measured using several microphones. These measurements are the input for a one-dimensional acoustic model of the system with as unknown the flame transfer function. The equations of the model are solved with as result the flame transfer function. The basis for this method is the well known two-microphone technique from acoustics (appendix G) and the transfer matrix method (appendix D).

To be able to use the acoustic method to measure the flame transfer function, it is necessary to make the acoustic system of the combustion rig as simple as possible. This is achieved by making the maximum diameter of the system so small that it can acoustically be treated as being one-dimensional for the frequencies of interest. Because of this one-dimensional behavior the propagation of sound can be described with plane waves, what results in a set of 4 pole transfer matrices to describe the whole system. This set of 4 pole
4.5 Measurement of the flame transfer function

matrices together with the results from the measurements gives a system matrix, which is solved numerically. From the solution the flame transfer function is calculated.

A schematic picture of the experimental set up is given in figure 4.3. This is the most simple situation, for more complex burner and/or combustion chambers it is possible to incorporate more elements (matrices). In this picture also the burner is shown (light gray). For now it is assumed that the transfer matrix of the burner is known, either from a model or from measurements. Later on it will be discussed how this burner transfer matrix is determined from cold flow measurements. The method to do this is very similar to the measurement of the flame transfer function.

An oscillation is imposed on the flow by a membrane that is placed perpendicular to the flow. The fluctuation can be imposed either on the fuel flow or the air flow. The membrane is driven by an exciter. It is chosen to use an exciter in combination with a membrane instead of a conventional loudspeaker because the sealing is easier and the acoustic power that can be supplied is much larger. The signal for the exciter is generated by the signal generator of the FFT analyzer. Sound is generated because of the vibration of the membrane. The different acoustic waves will arise $(\vec{p}_1, \vec{p}_2, \ldots, \vec{p}_5, \vec{p}_6)$.

The transfer function between the pressure measured by $\text{mic}_b$ and the pressure measured by $\text{mic}_a$ is determined with the FFT-analyzer, this is transfer function $T_I$. Also the transfer function between the pressure at $\text{mic}_a$ and the pressure at $\text{mic}_a$ is determined $(T_{II})$. The reference microphone for both transfer functions is $\text{mic}_a$. The acoustical experimental setup is described in appendix B.
The acoustic model of the combustion rig

There are twelve pressure components \( (p_1^+ \ldots p_9^-) \) in the acoustic model of the combustion rig, see figure 4.3. All pressure components are linear functions of each other. One pressure component is chosen as the independent pressure component e.g. \( p_1^+ \). This pressure component is written on the right hand side of the matrix equation to be solved. With this procedure the number of pressure components, which has to be solved, is 11.

The acoustic model contains several matrices. The matrix, that describes the propagation of sound in the supply system \((A)\), and the matrices for the sound propagation in the combustion chamber \((D \text{ and } E)\) are the standard matrices for the propagation of sound. The burner matrix \((B)\) is known in advance. These four matrices give eight equations. Two additional equations are known from the measurement of the transfer functions:

\[
p_1^- + p_1^+ = T_I (p_1^- + p_1^+)
\]

\[
p_6^- + p_6^+ = TII (p_6^- + p_6^+)
\]

The matrix that has to be determined is the flame transfer matrix. This matrix has 4 independent components \((F_1, F_2, F_3, F_4)\), this gives 2 extra equations with 4 new unknowns:

\[
p_1^- = F_1 p_3^- + F_2 p_3^+
\]

\[
p_4^+ = F_3 p_3^+ + F_4 p_3^-
\]

Together there are 15 unknowns (11 pressure components and 4 flame matrix components) and 12 equations, showing that it is impossible to determine all unknowns. There are two ways to overcome this problem. The first way is to measure 4 independent test states, this can be done by attaching four different acoustic dampers at the exit of the combustion chamber or using combustion chambers with different lengths. For these four situations the flame matrix components remain the same, this means that there are 48 unknowns (44 pressure components and 4 flame matrix components). There are also 48 equations, what makes this matrix equation solvable. This method has the disadvantage that it is difficult to create four independent test states. If the differences between the test states are small, it is hard to get accurate results.

The other approach, which is used in this thesis, is to make some assumptions about the flame transfer matrix. These assumptions have already been described in section D.3.2. Instead of the flame transfer matrix the flame transfer function is determined. An extra model equation is used, which is the linearized Bernoulli’s equation (D.8). This equation states that the acoustic pressure is constant over the flame. This assumption is allowed if the flame is compact compared to the acoustic wave length. With this assumption the total number of equations becomes 11 what equals the number of unknown pressure components.

The known model matrices \((A, B, D \text{ and } E)\) together with equations (4.7)and (4.8) and the linearized Navier Stokes equation give a matrix called \(M_{system}\), which is a function of
the frequency. The set of equations can be written as:

\[
[M_{\text{system}}] \cdot \begin{bmatrix}
p_1^+ \\
p_1^- \\
p_2^+ \\
p_2^- \\
p_3^+ \\
p_3^- \\
p_4^+ \\
p_4^- \\
p_5^+ \\
p_5^- \\
p_6^+ \\
p_6^- \\
\end{bmatrix} = \begin{bmatrix}
0 & & & & & & & & & & & \\
0 & & & & & & & & & & & \\
-B_3 & & & & & & & & & & & \\
-B_7 & & & & & & & & & & & \\
-C_1 & & & & & & & & & & & \\
-C_5 & & & & & & & & & & & \\
0 & & & & & & & & & & & \\
& & & & & & & & & & & \\
0 & & & & & & & & & & & \\
\end{bmatrix} \begin{bmatrix}
p_3^+ \\
p_3^- \\
p_3^+ \\
p_3^- \\
\end{bmatrix}
\] (4.11)

This equation is solved by multiplying both the left hand side and the right hand side with \( M_{\text{system}}^{-1} \). All pressure components are known then as function of \( p_3^+ \). The flame transfer function is then calculated from the following equation, using the equations (D.14) and (D.15):

\[
H_{\text{flame}} = \frac{\bar{p}_c c_2}{p_3^* - p_3^*} \left[ \frac{S_4}{c_4} \left( p_1^+ (1 + M a_4) + p_1^- (M a_4 - 1) \right) \\
- \frac{S_3}{c_3} \left( p_3^+ (1 + M a_3) + p_3^- (M a_3 - 1) \right) \right] 
\] (4.12)

Because \( M_{\text{system}} \) is a function of the frequency, this calculation should be repeated for all frequencies. The matrix equation is solved by Matlab®.

A sensitivity analysis has been performed for this transfer function method in appendix F.

### 4.5.3 The burner transfer function

For the measurement of the burner transfer function, the same set up and method is used as for the measurement of the flame transfer function, see figure 4.3. Instead of a flame a cold flow is used, the matrix of the flame (C) is also not applicable. There are also 9 unknown pressure components (expressed as function of e.g. \( p_3^+ \)) and 8 equations (6 from the matrices \( A, D \) and \( E \) and 2 from the transfer function measurements). Similar to the measurement of the flame transfer matrix, not all four components of the burner transfer matrix are determined but an extra equation is used instead. This equation is based on the linearized one-dimensional mass conservation equation integrated over the burner, equation (D.7). If the length of the burner is small compared to the acoustic wavelength, the accumulation term in this equation can be neglected. This extra equation makes the set of equations solvable.

The burner transfer function is then calculated from (see also equation (D.11)):

\[
H_{\text{burner}} (\omega) = -\frac{p_3 S_3 - p_2 S_2}{u_2} = -\bar{p}_2 c_2 \left( p_1^+ + p_1^- \right) - S_2 \left( p_3^+ + p_3^- \right) 
\] (4.13)

The burner transfer function gives the deviation from the ideal acoustic situation. In the ideal situation the acoustic pressure difference across the burner should be zero and the burner transfer function is then zero.
Chapter 5

Experimental results for a bluff body burner

5.1 Introduction

In this chapter the experimental results for the bluff body burner, described in appendix B.2, are discussed. Both the flame transfer function and the burner transfer function have been determined. The methods for this have been presented in section 4.5.

The influence of the following parameters has been studied for both transfer functions:

- The linearity of the transfer function by using white noise and pure tone excitation.
- Excitation of the fuel flow.
- Excitation of the air flow.

For the flame transfer function the following aspects have further been studied:

- The effect of a variation in the air factor
- The effect of a variation in the thermal power.
- The effect of a variation in the chemical composition of the fuel.

Besides the flame and burner transfer function the acoustic system behavior has been studied. For this study the measured burner and flame transfer function are used in an acoustic model of the combustion rig. The results from this model are compared with measurements.

The flames in this chapter are similar to the flames studied in chapter 3. The properties of the different fuels and the inlet conditions are summarized in the tables 3.1 and 3.2.

The microphones have been calibrated before each set of experiments. In this calibration the effect of the connection between the microphone and the combustion chamber has been included.
5.2 The burner transfer function

For the measurement of the burner transfer function the fuel flow has been replaced by a nitrogen flow with identical volume flow rate. It is seen from the measurements that the determination of the burner transfer function is very sensitive to acoustic resonance in the system and to pressure nodes on the microphones, this is because of the small deviation from the ideal acoustic situation. It has been attempted to minimize these effects.

The burner transfer function has been measured both with white noise excitation and with pure tone excitation. The burner transfer functions for the fuel side of Flame 3.4 at 16.6 kW and 40 kW are shown in figure 5.1. It is seen from this figure that, except for the higher frequencies, pure tone excitation and white noise excitation give identical results. This illustrates that (as assumed) the burner transfer function is a linear function.

In the amplification factor plot of figure 5.1 there is a small peak around 150 Hz and a large peak around 250 Hz. These peaks are caused by resonance in the combustion chamber (length 1.3 m corresponds to eigenfrequencies of 130 Hz (1/2λ) and 260 Hz (λ)) and resonance in the fuel supply system (eigenfrequency ≈ 160 Hz). The measurements at these frequencies are not reliable. Apart from the resonance frequencies the amplification factor is about $1 \times 10^{-2}$ for both powers and the phase is about 0° for the 40 kW case and increasing with the frequency for the 16.6 kW case for low frequencies ($f < 150$ Hz).

In the ideal acoustic situation the acoustic pressure difference across the burner is zero. The burner transfer function gives the deviation from this ideal acoustic situation. The importance of the burner transfer in the total acoustic system can be estimated from equation (4.13): $H_{burner}/\pi c_1$ should be compared with $S_1$. This gives that $|H_{burner}/\pi c_1|$ is just around 4 % of $S_1$ for the 16.6 kW case and around 8 % for the 40 kW case. It can be concluded that the importance of the burner transfer function is small and that the velocity fluctuation in the burner mouth for the largest part can be described with the standard
5.3 The flame transfer function for Flame 0

acoustic equations. This small deviation from the ideal acoustic conditions explains also why the measurement of the burner transfer function is very sensitive to resonance in the system.

The burner transfer function for the air side has also been determined. A similar order of magnitude estimate as for the fuel side has been performed. It was found that the importance of the burner transfer function for the air side was even smaller than of that for the fuel side \((|\mu_{\text{burner}}|/\mu_c)\) was about 2% of the \(S_1\) for the 16.6 kW case.

The experiments for the burner transfer function have been repeated for the other flow rates. For all flow rates, it showed that the burner transfer function was not important. In the remainder of this chapter the burner transfer function is set to zero.

5.3 The flame transfer function for Flame 0

In this section the following results for Flame 0 will be discussed: the fuel side flame transfer function, the air side flame transfer function, the linearity of the flame transfer function, the influence of the air factor and the flame transfer function as function of the thermal power of the flame.

In the calculation of the flame transfer function the following value has been used for the speed of sound in the combustion chamber: \(c_{\text{comb, chamber}} = 600 \text{ m/s}\), which correspond to a temperature of 1000 K, a typical value for the studied flames.

5.3.1 Fuel side excitation

To study the (assumed) linearity of the flame transfer function, the flame transfer function determined with pure tone excitation and with white noise excitation are compared. Both plots are shown in figure 5.2.

The phase plots for both types of excitation go to 180° for low frequencies (below 20 Hz). This is expected to be an effect of the inaccuracies of the method at low frequencies. For higher frequencies (from 20 Hz to 70 Hz) the phase plot follows a line with constant negative inclination: this indicates a constant time delay. If this line is interpolated to 0 Hz, it crosses the y-axis at 0°. At higher frequencies (above 70 Hz) the phase becomes constant and equal to (nearly) \(\pm 180°\). The phase plots for white noise excitation and pure tone excitation nearly coincide. This confirms that the phase shift of the flame transfer function is a linear function. This can be explained by the fact that the phase shift is mainly a function of the time delay, which is determined by the convection of the fluctuations by the mean flow, which is a linear function.

The amplitude plots decrease both with the frequency up to 140 Hz, after 140 Hz the amplitude reaches a constant value \((2.8 \times 10^{-4})\). Between 30 Hz and 70 Hz and between 100 Hz and 140 Hz, the amplitude from the white noise excitation is a little higher than that from the pure tone excitation. These differences are probably caused by the interaction between the turbulence and the acoustic fluctuations. The non-linearity of the turbulence explains then the difference between white noise excitation and pure tone excitation. This effect has been verified by imposing pure tones with different amplitudes, in the case of small amplitude pure tones the results from the white noise measurements were found.

A constant phase of \(\pm 180°\) indicates that the flame is insensitive for the velocity fluctuations in the burner mouth. This can be seen from the linearized integrated 1D mass
Figure 5.2: Bode plot of the fuel side flame transfer function for Flame 0 at 16.0 kW, $\lambda = 1.3$.

conservation equation (D.14) (the Mach number is assumed to be small, location 1: upstream of the flame, location 2: downstream of the flame):

$$\bar{p}_g S_2 \ddot{u}_2 = \ddot{u}_1 (H_{\text{flame}} + \bar{p}_1 S_1)$$

At a phase of $\pm 180^\circ$, $H_{\text{flame}}$ is a negative real, and the right hand side of equation (5.1) becomes smaller than it would be without a flame. This means that the acoustic (velocity) coupling between the burner mouth and the combustion chamber is overpredicted by the present model. This effect can partly be explained by the negligence of the burner transfer function in the calculation of the flame transfer function, but even with the inclusion of the burner transfer function this effect is still present. An explanation for this phenomenon is that with flame the jump in the Bernoulli constant at the edge of the burner is larger than without flame, see section 4.4.1, what will increase the (acoustic) pressure drop across the burner. By neglecting this effect the calculated velocity fluctuations in the burner mouth will be too high, which is compensated by the phase of $\pm 180^\circ$ in the flame transfer function from the present measurements.

The amplitude of the flame transfer function at 0 Hz (steady state condition) should be of the order, see equation (4.1):

$$\frac{\gamma - 1}{c^2} \frac{Q_{\text{total}}}{U_{\text{burner}}}$$

where $Q_{\text{total}}$ is the total heat release by the flame and $U_{\text{burner}}$ the mean velocity in the burner mouth. For this flame is this factor $4 \cdot 10^{-3}$, which is the same order of magnitude as the value of the amplitude of the flame transfer function at 0 Hz. This gives confidence in the value of the measured amplification factor of the flame transfer function.
### 5.3 The flame transfer function for Flame 0

![Bode plot](image)

Figure 5.3: Bode plot of the air side flame transfer function for Flame 0 at 16.0 kW, $\lambda = 1.3$.

#### 5.3.2 Air side excitation

In figure 5.3 the air side flame transfer functions are plotted. Shown are the transfer functions determined with white noise excitation and with pure tone excitation. Above 100 Hz the white noise curve coincides with the pure tone curve both in the amplitude plot and in the phase plot. The difference between the two curves below 100 Hz is ascribed to the interaction with turbulence just as with the fuel transfer function.

The phase of the pure tone excitation is $\pm 180^\circ$ over the whole frequency domain, this indicates that the heat release by the flame is insensitive to these air flow fluctuations. It further indicates that the acoustic (velocity) coupling between the burner mouth and the combustion chamber is overpredicted by the model of the acoustic system. This effect has been explained in the previous section.

For all bluff body burner flames, studied in this thesis, the air side flame transfer function showed a similar behavior as plotted in figure 5.3, which means that the flame is not sensitive to these air side fluctuations. Therefore the study is restricted to the fuel side transfer functions.

#### 5.3.3 The influence of the air factor

The air factor has no effect on the flame transfer function as can be seen in figure 5.4. Also for the air side transfer function no influence of the air factor could be observed. The other bluff body burner flames, which have been studied in this research, also showed no influence (or a minor influence) of the air factor on the flame transfer function.
Figure 5.4: Bode plot of the fuel side flame transfer function for Flame 0 at 16.0 kW, different air factors.

Figure 5.5: Bode plot of the fuel side flame transfer function for Flame 0 at 16.0 kW, 24.0 kW and 39.0 kW, $\lambda = 1.3$. On the x-axis the Strouhal number is plotted.
5.3.4 The influence of the thermal power

The flame transfer function for Flame 0 has been determined at different thermal powers: 16 kW, 24 kW and 39 kW. To be able to compare these three experiments, the frequency is written as the Strouhal number:

\[ St = \frac{fD}{U} \]  

(5.3)

where \( U \) is the exit velocity of the fuel (3.7 m/s, 6.5 m/s and 9.3 m/s respectively for 16 kW, 24 kW and 39 kW) and for \( D \) the diameter of the combustion chamber (0.1 m) is taken. The bode plots are shown in figure 5.5.

For the amplitude plot the agreement between the three thermal powers is excellent above a Strouhal number of 0.8 (30 Hz for the 16 kW flame), the difference at lower Strouhal numbers can be contributed to the inaccuracy of the method at low frequencies. The agreement between the three powers is even better in the phase plot. This observation shows that the Strouhal number is a good way to scale these flames. As the characteristic dimension \( D \) was constant in the Strouhal number, it can be concluded that the length scales of the flame at the three powers are identical. This is in good agreement with standard turbulent jet diffusion flame theory, which states that the length of the flame is not a function of the power at sufficiently high Reynolds number (Warnatz et al. (1998)).

From the slope of the phase plot the time delay necessary for a fluctuation to travel from the burner mouth to the flame interface can be calculated. The time delays for the different powers are 6.9 ms (16.0 kW), 4.6 ms (24.0 kW) and 2.8 ms (39.0 kW). The fluctuation is transported with the fuel flow velocity to the flame interface. The distance between the burner mouth and the flame interface is 2.6 cm (calculated using the fuel flow velocity and the time delays mentioned before). This is about the distance between the burner mouth and the bluff body.

5.4 The flame transfer functions for Flame 3.2, 3.4 and 3.5

In this section the measured flame transfer functions for the flames Flame 3.2, Flame 3.4 and Flame 3.5 are described (see section 3.7 for the composition of these fuels). The following aspects have been studied:

- The influence of the type of excitation: white noise or pure tone excitation.
- Air side excitation and fuel side excitation.
- The influence of the air factor.
- The influence of the chemical composition. All the fuels of serie 3 have the same stoichiometric mixture fraction. The main difference between the different fuels is the chemical composition.
- The influence of the thermal power.

The influences of the type of excitation (white noise/pure tone), of air side excitation and of the air factor are very similar to the influences of these aspects for Flame 0: The type of
excitation has only influence on the amplification factor of the flame transfer function and in the frequency band where the turbulence is active. The flames Flame 3.2, 3.4 and 3.5 are all not sensitive to air side excitations. The air factor only has a minor effect on the flame transfer function. The details of these aspects are not presented here.

5.4.1 The influence of the chemical composition

In figure 5.6 the flame transfer functions of Flame 3.2 (12.5 kW), Flame 3.4 (16.6 kW) and Flame 3.5 (14.4 kW) are plotted. The experimental conditions of these flames are the same as those for the noise generation studies, see table 3.1 and table 3.2. The inlet velocities are identical for these three flames.

At 180 Hz there is a dip visible in the flame transfer function of Flame 3.4, this dip can also be seen in the phase plot as a phase jump. This behavior of the flame transfer function of Flame 3.4 at 180 Hz is very similar to the behavior of the model flame transfer function shown in figure 4.2 ($l = 0.04$) at 250 Hz. This behavior of the flame transfer function indicates a distributed heat release for Flame 3.4.

The amplification factors of the flame transfer functions of Flame 3.2 and Flame 3.4 are almost identical up to 150 Hz. At high frequencies the amplification factors of all three fuels go to about the same value ($2 \cdot 10^{-4}$). This indicates again that the acoustic coupling between the fuel inlet and the combustion chamber is less strong than predicted by the model. For low frequencies (below 60 Hz) the amplification factor of the flame transfer function of Flame 3.5 is the same as that of Flame 3.2 and Flame 3.4. Above 60 Hz the amplification factor of Flame 3.5 drops strongly to the end value around of $2 \cdot 10^{-4}$. Hence Flame 3.5 is only active at low frequencies, this agrees well with the observations made about the noise generation by Flame 3.5.

The phase plots of all three flames start at about $90^\circ$ at 0 Hz, this differs from the phase
plot for Flame 0, which starts at 0°. This phase of 90° indicates that the fluctuations in the heat release from these flames are caused by fluctuations in the radial velocity. These radial velocity fluctuations are a result of the vorticity generation at the edge of the burner mouth, see section 4.4.1. The phase between the radial velocity fluctuation and the axial velocity fluctuation can be calculated from the continuity equation: The axial velocity fluctuation \( u'(x, r, t) \) downstream of the burner mouth is written as (in cylindrical co-ordinates):

\[
u'(x, r, t) = \bar{u}(r) \cdot e^{-i\omega t} \cdot e^{i\omega t}\]

(5.4)

this equation gives the convective propagation of the axial velocity fluctuation with the mean velocity \( \bar{u} \).

The radial velocity fluctuation \( \nu \) can then be calculated using the (incompressible) continuity equation:

\[
u(x, r, t) = \frac{i\omega}{\nu} \left( \int_0^r u(r') \, dr' \right) \cdot e^{-i\omega t} \cdot e^{i\omega t}\]

(5.5)

this equation indicates that the phase between \( \nu \) and \( u' \) is 90°. This has been verified experimentally by Priesmüller (1986).

The fluctuations in the radial velocity \( \nu \) affects the amount of the fuel, that is transported to the flame interface. In the case of a positive \( \nu \), more fuel is brought into the flame interface, and vice versa. This means that the phase between the radial velocity fluctuation and the heat release fluctuation is 0° (the time scale of the combustion is assumed to be much faster than the time scale of \( \nu \)). The phase between the heat release and the axial velocity fluctuation is therefore 90°.

The fact that these flames are very sensitive to the radial velocity fluctuations is probably caused by the fact that these flames are close to the burner mouth, where the vorticity, generated at the edge of the burner, is strong.

The phase plots of Flame 3.2 and Flame 3.4 have nearly the same slope up to 100 Hz, the phase plot of Flame 3.2 is just a little steeper. After the phase plot of Flame 3.2 has reached -180° the phase stays constant (±180°) as function of the frequency. The phase of Flame 3.4 shows almost a linearly decay with the frequency up to 300 Hz. The phase plot of Flame 3.5 is much steeper than those of Flame 3.2 and Flame 3.4. It reaches -180° at 55 Hz and remains constant ±180° above this frequency. The time delays of Flame 3.2 and Flame 3.4 are nearly the same (3 ms and 4 ms respectively), the time delay of Flame 3.5 is 6 ms. This means that the location of largest heat generation is at about 2 cm downstream of the burner mouth for Flame 3.2 and Flame 3.4 and at about 4 cm for Flame 3.5. Hence, the heat release location for Flame 3.2 and Flame 3.4 is upstream of the bluff body. The heat release location for Flame 3.5 is downstream of the bluff body. The small difference in the time delay between Flame 3.2 and Flame 3.4 might be explained by the high hydrogen content in Flame 3.2. Hydrogen has a much higher diffusion coefficient than the other species and will mix faster with the air.

### 5.4.2 The influence of the thermal power

For Flame 3.4 a study of the influence of the thermal power on the flame transfer function has been performed. The results are plotted in figure 5.7. In this figure the frequency is plotted along the x axis and not the Strouhal number (in contrast with figure 5.5) because the scaling with the Strouhal number does not work for Flame 3.4.
The maximum value of the amplification factor is the same for all powers. This value $(1 \cdot 10^{-3})$ is compared, just as for Flame 0, with $\frac{2 - 1}{\sqrt{\frac{1}{Q_{fuel}}} = 1.9 \cdot 10^{-3}}$. The fact that the maximum value of the amplification factor is the same for all powers, means that the factor $\frac{2 - 1}{\sqrt{\frac{1}{Q_{fuel}}}}$ can be used to scale the amplification factor. The frequency band in which the flame amplification factor is near its maximum becomes wider with increasing thermal power. The broadening of the peak of the amplification factor can be explained with the increasing flow velocity and a distributed heat release (see section 4.4.2).

The phase delay becomes less steep with increasing thermal power. This indicates a decrease in time delay with the power. The time delays for the different powers are: 4 ms (16.6 kW), 3 ms (25 kW), 2.5 ms (33.2 kW) and 2 ms (40 kW). The location of the average heat release fluctuations are about: 2.5 cm, 3 cm, 3.3 cm and 3.5 cm downstream of the burner exit. For the higher powers the flame shifts more downstream. Obviously the location of the heat release shifts with the thermal power, making it impossible to use Strouhal number scaling using the diameter of the burner as the characteristic length scale. The reason why the location of the heat release shifts with the thermal power is not clear. One reason could be that these flames are much shorter than Flame 0, which means that the inlet conditions are more important for these flames. In that case the characteristic length of the flame will change with the inlet conditions. For this bluff body burner the inlet conditions are not well fixed by the geometry.

### 5.5 Conclusions for the flame and burner transfer function

Experiments to determine the flame transfer function have been performed with the bluff body burner. The following aspects are found for all flames:
5.6 The acoustic system behavior

- The burner transfer function contributes little to the acoustic behavior of the burner mouth. The bluff body burner can well be described with the standard acoustic relations.

- The sensitivity of the flames to air flow fluctuations is very small. Only a weak influence can be found at the frequency band where the turbulence is active.

- The difference between white noise excitation and pure tone excitation is small. Only in the low frequency range there is a small difference. Hence, the flame transfer is a linear function.

- All measured flame transfer functions for the bluff body burner do not change with the air factor.

- The phase shows a typical time delay behavior: a nearly linear decrease with the frequency.

- The value of the amplification factor of the flame transfer function can be scaled with $\frac{\omega}{\phi}$.

For Flame 0 it was further found that it is well possible to scale the flame with the Strouhal number. The location of heat release fluctuations does not change with the thermal power.

For the Flames 3.2, 3.4 and 3.5 it is not possible to use the Strouhal number to scale the frequency, this is because of the shift of the location of heat release with the thermal power. It was further found that the phase at $0$ Hz equals $90^\circ$ for these flames, this is because the heat release by the flame fluctuates especially because of the radial velocity fluctuations.

5.6 The acoustic system behavior

With the measured flame transfer functions, studies of the acoustic system have been performed. This has been done to demonstrate that it is possible to use a flame transfer function to describe the role of a flame in an acoustic system and to show the influence of the flame transfer function on the acoustic behavior.

The acoustic system behavior has been studied in two ways. In the first way the damper was removed from the exit of the combustion chamber and the resulting pressure spectrum was measured. From the measured spectrum the instability frequency, if present, was determined. The occurrence of instability and this instability frequency were compared with the instability frequency as predicted by the acoustic model of the system including the measured flame transfer function. The results of this study are presented in section 5.6.2.

In the other study of the acoustic system behavior, the acoustic damper remained on the exit of the combustion chamber. Sound was generated by a membrane, driven by an exciter, at the exit of the combustion chamber. The response of the system was measured by two microphones located in the combustion chamber. With these two microphones it is possible to distinguish between the upstream going wave and the downstream going wave (one-dimensional acoustics). The reflection factor at the burner face was determined from the ratio of the downstream going wave over the upstream going wave. This measured reflection factor has been compared with the calculated reflection factor from an acoustic
Figure 5.8: A schematic picture of the combustion test rig. The numbers refer to the pressure components at different locations. Microphones are mounted at the locations 6, 10 and 11. A high pressure drop is created upstream of the burner ($\Delta p$). The black piece in the burner supply is the rod which supports the bluff body ($d=6$ mm). The dimensions of the rig are given in table 5.1.

system model including the flame transfer function, measured in a previous experiment. By exciting the membrane with pure tones, it is possible to scan a frequency band. This is also the main advantage of this method over the first method, where only the response of the system at the acoustic eigenfrequency of the system is measured. This second method is called the measurement of the flame reflection factor.

It appeared from both studies that one of the most important parts of the acoustic system is the acoustic boundary condition at the upstream end of the burner. This condition namely has a strong influence on the admittance of the burner mouth (the velocity fluctuation in the burner mouth as function of the acoustic pressure in the combustion chamber). This upstream boundary condition has been derived both from an acoustic model and from measurements.

### 5.6.1 The flame reflection factor

A schematic picture of the experimental set up is given in figure 5.8, the dimensions are given in table 5.1. In this series of experiments only the influence of the fuel supply has been studied. Upstream of the T-piece in the fuel supply a high pressure drop has been created to decouple the acoustic system of the combustion rig from the upstream acoustic system. In the air inlet a similar pressure drop has been created, upstream of the air exit of the burner.
5.6 The acoustic system behavior

<table>
<thead>
<tr>
<th>$d$</th>
<th>0.015 m</th>
<th>$S_1 = S_2$</th>
<th>$(\pi/4) \cdot 0.12 \ m^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_a$</td>
<td>0.1 m</td>
<td>$S_3 = S_4$</td>
<td>$(\pi/4) \cdot 0.024 \ m^2$</td>
</tr>
<tr>
<td>$l_b$</td>
<td>0.2 m</td>
<td>$S_5 = S_6 = S_7$</td>
<td>$(\pi/4) \cdot (0.024^2 - 0.00002^2) \ m^2$</td>
</tr>
<tr>
<td>$l_c$</td>
<td>0.145 m</td>
<td>$S_8$</td>
<td>$S_7 = 1.14 \cdot 10^{-4} \ m^2$</td>
</tr>
<tr>
<td>$l_d$</td>
<td>0.42 m</td>
<td>$S_9 = S_{10} = S_{11} = S_{12}$</td>
<td>$(\pi/4) \cdot 0.1^2 \ m^2$</td>
</tr>
<tr>
<td>$l_e$</td>
<td>0.82 m</td>
<td>$l_f$</td>
<td>1.3 m</td>
</tr>
<tr>
<td>$R_1$</td>
<td>1.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.1: The dimensions of the used combustion rig. For the supply system only the fuel supply is considered. The area $S_8$ is smaller than $S_7$ because of the supports of the rod for the bluff body. $R_1$ is the acoustic reflection factor at location 1.

Measurement of the flame reflection factor

The flame reflection factor is measured by generating a pure tone at the exit of the combustion chamber. The standard acoustic damper is attached to the combustion chamber to prevent resonance. The transfer function between the signals from the two microphones at the locations 11 and 10 is determined with a FFT analyzer, the microphone at location 10 is the reference microphone. From this transfer function the ratio between the upstream traveling wave and the downstream traveling wave is calculated (see appendix G), this gives the reflection factor at location 9, the upstream boundary of the combustion chamber. If this upstream end would be a closed end, the reflection factor would be 1 (phase = 0°). If the system upstream of location 9 is passive, the reflection factor is always smaller than or equal to 1. Only if extra energy is added to the acoustic wave, the reflection factor will be larger than 1. This energy addition can be supplied from a flame between location 8 and 9 that fluctuates with the incoming acoustic wave.

With this method it is possible to measure the acoustic response of the combination of the flame, burner, supply system for a number of frequencies. For the frequencies where the reflection factor is larger than one, instability may occur if this combination of supply, burner, flame is positioned in a combustion chamber with an eigenfrequency close to that frequency. A shift of the instability frequency with respect to the acoustic eigenfrequency of the combustion chamber will occur if the phase shift of the flame reflection factor differs from 0°.

Calculation of the flame reflection factor

For the calculation of the flame reflection factor a good acoustic description of the supply system/burner is necessary. Two methods have been used, in the first method an acoustic model has been derived according to figure 5.8 and table 5.1 using the pressure components at location 1 till 9, this method is referred to as pre-system model 1. In the second method, post-system model 2, the reflection factor at location 5 is determined from cold flow measurements and an acoustic model is derived for the pressure components at the locations 5 till 9. The result of both methods is the flame reflection factor defined by:

$$R_{flam,e} = \frac{p_0^+}{p_0^-} \quad (5.6)$$
In the calculation of the flame reflection factor the influence of the air supply has been neglected.

The matrices between 1 and 2, 3 and 4, 5 and 7 are all standard sound propagation matrices, the matrices between 2 and 3, 4 and 5 are matrices that describe joints. The matrix between 7 and 8 is the burner transfer matrix; it is assumed here that the acoustic pressure difference across the burner is zero ($H_{\text{burner}} = 0$). The matrix between 8 and 9 is the flame transfer matrix, in this matrix the measured flame transfer function is used. With the set of matrices a matrix equation is derived with $p_\text{in}^-$ at the right hand side. This matrix equation is solved numerically in a similar way as in the calculation of the flame transfer function.

**The reflection factor at the burner inlet**

In figure 5.9 the reflection factor at location 5 ($p_\text{in}^-/p_\text{in}^+$) calculated with the acoustic model is compared with the measured reflection factor at this location. For the measurement of the reflection factor the microphones at location 6 and 10 have been used. With an acoustic model the reflection factor has been calculated from this experiment. The experiments were performed under no flow conditions. It has been tested, in a similar way as in appendix F, that the determination of the burner inlet reflection factor from measurements is very sensitive to inaccuracies in the experiments especially at low frequencies. Hence care should be taken with the interpretation of these measurements.

The phase plots from the model and the measurements follow the same straight line decreasing with the frequency, which is a typical time delay behavior. Below 120 Hz there are large deviations between the experiments and the model, these differences are probably caused by both inaccuracies in the measurements and the errors in the model (the influence of the fuel supply tube upstream of the pressure drop has for example not been taken into account in the acoustic model). The deviation around 210 Hz is caused by a pressure node.
at the microphone at location 10.

The amplification factor plot of the modeled reflection factor is constant one, this again indicates a pure, passive time delay behavior. The measured reflection factor is nearly one up to 150 Hz (except the peak at 45 Hz). The peak at 45 Hz is probably caused by resonance in the tubing upstream of the T piece in the fuel supply.

Results

In figure 5.10 the measured and calculated flame reflection factors for Flame 3.4 (16.6 kW) are plotted. In the presystem model 2 the measured burner inlet reflection factor is used instead of an acoustic model that describes the complete supply system.

The measured flame reflection factor is larger than one at 60 Hz, between 75 Hz and 150 Hz and above 200 Hz. This means that for these frequencies the sound is amplified by the flame. Between 150 Hz and 200 Hz there is a large dip in the flame reflection factor, this means that the flame diminishes the sound.

The shape of the amplification factor curve is in general well caught by both acoustic models. At details the presystem model 2 performs better, especially at the frequencies below 100 Hz. The shape of the dip between 150 Hz and 200 Hz is not well predicted by both acoustic models. This dip is caused by resonance in the supply system. This resonance is not well included in both models.

From the phase plot it is seen that the measured phase is around zero below 50 Hz and above 200 Hz. There is a strong jump in the phase at 160-170 Hz, this is typical the phase jump which can be observed in cases of resonance. The general shape of the phase plot is well predicted by both models. The total acoustic model overpredicts the phase jump at 160 Hz. The phase jump at 160 Hz, predicted with the presystem model 2 corresponds well with the measured phase jump. The low frequency peaks (10 Hz, 45 Hz) in the presystem
Experiments for a bluff body burner

![Graphs showing amplification factor and phase](image)

(a) Amplification factor  
(b) Phase

Figure 5.11: The measured and calculated flame reflection factor for Flame 3.5 at 14.8 kW ($\lambda = 1.3$). For the calculations two different models are used, see text.

The model 2 plot are probably caused by an ill measurement of the burner inlet reflection factor.

The fact that the phase differs from zero means that if this combination of supply, burner and flame is put into a combustion chamber, the instability frequency (if present) will be different from the acoustic eigenfrequency of the combustion chamber.

In figure 5.11 the measured and calculated flame reflection factors for Flame 3.5 at 14.8 kW are plotted. The main difference in the flame transfer functions of Flame 3.5 and Flame 3.4, see figure 5.6, is that the phase plot of Flame 3.5 is steeper than that of Flame 3.4 and that after 55 Hz the phase of Flame 3.5 remains constant at ±180°, what indicates that the flame has become insensitive for the fuel flow fluctuations above this frequency. The effect of these differences on the flame reflection factor can be seen from the comparison of the figures 5.10 and 5.11.

Both the phase plot and the amplification factor plot of Flame 3.5 are flatter than those of Flame 3.4. Especially the large jump around 160 Hz has decreased. This is because of the fact that Flame 3.5 is not active at these frequencies.

Again the general shape of both plots for Flame 3.5 is well predicted by the models. At details the presystem model 2 performs better, especially around 160 Hz. Very important is that the differences in the flame reflection factor between Flame 3.5 and Flame 3.4, because of the different flame transfer functions, are well predicted by the acoustic models. This indicates that the general influence of the flame on the acoustic system behavior is well described with the measured flame transfer function.

### 5.6.2 Instability and instability frequency

In this section the occurrence of instability and the instability frequency is calculated from the acoustic system matrix, using the Nyquist criterion (see appendix D.6). The same setup and dimensions have been used as in the determination of the flame reflection factor, see
5.6 The acoustic system behavior

<table>
<thead>
<tr>
<th>Flame 0 (16 kW)</th>
<th>Meas. Freq</th>
<th>Inst.</th>
<th>Presystem model 1 Freq</th>
<th>R</th>
<th>Presystem model 2 Freq</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flame 3.2 (12.5 kW)</td>
<td>115 Hz</td>
<td>No</td>
<td>105 Hz</td>
<td>1.15</td>
<td>100 Hz</td>
<td>1.23</td>
</tr>
<tr>
<td>Flame 3.4 (16.6 kW)</td>
<td>118 Hz</td>
<td>Yes</td>
<td>116 Hz</td>
<td>0.92</td>
<td>121 Hz</td>
<td>1.4</td>
</tr>
<tr>
<td>Flame 3.4 (25.0 kW)</td>
<td>121 Hz</td>
<td>Yes</td>
<td>122 Hz</td>
<td>1.25</td>
<td>120 Hz</td>
<td>2.28</td>
</tr>
<tr>
<td>Flame 3.4 (33.2 kW)</td>
<td>134 Hz</td>
<td>Yes</td>
<td>X</td>
<td>X</td>
<td>135 Hz</td>
<td>3.17</td>
</tr>
<tr>
<td>Flame 3.4 (40.0 kW)</td>
<td>143 Hz</td>
<td>Yes</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Flame 3.5 (14.8 kW)</td>
<td>147 Hz</td>
<td>Yes</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

Table 5.2: The measured instability/resonance frequencies, the occurrence of instability (yes/no), the predicted instability frequency and the predicted amplification factor. Length of the combustion chamber is 1.3 m, acoustic eigenfrequency 106 Hz. The X indicates that no value has been found in the frequency domain of interest.

Figure 5.8 and table 5.1. The average speed of sound in the combustion chamber is set to 550 m/s, this is a lower value than used for the calculation of the flame transfer function. This is done because the sound propagation in the whole combustion chamber should now be taken into account, which has a lower average temperature (∼850K). The length of the combustion chamber is 1.3 m and the acoustic end condition is an open end. The acoustic losses are neglected.

The presence of a spike in the amplitude spectrum of the sound in the combustion chamber indicates the occurrence of instability, see also figure 2.1 for the difference between resonance and instability.

In table 5.2 the measured instability/resonance frequencies are given, just as the occurrence of the instability in the experiments (indicated with yes or no), the model predictions of the instability frequency and the amplification factor of the transfer function of the open system, indicated with R, which gives the occurrence of instability (R > 1). In the case of no instability (Flame 0) the center frequency of the measured resonance peak is mentioned. The same two acoustic models have been used as in the calculation of the flame reflection factor, which differ in the acoustic description of the presystem.

All flames, except Flame 0, show instability in the experiments. In the calculations no instability frequency could be found for Flame 3.4 (33 kW) and (40 kW) with both models and also for Flame 3.4 (25 kW) with the presystem model 1. This is indicated in the table with an 'X'.

Flame 0

With both acoustic models for the presystem, instability is predicted for Flame 0, although this does not occur in reality. This misprediction could either be caused by the ill modeling of the burner flame combination or by the negligence of acoustic losses in the model. It will be shown in the discussion of Flame 3.5 that the acoustic losses in the system are very small. The most probable explanation for the misprediction is an ill modeling of the flame burner combination. This will be discussed below in more detail in the section about the total phase method.
Flame 3.2 and 3.4

With both acoustic models very good predictions are made for Flame 3.2 and Flame 3.4 at 6.6 kW, both the occurrence of instability and the instability frequency are well predicted. With the presystem model 2 good results are also obtained for Flame 3.4 at 25 kW.

In the calculations of the higher powers of Flame 3.4 no instability frequency could be found because the phase of the transfer function of the open system differs strongly from zero. The problem with these flames is probably that the frequencies of interest (around 160 Hz) are very close to the resonance frequency of the presystem. That it is difficult to get good predictions around this frequency can also be seen from the pictures of the flame reflection factor, e.g. figure 5.10. A variation of the Nyquist method has been developed to overcome this problem, called the total phase method, which will be discussed below.

Flame 3.5

The frequency for Flame 3.5 is well predicted with both acoustic models for the presystem, the occurrence of instability is predicted, however R is very close to one (1.06 for the presystem model 1 and 1.00 for the presystem model 2). R could be smaller than one if the acoustic losses would be taken into account.

To study the correctness of the prediction of the very small R for Flame 3.5 by both acoustic models, the measured and calculated flame reflection factors for this flame are analyzed, see figure 5.11. Both the measured and the calculated flame reflection factors vary around one at 100 Hz. It is not possible to predict from the measured flame reflection factor if instability will occur. Apparently the acoustic losses in the system are very small and the amplification factor is a little above one at the instability frequency. The predicted value of the amplification factor for the open system seems therefore to be correct.

The influence of the presystem

From the comparison of the results from the presystem model 2 with those from the presystem model 1 it follows that the upstream boundary conditions have a strong influence on the predictions of instability. This is logical because the occurrence of instability is determined by the phase between the pressure and the heat release (Rayleigh's criterion). This phase is a function of the phase between the heat release and the velocity fluctuation in the burner mouth (the flame transfer function) and the phase between the pressure in the combustion chamber and the velocity fluctuation in the burner mouth (the burner admittance). The upstream boundary conditions have a strong influence on the admittance of the burner.

The total phase method

With the standard Nyquist method no instability frequencies are found for Flame 3.4 at the higher thermal powers. Therefore another method to predict the instability has been used. This method uses the flame reflection factor, defined by equation (5.6). The method is based on the same idea as the Nyquist criterion: the total phase of a sound wave that travels from the exit of the combustion chamber to the flame, reflects at the flame, returns to the exit and reflects at the boundary should be $\pm 0^\circ + n \cdot 360^\circ$ (positive feedback). This phase is the sum of the phase from the flame reflection factor ($\phi_{\text{flame refi}}$), the phase shift
caused by the acoustic time delay, necessary to travel from the downstream boundary to the burner and back (\(\phi_{ac, delay}\)), and the phase from the reflection at the downstream boundary (180° for an open end):

\[
\phi_{total} = \phi_{flame refl} + \phi_{ac, delay} + \phi_{refl end} = \phi_{flame refl} - 360\degree \cdot \frac{2 \cdot f \cdot l_{ac}}{\alpha_{ac}} + 180\degree = \pm 0\degree
\]

In figure 5.12 the total phase and the amplitude of the flame reflection factor have been plotted for the measured and calculated flame reflection factors of Flame 3.4 (40 kW). Also the line of the total phase is plotted if the phase shift by the flame reflection factor would be zero (No Flame).

Up to 90 Hz and above 250 Hz the total phase for all plots follow the line for 'No Flame'. Above 90 Hz the three plots based on the flame reflection factor start to deviate from the 'No Flame' line. The difference between the total phase from the measured flame reflection factor and the total phase based on the acoustic models is not too large except around 160 Hz. At this frequency the total phases from both acoustic models make a jump of about 360°.

To determine the possible instability frequency one has to look where the plot crosses the 0° line. The total phase from the experiments crosses this line between 140 and 155 Hz. The total phase from the acoustic models only crosses the line of 0° because of the jump of 360° at 160 Hz. This frequency is taken as the instability frequency. The instability frequency is strongly shifted with respect to the acoustic eigenfrequency (109 Hz). The amplification factors for the two acoustic models are 2.21 (pre system model 2) and 1.14 (pre system model 1), hence instability will occur.

For the flames where it has been possible to determine the instability frequency with the standard Nyquist criterion the total phase method gives the same results. Only for the flames where no instability frequency is found using the Nyquist criterion the total phase
Table 5.3: Comparison of the prediction of the occurrence of instability and the instability frequency using the two acoustic models and the total phase method with experiments.

<table>
<thead>
<tr>
<th>Flame 3.4 (25.0 kW)</th>
<th>Meas. Freq</th>
<th>Inst.</th>
<th>Presystem model 1 Freq</th>
<th>R</th>
<th>Presystem model 2 Freq</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>134 Hz</td>
<td>Yes</td>
<td>159 Hz</td>
<td>1.05</td>
<td></td>
<td>135 Hz</td>
<td>3.17</td>
</tr>
<tr>
<td>143 Hz</td>
<td>Yes</td>
<td>159 Hz</td>
<td>0.99</td>
<td></td>
<td>145</td>
<td>2.6</td>
</tr>
<tr>
<td>147 Hz</td>
<td>Yes</td>
<td>159 Hz</td>
<td>1.14</td>
<td></td>
<td>158</td>
<td>2.29</td>
</tr>
</tbody>
</table>

Figure 5.13: The calculated total phase based on the measured and calculated flame reflection factor for Flame 0 (16 kW, $\lambda = 1.3$). Note that the reflection factor is plotted linear.

Method may help. The results of the total phase method for Flame 3.4 at powers above 25.0 kW are summarized in Table 5.3. The calculations with the presystem model 2 correctly predict the occurrence of instability, instability is not clearly predicted by the presystem model 1.

The total phase method has also been applied to Flame 0. The results are plotted in Figure 5.13. The possible instability frequency from the experimentally determined flame reflection factor is 128 Hz. The magnitude of the measured flame reflection factors at this frequency is 0.99. According to this flame reflection factor measurement the acoustic system should be stable.

For this flame the difference between the measured total phase and the predicted one is rather large, also the prediction of the amplitude of the flame reflection factor is not too good. This deviation between experiments and predictions explains why instability is predicted by the calculations although it does not occur in the experiments. Apparently the transfer function measurements, which were fed into the calculations, lacked sufficient accuracy.
5.6.3 Conclusions for the system analysis

The calculation of the flame reflection factor and the prediction of instability are very sensitive to the acoustic modeling of the presystem, especially in the neighborhood of the eigenfrequency of the presystem. Besides a good acoustic description of the flame, a very accurate modeling of the presystem is necessary. In this study two acoustic models of the presystem have been used. In the first model the complete presystem is modeled with acoustic elements (presystem model 1), in the second model part of the presystem acoustic model is derived from measurements, this second model is referred to as the presystem model 2.

The shape of the flame reflection factor is well predicted using both acoustic models. On details the presystem model 2 performs better, especially around the acoustic eigenfrequency of the presystem.

The instability frequency is well predicted with the two acoustic models, there can be a large shift of this frequency with respect to the acoustic eigenfrequency of the combustion chamber.

The occurrence of instability is reasonably predicted with the presystem model 1 and a little better with the presystem model 2. In certain cases, if the instability frequency is in the neighborhood of the resonance frequency of the presystem, no instability frequency could be found using the standard Nyquist criterion, although it was measured. A small modification has been made to the Nyquist method, yielding the total phase method. With this method the occurrence of instability was predicted.

If the predicted amplification factor of the open system is close to one, it is difficult to tell if instability will occur, especially if the amplification factor plot (as function of the frequency) has a steep gradient around the eigenfrequency of the system.
Chapter 6

Experimental results for swirl burners

6.1 Introduction

In this chapter the acoustic behavior of two swirl burners is studied experimentally. The design of these burners is based on the Siemens V94.2 coal gas burners. The cross sectional area of these model burners is about 4% of that of a real gas turbine burner (the diameter of the model burners is about 1/5th of that of the real burner). The main difference between the two model burners is the swirl angle of the syngas swirlers.

The burner transfer function and the flame transfer function have been determined experimentally for these burners. With these transfer functions and an acoustic model of the presystem the flame reflection factor is calculated and compared with measurements.

The thermal power for the reference experiments is based on a constant Strouhal number scaling: because the diameter of the model burner is 1/5th of that of the real burner, the flow velocity should also be 1/5th to keep the Strouhal number constant (at the same frequency). This scaling corresponds to a constant time delay scaling if the flame length scales linearly with the diameter of the burner ($\tau \propto l_{ame}/U \propto D_{burner}/U_{fuel} = St/f$). The fuel flow velocity in the model burner at the reference power is 1/5th of the fuel flow velocity in the real burner under 100% load conditions.

The experiments are performed with a fuel with Demokleen composition: 28.7% CO, 10.6% H2, 44.6% N2, 16.1% H2O. The air flow and the fuel flow are preheated to respectively 373 K and 420 K. The (arbitrary) air factor ($\lambda$) in the reference experiments is 1.75.

6.2 The experimental set up

The experiments have been performed in the same combustion rig as the experiments with the bluff body burner, only the burner and the connection to the fuel and air system have been replaced. A drawing of the swirl burner is given in figure 6.1. Two different burners have been studied, GT1 and GT2. The main difference between the two burners is the angle of the swirl vanes of the fuel swirler, which gives different swirl numbers. For GT1 the swirl...
number of the fuel flow is 1.4. For GT2 this swirl number is 0.4. The swirl numbers of the air flow are comparable for both burners.

The burners have three passages: The inner passage is the axial (air) swirler. Just upstream of the swirler in this passage is the natural gas injector. The natural gas flame using this injector is a diffusion flame. The outer passage of the burner is the diagonal (air) swirler, the bulk of the mass flow of air goes through this passage. The central passage is the syngas swirler.

Two concentric tubes are attached to the burner to deliver the natural gas (through the inner tube) and the syngas (through the outer tube). The burner is mounted in an air dome. The air is supplied to this air dome by six small flexible tubes. The air for the axial swirler is fed from the air dome through small tubes in the burner. The design of the air and fuel supply has been described by Vrielink (1998).

The stabilization of the flame from this burner is based on the recirculation zone generated by the (three) swirling flows. Hot flue gases are fed back in this recirculation zone to the burner where they ignite the fresh fuel and air mixture.

### 6.3 The burner transfer function

In the calculation of the burner transfer function from the measurements, assumptions should be made about the location of the burner mouth. The burner mouth for the air exit is the air swirler (a). For the burner transfer function of the fuel passage the part between (b) and (c) is modeled as burner mouth, see figure 6.1. The part between (b) and (c) is however relatively long: for higher frequencies there will be accumulation of mass in this piece due
6.3 The burner transfer function

![Graphs showing real and imaginary parts of burner transfer function.](image)

**Figure 6.2:** The dimensionless real and imaginary part of the burner transfer function for the GT2 burner at a fuel and air velocity belonging to a thermal power of $2/3^d Q_{ref}f$.

Incompressibility. This has been neglected in the definition of the burner transfer function. The use of the burner transfer function for the fuel side is for that reason only correct for low frequencies (below about 300 Hz).

As in the experiments with the bluff body burner, the determination of the burner transfer function is very sensitive to inaccuracies in the measurements because of resonance in the system and pressure nodes on the microphones. These effects are visible in the burner transfer function figures as peaks. In the experiments the fuel flow is replaced by a preheated nitrogen flow.

The general behavior of the burner transfer function is given by equation (D.13): the imaginary part of the burner transfer functions is mainly determined by the first term at the left hand side in equation (D.13), the real part of the burner transfer function by the other terms. According to equation (D.13) the imaginary part should increase linearly with the frequency.

In figure 6.2 the imaginary part and the real part of the burner transfer function of the GT2 burner for respectively the fuel side and the air side are plotted. The plots have been made dimensionless by the corresponding mass flow of fuel and air. For both the fuel and the air side, it can be seen that besides the peaks, the imaginary part shows a linear increase with the frequency. From the slope of this line the factor $l_{burner}S_{burner}$ in equation (D.13) can be calculated. Both for the fuel side and the air side this value corresponds very well with the physical length of the burner mouth: for the air side $l_{burner}$ is very short, because the burner mouth is only formed by the swirler. For the fuel side $l_{burner}$ is much longer because the part between (b) and (c) forms the burner mouth, see figure 6.1. It appears (not shown here) that the behavior of the imaginary part is almost independent of the flow velocity.

The real part in the figure 6.2 shows a nearly linear decrease with the frequency. At 0 Hz the real part differs from zero, this is because of the fact that there is also a static pressure
Figure 6.3: The dimensionless burner transfer function for the fuel side at different fuel flow velocities.

To estimate the importance of the burner transfer function in the acoustic system, the amplification factor of it should be compared to $S_{burner} \cdot \rho \cdot c_{burner}$. It appears that for the air side the influence of the burner transfer function is small for all cases (about 5 %). For the fuel side the influence of the burner transfer function on the acoustic behavior is large. In the remainder of this chapter only the fuel side burner transfer function has been taken into account.

### 6.3.1 The influence of the flow velocity

In this set of experiments the fuel and air flow have been varied, without flame. A variation of the fuel flow velocity corresponds to a variation in thermal power (if there would be a flame). The ratio of the air flow over the fuel flow has been kept constant (air factor constant). The experiments have been performed from $2/3^d$ to $4/3^d$ of the reference fuel velocity. The amplification factors for the different fuel velocities have been made dimensionless by the corresponding mass flow of the fuel.

In figure 6.3 the burner transfer functions for the different fuel flow velocities are plotted. The peaks due to experimental inaccuracies are present both in the amplification factor plot and the phase plot, around 140 Hz and 280 Hz. For the high flow rate ($4/3^d$) the amplification factor is almost flat over a wide frequency range. For the lower flow rates the amplification factor increases with the frequency. For these lower flow rates the first term at the left hand side of equation (D.13), the accumulation term, is relatively more important than the other terms, what explains this increase with the frequency. This effect can also be seen in the phase plot: For the lower flow rate the phase is close to 90°, what indicates a relatively large imaginary part.

In the amplification factor plot the dimensionless amplification factor nearly coincides at
low frequencies (< 150 Hz), besides the peaks, for the three different fuel flow velocities. Because the three plots have been made non-dimensional with their own fuel mass flow, this means that there is a linear correlation between the fuel mass flow and the burner transfer function amplification factor. The amplification factor of the burner transfer function can also be expressed as a function of the static pressure drop across the burner \( \Delta p_{\text{static}} \propto p U_1^2 \) (the primes (') indicate an acoustic variable, \( S_1 = S_2 \), see equation (D.13) for the definition of 1 and 2):

\[
H_{\text{burner}} = \frac{\Delta p S_1}{u_1} \propto \frac{\Delta p_{\text{static}}}{U_1} \propto \frac{2 \Delta p_{\text{static}}}{U_1} \tag{6.1}
\]

In the experiments the value of the burner transfer function has been compared with the static pressure drop and it appears that a reasonable estimation for the magnitude of the amplification factor of the burner transfer function at low frequencies can be calculated from \( 2S_1 \Delta p_{\text{static}} / U_{\text{fuel}} \).

### 6.3.2 The influence of the swirl

The influence of the swirl on the burner transfer functions is only very small as can be expected (if the pressure drop across the burner mouth does not change significantly). The results with the reference fuel flow for the GT1 burner and the GT2 burner are plotted in figure 6.4. For the phase the results for both burners are nearly the same. For the amplification factor the differences between the two burners are a little larger, but these differences are mainly caused by the experimental inaccuracies.
6.3.3 The influence of a swirl perturbator in the fuel swirler

One of the geometrical changes that has been studied is the introduction of a swirl perturbator in the fuel swirler. One of the effects of this swirl perturbator is that it increases the mean velocity in the fuel swirler and therefore increases the static pressure drop across the burner. The burner transfer function with and without perturbator is plotted in figure 6.5.

The phase with and without swirl perturbator is almost identical, the influence of experimental inaccuracies is a little smaller with swirl perturbator. The amplification factor for the case with swirl perturbator is higher than without, as expected. The influence of experimental inaccuracies is smaller for the measurements with swirl perturbator, because the deviation from the ideal acoustic situation becomes larger by the introduction of the swirl perturbator. The introduction of a swirl perturbator will decrease the magnitude of the admittance of the burner in an acoustic system.

6.4 The flame transfer function

The flame transfer function has been measured both for the fuel and the air side. It appears that the phase in the air side flame transfer function is for all situations ±180°. This indicates that the flames are insensitive for air side fluctuations. In this section only the fuel side flame transfer function is presented.

The burner transfer function of the fuel side has a strong influence on the acoustic behavior of the fuel side burner mouth. This burner transfer function should also be taken into account in the calculation of the flame transfer function from the measurements. To minimize the effects of the non-physical peaks in the burner transfer function, a modeled burner transfer function has been used. This model is made by linear interpolation of both
the imaginary part and the real part of the burner transfer function.

The effect of the following parameters on the flame transfer function has been studied:

- The influence of the thermal power.
- The influence of the swirl
- The influence of the chemical composition
- The influence of a modified syngas swirler (with swirl perturbator)

The amplification factor of the flame transfer function has been made non-dimensional with $\frac{2}{3} Q_{\text{ref}}$, where $Q_{\text{ref}}$ is the thermal power of the flame and $U_{f.a}$ the fuel flow velocity.

### 6.4.1 The influence of the power

Experiments have been performed at $2/3^d$ and $4/3^d$ of the reference power, just as with the burner transfer function. The air factor was constant during these experiments ($\lambda = 1.75$). Only the results for the burner GT2 are shown. The results for the GT1 burner show similar behavior.

In figure 6.6 the flame transfer function has been plotted, with on the axis the Strouhal number instead of the frequency. The velocity in the Strouhal number is $U_{f.a}$, the length scale is a typical length scale of the burner.

It shows from the phase plot that the phase scales very well with the Strouhal number, which means that the time delay scales with the fuel velocity; the length scale in the flame (the position of maximum heat release) is also constant for the three studied cases.
Figure 6.7: The influence of the swirl on the flame transfer function at $Q_{ref}$ (GT1: high swirl, GT2: low swirl).

In the amplification factor plots there are some peaks visible; these peaks are not caused by experimental inaccuracies, they occur at different frequencies for the different cases. They may be caused by flow instabilities. The value of the dimensionless amplification factor fluctuates around one, which means that the order of magnitude of the flame transfer function can be scaled with $\frac{A_{f}}{Q_{ref}}$. The fact that the peaks are larger than one indicates non-linear processes.

For the amplification factor the Strouhal number scaling works well with the $2/3^{rd}$ case in comparison to the reference case. For the $4/3^{rd}$ case it is apparently not possible to scale the frequency with the Strouhal number.

### 6.4.2 The influence of the swirl

The results for the two different burners are compared in this section. In this section only results at the reference power are presented, the comparison between the two burners at the other powers gives similar results. In figure 6.7 the flame transfer function for the flame with the high swirl velocity (GT1) and that with the lower swirl velocity (GT2) are presented.

The phase plot of the GT2 burner is much steeper than that of the GT1 burner. The flame in the GT2 burner is also located further downstream for this burner. This agrees well with the expected influence of the swirl: a higher swirl increases the mixing and creates a stronger recirculation zone. This means that a flame with a high swirl number is in reality shorter than a flame with a low swirl number. This effect has also been studied with CFD calculations by Hekkens (1998), which showed that the flame length decreases with increasing swirl.

There is a large difference in the amplification factors of GT1 and GT2. The amplification factor of the GT1 burner is much higher than that of the GT2 burner. The difference is maximum a factor two. This agrees again well with the expected effect of the swirl: the
shorter the flame the higher the magnitude of the flame transfer function, see also equation (4.6). The high value of the amplification factor for GT1 is probably not only caused by the shorter flame length but also by flow instabilities. A good indication for this is that the non-dimensional amplitude of the flame transfer function is larger than unity.

### 6.4.3 The influence of a swirl perturbator in the fuel swirler

In this section the influence of a modified fuel swirler of the GT2 burner on the flame transfer function is studied. A swirl perturbator is introduced in the fuel swirler. Only the results for the reference power are presented, the results at the other powers show the same trend. The flame transfer functions for this flame with and without swirl perturbator are shown in figure 6.8.

The general differences between the flame transfer functions with and without swirl perturbator are: (i) The slope of the phase for the flame with swirl perturbator is less steep than that of the flame without swirl perturbator. (ii) The amplification factor is in general a little lower with swirl perturbator. The main effect of the swirl perturbator on the amplification factor is however that the amplification factor is much flatter over the frequency domain. Peaks which occur in the amplification factor for a flame without swirl perturbator have disappeared after the introduction of the swirl perturbator.

The less steep slope of the phase for the flame with swirl perturbator indicates a shorter time delay. This shorter time delay can either be caused by the higher fuel flow velocity in the burner mouth because of the introduction of the swirl perturbator or by the fact that the flame has moved closer to the burner. It is not clear what is the main reason.

The flatter amplification factor over a large frequency range for the flame with the swirl perturbator indicates that the instabilities have been decreased by the introduction of the swirl perturbator. The swirl perturbator probably stabilizes the swirling flow.
Table 6.1: The compositions of the used fuels. In case 1 the ratio \( \text{CO}/H_2 \) has been varied keeping the calorific value of the fuel (in \( J \cdot m^{-3} \)) (nearly) constant. In case 2 \( CH_4 \) has been added keeping the calorific value (in \( J \cdot m^{-3} \)) (nearly) constant.

<table>
<thead>
<tr>
<th></th>
<th>( CO )</th>
<th>( H_2 )</th>
<th>( N_2 )</th>
<th>( H_2O )</th>
<th>( CH_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference</td>
<td>28.7 %</td>
<td>10.6 %</td>
<td>44.6 %</td>
<td>16.1 %</td>
<td>0 %</td>
</tr>
<tr>
<td>Case 1a</td>
<td>31.9 %</td>
<td>7.1 %</td>
<td>44.8 %</td>
<td>16.2 %</td>
<td>0 %</td>
</tr>
<tr>
<td>Case 1b</td>
<td>25.6 %</td>
<td>14.0 %</td>
<td>44.3 %</td>
<td>16.1 %</td>
<td>0 %</td>
</tr>
<tr>
<td>Case 2a</td>
<td>25.8 %</td>
<td>8.1 %</td>
<td>51.2 %</td>
<td>15.9 %</td>
<td>3.0 %</td>
</tr>
<tr>
<td>Case 2b</td>
<td>17.0 %</td>
<td>6.3 %</td>
<td>55.6 %</td>
<td>15.9 %</td>
<td>5.2 %</td>
</tr>
</tbody>
</table>

Figure 6.9: The influence of the \( CO \) and \( H_2 \) content in the fuel on the flame transfer function. The volume flow rate and thermal power have been kept constant in this series.

6.4.4 The influence of the chemical composition

In this section the influence of the chemical composition of the fuel on the flame transfer function is described. Two different parameter variations have been applied: (i) Change of the ratio \( CO/H_2 \) with keeping the thermal power and the flow rates constant and (ii) Addition of \( CH_4 \) to the fuel, the power is kept constant by decreasing the \( CO \) and \( H_2 \) flow, the total volume flow is kept constant by increasing the \( N_2 \) flow. In table 6.1 the different studied compositions are summarized. During the experiments the volume flow of air and fuel and the preheat temperatures have been kept constant. The experiments presented here have been performed with the GT2 burner at the reference thermal power.

In figure 6.9 the influence of the \( CO \) and \( H_2 \) on the flame transfer function is shown. The difference between the reference case and case 1a (more \( CO \)) is very small both for the amplification factor and the phase. For case 1b (more \( H_2 \)) the amplification factor is a little higher at high frequencies and the slope of the phase is a bit less steep. The less steep phase indicates a shorter time delay for case 1b (a shorter flame) than the reference case, this is
explained by the high content of \( H_2 \) in case 1b. The fact that the amplification factor is a little higher for case 1b can also be explained by the shorter flame length.

In figure 6.10 the influence of the content of \( CH_4 \) in the fuel on the flame transfer function is shown. The difference in the flame transfer function between case 2a and the reference case is very small, both for the amplification factor and the phase. Only for high frequencies the case 2a flame shows a steeper time delay behavior than the reference flame. Apparently 3 \% \( CH_4 \) does not have much effect.

For the flame with the higher \( CH_4 \) content, case 2b, the difference is much larger. The phase of case 2b is steeper (longer time delay). Further, the amplification factor for case 2b is much lower at high frequencies. Both aspects indicate that the case 2b flame is longer than the reference flame. It seems that \( CH_4 \) decreases the flame speed and thereby increases the flame length. The much lower amplification factor for this flame is very attractive with respect to the avoidance of combustion instabilities.

### 6.5 System behavior, the flame reflection factor

For the different burners and flames, the flame reflection factor has been measured and calculated as described in section 5.6.1. The flame reflection factor is treated here to point out the validity of the measured flame transfer functions and burner transfer functions in an acoustic system analysis. In the calculation of the flame reflection factor the presystem is described by splitting it in its acoustic components for which standard transfer matrices are used (presystem model 1, see section 5.6.1), so no experimental data have been used for this.

In figure 6.11 the measured and calculated flame reflection factors for the GT2 burner with and without swirl perturbator in the fuel swirler at the reference power are presented.
Figure 6.11: The measured and calculated flame reflection factor for the GT2 burner with and without swirl perturbator at $Q_{ref}$.

The drawn lines are the lines for the cases without swirl perturbator, the dotted lines are used for the flames with swirl perturbator.

The general comparison for the amplification factor between the calculations and the measurements for both cases is very good. In general the amplification factor of the flame without swirl perturbator is higher than that of the flame with swirl perturbator. This is a combined effect of the higher magnitude of the burner transfer function and the lower magnitude of the flame transfer function for the flame with swirl perturbator.

The flame without swirl perturbator has a maximum amplification factor at 140 Hz of about 4. This is very well predicted by the calculation. For lower frequencies, between 60 Hz and 120 Hz the flame reflection factor is underpredicted for the flame without the swirl perturbator. For higher frequencies, above 200 Hz, an amplification factor of about one (the flame is inactive) is predicted for the flame without swirl perturbator although it is measured that the flame works as a strong damper in this frequency band. It should be mentioned that the measured coherence in the measurement of the flame reflection factor was relatively low above 200 Hz (0.6 till 0.85), what makes these measurements less reliable.

The change in the frequency (from 130 Hz to 170 Hz) and the amplitude of maximum flame reflection factor (from 4 to 2) from the flame without swirl perturbator to the flame with the swirl perturbator is very well predicted by the calculations. For the flame with the swirl perturbator the overall comparison of the amplification factor of the flame reflection factor is even better than that for the flame without swirl perturbator. This is probably due to the fact that for the flame with swirl perturbator the acoustic description of the presystem is less important in the total acoustic system because of the larger influence of the burner transfer function.

For the phase plot the comparison between the calculations and experiments is very good for the flame with swirl perturbator. For the flame without swirl perturbator the comparison is a little worse, especially at high frequencies (this can partly be explained by
the bad coherence in the measurements at the high frequencies), but the general shape is very well predicted.

In this section just one example of the flame reflection factor has been presented but the comparison between the measured and calculated flame reflection factor showed similar results for the other cases. This shows that the measured flame transfer function and burner transfer function have great value in an acoustic system analysis.

6.6 Conclusions

In this chapter the experimental results for two model gas turbine burners are shown. Both the burner transfer function and the flame transfer function have been measured. It appears that the imaginary part of the burner transfer function increases linearly with the frequency, as given by the linearized Navier Stokes equations. The slope of this line gives the volume of the burner mouth. The value of it corresponds well with the physical volume of the burner mouth. The real part of the burner transfer function decreases with the frequency. For the air side the burner transfer function only has a small influence, for the fuel side the burner transfer function has a significant influence on the acoustic burner behavior.

In the flame transfer function experiments it appears that the air side fluctuations do not influence the flame. For this non-premixed flame apparently only the fuel side fluctuations are important.

The influence of the chemical composition on the flame transfer function is studied. The effect of the CO/H₂ ratio on the flame transfer function is marginal if the concentration of H₂ does not become too large. A significant addition of CH₄ to the fuel has a strong influence on the flame transfer function: the time delay becomes longer and the amplitude of the flame transfer function decreases.

An attractive flame transfer function has a low amplification factor. It showed from the experiments that the amplification factor decreases with: (i) decreasing swirl number, (ii) introduction of a swirl perturbator and (iii) the addition of CH₄ to the fuel.

A system analysis has been performed with the measured burner and flame transfer functions. The results of this analysis are compared with measurements of the flame reflection factor. The model predictions compare very well with the measured flame reflection factor.
Chapter 7

System study

7.1 Introduction

In this chapter a study of the acoustic system behavior of a realistic gas turbine situation is performed. The modeling of the geometry is based on the Demkolec gas turbine (Siemens V94.2). The burner transfer functions and flame transfer functions in this system analysis are derived from the transfer functions described in chapter 6. The study in this chapter will be performed solely to show the possibilities of the acoustic system analysis of a gas turbine using the transfer matrix method and the mode analysis method. The reader should be aware that the burners used in the model experiments are not identical to the burners in the real gas turbine. From the results presented in this chapter no conclusions may be drawn for the real Demkolec gas turbine.

Two methods to analyze the system behavior have been used in this chapter. The first method is based on the description of the acoustic system with transfer matrices. The occurrence of instability is investigated using the Nyquist criterion (see appendix D.6). The other method for the system study is based on the mode analysis method (section 2.5). In this latter method the mode analysis method is only used to describe the acoustic behavior of the combustion chamber including the flame, the acoustic behavior of the burner and the supply system is again described with transfer matrices.

7.1.1 The dimensions of the gas turbine

The Demkolec gas turbine has two silo combustion chambers, which join at the turbine inlet. A drawing of one combustion chamber is shown in figure 7.1. In this study only one combustion chamber is taken into account and symmetry conditions are used at the joint of the two combustion chambers. Each combustion chamber is equipped with 8 burners. The combustion air is supplied from the compressor via the gap between the flame cylinder and the casing to the air dome. The syngas comes from the fuel manifold, this is a ring tube. Each burner is connected with this manifold (the fuel manifold is not shown in figure 7.1). The burner is a swirl burner, similar to the burners described in chapter 6. The diameter of the gas turbine burner is about 5 times as large as the model burner used in our laboratory.
Figure 7.1: A figure of the combustion chamber of the Siemens V94.2 gas turbine. Not shown is the fuel manifold.

Figure 7.2: A schematic drawing of the V94.2 gas turbine combustion chamber. At the left hand the connections of the burners to the fuel manifold is shown. The dimensions are presented in table 7.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_{cc}$</td>
<td>1.9 m</td>
</tr>
<tr>
<td>$l_{tran}$</td>
<td>3.1 m</td>
</tr>
<tr>
<td>$l_{con}$</td>
<td>3.1 m</td>
</tr>
<tr>
<td>$l_{tube}$</td>
<td>2.5 m</td>
</tr>
<tr>
<td>$T_{cc}$</td>
<td>1000 K</td>
</tr>
<tr>
<td>$D_{cc}$</td>
<td>2.1 m</td>
</tr>
<tr>
<td>$T_{tran}$</td>
<td>1300 K</td>
</tr>
<tr>
<td>$S_{b}$</td>
<td>0.015 m²</td>
</tr>
<tr>
<td>$D_{mani}$</td>
<td>3.5 m</td>
</tr>
<tr>
<td>$S_{mani}$</td>
<td>0.2 m²</td>
</tr>
</tbody>
</table>

Table 7.1: The main (estimated) dimensions and conditions of the gas turbine combustion chamber. The meanings of the subscripts are: $cc$: combustion chamber, $con$: connection part, $tran$: transition part, $b$: burner, $tube$: supply tube of the burner and $mani$: fuel manifold.
7.2 Methodology

7.2.1 Scaling rules

The results obtained in the model experiments are scaled up in order to be used in the real scale gas turbine system study. The main differences between the model experiments and the gas turbine situation are:

- The mean pressure. The model experiments have been performed under atmospheric conditions. The gas turbine pressure is 11 bar. It is assumed here that the mean pressure does not influence the main chemistry in the flame. The only effect of the pressure is the higher density (and the higher heat release). As was shown in section 3.9, it is well possible to scale the fluctuating heat release linearly with the mean pressure.

- The inlet temperature of fuel and air. The main effect of the higher inlet temperatures on the flame transfer function is the higher inlet velocity and therefore a shorter time delay and a lower amplification factor.

- The dimensions. The dimensions of the model burner are about five times smaller than the real gas turbine burner. Both the length scale of the flow and the flame length scale with the dimension of the burner. The acoustic length scale is however independent of the burner dimension and depends only on the speed of sound and the frequency.

- The inlet flow velocities. The inlet flow velocities in the model experiments are smaller than the real inlet velocities.

Both the burner transfer function and the flame transfer function for the model burners have been determined up to a frequency of 300 Hz. For the system study they should be known up to about 800 Hz. The values for the higher frequencies are obtained by interpolation. The lowest eigen-frequency for the cylindrical modes is about 340 Hz, this means that for these modes both the flame transfer function and the burner transfer function are derived from linear interpolation.

**The burner transfer function**

The real part and the imaginary part of the burner transfer function are scaled independently. From the experiments with the model burner it appeared that the imaginary part of the burner transfer function is independent of the velocity. The imaginary part of the burner
transfer function is scaled using the first term at the right hand side of equation (D.13) (the subscript _0 refers to the model experiments, the subscript _1 to the real gas turbine situation, \( l_{burner} \) is the length of the burner mouth):

\[
H_{burner,im} = H_{burner,im,0} \frac{\bar{p}_1 S_1 l_{burner,1}}{\bar{p}_0 S_0 l_{burner,0}} 
\]

(7.1)

The real part of the burner function can be scaled with the mass flow of the fuel (\( \dot{m} \)) as was shown in the experiments, it is scaled with:

\[
H_{burner,rc} = H_{burner,rc,0} \frac{\dot{m}_1}{\dot{m}_0} 
\]

(7.2)

At low frequencies the real value of the burner transfer function should be of the order of \( S_{burner} \frac{\Delta p_{static}}{U_{fuel}} \). The value of \( \Delta p_{static} \) has been calculated from the scaled burner transfer function, its value is 3.5 % of the mean pressure in the combustion chamber. This is of the same order as the measured static pressure drop across the fuel swirler in the Dnokolec gas turbine (Vortmeyer et al. (1995)). In a real gas turbine the static pressure drop is often increased by additional measures.

Both the real part and the imaginary part of the burner transfer function scale with the length scale of the burner. As long as the length scale of the burner is much smaller than the acoustic length scale, this scaling is allowed because the flow in the burner can be treated as incompressible and the acoustics are essentially one-dimensional.

The experimental results for the burner transfer function of the model burner contain many errors caused by experimental inaccuracies. A model burner transfer function has been derived from these experimental results using linear interpolation for the real part and the imaginary part (see section 6.4). This linear interpolation is used here for the whole studied frequency domain.

The flame transfer function

For the flame transfer function, the amplification factor and the phase are scaled independently. In the model experiments it has been shown (see section 6.4.1) that the phase of the flame amplification factor scales well with the Strouhal number based on the burner diameter (\( D \)) and the fuel flow velocity (\( U_{fuel} \)). The scaling of the phase of the flame transfer function is then:

\[
\phi_{flame} = -\omega \cdot \tau = -\omega \cdot \frac{\tau_0 U_{fuel,0}}{D_0 U_{fuel,1}} 
\]

(7.3)

The time delay in the reference model experiments is nearly the same as the time delay in the gas turbine flame (\( D_0 \tau_0 U_{fuel} \approx 1 \)). The scaling for the phase with the time delay can also be used for higher frequencies than for which the flame transfer function has been measured.

The frequency of the amplification factor is scaled with the Strouhal number and the magnitude of it with \( \frac{\omega_0 D_1 U_0}{U_1} \) :

\[
R_{flame}(f) = R_{flame,0} \left( \frac{D_1 U_0}{D_0 U_1} \right) \left[ \frac{\gamma^2}{\gamma - 1} \frac{U_0}{Q_{stat,0}} \right] \frac{\gamma - 1 Q_{stat,1}}{2} \frac{U_1}{U_1} 
\]

(7.4)

It is difficult to interpolate the measured amplification factor to higher frequencies. Especially the occurrence of peaks in the amplification factor, caused by flow instabilities makes
Figure 7.3: The description of the presystem with transfer matrices. $J_1 \cdots J_8$ and $J$ are matrices for joints, $T_1 \cdots T_8$, $T_a$ and $T_b$ are matrices for the propagation of sound, $B_T$ is the burner transfer matrix.

this difficult. Fortunately these peaks occur at the lower frequencies. For the higher frequencies (around 250-300 Hz) the maximum value of the amplification factor is about $\frac{\Delta L_{max}}{2\pi \Delta f}$. This value of the amplification factor is used for the frequencies above 300 Hz.

The flame is treated in this thesis as a point source of sound, what means that this scaling for the flame is only valid when the flame is compact compared to the acoustic length scale. The dimension of the flame in axial direction is longer than the dimension in radial direction. For the axial modes the present scaling is valid up to a frequency of about 400 Hz, for the cylindrical modes the maximum frequency is about 800 Hz.

### 7.2.2 Acoustic system description

The acoustic system is split into three parts: (i) the presystem (supply), (ii) the flame and (iii) the combustion chamber. With both methods used, the transfer matrix method and the mode analysis method, the presystem and the flame are described in the same way. Both methods differ only in the description of the combustion chamber.

**The presystem**

The presystem is in both methods described with transfer matrices. A scheme of the matrix system is given in figure 7.3. The ring for the fuel supply is modeled with a set of transfer matrices ($T_1 \cdots T_8$) and a set of joint matrices ($J_1 \cdots J_8$). The transfer matrices all describe the propagation of a one-dimensional acoustic wave in a tube with length $(\pi/8)D_{mani}$. At
the joints the ring is connected with the supply tubes for the different burners (S1 ··· S8).

At the joints of the supply tubes to the manifold the standard conditions for a joint should be satisfied: the velocity potential is constant across the joint and the acoustic mass flow should be conserved. This gives a set of three equations (two for the pressure and one for the acoustic mass flow).

The supply system of the burner (see enlargement in figure 7.3) is modeled with 2 matrices that describe the propagation of sound (Ta and Tb), a joint matrix (J) and the burner transfer matrix (BT). The matrix Ta describes the propagation of sound from the ring to the burner over a length $l_{tube}$. Tb describes the propagation within the burner (over a length $l_b$). The burner transfer matrix (BT) contains the burner transfer function and describes the acoustic behavior of the burner mouth. The burner transfer function in BT is known from experiments or should be modeled. The acoustic output variables of the burner transfer matrix are the acoustic velocity ($u$) and the acoustic pressure ($p$). This differs from the standard situation in this thesis where $p^+$ and $p^-$ are chosen as acoustic variables. This is done to be able to use this matrix description of the presystem in the mode analysis method.

To reduce the number of degrees of freedom in the system the set of matrices that describes the supply system (Ta, J, Tb and BT) is written as one matrix ($S$). This matrix is the same for all burners. A matrix equation is now derived using T1 ··· T8, J1 ··· J8 and S1 ··· S8, with the vector $p_1 ··· p_8$ at the right hand side:

$$ [M_{system}] \cdot \mathbf{u} = [M_p] \cdot \begin{bmatrix} p_1 \\ \vdots \\ p_8 \end{bmatrix} \quad (7.5) $$

The vector $\mathbf{u}$ contains the amplitude of velocity fluctuation at the burner mouth ($u_1 ··· u_8$) and all pressure amplitudes in the system (except $p_1 ··· p_8$). The matrix equation is solved by multiplying both the left hand side and the right hand side with $[M_{system}]^{-1}$. This solution can be rewritten to:

$$ \begin{bmatrix} u_1 \\ \vdots \\ u_8 \end{bmatrix} = [M_{UP}] \cdot \begin{bmatrix} p_1 \\ \vdots \\ p_8 \end{bmatrix} \quad (7.6) $$

where $[M_{UP}]$ contains the relevant elements of $[M_{system}]^{-1} \ast [M_p]$. The matrix $[M_{UP}]$ is a 8$^8$ matrix, it describes the acoustic behavior of the presystem.

The description of the presystem is based on one-dimensional acoustic transfer matrices. This is not a problem for the supply tubes and the burner, the fuel manifold ring has however a high inner diameter which means that for higher frequencies (above about 400 Hz) it can not be treated as one-dimensional. From test calculations it appears that the joint of the supply tube to the fuel manifold can be treated as an open end, the 2D acoustic effects in the fuel manifold will also have a minor influence on the acoustic behavior of the presystem.

The flame

The acoustic behavior of the flame is described with the flame transfer function. The flame transfer function gives the heat release by the flame as function of the inlet velocity of the burner:

$$ \frac{\gamma - 1}{c^2} Q_{flame} = H_{flame} \cdot \bar{u}_{burner} \quad (7.7) $$
For the transfer matrix method this flame transfer function is used in the flame transfer matrix. In the mode analysis method the flame transfer function is used directly.

The combustion chamber

The transfer matrix method  In the transfer matrix method the combustion chamber is described with a set of three transfer matrices: one for the cylindrical part, one for the conical part and one for the transition part to the turbine. The transfer matrices for the cylindrical part and the transition part are standard transfer matrices for the propagation of sound. The transfer matrix for the conical part is composed of a large number (20) transfer matrices for the propagation of sound and matrices that describe joints of tubes with different cross sectional areas. In this way it is possible to include the effect of the varying cross sectional area (from \( d_{cc} \) to \( d_{trans} \), the varying speed of sound and the varying Mach number. The effect of the changing Mach number, due to the addition of dilution air in the conical part, on the acoustic propagation is incorporated.

The boundary condition at the downstream end of the transition part is a symmetry boundary condition (there are two combustion chambers). The condition can either be a ‘closed’ end or an ‘open’ end.

For the axial modes the acoustic pressure is the same at all burners \((p_1 = p_2 = \cdots = p_8 \text{ in equation (7.6)})\). The velocity fluctuations in the burners \((u_1 = u_2 = \cdots = u_8)\) and the heat release by the different flames are also identical. For these modes it is possible to treat the eight burners as one single burner.

The mode analysis method  In the mode analysis method the acoustic eigenfunctions \((\psi_{mno})\) should satisfy the following equation in cylindrical co-ordinates:

\[
\frac{\partial^2 \psi_{mno}}{\partial r^2} + \frac{1}{r} \frac{\partial \psi_{mno}}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \psi_{mno}}{\partial \theta^2} + \frac{\partial^2 \psi_{mno}}{\partial z^2} + \left( \frac{\omega_{mno}}{c_{cc}} \right)^2 \psi_{mno} = 0 \quad (7.8)
\]

The solution for \(\psi_{mno}\) is:

\[
\psi_{mno}(x, r, \theta) = \cos (k_m \cdot x) \cdot J_n (\lambda_{no} \cdot r) \cdot \cos (n \cdot \theta) \quad (7.9)
\]

In this equation \(m\) refers to the axial modes, \(n\) to the tangential modes and \(o\) to the radial modes. \(J_n\) is the \(n^{th}\) order Bessel function of the first kind. The values of \(k_m\) can be calculated from:

\[
k_m = \begin{cases} 
(m + \frac{1}{2}) \frac{n}{l_{total}} & m = 0, 1, \cdots \text{‘open’ end} \\
\frac{mn}{l_{total}} & m = 0, 1, \cdots \text{‘closed’ end}
\end{cases} \quad (7.10)
\]

where \(l_{total} = l_{cc} + l_{con} + l_{trans}\).

For the radial part of the eigenfunction the symmetry conditions \((\frac{\partial \psi_{mno}}{\partial r} = 0)\) must be satisfied at \(r = 0\) and at \(r = r_{cc}\). The symmetry condition at \(r = 0\) implies that for \(n\) the following values are valid: \(n = 0, 2, 3, \cdots\). To determine the value of \(\lambda_{no}\) the root of the following expression should be calculated:

\[
\frac{dJ_n}{dr} (\lambda_{no} r_{cc}) = 0 \quad (7.11)
\]
The natural frequency of the eigenmode is calculated from:

$$\omega^2_{mno} = (\lambda^2_{no} + k^2_m) \cdot c^2_{cc}$$

(7.12)

In this thesis only single axial modes and single cylindrical modes (combination of radial and tangential modes) are studied with the mode analysis method, so the acoustic eigenfrequency is the acoustic wave number ($k_m$ or $\lambda_{no}$) times the speed of sound.

The acoustic coupling of the flame and the presystem to the combustion chamber is in the mode analysis method described with the function $T_{\text{flame}}$, see equation (2.23). This function $T_{\text{flame}}$ is for this situation:

$$T_{\text{flame}} = \frac{\epsilon^2}{\int \psi_{mno}^2 dV} \left( [MUP] \cdot \begin{bmatrix} \psi_{mno,1}^2 \\ \vdots \\ \psi_{mno,8}^2 \\ H_{\text{flame}} \\ \cdots \\ H_{\text{flame}} \end{bmatrix} \right)$$

(7.13)

The pressure amplitudes at the eight burners are described by the vector $[\psi_{mno,1} \ldots \psi_{mno,8}]$. The flame transfer function, $H_{\text{flame}}$, is identical for all burners, this is described in the vector $[H_{\text{flame}} \ldots H_{\text{flame}}]$. The function $T_{\text{flame}}$ is also the summation of the influence of the different burners.

The equation for $H_{\text{flame}}$, using equations (7.3) and (7.4), is substituted in equation (7.13). With the solution for $\psi_{mno}$ the instability frequency and the growth rate of the instability at the mode $mno$ are found.

### 7.3 Results

#### 7.3.1 The axial modes

For the calculation of possibly unstable axial modes both the transfer matrix method and the mode analysis method have been used. In the mode analysis method the combustion chamber has been modeled as a cylinder (without contraction), the influence of the mean flow was neglected and the speed of sound was assumed to be constant. The results of the mode analysis method have been compared with the results from the transfer matrix method (under the same conditions). The comparison between the two methods is very good. This comparison is not shown here.

The results for the axial modes presented in this chapter are all obtained using the transfer matrix method, because it is easier to include the effects of the conical shape of the combustion chamber, the mean flow and the spatial variations in the speed of sound.

In the acoustic models no damping has been included. In reality damping will occur in the combustion chamber due to turbulence and non-ideal reflection at the transition part. For the Siemens V94.2 combustion chamber there is information available about the total amount of damping at one fundamental mode in the paper by Hobson et al. (1999). Hobson measured the autospectrum of the noise from the combustion chamber, from the shape of the spectrum he calculated the modal damping factor $\zeta$, using the method of least squares.
7.3 Results

Figure 7.4: The amplification factor of the open system for the GT1 and GT2 burner at 100% load. On the x-axis are plotted the possible instability frequencies (determined from the phase plot). The meaning of the line at 1.1 is explained in the text.

For the fundamental frequency of 78 Hz, $\zeta$ equals about 0.01. This value of $\zeta$ corresponds to an amplification factor for the transfer function of the open system of about 0.92 at 78 Hz. This means that the amplification factor of the open system without damping should be at least $1 / 0.92 \approx 1.09$ to give instability. This minimum amplification factor is actually only valid at 78 Hz, but because no other data is available, this value is used for the whole frequency domain.

In this section the influence of the following parameters on the system behavior will be studied (see chapter 6 for an explanation):

- The influence of the swirl: burner GT1 versus burner GT2.
- The influence of a swirl perturbator in the fuel swirler of GT2.
- The influence of the burner transfer function.

In figure 7.4 the amplification factor of the open system is plotted against the frequency for the GT1 and the GT2 burner (on real scale) to study the influence of the swirl. The possible instability frequencies are all (nearly) a multiple of 27 Hz (determined from the phase plot of the open system), these frequencies are plotted on the x-axis. The frequencies on the x-axis are the instability frequencies for both situations of the reflection at the transition piece: 'open' end or 'closed' end. In the plot the line with the amplification factor of 1.1 is included, above this line the amplification factor of the open system is higher than the damping from turbulence and non-ideal reflection. The phase plot of the open system is not
shown here, it is only used to determine the instability frequencies, which are plotted on the x-axis of the amplification factor plot.

The curve of the GT1 burner (high swirl) in figure 7.4 shows much larger variations than the curve of the GT2 burner (low swirl). This is mainly because of the higher amplification factor of the GT1 burner. For the GT1 burner there is a large probability for instability around 160 Hz, the amplification factor is very high there. For the GT2 burner there is a probability for instability at 325 Hz, the amplification factor is a little higher here than the minimum of 1.1. Between 54 Hz and 136 Hz the flame from the GT1 burner works as a damper, that of the GT2 burner as an amplifier. This is caused by the difference in the phase of the two flame transfer functions. It should be mentioned that above 300 Hz the accuracy becomes less because both the flame transfer function and the burner transfer function are described fully by (extrapolated) models.

In figure 7.5 the influence of a swirl perturbator in the syngas swirl on the amplification factor of the open system is shown. The burner with swirl perturbator may cause combustion driven instabilities between 244 Hz and 271 Hz. The peak at 325 Hz is shifted because of the swirl perturbator and its bandwidth has become much smaller. At low frequencies, below 136 Hz, the amplification factor with swirl perturbator is much smaller. The global effect of the swirl perturbator on the transfer function of the open system is much smaller than the effect of the swirl.

The burner transfer function has a large effect on the amplification factor of the open system as can be seen in figure 7.6. In this figure the amplification factor of the open system with and without inclusion of the burner transfer function is shown. The peaks in the amplification factor occur at about the same frequency with and without burner transfer.
7.3 Results

Figure 7.6: The amplification factor of the open system for the GT1 burner (100% load). In one case the burner transfer function is included and the other case it is neglected ($H_{burner} = 0$). On the x-axis are plotted the possible instability frequencies.

The amplitude of the peaks has increased significantly if the burner transfer function is omitted. Hence significant changes in the acoustic behavior of the system can be obtained by changing the acoustics of the presystem and the burner.

7.3.2 The cylindrical modes

The cylindrical modes are studied using the mode analysis method. With the mode analysis method only the behavior of the system around the eigenfrequency is studied, this means that only one eigenfunction ($\psi_{Span}$) is taken into account. The modes presented here are all pure cylindrical modes, $m = 0$ for all cases. Because all cylindrical modes have an eigenfrequency above 300 Hz, it is necessary to use a modeled burner and flame transfer function.

Calculations have been performed for the three different burners (GT1, GT2 without swirl perturbator and GT2 with swirl perturbator (referred to as GT2Pert) at different eigenmodes. Per mode the calculated growth rate $\alpha$ and the calculated instability frequency $f_{inst}$ are tabulated in table 7.2, a negative growth rate indicates instability. The two-dimensional mode shapes for the different modes are plotted in figure 7.7. In this figure the location of the burners in the combustion chamber is indicated with black circles.

The results presented in table 7.2 should be treated with greatest caution, because only the growth rate at the instability frequency is given. In certain cases the growth rate varies strongly with the frequency and in such cases it is dangerous to draw conclusion from this table.
Figure 7.7: The mode shapes for the Siemens V94.2 combustion chamber. The black circles indicate the location of the burners.
### 7.4 Conclusions

<table>
<thead>
<tr>
<th>Mode</th>
<th>( \alpha )</th>
<th>( f_{\text{inst}} )</th>
<th>Mode</th>
<th>( \alpha )</th>
<th>( f_{\text{inst}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>001</td>
<td>GT1</td>
<td>0</td>
<td>022</td>
<td>GT1</td>
<td>-8</td>
</tr>
<tr>
<td></td>
<td>GT2</td>
<td>0</td>
<td></td>
<td>GT2</td>
<td>-4</td>
</tr>
<tr>
<td></td>
<td>GT2Pert</td>
<td>0</td>
<td></td>
<td>GT2Pert</td>
<td>-9</td>
</tr>
<tr>
<td>002</td>
<td>GT1</td>
<td>-3</td>
<td>031</td>
<td>GT1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>GT2</td>
<td>-4</td>
<td></td>
<td>GT2</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>GT2Pert</td>
<td>-7</td>
<td></td>
<td>GT2Pert</td>
<td>-8</td>
</tr>
<tr>
<td>021</td>
<td>GT1</td>
<td>-2</td>
<td>041</td>
<td>GT1</td>
<td>-5</td>
</tr>
<tr>
<td></td>
<td>GT2</td>
<td>-10</td>
<td></td>
<td>GT2</td>
<td>-1.5</td>
</tr>
<tr>
<td></td>
<td>GT2Pert</td>
<td>7</td>
<td></td>
<td>GT2Pert</td>
<td>-1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mode</th>
<th>( \alpha )</th>
<th>( f_{\text{inst}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>001</td>
<td>-3</td>
<td>784 Hz</td>
</tr>
<tr>
<td>002</td>
<td>-7</td>
<td>783 Hz</td>
</tr>
<tr>
<td>021</td>
<td>7</td>
<td>341 Hz</td>
</tr>
</tbody>
</table>

Table 7.2: Calculated instability frequency and growth rate for the different modes and the different burners. GT2Pert is the GT2 burner with a swirl perturbator in the syngas swirler.

The growth rate for mode 001 (the first radial mode) is about zero for all three burners. This can be explained by the mode shape of this mode (see figure 7.7). The burners are located close to a pressure node, this means that \( \psi_{\text{inst}} \) is nearly zero and \( T_{\text{flame}} \) is very small. For mode 002 the growth rate is negative for all three burners, which indicates instability. For this mode the burners are located near a pressure antinode and the flame also has a large influence on the acoustic system. Very large differences for the three burners are found in mode 031, the growth rate for the burners GT1 and GT2 is zero, the growth rate of GT2 with swirl perturbator is high, -8.

The different results for the different burners are caused by the differences in the phase delay for the three flame transfer functions (the amplification factor of the flame transfer function has been set to the same value for the three burners) and the different burner transfer functions. The influence of the burner transfer function on the results appears to be large (not shown here). It is however difficult to determine the burner transfer function with great accuracy for the high frequencies of the cylindrical modes. At these high frequencies compressibility effects and 2D/3D effects might become important.

From the mode shape figure, figure 7.7, it can be concluded that for mode 021 and mode 022 four burners contribute to the instability behavior, the other burners are located at a pressure node. For mode 041 all burners are located near pressure antinodes, what makes that they may contribute strongly to the acoustic system behavior.

### 7.4 Conclusions

Within certain limits it is possible to predict the acoustic behavior of the gas turbine combustion chamber with the models presented in this thesis. More advanced models may be necessary because of the dimensions of the burner and flame: for high frequencies these are not compact compared to the acoustic wave length and compressibility and two-dimensional and three-dimensional effects should be taken into account. This is also the situation if the results of a small scale test burner are scaled up to the large scale gas turbine burner, the scaling used in this chapter is only possible as long as the compressibility effects and two and three-dimensional effects can be neglected.

Both the transfer matrix method and the mode analysis method have been used to model
the occurrence of instabilities. The big advantage of the mode analysis method is that it is possible to include two-dimensional effects (cylindrical modes), what is not possible with the (one-dimensional) transfer matrix method. The disadvantage of the mode analysis method is that it only gives results at the instability frequencies, what makes it difficult to judge the accuracy of the predictions.

From the results, it is seen that the burner transfer function has a large influence on the occurrence of instability. The burner transfer functions used in this chapter all represented a very low pressure drop across the burner (about 3.5 % of the mean pressure). It is known from practice that the pressure drop across the burner has a large influence on the occurrence of instabilities (Vortmeyer et al. (1995)). According to Vortmeyer et al. the fuel side pressure drop should be at least 5 % of the mean pressure to prevent instabilities for a syngas fired gas turbine.

It appears that for the axial modes the swirl of the burner (GT1 versus GT2) has a large influence on the occurrence of combustion driven instabilities. For the studied situation a combustion driven instability is predicted for the burner GT1 around 160 Hz and at about 325 Hz for the burner GT2. The influence of a swirl perturbator in the fuel swirler is much smaller.

The accuracy of the predictions could be enhanced significantly by performing experiments on a real scale burner up to the frequencies of interest. These experiments will however be very complicated and expensive. As a first step, experiments should be performed to validate the scaling rules used in this chapter.

Improvements should be made in the measurement of the burner transfer function, because of its big influence on the acoustic system behavior. This can be achieved by a cold flow test rig, which is especially designed for this type of measurements.
Chapter 8

Conclusions

In this chapter the main conclusions of this thesis are presented in a condensed form. Behind each conclusion the relevant reference is stated.

8.1 General

Unsteady combustion induces a monopole source term in the acoustic wave equation, via the fluctuating heat release by the flame (appendix C). Because the acoustic wave equation is a linear equation, this heat release source term can be written as the sum of two source terms (chapter 2):

1. A source term that is independent of the acoustic pressure field (the flame as autonomous source of sound)

2. A source term that is a function of the acoustic pressure field (the flame as amplifier of sound).

These two contributions of the flame are investigated and discussed separately.

8.2 The flame as source of sound

The main general conclusions for a turbulent non-premixed flame as a source of sound are:

- The sound spectrum from a turbulent non-premixed flame with infinitely fast chemistry can be written in terms of the turbulent mixture fraction spectrum at the flame front. (section 3.3)

- For frequencies \( \omega > 2U k_c \) where \( k_c \) is the integral wave number and \( U \) a typical velocity scale, the one-dimensional sound spectrum for a turbulent non-premixed flame with infinitely fast chemistry has a fall off with frequency of \( \omega^{-10/3} \). (section 3.3)

- The predicted sound spectrum of flames, in which the chemistry can be assumed to be infinitely fast, compares well with the measured sound spectrum, both for the sound level and the shape of the spectrum. (section 3.7)
The sound spectrum of flames with finite rate chemistry shows a steeper fall off with frequency than that of flames with infinitely fast chemistry. (section 3.7)

The sound level scales quadratically with the thermal power of the flame, as predicted by the theory. This means that the sound level scales quadratically with the inlet velocity of the fuel (constant mean pressure and temperature) and with the mean pressure (constant inlet velocities and temperature). (sections 3.4 and 3.9)

The sound spectrum with finite rate chemistry is not affected by elevation of the mean pressure to 4 bar. (section 3.9)

The sound spectrum with finite rate chemistry is shown to reduce to an infinitely fast chemistry spectrum by a sufficient increase of the inlet temperatures. (section 3.7)

8.3 The flame as amplifier of sound

The main general conclusions for the flame as an amplifier of sound are:

- The acoustic behavior of the flame can be represented with the flame transfer function. (chapter 4)
- The flame transfer function can be determined from measurements using only microphones, for an acoustically one-dimensional combustion rig. (section 4.5)
- For syngas flames the air side flame transfer function is not important. (chapters 5 and 6)
- The phase of the flame transfer function shows for all cases a time delay behavior. (chapters 5 and 6)
- The average value of the amplitude of the flame transfer function is a function of the thermal power and the inlet velocity. Peaks may occur in the amplitude plot of the flame transfer function due to interaction with turbulence and/or flow instabilities. (chapters 5 and 6)
- An acoustically one-dimensional combustion installation can be modeled with transfer matrices. For acoustically more-dimensional systems other methods, like the mode analysis method, should be used. (appendix D and section 2.5)
- In the analysis of the acoustic system of a combustion installation an accurate acoustic modeling of the (fuel) supply system is very important. (chapters 5 and 7)
- For realistic burners the pressure drop across the fuel injector must be taken into account in the acoustic model. (chapter 6)
- For most flames being tested it appears to be possible to scale the frequency with the Strouhal number. The validity of Strouhal number scaling depends on the mechanism of flame stabilization. (chapters 5 and 6)
- Within certain limits the experimental results obtained with a scaled burner can be used for an analysis of the real (full scale) situation. (chapter 7)
Bibliography


and low-dimensional manifold mechanisms to reduce reaction mechanisms, *Combustion Science and Technology* 123, 347.


the acoustic characteristics of turbomachinery cavities, ASME Press, New York.
Matsui, Y. (1981) An experimental study on pyro-acoustic amplification of premixed laminar flames, 
Combustion and Flame 43, 199.
Strahle, W.C., Muthukrishnan, M., Neule, H. (1979) Experimental determination of noise source


### Appendix A

### Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Constant in sound generation equation ($A = A^+ \Delta g_3$) [m]</td>
</tr>
<tr>
<td>$A^+$</td>
<td>Constant in sound generation equation [-]</td>
</tr>
<tr>
<td>$B$</td>
<td>Constant from scalar dissipation [1/m²]</td>
</tr>
<tr>
<td>$c$</td>
<td>Modelling constant for the turbulent scalar time scale [-]</td>
</tr>
<tr>
<td>$c_p$</td>
<td>Speed of sound [m/s]</td>
</tr>
<tr>
<td>$c_v$</td>
<td>Specific heat at constant pressure [J/kg · K]</td>
</tr>
<tr>
<td>$D, d$</td>
<td>Specific heat at constant volume [J/kg · K]</td>
</tr>
<tr>
<td>$D$</td>
<td>Diameter [-]</td>
</tr>
<tr>
<td>$D_d$</td>
<td>Diffusion coefficient [m²/s]</td>
</tr>
<tr>
<td>$D_a$</td>
<td>Damköhler number [-]</td>
</tr>
<tr>
<td>$E_{\xi\xi}(k)$</td>
<td>Turbulent energy spectrum of $\xi$ [m]</td>
</tr>
<tr>
<td>$F^\omega$</td>
<td>Fourier transform: $F^\omega(m) = \int_{-\infty}^{\infty} m(t)e^{i\omega t} dt$</td>
</tr>
<tr>
<td>$F^k$</td>
<td>Fourier transform: $F^k(m) = \int_{-\infty}^{\infty} m(x)e^{ikx} dx$</td>
</tr>
<tr>
<td>$F_{\mathbf{m}}$</td>
<td>Fourier transform: $F_{\mathbf{m}}(m) = \int_{-\infty}^{\infty} m(x)e^{i\mathbf{k} \cdot \mathbf{x}} d\mathbf{x}$</td>
</tr>
<tr>
<td>$f$</td>
<td>Force on flow in burner [N]</td>
</tr>
<tr>
<td>$g$</td>
<td>Frequency [Hz]</td>
</tr>
<tr>
<td>$G$</td>
<td>Green’s function [1/m · s]</td>
</tr>
<tr>
<td>$g$</td>
<td>Variance of $\xi (=\xi^2)$ [-]</td>
</tr>
<tr>
<td>$H$</td>
<td>Heaviside step function [-]</td>
</tr>
<tr>
<td>$H_{\text{burner}}$</td>
<td>Burner transfer function [kg/s]</td>
</tr>
<tr>
<td>$H_{\text{closed}}$</td>
<td>Transfer function of the closed system in the Nyquist analysis [-]</td>
</tr>
<tr>
<td>$H_{\text{flame}}$</td>
<td>Flame transfer function [kg/m]</td>
</tr>
<tr>
<td>$H_{\text{open}}$</td>
<td>Transfer function of the open system in the Nyquist analysis [-]</td>
</tr>
<tr>
<td>$h$</td>
<td>Enthalpy [J/kg]</td>
</tr>
<tr>
<td>$h_i^0$</td>
<td>Enthalpy of formation of species $i$ [J/kg]</td>
</tr>
<tr>
<td>$i$</td>
<td>Jacobian [-]</td>
</tr>
<tr>
<td>$J_n$</td>
<td>$n^{th}$ order Bessel function of the first kind [-]</td>
</tr>
<tr>
<td>$k$</td>
<td>Wave number (vector) [1/m]</td>
</tr>
<tr>
<td>$k^+$</td>
<td>Wave number of the acoustic wave in the positive axial-direction [1/m]</td>
</tr>
<tr>
<td>$k^-$</td>
<td>Wave number of the acoustic wave in the negative axial-direction [1/m]</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$k_e$</td>
<td>Integral wave number [1/m]</td>
</tr>
<tr>
<td>$k_m$, $k_n$</td>
<td>Wave number acoustic eigenfunction [1/m]</td>
</tr>
<tr>
<td>$k_{Kd}$</td>
<td>Kolmogorov wave number [1/m]</td>
</tr>
<tr>
<td>$K_a$</td>
<td>Karlovitz number [-]</td>
</tr>
<tr>
<td>$l$</td>
<td>Length (scale) [m]</td>
</tr>
<tr>
<td>$l_e$</td>
<td>Integral length scale [m]</td>
</tr>
<tr>
<td>$l_{corr}$</td>
<td>Correlation length in the $y_2$ direction [m]</td>
</tr>
<tr>
<td>$l_{flame}$</td>
<td>Flame length [m]</td>
</tr>
<tr>
<td>$Ma$</td>
<td>Mach number [-]</td>
</tr>
<tr>
<td>$P$</td>
<td>Acoustic power [W]</td>
</tr>
<tr>
<td>$p$</td>
<td>(Acoustic) pressure [Pa]</td>
</tr>
<tr>
<td>$p^-$</td>
<td>Complex amplitude of the acoustic wave in the negative axial-direction [Pa]</td>
</tr>
<tr>
<td>$p^+$</td>
<td>Complex amplitude of the acoustic wave in the positive axial-direction [Pa]</td>
</tr>
<tr>
<td>$pp(\omega)$</td>
<td>Sound spectrum [Pa$^2$ s rad]</td>
</tr>
<tr>
<td>$pp(f)$</td>
<td>Sound spectrum [Pa$^2$/Hz]</td>
</tr>
<tr>
<td>$\bar{p}$</td>
<td>Total sound emission [Pa$^2$]</td>
</tr>
<tr>
<td>$Q_{amp}$</td>
<td>Source due to coupling with acoustic pressure [W]</td>
</tr>
<tr>
<td>$Q_{flame}$</td>
<td>Heat release by the flame [W]</td>
</tr>
<tr>
<td>$Q_{noise}$</td>
<td>Autonomous noise source term [W]</td>
</tr>
<tr>
<td>$Q_{total}$</td>
<td>Thermal power of the flame [W]</td>
</tr>
<tr>
<td>$q$</td>
<td>Volumetric heat release rate [W/m$^3$]</td>
</tr>
<tr>
<td>$\bar{q}$</td>
<td>Heat release correlation function [W$^2$/m$^6$]</td>
</tr>
<tr>
<td>$R$</td>
<td>Amplification factor in transfer function</td>
</tr>
<tr>
<td>$R$</td>
<td>Gas constant [m$^2$/s$^2$ K]</td>
</tr>
<tr>
<td>$R_{flame}$</td>
<td>Reflection factor at Rijke tube boundary [-]</td>
</tr>
<tr>
<td>$r$</td>
<td>Free space distance from noise source [m]</td>
</tr>
<tr>
<td>$r$</td>
<td>Radial co-ordinate [m]</td>
</tr>
<tr>
<td>$r(k, \omega)$</td>
<td>Wave number frequency correlation function [s]</td>
</tr>
<tr>
<td>$r(k, \Delta t)$</td>
<td>Wave number time correlation function [-]</td>
</tr>
<tr>
<td>$r_{st}$</td>
<td>Radial location of the stoichiometric interface [m]</td>
</tr>
<tr>
<td>$Re_t$</td>
<td>Turbulent Reynolds number [-]</td>
</tr>
<tr>
<td>$S$</td>
<td>Cross section area [m$^2$]</td>
</tr>
<tr>
<td>$S$</td>
<td>Acoustic source term in mode analysis method [1/s$^2$]</td>
</tr>
<tr>
<td>$\triangle S$</td>
<td>Noise source term [Pa]</td>
</tr>
<tr>
<td>$St$</td>
<td>Strouhal number [-]</td>
</tr>
<tr>
<td>$\Delta S$</td>
<td>Source of matrix equation</td>
</tr>
<tr>
<td>$St_L$</td>
<td>Laminar flame speed [m/s]</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature [K]</td>
</tr>
<tr>
<td>$T$</td>
<td>Time interval [s]</td>
</tr>
<tr>
<td>$T$</td>
<td>Modified Lighthill stress tensor [kg/s$^2$, m]</td>
</tr>
<tr>
<td>$\tilde{\tau}_{flame}$</td>
<td>Coupling function in mode analysis method [W/Pa]</td>
</tr>
<tr>
<td>$t$</td>
<td>Time [s]</td>
</tr>
<tr>
<td>$t_{ke}$</td>
<td>Turbulent kinetic energy [m$^2$/s$^2$]</td>
</tr>
<tr>
<td>$U$</td>
<td>Mean velocity (scale) [m/s]</td>
</tr>
<tr>
<td>$u$</td>
<td>Velocity [m/s]</td>
</tr>
</tbody>
</table>
\( \bar{\pi} \) Mean velocity in axial direction [m/s]
\( V, V_{\text{Vol}} \) Volume [m³]
\( \bar{x}, \bar{z}, \bar{z}_i \) Co-ordinate [m]
\( Y_i \) Mass fraction of species \( i \) [-]
\( y_1, y_2, y_3 \) Flame co-ordinate system in chapter 3 [m]
\( Z_i \) Mass fraction of element \( i \) [-]

Greek symbols
\( \alpha \) Growth rate [1/s]
\( \beta \) Heat release from heated gauze in Rijke’s tube [kg·m·s⁻²]
\( \Gamma \) Correction factor for wave number to include damping [-]
\( \gamma \) Poisson’s constant [-]
\( \Delta t \) Time delay [s]
\( \Delta x \) Axial distance [m]
\( \Delta y_i \) Thickness of the mixing layer [m]
\( \delta \) Dirac delta function
\( \delta_f \) Laminar flame thickness [m]
\( \delta_{ij} \) Kronecker delta [-]
\( \epsilon \) Dissipation of turbulent energy [m²/s³]
\( \zeta \) Modal damping [-]
\( \eta \) Kolmogorov length scale [m]
\( \eta_0 \) Time dependent function in mode analysis [-]
\( \theta \) Tangential co-ordinate [rad]
\( \lambda \) Air factor [-]
\( \lambda \) Heat conductivity [W · m / K]
\( \lambda \) Wave length [m]
\( \lambda, \beta \) Wave number [1/m]
\( \lambda_{\text{max}} \) Cylindrical wave number of eigen function \( \psi_{\text{max}} \) [1/m]
\( \mu \) Dynamic viscosity [kg/m · s]
\( \nu \) Kinematic viscosity [m²/s]
\( \xi \) Mixture fraction [-]
\( \xi_{\text{st}} \) Stoichiometric mixture fraction [-]
\( \rho \) Density [kg/m³]
\( \rho_e \) Excess density [kg/m³]
\( \tau \) Time delay [s]
\( \overline{\tau_{\text{urb}}} \) Viscous stress tensor [kg/s², m]
\( \phi \) Phase of transfer function [*]
\( \phi \) Scalar [-]
\( \phi \) Velocity potential [m²/s]
\( \chi \) Scalar dissipation rate [1/s]
\( \psi \) Stream function [m²/s]
\( \psi_n, \psi_n, \psi_{\text{max}} \) Mode shape, acoustic eigen function [-]
\( \omega \) Vorticity [1/s]
\( \omega \) Frequency [rad/s]
\( \omega_i \) \hspace{1cm} \text{Chemical source term of species } i \ [1/\text{s}]

\( \omega_{\text{m}}, \omega_{\text{mono}} \) \hspace{1cm} \text{Eigenfrequency in mode analysis method} \ [\text{rad/s}]

Subscripts

- 0 \hspace{1cm} \text{Far field in case of noise generation}
- 6 \hspace{1cm} \text{Situation for the model burner in chapter 7}
- 7 \hspace{1cm} \text{Situation for the gas turbine burner in chapter 7}
- 1 \hspace{1cm} \text{Upstream part of an acoustic 1D element}
- 2 \hspace{1cm} \text{Downstream part of an acoustic 1D element}
- cc \hspace{1cm} \text{Combustion chamber}
- con \hspace{1cm} \text{Conical part of gas turbine}
- im \hspace{1cm} \text{Imaginary}
- Kol \hspace{1cm} \text{Kolmogorov}
- L \hspace{1cm} \text{Laminar}
- ref \hspace{1cm} \text{Reference}
- re \hspace{1cm} \text{Real}
- t, turb \hspace{1cm} \text{Turbulent}
- trans \hspace{1cm} \text{Transition part of gas turbine}

Operators

- \text{\textquoteleft\textquoteleft} \hspace{1cm} \text{Favre decomposition}
- \text{\textquoteleft\textquoteleft} \hspace{1cm} \text{Reynolds decomposition}
- \text{\textquoteleft\textquoteleft} \hspace{1cm} \text{Acoustic fluctuation}
- \text{\textquoteleft\textquoteleft} \hspace{1cm} \text{Reynolds (time) averaged}
- \text{-} \hspace{1cm} \text{Favre averaged}
- \text{-} \hspace{1cm} \text{Fourier transform in time, complex amplitude}
Appendix B

The experimental setup

A new laboratory facility has been built for the research project described in this thesis. A scheme of the experimental setup is given in figure B.1.

B.1 The fuel and air supply

The main components in syngas are: carbon monoxide (CO), hydrogen (H₂), nitrogen (N₂) and steam (H₂O). In this setup the choice has been made to mix the syngas from its pure components.

The fuel system can supply a mixture of the following components: CO, H₂, CH₄, N₂ and H₂O. The first four are stored in gas bottles, the H₂O is generated by a steam generator. The fuel gases CO, H₂ and CH₄ are stored in bottles in special safety lockers. The amount of bottles in the different lockers is enough to deliver fuels with a thermal heating power of 50 kW (based on the Demkolec composition) for 4 hours. N₂ is supplied by the laboratory nitrogen system.

The mass flow of the components is controlled with mass flow controllers (MFC’s), for each component one. There are two sets of mass flow controllers, one set is designed to deliver fuel up to a thermal power of 50 kW. The other set is designed to deliver fuel up to a thermal power of 100 kW. In principle the fuel system can deliver fuels up to a thermal power of 150 kW.

Downstream of the mass flow controllers the components are mixed and fed to an electrical heater. The heater is a tube with a length of 2 m and a diameter of 10 cm. The heater can preheat the gas to 200°C. The other function of the heater is to mix the components.

Downstream of the heater the steam is added to the fuel flow. The steam is generated by a steam generator, the mass flow of it is set with a calibrated needle valve. The mass flow from the steam generator is monitored with an electronic mass balance.

The heated gas mixture is fed with a flexible metal tube to the combustion installation. Upstream of the burner there is a pneumatic controlled 3 way valve. Normally the gas mixture is fed to the burner. In case of an emergency the route is changed automatically by the 3 way valve and the gas is led to the outside environment.

The air necessary for the combustion is supplied by the pressurized air system of the laboratory. The air is delivered at a pressure of 6 bar. The air pressure is reduced to control
Figure B.1: A scheme of the experimental setup.

Figure B.2: Drawing of the bluff body burner. The dimensions are in mm.
the air mass flow using a sonic nozzle. For the different power ranges, different nozzles are used. The air, which is now at atmospheric pressure, is led to an electrical heater. This heater can preheat the air to 150°C. After the heater the air is led to the burner using a flexible metal tube.

B.2 The combustion rig

The combustion rig consists of a vertically placed combustion chamber and a burner. A schematic picture of it is shown in figure B.3. The combustion chamber is constructed of two concentric tubes. Combustion takes place in the inner tube of 100 mm diameter and a length of 1.3 m. Cooling air flows between the two tubes to cool the wall of the inner tube, similar to a gas turbine combustion chamber. At the exit of the combustion chamber the flue gases are mixed with the cooling air and escape via a chimney. Sample holes are placed at several axial distances along the combustor geometry. These holes can be used to insert a probe or thermocouple into the combustion chamber to take samples. In the acoustic experiments the holes are used to mount the microphones. This combustion chamber is the same as used by Louis (1997).

The burner that is used in most experiments, is a bluff body diffusion flame burner, see figure B.2. The burner consists of a central tube through which the fuel enters the combustor, surrounded by a concentric tube supplying the combustion air. The inner diameter of the air tube is 50 mm, half the diameter of the combustion chamber. A cylindrical flameholder is attached to the end of a rod extending axially through the burner. The rod is kept on the axis of the burner by three ribs in the fuel outlet. The axial position of the flameholder can be varied by screwing the rod in its suspension on the upstream end of the burner.

The burner has proved to be very useful for verification of combustion models. One serious flaw in the design is the distribution of the air flow. The bend in the air tube, pierced by the fuel tube, leads to suppression of the air flow at the inside bend (Louis (1997)). This ill distribution of air affects the flame behavior.

B.3 The acoustical experimental setup

For all acoustic measurements the combustion chamber should be as anechoic as possible. This is achieved by attaching several dampers: A high pressure drop is created in the fuel inlet and the air inlet, to decouple them acoustically from the upstream components of the system. This pressure drop is achieved by the introduction of mineral wool. At the end of the combustion chamber an acoustic damper is attached, consisting of a disk with a small hole. This is a well-known damper in acoustics (e.g. used in the piping system of compressors) (Hirschberg and Rienstra (1992) , Bechert (1980)). By attaching this damper it is attempted to create an anechoic exit condition in the combustion chamber, which means that there is only a downstream traveling wave and not a reflected upstream traveling wave. The working of the damper is discussed below.

The combustion chamber which is used in the experiments can be treated as acoustically one-dimensional for frequencies up to about 2000 Hz, because of its small diameter (100 mm). This implicates that the acoustics can be described with two traveling waves, one going upstream and one downstream.
Two microphones are mounted at different axial locations in the wall of the combustion chamber. Two of them are used to check the performance of the damper by measuring the transfer function between them. In the ideal situation of just a downstream traveling wave the amplification factor of the transfer function between the two microphones should be one and the phase should be a linear function of the frequency because of the time delay necessary for the acoustic wave to travel from one microphone to the other.

The working principle of the used acoustic damper is based on the sudden contraction and the presence of a mean flow: The acoustic wave that arrives at the damper induces a velocity fluctuation in the contraction. Due to this (acoustic) velocity fluctuation, vortex shedding occurs at the edges of the hole. These vortices are swept away by the mean flow and are dissipated. This means that energy is diverted from the acoustic system and that the reflection coefficient at the exit will be smaller than one (Hirschberg and Rienstra (1992), Bechert (1980)).

The performance of the damper is a function of the mean flow through the hole and the diameter of the hole. From Bechert (1980) it is known that such a damper is only active, this means that it has a small reflection factor, in a certain frequency range. To determine the optimum damper for the experiments, a separate set of experiments has been performed by Pronk (1999). The typical magnitude of the reflection factor at the exit of the combustion chamber is: 0.1 · · · 0.3.

The acoustic measurements are performed with a set of five microphones and a spectrum analyzer. The microphones are piezo-resistive differential pressure microphones from Enkevo, type 8610B1. Their sensitivity is ±1 Psi. The microphones are connected to a bridge amplifier, Enkevo 106 amplifier (analog) or Enkevo 306 amplifier (digital). The output from the amplifiers is fed to the digital FFT analyzer.

The FFT analyser is a DIFA APB200-8 analyser. This analyser has 8 input channels, which are scanned simultaneously. The analyser is mounted in a standard P.C. This FFT analyser has two signal generators.

To perform the flame transfer function measurements two exciters (shakers) are used. Both shakers have their own amplifier. The signal to drive these shakers is generated by the signal generator of the DIFA FFT analyser. The location of the shakers depends on the type of experiment. If the flame or burner transfer function (see section 4.5) is measured,
the shakers are positioned upstream of the burner to drive the membrane in the fuel or air supply. If the flame reflection factor is measured (see section 5.6.1), the shaker is located at the downstream end of the combustion chamber.
Appendix C

The thermo-acoustic source term

In chapter 2 the following wave equation has been derived:

\[ \frac{1}{c_0^2} \frac{\partial^2 p'}{\partial t^2} - \nabla^2 p' = \nabla \cdot (\nabla \cdot T) - \frac{\partial^2 \rho_e}{\partial t^2} \]

(C.1)

In equation (C.1) is \( p' \) the acoustic pressure fluctuation defined as \( p' = p - p_0 \), \( T \) is the modified Lighthill stress tensor, defined as \( \rho u u - \frac{1}{3} \rho \mathbf{u} \cdot \mathbf{u} \) and \( \rho_e \) is the 'excess' density defined as:

\[ \rho_e = \rho - \rho_0 - (p - p_0) / c_0^2 \]

(C.2)

This excess density is the thermo-acoustic source term, it is zero in the far field but can give a large contribution to the generated sound in the flame region. To evaluate this term the thermodynamics of the source region should be studied. Therefore the term \( \frac{\partial \rho_e}{\partial t} \) is written as:

\[ \frac{\partial \rho_e}{\partial t} = \frac{D \rho_e}{D t} - \mathbf{u} \cdot \nabla \rho_e = \frac{D \rho_e}{D t} - \frac{\rho_e}{\rho} \frac{D \rho}{D t} - \nabla \cdot (\rho_e \mathbf{u}) \]

(C.3)

where the equation for the conservation of mass is used to write \( \rho_e \mathbf{u} \cdot \mathbf{u} = -\frac{\rho_e}{\rho} \frac{D \rho}{D t} \). The definition for \( \rho_e \) (equation (C.2)) is substituted into the equation (C.3). This gives:

\[ \frac{\partial \rho_e}{\partial t} = \left( \frac{p - p_0}{\rho c_0^2} + \frac{\rho_0}{\rho} \right) \frac{D \rho}{D t} - \frac{1}{c_0^2} \frac{D \rho}{D t} - \nabla \cdot (\rho_e \mathbf{u}) \]

(C.4)

To evaluate the term \( \frac{D \rho}{D t} \) this term is taken to be a function of the pressure \( p \), the enthalpy \( h \) and the mass fractions of the species \( Y_i \). This is a similar approach as followed by Dowling in Crighton et al. (1992) (pp. 378). From thermodynamics it is known that all thermodynamic quantities can be expressed as a function of three independent variables. The chain rule is applied to \( \frac{D \rho}{D t} \) in equation (C.4):

\[ \frac{D \rho}{D t} = \left. \frac{\partial \rho}{\partial p} \right|_{h,Y_i} \frac{D p}{D t} + \left. \frac{\partial \rho}{\partial h} \right|_{p,Y_i} \frac{D h}{D t} + \sum_{i=1}^{n} \left. \frac{\partial \rho}{\partial Y_i} \right|_{h,p} \frac{D Y_i}{D t} \]

(C.5)
The three terms in the r.h.s of equation (C.5) are now discussed separately.

The first term
The partial derivative in the first term of equation (C.5) can be written as:

\[
\frac{\partial p}{\partial t} \bigg|_{h, Y_i} = - \frac{\partial}{\partial t} \left( \frac{\partial p}{\partial Y_i} \right)_{h,Y_i} - \frac{T}{\sum Y_i W_i} \frac{\partial T}{\partial t} \bigg|_{p, Y_i} + \sum_i h_i^0 \frac{\partial Y_i}{\partial t} + \sum_i h_i^0 \frac{\partial Y_i}{\partial t} = \frac{1}{c_p T} \frac{\partial}{\partial t} \left( \frac{\partial p}{\partial Y_i} \right)_{p,Y_i}
\]

(C.6)

where \( h_i^0 \) is the formation enthalpy and \( c \) is the local speed of sound.

In the equation (C.6) the ideal gas law is used:

\[
p = \frac{RT \rho}{W_0} = \frac{RT \rho}{\sum Y_i W_i}
\]

where \( R \) is the universal gas constant and \( W_0 \) is the average molecular weight.

The second term
The partial derivative in the second term in the r.h.s of equation (C.5) can be written as:

\[
\frac{\partial p}{\partial h} \bigg|_{p, Y_i} = - \frac{p}{W_0 RT^2} \frac{\partial T}{\partial h} \bigg|_{p,Y_i} = - \frac{p}{c_p T}
\]

(C.8)

The ideal gas, low Mach number approach for the enthalpy is used: the enthalpy is only a function of the temperature \( T \) and the mass fractions of the species. The enthalpy can then be written as:

\[
h = \sum_{i=1}^{n} h_i(T) Y_i = \sum_{i=1}^{n} \left[ h_i^0 Y_i + \int_{T_0}^{T} c_p(T) Y_i dT \right] = \sum_{i=1}^{n} h_i^0 Y_i + \int_{T_0}^{T} c_p(T) dT
\]

(C.9)

where \( c_p(T) = \sum c_{p,i}(T) Y_i \). The heat of formation of species \( i \) is \( h_i^0 \). The transport equation for the enthalpy is (Bird et al. (1960)):

\[
\frac{Dh}{Dt} = - \left( \nabla \cdot \mathbf{q} \right) - (\mathbf{\varepsilon} : \nabla \mathbf{u}) + \frac{Dp}{Dt}
\]

(C.10)

where \( D_h \) is the heat conduction vector. There are no heat release source terms in the enthalpy equation because this is the equation for the total enthalpy.

The third term
The third term in the right hand side of equation (C.5) is the change in density at constant \( h \) and \( p \) due to changes in the species concentrations. This term can be written as:

\[
\sum_{i=1}^{n} \frac{\partial p}{\partial Y_i} \bigg|_{h,p} \frac{DY_i}{Dt} = - \sum_{i=1}^{n} \frac{\partial}{\partial Y_i} \bigg |_{h,p} \frac{DY_i}{DT} =
\]

\[
- \sum_{i=1}^{n} \left[ \frac{\partial}{\partial Y_i} \bigg |_{h,p} \frac{DY_i}{DT} - \rho \frac{\delta h^0}{\delta Y_i} \right] \frac{DY_i}{DT}
\]

(C.11)
The material derivative of $Y_i$ in equation (C.11) is given by the transport equation $i$:

$$\rho \frac{DY_i}{Dt} = - (\nabla \cdot D_i) + \omega_i$$  \hspace{1cm} (C.12)$$

where $- (\nabla \cdot D_i)$ is the laminar diffusion of species $i$ and $\omega_i$ is the chemical source term.

As can be seen in equation (C.11) the effect of mixing and chemical reaction on the density is twofold: the first term between the square brackets describes the direct effect on the density of the change of the mass fraction of the species. However, if the average molecular weight remains constant, this term equals zero. The second term between the square brackets reflects the effect of chemical reaction: it is a heat release term.

The equations (C.6), (C.8), (C.10), (C.11) and (C.12) are now substituted into equation (C.4). This gives:

$$\frac{\partial \rho}{\partial t} + \frac{(\rho - \rho_0) \rho}{\rho_0} \frac{D \rho}{Dt} - \frac{1}{\rho_0} \left( \frac{\partial \rho}{\partial t} \right) \frac{D \rho}{Dt} + \frac{\partial \rho}{\partial t} \left( \frac{\rho - \rho_0}{\rho_0} \right) \frac{D \rho}{Dt} + \left( \nabla \cdot \frac{\rho \rho^2}{\rho_0} \frac{\rho}{\rho_0} \right) \sum_{i=1}^{n} h_i^0 \omega_i$$  \hspace{1cm} (C.13)$$

In the case of a burning flow with a high Reynolds number the viscous heating $(\frac{\rho \rho^2}{\rho_0} \frac{\rho}{\rho_0})$), the heat conduction $(\nabla \cdot \frac{\rho \rho^2}{\rho_0} \frac{\rho}{\rho_0})$ and the laminar diffusion of species $(\nabla \cdot D_i)$ from equation (C.13) can be neglected. The wave equation, equation (C.1), becomes then:

$$\frac{1}{\theta} \frac{\partial^2 \theta}{\partial t^2} - \nabla^2 \theta = \nabla \cdot \left( \nabla \cdot \frac{\rho \rho^2}{\rho_0} \frac{\rho}{\rho_0} \right) \sum_{i=1}^{n} h_i^0 \omega_i$$  \hspace{1cm} (C.14)$$

The free-space Green’s function $(G)$ can be used to write down the solution for equation (C.14) at location $z$. The free space Green’s function is:

$$G(z|y) = \frac{\delta \left( t - \frac{\left| y - z \right|}{\theta} \right)}{4\pi \left| y - z \right|}$$  \hspace{1cm} (C.15)$$

The free space solution of equation (C.14) is written in index notation:

$$p^I (z, t) = \frac{\delta}{\theta \rho \rho_0} \int \left[ \rho \rho^2 \omega - \tau \right] \frac{dV}{d\tau}$$

$$+ \frac{\delta}{\theta \rho \rho_0} \int \left[ \rho \rho^2 \omega \sum_{k=1}^{n} h_i^0 \omega_i \right] \frac{dV}{d\tau}$$

$$- \frac{1}{\theta \rho \rho_0} \int \left[ \frac{(\rho - \rho_0)}{\rho} \frac{D \rho}{Dt} \right] \frac{dV}{d\tau}$$

$$+ \frac{\delta}{\theta \rho \rho_0} \int \left[ \rho \rho^2 \omega \right] \frac{dV}{d\tau}$$  \hspace{1cm} (C.16)$$

where $r$ denotes the distance from the source to the listener at location $z$. In this equation the square brackets denote that the functions they enclose should be evaluated at the retarded time $t - r/c_0$. When $z$ is in the far field and the flame is a compact source of sound in the vicinity of $z = 0$, $r$ can be written as $\left| z \right|$. The $\nabla$ operator can then be written in index
notation as \(-x_i/(r_0) \frac{\partial}{\partial t}\). The solution in the far field is then:

\[
4\pi P^j(x_0, t) = \frac{x_i x_j}{2\pi r_0^2} \int \left[ \rho u_i u_j - \tau_{ij} \right] dV \\
+ \frac{\partial}{\partial t} \int \left[ \frac{\rho \partial u_j}{\partial x_i} \sum_{k=1}^n \beta_k \omega_k \right] dV \\
- \frac{1}{\rho c_s^2} \frac{\partial^2}{\partial t^2} \int \left[ \rho u_i \right] dV \\
- \frac{\partial^2}{\partial t^2} \frac{x_i}{r_0} \int \left[ \rho u_i \right] dV
\]  

(C.17)

In equation (C.17) the first term in the right hand side describes the sound generation by turbulence, this is a quadrupole term. The second term in the r.h.s is the combustion term, this a monopole term. The third term is a monopole and it is important if there are regions of unsteady flow with different mean density and sound speed from the ambient fluid. The last term in equation (C.17) is a dipole and describes the effect of momentum changes of density inhomogeneities. To evaluate the relative magnitude of these terms in the case of combustion the acoustic power is introduced. For a free space the acoustic power is defined by (Crighton et al. (1992) pp. 387):

\[
P = \frac{\int_{\Omega} 4\pi r^2}{\rho_0 c_s^2}
\]  

(C.18)

The relative magnitude of the terms in equation (C.17) is estimated with

\[
\left| \rho u_i u_j \right| \approx \rho_0 U U \\
\left| \frac{\partial}{\partial t} \right| \approx \frac{U}{T} \\
\left| dV \right| \approx L^3 \\
\left| \sum \beta_k \omega_k \right| \approx \frac{\rho_0 c_s^2 T}{\rho c_s^2} T \approx \frac{U}{T} \rho c_s T
\]  

(C.19)

Where \(U\) is a typical velocity of the flow and \(L\) is a length scale of the flow (e.g. the jet diameter), the viscous stress tensor \(\tau_{ij}\) has been neglected. The acoustic power from the first, second and last term in the right hand side of equation (C.17) becomes with these estimations (the third term is not treated here because it appears that this term only gives a contribution if two different phases are present with big differences in the speed of sound and the density):

\[
4\pi P \approx \frac{1}{\rho_0 c_s^2} \left[ \frac{U^2}{c_s^2} U^2 L^6 + \frac{U^2}{c_s^2} \rho_0 \frac{\partial u_i}{\partial x_j} U^2 L^6 + \frac{U^2}{c_s^2} + U^2 (\rho - \rho_0)^2 L^6 \right] = \\
\rho_0 M a L^2 U^3 \left[ Ma^4 + 1 + Ma^2 \left( \frac{c_s}{\rho_0} - 1 \right) \right]
\]  

(C.20)

The acoustic power from the combustion process is also of the order of \(1/Ma^4\) bigger than the acoustic power generated by the turbulence. The sound generation by the heat release is of the order of \(1/Ma^2\) stronger than the sound generation due to the acceleration of density inhomogeneities (the third term at the right hand side of equation (C.20)). For low Mach number flows \((Ma \ll 1)\) the first and the third term can be neglected compared to the combustion source term.
It can be concluded that the main sound output from a flame results from the fluctuating heat release \(q\). The wave equation with only this source term is

\[
\frac{1}{c_0^2} \frac{\partial^2 p'}{\partial t^2} - \nabla^2 p' = \frac{\partial}{\partial t} \left[ \frac{\rho_0}{\rho c_p T_0} \frac{\partial q'}{\partial x} \right] = \frac{\partial}{\partial t} \left[ \frac{\rho_0}{\rho} \frac{\gamma - 1}{c_s^2} q' \right]
\]

(C.21)

The last term at the right hand side of equation (2.7) is the most usual shape of the source term to describe combustion noise.
Appendix D

The transfer matrix method

In this appendix the construction and use of the transfer matrix method for system analysis will be discussed.

From a physical point of view four types of acoustic elements can be distinguished:

1. Element for the propagation of sound.

2. Element that describes the joint of two (or more) elements.

3. Special elements: An element for the flame and an element for contractions in the flow, like the burner mouth.

4. Elements that describe acoustic boundaries.

The general shape of the matrix equation for the element types 1, 2 and 3 is:

\[
\begin{bmatrix}
    M_1 & M_2 & M_3 & M_4 \\
    M_5 & M_6 & M_7 & M_8
\end{bmatrix}
\begin{bmatrix}
    p_1^+ \\
    p_2^+
\end{bmatrix}
= S
\]  

(D.1)

\(S\) is the source term vector.

The number of degrees of freedom of this matrix element is two, the four elements in the \(p\) vector in the left hand side of equation (D.1) might be confusing. It is written in this way because it is now easier to construct the system matrix.

The general shape for the matrix equation of an acoustic boundary is:

\[
\begin{bmatrix}
    E_1 E_2
\end{bmatrix}
\begin{bmatrix}
    p_1^+ \\
    p_1^-
\end{bmatrix}
= \Omega
\]  

(D.2)
Figure D.1: The definition of the joints 1 and 2 of the duct. At these joints the duct is coupled to other acoustic elements.

D.1 Element for the propagation of sound

For a simple duct with constant cross sectional the following matrix equation can be derived using equation (2.13):

\[
\begin{pmatrix}
e^{-ik^+l_{duct}} & 0 & -1 & 0 \\
0 & e^{ik^-l_{duct}} & 0 & -1 \\
\end{pmatrix}
\begin{bmatrix}
p_1^+ \\
p_1^- \\
p_2^+ \\
p_2^- \\
\end{bmatrix} = \mathbf{0}
\]  

(D.3)

The definitions of 1, 2 and \(l_{duct}\) are given in figure D.1. \(k^+\) is defined as \(k^+ = \omega/(c_0 + U)\) and \(k^-\) as \(k^- = \omega/(c_0 - U)\).

The viscous effects are neglected in equation (D.3). The key quantity which tells if it is allowed to neglect the viscous effects is the shear wave number \((s)\) (Beltman (1998)):

\[
s = l \sqrt{\frac{\rho_0 \omega}{\mu}}
\]  

(D.4)

, in this equation is \(l\) a length scale. For a circular tube it is the diameter of the tube. The shear wave number is the ratio of the diameter of the tube over the thickness of the viscous boundary layer.

It is allowed to use the wave equation if the shear wave number is much larger than one \((s >> 1)\), which means that the boundary layer thickness is much smaller than the diameter of the tube. Typical values for the combustion rig in this thesis are: \(l = D_{min} \approx 0.02\) m, \(\rho_0 \approx 0.5\) kg m\(^{-3}\), \(\omega \approx 10^5 \cdots 10^7\) rad/s, \(\mu \approx 10^{-5}\) kg m\(^{-3}\) s\(^{-1}\). For these values \(s\) has a value between 14.4 and 1.44 \(\times 10^4\). Hence it is allowed to neglect the viscous effects in this test rig. For a gas turbine (where \(\rho \) and \(l\) are much larger) it will in general also be allowed to neglect the viscous effects.

D.2 Connection element

At the joint of two elements conservation of the acoustic mass flow and the acoustic potential should be fulfilled, given by the linearized one-dimensional mass conservation equation
integrated over the joint. This equation is (see figure D.2 for the definitions):

\[ i\omega \int_{Vol} \rho \hat{d}V + S_2 \left( \bar{p}_2 \bar{u}_2 + \bar{u}_2 \frac{\bar{p}_2}{c_2^2} \right) - S_1 \left( \bar{p}_1 \bar{u}_1 + \bar{u}_1 \frac{\bar{p}_1}{c_1^2} \right) = 0 \]  

(D.5)

The volume of the joint, \( Vol \) in the first term at the left hand side, can be estimated from \( S_2 \cdot l_{joint} \).

The acoustics are treated as two traveling waves, the amplitude of the acoustic velocity can then be written as:

\[ u = \frac{p^+ - p^-}{\rho_c} \]  

(D.6)

With this equation for the acoustic velocity, equation (D.5) is rewritten to:

\[ i\omega \left[ \frac{1}{c} \right] S_2 l_{joint} + \frac{S_2}{c_2^2} \left( p_2^+ (1 + Ma) + p_2^- (Ma - 1) \right) - \frac{S_1}{c_1^2} \left( p_1^+ (1 + Ma) + p_1^- (Ma - 1) \right) = 0 \]  

(D.7)

A spatially averaged value for \( \bar{p} \) and \( c \) should be taken in the term between the square brackets. The relative magnitude of the accumulation term (the first term in the left hand side) to the other terms is \( \frac{\omega l_{joint}}{c} = k \cdot l_{joint} = \frac{2\pi l_{joint}}{\lambda} \), with \( \lambda \) the acoustic wave length. In general the length of the joint is much smaller than the acoustic wave length term and the first term in the left hand side can be neglected.

Because the acoustic velocity is irrotational and viscous effects can be neglected, it can be described with Bernoulli’s equation (Hirschberg and Rienstra (1992)). The Fourier transform in time of the linearized one-dimensional version of Bernoulli’s equation is used to calculate the acoustic pressure difference across the joint:

\[ i\omega \nabla \left( \phi_2 - \phi_1 \right) + \left( \bar{u}_2 \bar{u}_2 + \bar{p}_2 \rho_2 - \bar{p}_1 \bar{u}_1 + \bar{p}_1 \rho_1 \right) \bar{u}_2 - \bar{p}_1 = 0 \]  

(D.8)

where \( \phi \) is a scalar velocity potential (\( \nabla \phi = \mathbf{u} \)).

The first term at the left hand side of equation (D.8) can be written as \( i\omega \nabla \phi_{joint} \bar{u}_2 \).

The relative magnitude of this term is again \( \omega l_{joint}/c \). This term can be neglected for low frequencies. Equation (D.8) states that, in the absence of a mean flow, the acoustic pressure is constant across the joint.
The matrix for the joint of two tubes is:

\[
\begin{pmatrix}
\frac{S_2}{S_1} (-1 - Ma_1) & \frac{S_2}{S_1} (1 - Ma_1) & \frac{S_2}{S_1} (1 + Ma_2) & \frac{S_2}{S_1} (Ma_2 - 1) \\
-1 - Ma_1 - Ma_1^2 & -1 + Ma_1 - Ma_1^2 & 1 + Ma_2 + Ma_2^3 & 1 - Ma_2 + Ma_2^3
\end{pmatrix}
\]  \hspace{1cm} (D.9)

In this discussion of the joint element it is assumed that the flow follows the expansion (or contraction) smoothly. This will only be the case if the cross sectional area changes gradually. For a sudden change of cross sectional area, the flow is not smooth and the acoustics will be different from the case of gradual changes.

D.3 Special elements

D.3.1 Burner element

The standard acoustic elements are derived for the case that the mean flow affects only the propagation of sound. Such an approach is not valid for all parts of the combustion installation. Especially for parts with a sudden contraction (or expansion) or parts where the direction of the flow is changed (e.g. a swirler) the mean flow can affect the acoustics strongly. A typical component in the combustion installation with such a behavior is the burner mouth (the fuel/air injector or the swirler). A special acoustic element will be defined, which is called the burner element. To quantify this element, measurements are generally necessary.

The element is based on the integrated linearized versions of the one-dimensional mass conservation equation and the one-dimensional Navier Stokes equation. The integrated one-dimensional equation for the conservation of mass is given in equations (D.5) and (D.7). It is assumed that the length of the burner mouth is short. The accumulation term in the mass conservation equation can then be neglected, see also section D.2. The integrated one-dimensional Navier Stokes equation is (the density fluctuation in the first term in the left hand side is neglected):

\[
\begin{align*}
\quad & i\omega S_{burner} \bar{p} \bar{u}_{burner} + 2\bar{p}_1 S_1 (\bar{u}_2 - \bar{u}_1) \bar{u}_1 - F(\bar{u}_{burner}) = -(\bar{p}_2 S_2 - \bar{p}_1 S_1)
\end{align*}
\]  \hspace{1cm} (D.10)

where \( \bar{u}_{burner} \) is the (spatially) averaged velocity fluctuation in the burner, \( \bar{p} \) is the average density in the burner and \( F(\bar{u}_{burner}) \) is the force on the flow in the burner. This force can be caused by viscous friction, swirl generation or a sudden expansion. It is assumed that this force is only a function of the velocity fluctuation.
Figure D.4: A schematic picture of the acoustic flame element. The control volume over which is integrated is indicated with the dotted line.

Now a burner transfer function is defined as:

\[
H_{burner}(\omega) = -\frac{p_2 S_2 - p_1 S_1}{u_1}
\]  

(D.11)

where the velocity fluctuation in the burner \((\bar{u}_{burner})\) is written as the velocity fluctuation upstream of the burner \((\bar{u}_1 = \bar{u}_{burner})\).

The matrix in equation (D.1) for the burner is:

\[
\begin{bmatrix}
\frac{S_2}{c_2}(-1 - M_{a1}) & \frac{S_1}{c_2}(-1 - M_{a1}) & \frac{S_2}{c_2}(1 + M_{a2}) & \frac{S_2}{c_2}(M_{a2} - 1) \\
\frac{S_2}{c_2}(-1 - M_{a1}) & \frac{S_1}{c_2}(-1 - M_{a1}) & \frac{S_2}{c_2}(1 + M_{a2}) & \frac{S_2}{c_2}(M_{a2} - 1) \\
\frac{S_2}{c_2}(-1 - M_{a1}) & \frac{S_1}{c_2}(-1 - M_{a1}) & \frac{S_2}{c_2}(1 + M_{a2}) & \frac{S_2}{c_2}(M_{a2} - 1) \\
\frac{S_2}{c_2}(-1 - M_{a1}) & \frac{S_1}{c_2}(-1 - M_{a1}) & \frac{S_2}{c_2}(1 + M_{a2}) & \frac{S_2}{c_2}(M_{a2} - 1)
\end{bmatrix}
\]  

(D.12)

The source vector in the matrix equation equals zero.

From the integrated one-dimensional Navier Stokes equation it follows that the (modeled) burner transfer function will show the following behavior:

\[
H_{burner}(\omega) = i\omega S_{burner} l_{burner} \bar{\rho} + 2\bar{\rho}_1 S_1 (\bar{u}_2 - \bar{u}_1) - \frac{dF}{du}
\]  

(D.13)

D.3.2 Flame element

The acoustic behavior of the flame in the system can also be represented by a transfer matrix. A distinction is made between the flame as a source of sound and the flame as an amplifier of sound. For the flame as amplifier of sound the flame transfer function is introduced, more details about this flame transfer function can be found in section 4.4.

The flame matrix is derived from the integration of the one-dimensional linearized mass conservation equation over the control volume and from the linearized Bernoulli’s equation, see figure D.4. The linearized one-dimensional Bernoulli equation is given in equation (D.8). The mass conservation equation is repeated here because the heat release by the flame adds a source term to this equation. The integrated version of the linearized one-dimensional mass conservation equation is:

\[
i\omega \tilde{\rho}_{flame} S_2 + S_2 \left( \frac{\tilde{p}_2 \bar{u}_2 + \bar{p}_2}{c_2^2} \right) - S_1 \left( \frac{\tilde{p}_1 \bar{u}_1 + \bar{p}_1}{c_1^2} \right) = \frac{\gamma - 1}{c_2^2} \dot{Q}
\]  

(D.14)
The source term $\hat{Q}$ is the complex amplitude of the total fluctuating heat release, defined by: $\hat{Q} = \int_{V_m} q dV$. If the flame region is small compared to the acoustic wave length ($l_{\text{flame}} << \lambda$) the first term at the left hand side of the equations (D.14) and (D.8) can be neglected.

The source term $\hat{Q}$ is split into two parts: one that describes the noise generation (the flame as source), $Q_{\text{noise}}$, and the other that describes the flame as amplifier ($Q_{\text{amp}}$). For the latter the flame transfer function is used. In section 4.2 several mechanisms are mentioned which describe the coupling between the acoustic variables and the heat release by the flame. It appears from that section that the flucutating heat release by the flame is mainly a function of the velocity fluctuation in the burner mouth. The flame transfer function is therefore defined as:

$$\gamma - \frac{1}{c^2} Q_{\text{amp}} = H_{\text{flame}} \cdot \bar{u}_1$$  \hspace{1cm} (D.15)

With this flame transfer function and the autonomous noise generation by the flame ($Q_{\text{noise}}$) the following matrix in equation (D.1) for the flame is found:

$$\begin{bmatrix}
\begin{array}{ccc}
\frac{\rho_1}{c_1^2} & -\frac{\rho_1}{c_1^2} & \frac{\rho_2}{c_2^2} (Ma_2 + 1) & \frac{\rho_2}{c_2^2} (Ma_2 - 1) \\
\frac{H_{\text{flame}}}{\rho_1 c_1^2} & \frac{H_{\text{flame}}}{\rho_1 c_1^2} & 0 & 0 \\
-1 + Ma_1 - Ma_2 & -1 + Ma_1 - Ma_2 & 1 + Ma_2 + Ma_2^2 & 1 - Ma_2 - Ma_2^2
\end{array}
\end{bmatrix}$$  \hspace{1cm} (D.16)

The source vector in equation (D.1) equals:

$$\begin{bmatrix}
\frac{\gamma - 1}{c^2} Q_{\text{noise}} \\
0
\end{bmatrix}$$  \hspace{1cm} (D.17)

D.4 Elements for acoustic boundaries

D.4.1 An open end

At an acoustic open end the total acoustic pressure should vanish. The total acoustic pressure can be calculated from the Bernoulli’s equation. The boundary condition for an open end is:

$$\hat{p}_{\text{total}} = \hat{p} + \bar{\rho} \bar{u} \bar{a} = 0$$  \hspace{1cm} (D.18)

, with $\bar{u}$ the mean axial ($u$) velocity at the open end.

In reality the open end is not at the exit of the tube but at some distance from it because of the effect of entrained flow outside the tube. Therefore a correction length is introduced. The correction length for an unflanged circular pipe without mean flow is (Munjal (1987)):

$$\Delta L = 0.61 r_s$$  \hspace{1cm} (D.19)

, where $r_s$ is the radius of the pipe. In the presence of the mean flow an extra correction should be used. If the open boundary condition is at the exit of the pipe this correction is small and can be neglected. If the open boundary is at the entrance of the pipe, this extra correction can be large as given by Davies (1988). This extra correction for the mean flow is neglected here.
The matrix equation for an open end is:

\[
\begin{bmatrix}
(1 + Ma)e^{-ik^+\Delta L} & (1 - Ma)e^{ik^-\Delta L}
\end{bmatrix} \cdot \begin{bmatrix}
p^+ \\
p^-
\end{bmatrix} = 0
\] (D.20)

D.4.2 A closed end

At a closed end, a wall, the acoustic velocity fluctuations normal to the end should disappear. The boundary condition can be expressed as:

\[
\ddot{u} = \frac{1}{\rho_c} (p^+ - p^-) = 0
\] (D.21)

This gives in the following matrix equation:

\[
\begin{bmatrix}
1 & -1
\end{bmatrix} \cdot \begin{bmatrix}
p^+ \\
p^-
\end{bmatrix} = 0
\] (D.22)

D.4.3 A choked end

If the mean flow at the exit is choked by a sonic nozzle, the mass flow is constant through the exit. The acoustic mass flow fluctuation should also be zero:

\[
\ddot{\rho} + \ddot{u} = p^+ (1 + Ma) - p^- (1 - Ma) = 0
\] (D.23)

This equation should be evaluated at the beginning of the sonic nozzle \((Ma << 1)\). It results in the next matrix equation:

\[
\begin{bmatrix}
1 + Ma & Ma - 1
\end{bmatrix} \cdot \begin{bmatrix}
p^+ \\
p^-
\end{bmatrix} = 0
\] (D.24)

If there is no mean flow \((Ma = 0)\) this equation equals the equation for a closed end, equation (D.22).

D.5 The matrix description of the system

The different elements should be joined to get the matrix description of the system. The general shapes for the 2 x 4 matrices which describe the propagation of sound and the matrices for a flame and a burner are given in equation (D.1). These matrices have the elements \(M_1\) to \(M_8\). The 1 x 2 matrix that describes an acoustic boundary is given in equation (D.2). This matrix has the elements \(E_1\) and \(E_2\).

As an illustration of the joining of the different components an example, shown in figure D.5, is discussed. The system consists of combustion chamber \((M^4)\) with a flame \((M^3)\). At the exit of the combustion chamber there is an acoustic boundary \((E^5)\). Upstream of the combustion chamber is a supply tube \((M^2)\), with another diameter. At the upstream end of this supply tube is another acoustic termination \((E_1)\). The matrix equations for the
different components can be combined to a system matrix equation:

\[
\begin{bmatrix}
E_1^1 & E_2^1 & 0 & 0 & 0 & 0 & 0 & 0 \\
M_1^1 & M_2^1 & M_3^2 & M_4^2 & 0 & 0 & 0 & 0 \\
M_5^2 & M_6^2 & M_7^2 & M_8^2 & 0 & 0 & 0 & 0 \\
0 & 0 & M_1^3 & M_2^3 & M_3^3 & M_4^3 & 0 & 0 \\
0 & 0 & M_5^3 & M_6^3 & M_7^3 & M_8^3 & 0 & 0 \\
0 & 0 & 0 & M_1^4 & M_2^4 & M_3^4 & M_4^4 & 0 \\
0 & 0 & 0 & 0 & M_5^4 & M_6^4 & M_7^4 & M_8^4 \\
0 & 0 & 0 & 0 & 0 & M_1^5 & M_2^5 & M_3^5 & M_4^5 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
p_1^+ \\
p_1^- \\
p_2^+ \\
p_2^- \\
p_3^+ \\
p_3^- \\
p_4^+ \\
p_4^-
\end{bmatrix} = \frac{1}{\zeta - Q_{noise}} \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\] (D.25)

Many elements in this system matrix are zero. The matrix is normally a band matrix, just as here.

### D.6 The solution of the matrix equation

The solution of the matrix equation (D.25) can be separated into two parts: 1. the solution of the homogeneous equation \(Q_{noise} = 0\) and 2. the solution of the inhomogeneous equation \(Q_{noise} \neq 0\). The solution of the inhomogeneous equation gives the response of the system driven by a source of sound, this is not discussed here.

The solution of the homogeneous part of the matrix equation is most interesting. If such a solution exists there may be sound waves in the system without forcing by an external source term. The matrix equation (D.25) has solutions of the homogeneous equation if the determinant of the matrix equals zero. The frequencies for which the determinant equals zero are the eigenfrequencies of the system. In general these eigenfrequencies have a real and an imaginary part \((\omega = \omega_r + i\alpha)\). The real part \((\omega_r)\) gives the frequency of the oscillation in rad/s, the imaginary part \((\alpha)\) the growth rate \((1/s)\). Both parts of the eigenfrequency can be determined from the solution of the equation for the determinant (which also has a real and an imaginary part). The sign of \(\alpha\) gives the occurrence of instability: if \(\alpha\) is positive the oscillation will decrease in time and an instability does not exist at that frequency. If \(\alpha\) is negative the amplitude of the oscillation will grow exponentially in time and there is an instability.

The disadvantage of the determinant method is that one needs to search for the solution of the determinant equation. The plot of the determinant is very steep near the eigenfrequency, what makes it difficult to find the solution both to the real part and the imaginary part.
simultaneously. This is especially the case if the transfer matrices are only known at the number of discrete frequencies, for example if they are known from measurements.

Another method is therefore used to study the acoustic system behavior. This method is based on the Nyquist criterion from control theory. This method is very similar to the method used by Deuker (1995) to study the acoustics of a combustion installation.

The Nyquist criterion is used to study the stability behavior from systems with a feedback loop. A combustion installation can also be treated as a system with a feedback loop, see figure D.6 for a simplified control scheme of the combustion installation. A sound wave is generated by the flame ($p_1^+$), propagates to the end of the combustion chamber ($p_2^+$) and is reflected at the exit of the combustion chamber ($p_2^-$). The reflected sound wave travels upstream to the flame ($p_1^-$) where it affects (together with $p_1^+$) the sound generation by the flame. It is possible to split the transfer matrices that describe the propagation of sound into a number of one-dimensional transfer functions. Figure D.7 shows how the system can be treated as a control loop with unit feedback. The transfer function of the open system (without feedback) from figure D.7 is:

$$H_{\text{open}}(\omega) = \frac{p_{2a}}{p_{2a}}$$  \hspace{1cm} (D.26)

The transfer function of the closed system (with unit feedback, the phase shift over the feedback loop is zero) is then:

$$H_{\text{closed}}(\omega) = \frac{H_{\text{open}}}{1 - H_{\text{open}}}$$  \hspace{1cm} (D.27)

According to Nyquist’s criterion, instability occurs if in a polar plot the transfer function of the open system ($H_{\text{open}}$) crosses the real axis right from the point 1 (positive feedback). In a bode plot, this means that the amplification factor of $H_{\text{open}}$ should be larger than one for frequencies that the argument (phase shift) of $H_{\text{open}}$ crosses the zero-axis. The instability
frequency is found from the point of $H_{\text{open}}$ in the polar plot with the minimum distance to the point 1 on the real axis (Polišček et al. (1997)).

To transform the matrix equation (e.g. equation (D.25)) to a transfer function equation to be able to use the Nyquist criterion, the matrix equation is written in a somewhat different form. This is illustrated with the figures D.6 and D.7. The matrix equation for figure D.6 is:

$$[M] \cdot \begin{bmatrix} p_1^i \\ p_1^1 \\ p_2^1 \\ p_2 \end{bmatrix} = 0 \quad (D.28)$$

This equation is rewritten to the following equation, in which the incoming variable of $H_{\text{open}}$ ($p_{\text{2a}}^i$) is moved to the right hand side (see figure D.7 for the definitions):

$$[M^*] \cdot \begin{bmatrix} p_1^i \\ p_1^1 \\ p_2^1 \\ p_{\text{2b}} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot p_{\text{2a}} \quad (D.29)$$

The matrix $[M^*]$ is another matrix than $[M]$ in equation (D.28). Multiplying the left hand side and the right hand side of equation (D.29) with $[M^*]^{-1}$ gives the outgoing variable $p_{\text{2b}}$ as function of the incoming variable $p_{\text{2a}}^i$ ($H_{\text{open}}$). With this $H_{\text{open}}$ the Nyquist criterion can be checked.

The growth rate ($\alpha$) of the amplitude of the pressure fluctuations in the system can be calculated from $H_{\text{open}}$. In the time domain there is a time delay, say $\Delta t$, between the input ($p_{\text{2a}}^i$) and the output ($p_{\text{2b}}$), mainly due to the propagation of sound. The output after this time delay is $[H_{\text{open}}] \cdot p_{\text{2a}}^i$, which is the new input for the system. So after a time of $n \cdot \Delta t$ the amplitude of the input has grown by a factor $[H_{\text{open}}]^n$. The growth rate from $H_{\text{open}}$ is also:

$$\alpha = -\frac{\ln([H_{\text{open}}])}{\Delta t} \quad (D.30)$$
Appendix E

Comparison of the transfer matrix method with the mode analysis method

In this appendix the transfer matrix method, described in appendix D is compared with the mode analysis method, explained in section 2.5. Two quantities are compared: the instability frequency and the growth rate. The comparison is performed for a Rijke tube. The same case was used by Deuker (1995) to test the Nyquist criterion for the matrix method.

The equations for the Rijke tube are based on the paper by Heekl (1988), who performed a study on active control of the noise from a Rijke tube. The equations will not be repeated here in detail. Heekl determined the eigenfrequencies of the system from the system matrix. Instead of using the Nyquist criterion she determined for which (complex) frequency the determinant of the system matrix was zero.

For the Rijke tube three different models will be compared: (i) The matrix method with the Nyquist criterion (see appendix D), (ii) The method based on the determinant of the matrix and (iii) The mode analysis method (see section 2.5). The influence of damping is also studied, this gives three different situations for which the three models can be compared: (i) No damping, (ii) Damping due to wall friction and (iii) Damping due to non ideal reflection at the boundaries.

The Rijke tube is a straight tube with two open ends and a heat source, which can be a hot gauze. There is a coupling between the pressure in the tube and the heat transfer from the gauze. In a number of situations this coupling results in pressure oscillations with a large amplitude. Rijke was the first to describe this phenomenon. The requirements for the instability are:

1. There must be an air flow through the tube (which can be caused by natural convection from the hot gauze if the tube is held vertically). In the situation studied by Heekl an air flow is forced through the tube by sucking the air in.

2. The gauze has to be in the upstream half of the tube. This can be explained by the phase between the heat release and the pressure.
3. The gauze should be so hot that the energy transfer from the heated gauze to the acoustic system overcomes the damping in the system.

The fluctuating heat release from the gauze as function of the velocity fluctuation at the time $t - \tau$ can be written as:

$$Q'(t) = \beta u'(t - \tau)$$  \hspace{1cm} (E.1)

with

$$\beta = L_{gauze} (T_{gauze} - T_{air}) \sqrt{\pi \lambda_C \rho d/2|/u_{air}}$$  \hspace{1cm} (E.2)

where $d$ is the diameter of the gauze wire, $L_{gauze}$ the total length of the gauze wire and $\lambda$ the heat conduction coefficient of air.

The 'gauze transfer function', the transfer function between the heat transfer from the gauze and the velocity fluctuation at the gauze is:

$$H_{gauze}(\omega) = \frac{\gamma - 1}{c^2 \rho S} \frac{1}{u(\omega)} = \frac{\gamma - 1}{c^2 \rho S} \beta e^{-i\omega \tau}$$  \hspace{1cm} (E.3)

In addition to this transfer function, it is assumed that the pressure is constant over the gauze. This assumption is based on the fact that the gauze is very thin and that the static pressure drop across the gauze is small.

The parts upstream and downstream of the gauze can be described with the equations for the propagation and the reflection of sound.

The matrix for the Rijke tube is:

$$\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & e^{-i|f \omega / c|} & -\beta_0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & e^{-i|f \omega / c|} & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & e^{-i|f \omega / c|} & 0 & -1 & 0 & 0 \\
0 & 0 & -1 & e^{-i|f \omega / c|} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & e^{-i|f \omega / c|} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & e^{-i|f \omega / c|} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & e^{-i|f \omega / c|} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{-1} R_L \end{bmatrix}$$  \hspace{1cm} (E.4)
\[
\frac{d^2 \eta_n}{dt^2} + \left( 2 \zeta \omega_n - \frac{c^2}{\int \psi_n^2 dV} \frac{\gamma - 1}{c^2} \psi_n(l) \frac{d \psi_n / dx}{\rho \omega} - \beta \omega \right) \frac{d \eta_n}{dt} + \omega^2_n \eta_n = 0
\] (E.5)

Because the tube has an open acoustic end at the upstream boundary \((x = 0)\) the following weighting function \(\psi_n\) is used:

\[
\psi_n = \sin (k_n x)
\] (E.6)

The different parameters in equation (E.4) and (E.5) are defined in table E.1.

The following situations are studied:

1. No damping: \(R_0 = R_L = -1\). For the mode analysis method: \(\zeta = 0\).

2. Damping due to wall friction. For the computations with the matrix methods the next equation is used for the wave number: \(k = \Gamma \omega / c = \left( 1 + 0.005903k \right) \omega / c\). For the mode analysis method the damping factor is: \(\zeta = 0.005903\).

3. Damping due to non-ideal reflection at the boundaries: \(R_0 = R_L = -0.986 + 0.12 \cdot i\).

This reflection factor can also be treated (at \(\omega_n\)) as an elongation of the tube of \(\Delta L = 0.019\) m together with non ideal reflection \(R_0^+ = R_L^+ = |R_0| = 0.993\). This gives a damping factor for the mode analysis method of: \(\zeta = ln(0.993^2) / (2\pi)\).

It appears that for the mode analysis method, it is not allowed to calculate the instability frequency and growth rate using the equations (2.28) and (2.29): the assumptions of a small deviation from the isentropic situation are not valid for the present case. The instability frequency and the growth rate for the mode analysis method should be calculated from equation (E.5) by means of iteration.

The results of the three methods are presented in table E.2. In the calculation with the mode analysis method only one eigenfunction is used \((n = 1)\). This table shows that the results of both matrix methods are very similar, there is a small deviation in \(\alpha\) for both methods.

The difference between the two matrix methods and the mode analysis method is due to the use of an approximate profile for the pressure in the mode analysis method. This gives a different (wrong) velocity fluctuation at the gauze and therefore a slightly different behavior. This could be improved by the use of more than one eigenfunction.

The comparison of the three methods gives good confidence that each of these methods is suitable to model the thermo-acoustic behavior of a cavity with a heat source.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta)</td>
<td>286.7 kg m s(^{-2})</td>
<td>(c)</td>
<td>358 m s(^{-1})</td>
</tr>
<tr>
<td>(\tau)</td>
<td>0.15 ms</td>
<td>(A)</td>
<td>0.00156 m(^2)</td>
</tr>
<tr>
<td>(\omega_n)</td>
<td>1250 rad s(^{-1})</td>
<td>(l)</td>
<td>0.3 m</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>1.4</td>
<td>(L)</td>
<td>1.0 m</td>
</tr>
</tbody>
</table>

Table E.1: The parameters used in the calculations for the Rijke tube.
### Comparison of the transfer matrix method with the mode analysis method

<table>
<thead>
<tr>
<th>No Damping</th>
<th>Matrix Nyquist</th>
<th>Matrix determinant</th>
<th>Mode analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f = 186.4$ Hz</td>
<td>$f = 186.4$ Hz</td>
<td>$f = 187.1$ Hz</td>
<td></td>
</tr>
<tr>
<td>$\alpha = -7.7$</td>
<td>$\alpha = -7.2$</td>
<td>$\alpha = 9.3$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\Gamma = 1 + 0.006i$</th>
<th>$f = 186.4$ Hz</th>
<th>$f = 186.4$ Hz</th>
<th>$f = 187.1$ Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = 1.3$</td>
<td>$\alpha = 1.3$</td>
<td>$\alpha = 3.3$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$R = -0.986 + 0.12i$</th>
<th>$f = 179.5$ Hz</th>
<th>$f = 179.5$ Hz</th>
<th>$f = 180.1$ Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = 4.5$</td>
<td>$\alpha = 4.6$</td>
<td>$\alpha = 5.93$</td>
<td></td>
</tr>
</tbody>
</table>

Table E.2: The results for the instability frequency and the growth rate with the three different methods, $\alpha$ is the imaginary part of the frequency.
Appendix F

Analysis of the experimental method

Because the experimental method to determine the flame transfer function is a new method, the sensitivity of the method to errors in the input values is studied. These errors can be caused by inaccuracies in the experimental equipment or they are caused by ill estimations of certain parameters, like the average speed of sound.

The influence of the following errors is studied:

1. Error in the estimated average speed of sound.
2. Error in the estimated Mach number.
3. Inaccuracies in one of the measured transfer functions.

The effect of errors in the geometry, like the cross sectional area and the axial location of the microphones is not studied because these values can be determined with great accuracy.

To analyze the influence of the different errors a representative case for the used experimental setup is studied. For this case a flame transfer function is assumed. This assumed flame transfer is the input for an acoustic model which gives the transfer functions that should be measured ($T_f$ and $T_{II}$). These transfer functions ($T_f$ and $T_{II}$) are the input for

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_{burner}$</td>
<td>430 m/s</td>
</tr>
<tr>
<td>$c_{uc}$</td>
<td>700 m/s</td>
</tr>
<tr>
<td>$x_a$</td>
<td>0.22 m</td>
</tr>
<tr>
<td>$x_b$</td>
<td>0.805 m</td>
</tr>
<tr>
<td>$x_c$</td>
<td>1.0 m</td>
</tr>
<tr>
<td>$A_{burner}$</td>
<td>$4.24 \cdot 10^{-4}$ m$^2$</td>
</tr>
<tr>
<td>$A_{uc}$</td>
<td>$7.85 \cdot 10^{-3}$ m$^2$</td>
</tr>
<tr>
<td>$U_{burner}$</td>
<td>46 m/s</td>
</tr>
<tr>
<td>$U_{uc}$</td>
<td>18 m/s</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1.3</td>
</tr>
<tr>
<td>$R$</td>
<td>$6.13 \cdot 10^{-4}$ kg/m</td>
</tr>
<tr>
<td>$\phi$</td>
<td>$-\omega \cdot \tau (\tau = 2 \text{ ms})$</td>
</tr>
<tr>
<td>$Refl$</td>
<td>0</td>
</tr>
</tbody>
</table>

Table F.1: The parameters used in the acoustic model to study the sensitivity of the flame transfer function calculation. $Refl$ is the reflection factor at the exit of the combustion chamber.
Figure F.1: The influence of a wrong estimation of the speed of sound on the flame transfer function.

The acoustic calculation of the flame transfer function as described in the previous section. To study the sensitivity, inaccuracies are introduced in $T_I$ and/or $T_{II}$ or wrong values are set for the speed of sound and the Mach number in the calculation of the flame transfer function from $T_I$ and $T_{II}$. The burner transfer function is set to zero.

The parameters of the studied cases are given in table F.1. For the flame transfer function the following well known shape is taken: $H_{flame} = R \cdot e^{-kz}$.

### F.1 Error in the speed of sound

The influence of an error in the estimation of the average speed of sound is shown in figure F.1. Errors in the average speed of sound in the supply (burner) and the combustion chamber are studied separately. Variations of 10% in the average speed of sound are studied. In the figures the calculated amplification factor and phase delay of the flame transfer function are plotted.

The error in the amplification factor increases with increasing frequency. This is due to the fact that the acoustic wave number increases linearly with the frequency, for higher frequencies the influence of errors due to the term $e^{ikz}$ will become larger. For the given frequency range (0 .. 500 Hz) the relative error in the amplification factor is at maximum about 16%.

For the phase delay the error increases also with increasing frequency. Mainly errors in the average speed of sound in the combustion chamber have influence, this can be explained by the wrong estimation of the acoustic time delay between the microphones. Errors in the speed of sound of the burner don’t have a large influence on the phase delay.
F.2 Error in the Mach Number

The influence of the Mach number, both on the amplitude and the phase is very small for frequencies above 20 Hz, see figure F.2. An error of 10 percent in the Mach number gives at maximum an error of 1 percent in the amplification factor. The Mach number in the combustion chamber has the largest influence, especially on the phase plot.

F.3 Errors in the measured transfer function

Both the effects of inaccuracies in the phase and the amplification factor are studied. These inaccuracies may have their origin in the connection of the microphones to the combustion chamber, the microphones, the microphone amplifiers or the FFT analysis. These experimental inaccuracies can be reduced by calibration.

The maximum inaccuracy in the measured amplification factor is ±5%. The amplification factor of $T_f$ and $T_{II}$ are multiplied by 0.95 and 1.05, the resulting flame transfer function is plotted in figure F.3.

The calculation of the flame transfer function is especially sensitive to inaccuracies in the measured amplification factor at low frequencies. In this case the flame transfer function amplification factor is at maximum overpredicted by a factor 8 at 10 Hz (for $R_f = 0.95 \times R_f$ or $R_{II} = 1.05 \times R_{II}$), above 50 Hz the miss prediction is less than 25 % (1 dB) for all cases. The error in the phase of the flame transfer function is at largest at low frequencies (below 150 Hz) for all four cases. From both the phase plot and the amplitude plot it shows that the resulting flame transfer function for a measured transfer function $R_f = 0.95 \times R_f$ is very similar to that for a measured flame transfer function $R_{II} = 1.05 \times R_{II}$ and vice versa. This makes it easy to check if this kind of error occurs by interchanging the two microphones in
Figure F.3: The influence of an inaccuracy of 5% in the measured amplification factor on the flame transfer function.

Figure F.4: The influence of an inaccuracy of two degrees in the measured transfer function on the flame transfer function.
the combustion chamber.

To study the sensitivity to errors in the measured phase of the flame transfer function, two degrees are added to (or subtracted from) the 'measured' phase, see figure F.4. In reality this error will be smaller (at maximum 0.5 degrees). The maximum errors in the amplification factor are a factor 5 (+7 dB) at 10 Hz for $\phi_I = \phi_I + 2$ and $\phi_{II} = \phi_{II} - 2$ and a factor 0.06 (-12 dB) for $\phi_I = \phi_I - 2$ and $\phi_{II} = \phi_{II} + 2$ at 35 Hz. Above about 100 Hz the effect on the phase of the flame transfer function of the error in the measured phase becomes very small. The occurrence of this kind of error can again be checked by interchanging the two microphones in the combustion chamber.

### F.4 Conclusions

From all figures it shows that the present method to determine the flame transfer from measurements is especially sensitive to errors at low frequencies. The effect of the errors can be minimized by good calibration. The occurrence of most errors can easily be checked by interchanging the microphones.

A similar study has been performed to study the influence of inaccuacies on the determination of the burner transfer function. It showed that the calculation of the burner transfer function from the measurements is also sensitive to these errors and inaccuacies. The results are not presented here.
Appendix G

The two-microphone method

The two microphone method is applicable for one-dimensional acoustics. For this situation the acoustics can be described with two traveling waves with the amplitudes $p^+$ and $p^-$, see figure G.1. The (complex) pressure amplitude, that is measured by microphone $a$ is (the acoustic damping is neglected):

$$ \tilde{p}_a = p^+e^{-ik^+x_a} + p^-e^{-ik^-x_a} \quad (G.1) $$

where

$$ k^+ = \frac{\omega}{c(1-M_0)} \quad \text{and} \quad k^- = \frac{\omega}{c(1-M_0)} $$

with $c$ the speed of sound. For the pressure amplitude measured by microphone $b$ ($\tilde{p}_b$) a similar equation as that for microphone $a$ can be stated, only $x_a$ is replaced by $x_b$.

The transfer function between microphone $b$ and $a$ is measured:

$$ H_{meas} = \frac{\tilde{p}_b}{\tilde{p}_a} \quad (G.2) $$

From this measured transfer function the ratio of the two traveling waves at $x = 0$ can be calculated (if only reflection takes place at $x = 0$):

$$ \frac{p^+}{p^-} = e^{i(k^-+k^+)x} \frac{H_{meas} - e^{i\Delta x}}{e^{-i(k^+\Delta x} - H_{meas}} \quad (G.3) $$

![Figure G.1: The basis of the two microphone technique is that the acoustics can be described with two traveling waves.](image-url)

173
with $\Delta x = x_b - x_a$

The amplitudes of the two pressure waves ($p^+$ and $p^-$) can be determined from the autospectrum measurement of one of the two microphones, using the equations (G.3) and (G.1).
Curriculum Vitae

Sikke Ate Klein was born on September, 17th in Hengelo (Ov).

1983-1989  Gymnasium β at the Carolus Clusius College in Zwolle.

1989-1995  Student at the Department of Mechanical Engineering of the University of Twente, main subject Thermal Engineering.

1994      Practical training period at Siemens KWU gas turbines in Mülheim an der Ruhr (D) and at the test rig in Berlin.

1995-1999  PhD student at the University of Twente,
Laboratory of Thermal Engineering.

2000-  Employee of Akzo Nobel, business unit Energy.
Function: technologist.