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ON THE OPTICAL PERFORMANCE OF
THE LONG PULSE XECL* EXCIMER LASER

PROEFSCHRIFT

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Dr. ir. ing. F.A. van Goor
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Preface

This thesis is based on the results of four years of research work in the Quantum Electronics group of the Faculty of Applied Physics at the University of Twente. The work has been performed as a part of the research programme of the Netherlands Technology Foundation (STW). The goal of the work was to investigate and improve the beam quality of the long pulse XeCl* excimer laser, which has been developed in the Quantum Electronics group and which is currently being developed into a commercial laser system by Nederlands Centrum voor Laser Research (NCLR) B.V. This thesis describes the theoretical and experimental aspects of the beam quality improvement studies.

In the first chapter the basics of laser physics are reviewed. The keywords in laser physics that are important for this thesis are treated in brief. The second chapter describes the laser system used in the experiments: a long pulse, relatively low gain XeCl* excimer laser. This chapter is mainly based on the thesis of my predecessor, John Timmermans. When studying the spatial beam quality, it would be very practical to be able to describe the quality of the beam with some parameter. In chapter 3 an overview of different beam quality numbers and their background is given.

The final two chapters form the main part of this thesis. In these chapters the experimental work is described together with the theoretical background and some modelling. In chapter 4 different resonator configurations to improve the spatial beam quality are discussed. Chapter 5 describes the experiments that were performed to improve the spectral quality of the beam.

Ramon Hofstra
List of publications

This thesis is based on the following publications and conference presentations.


Chapter 1

Introduction

The history of the laser starts in the beginning of this century when Albert Einstein introduced the concept of stimulated emission in 1917 [1]. He showed that the process of stimulated emission must exist to balance in thermal equilibrium the processes of absorption and spontaneous emission. This process of stimulated emission is the basis for a laser. The word laser is an acronym for Light Amplification by Stimulated Emission of Radiation.

Despite the early theoretical basis, it lasted until 1953 before a collaboration between electrical engineers and physicists resulted in the realization of the first laser in the group of C.H. Townes [2]. According to Townes [3] this lasted so long because the physicists, knowing about the principle of stimulated emission since Einstein, were not acquainted with amplifiers and the feedback mechanism, i.e. resonators, and the electrical engineers, who had experience with amplifiers and feedback systems, lacked the knowledge of the quantummechanical ideas about the interaction between matter and photons. At that time, 'Townes' system was not called a laser, but a maser as it operated in the microwave range. In 1958 Schawlow and Townes published ideas to extend maser techniques to the infrared and optical region [4].

The first laser at visible wavelength saw the light, red light to be more precise, in 1960 when T.H. Maiman demonstrated his ruby laser [5]. Since then the developments in the field of quantum electronics have been extremely rapid. Although applications for lasers had a very slow start during their first decade, a large variety of applications for laser radiation is known nowadays, from the barcode cash-registers in the shopping centre to the highly advanced laser techniques used in medicine, e.g. for the correction of nearsightedness.

Nowadays many different types of lasers are known. The wavelengths of all these lasers vary from $\sim$100 nm in the vacuum ultraviolet to 1 mm in the microwave range. Also the output power of these lasers cover a wide range: from microwatts from small semiconductor lasers to terawatts from huge laser systems used in nuclear fusion experiments. However the basic properties of the light, or better: the radiation, from all these different sources are more or less equal:

**Directionality** Due to the resonator used in a laser the output beam is strongly directed and not very divergent.
Monochromaticity The small amount of possible transitions and the feedback mechanism (i.e. the resonator) narrows the bandwidth of the laser light. Thus very narrow linewidths are possible. In practice this is usually between 1 MHz and 1 GHz for lasers in or near the visible spectrum. For some lasers a bandwidth of 1 Hz is known.

Coherence Laser light is coherent, i.e. there is a phase relation between the waves. Coherence is a necessary property for the performance of interference experiments. As lasers usually have a much better coherence than other light sources, lasers are much more useful for interference measurements than other light sources.

Brightness Laser light is strongly focusable, hence the amount of energy per unit of time per unit of area per unit of solid angle, which is called the brightness, is usually very high.

In this chapter, the principles of a laser will be treated, especially those relevant for this thesis. A rigorous analysis of the physics of the laser is quite complex, hence the treatment given in this chapter is therefore rather simplified.

1.1 Emission and absorption of radiation

If an electron in an atom undergoes a transition between two energy states or levels it can either emit or absorb a photon, whose energy is determined by the energy difference between the two energy levels or states concerned. The wavelength of this photon is determined by its energy according to \( E = h\nu \), where \( h \) is Planck’s constant and \( \nu \) the frequency of the radiation. Consider the electron transitions which may occur between the two energy levels of the hypothetical atomic system shown in figure 1.1. If an electron is in the lower level \( E_1 \), then it may be excited to the upper level \( E_2 \) by absorbing a photon of energy \( (E_2 - E_1) \). Alternatively, if the electron is in the level \( E_2 \) it may return to the ground state by emitting a photon. This emission process can occur in two ways: (a) the spontaneous emission process in which the electron drops to the lower level in an entirely random way and (b) the stimulated emission process in which the electron is ‘triggered’ to undergo the transition by the presence of photons of energy \( (E_2 - E_1) \). These processes are illustrated in figure 1.2.
Sec. 1.1 Emission and absorption of radiation

Stimulated emission results in coherent radiation: the waves associated with the stimulating and the stimulated photon have identical frequencies, they are in phase, they have the same polarisation and they travel in the same direction. Thus, with stimulated emission an incident wave can be amplified when passing though a medium where the stimulated emission is larger than the the absorption (which is of course the inverse process).

Einstein showed that the parameters for describing the aforementioned three processes are related through the requirement that for a system in thermal equilibrium the photon absorption should be equal to the photon emission (both spontaneous and stimulated). The total absorption depends on the number of atoms in the lower level $N_1$ and the photon energy density $\rho_\nu = \mathcal{N}h\nu$ where $\mathcal{N}$ is the number of photons per unit volume having frequency $\nu$.

\[
\text{absorption} = B_{12}N_1\rho_\nu \quad (1.1)
\]

Similarly the stimulated emission equals

\[
\text{stimulated emission} = B_{21}N_2\rho_\nu \quad (1.2)
\]

The spontaneous emission only depends on the number of atoms in the upper level

\[
\text{spontaneous emission} = A_{21}N_2 \quad (1.3)
\]

where $B_{12}, B_{21}$ and $A_{21}$ are the so-called Einstein coefficients. In thermal equilibrium the absorption should equal the emission, hence

\[
B_{12}N_1\rho_\nu = B_{21}N_2\rho_\nu + A_{21}N_2 \quad (1.4)
\]

Figure 1.2: Energy level diagrams illustrating (a) absorption, (b) spontaneous emission and (c) stimulated emission.

Initial state

\[
\begin{array}{c}
\text{Absorption} \\
(a) \\
\text{Spontaneous emission} \\
(b) \\
\text{Stimulated emission} \\
(c)
\end{array}
\]

Final state
With Boltzmann’s statistics and thermal equilibrium in mind, two relations between the three Einstein coefficients can be found

\[ g_1 B_{12} = g_2 B_{21} \]  

(1.5)

and

\[ \frac{A_{21}}{B_{21}} = \frac{8\pi \hbar \nu^3}{c^3} \]  

(1.6)

(1.5) and (1.6) are referred to as the Einstein relations. In (1.5) \( g_1 \) and \( g_2 \) denote the degeneracy of the energy levels \( E_1 \) and \( E_2 \) respectively. \( \hbar \) in (1.6) is Planck’s constant, and \( c \) is the speed of light in vacuo.

### 1.2 Spectral broadening

In the previous section it has been assumed that all the atoms in either the upper or lower levels would be able to interact with the (perfectly) monochromatic beam. However, spectral lines have a finite frequency spread. This can be seen both in emission and absorption.

The frequency spread is described by the lineshape function \( g(\nu) \). \( g(\nu)d\nu \) is the probability that a given transition between the two levels will result in the emission (or absorption) of a photon whose frequency lies between \( \nu \) and \( \nu + d\nu \). \( g(\nu) \) is normalised such that

\[ \int_{-\infty}^{\infty} g(\nu)d\nu = 1 \]  

(1.7)

The form of the lineshape function depends on the particular mechanism responsible for the spectral broadening in a given transition. The three most important mechanisms are natural (or lifetime) broadening, Doppler broadening and collision (or pressure) broadening.

Due to this lineshape function the Einstein coefficients become also frequency dependent

\[ A_{21}(\nu) = A_{21}g(\nu) \]  

(1.8a)

\[ B_{12}(\nu) = B_{12}g(\nu) \]  

(1.8b)

\[ B_{21}(\nu) = B_{21}g(\nu) \]  

(1.8c)

### 1.2.1 Natural broadening

During the spontaneous decay from the upper level \( E_2 \) to the lower level \( E_1 \) a photon with energy \( \hbar \nu_{21} \) is emitted. The transition rate is given by the Einstein coefficient \( A_{21} \). The lifetime of the upper level is given by

\[ \tau = \frac{1}{A_{21}} \]  

(1.9)
Sec. 1.2 Spectral broadening

This means that the intensity of the light emitted by a group of atoms or molecules in the upper level reduces exponentially with time according to

\[ I = I_0 e^{-t/\tau} \]  

(1.10)

The lifetime influences the spectral width of the spontaneously emitted light. The spectral distribution can be calculated both classically and quantummechanically. Both derivations lead to a Lorentzian lineshape function for the spontaneous emission:

\[ g(\nu, \nu_0) = \frac{\Delta \nu_N / 2\pi}{(\nu - \nu_0)^2 + (\Delta \nu_N / 2)^2} \]  

(1.11)

The full width at half maximum is called the natural linewidth, which is inversely proportional to the lifetime of the upper level:

\[ \Delta \nu_N = \frac{1}{2\pi \tau} \]  

(1.12)

Usually a difference is made between homogeneous and inhomogeneous line broadening. Homogeneous line broadening means that every atom can emit all frequencies of the line spectrum. Alternatively, inhomogeneous line broadening means that every atom can emit only at his own frequency. In natural line broadening every atom is capable of emitting at all frequencies, so natural broadening is a homogeneous broadening.

1.2.2 Doppler broadening

Doppler broadening is caused by the movement of the emitting atom. One finds from the Maxwell velocity distribution in thermal equilibrium and the Doppler shifted frequency the frequency distribution of the emitted light

\[ D(\nu, \nu_0) = \frac{c}{\nu_0} \left( \frac{m}{2\pi kT} \right)^{\frac{1}{2}} e^{-\frac{m\nu^2}{2kT}} \nu_0^2 (\nu - \nu_0)^2 \]  

(1.13)

where \( m \) is the mass of the emitting atom, \( k \) is Boltzmann's constant, \( T \) is the temperature of the gas, \( c \) the speed of light in vacuo and \( \nu_0 \) the central frequency of the spectrum (which is equal to the transition frequency in the case of an isotropic velocity distribution). The frequency distribution due to Doppler broadening results in a Gaussian function with a linewidth of

\[ \Delta \nu_D = 2\nu_0 \left( \frac{2kT}{mc^2} \ln 2 \right)^{\frac{1}{2}} \]  

(1.14)

As the velocity of the emitting atom determines the frequency shift, Doppler line broadening is an inhomogeneous broadening process.

1.2.3 Collision broadening

Collision broadening is caused by collisions of atoms emitting photons. If an atom collides with another atom just at the moment that it is emitting a photon, then the
phase of the wave train associated with this photon is altered. Such an event is called a dephasing event. These dephasing events lead to broadening of the spectral line. The collision broadened lineshape function is found to be a Lorentzian function

$$g(\nu, \nu_0) = \frac{\Delta \nu_C / 2\pi}{(\nu - \nu_0)^2 + (\Delta \nu_C / 2)^2}$$

(1.15)

with

$$\Delta \nu_C = \frac{1}{\pi \tau_{dp}}$$

(1.16)

where \(\tau_{dp}\) is the mean time between two dephasing events, i.e. the time between two collisions. In simplified gas kinetic theory \(\tau_{dp}\) is given by [6]

$$\tau_{dp} = \frac{1}{n \sigma \bar{u}}$$

(1.17)

with \(n\) the particle density of the gas, \(\sigma\) the collision cross-section and \(\bar{u}\) the average particle speed, given by \(\sqrt{8kT/m\pi}\) [7]. \(\tau_{dp}\) is inversely proportional to the gas pressure \(p\), which is related to the density \(n\) by the ideal gas law \(p = n k T\). Therefore, collision broadening is proportional to the pressure.

Collision broadening is just as natural broadening a homogeneous broadening process.

### 1.3 Small signal gain coefficient

The change in irradiance \(I\) of a monochromatic collimated beam passing though a homogeneous absorbing medium as a function of distance is given by

$$dI(x) = -\alpha I(x) dx$$

(1.18)

where \(\alpha\) is the absorption coefficient. Integrating this gives for the irradiance the Lambert-Beer law

$$I(x) = I_0 e^{-\alpha x}$$

(1.19)

with \(I_0\) the incident irradiance.

From the discussion of stimulated transitions (absorption is also a stimulated process as a photon is needed for occurance) above, one can write an expression for the net loss of photons per unit volume

$$-\frac{dN}{dt} = N_1 \rho_v B_{12} g(\nu) - N_2 \rho_v B_{21} g(\nu)$$

(1.20)

The spontaneously emitted photons are omitted because they are emitted in a random direction and therefore their contribution to the collimated beam can be neglected. Similarly scattering losses are omitted.
Sec. 1.4 Optical feedback

The irradiance of the beam, which is the energy crossing a unit area in a unit of time, is given by the energy density times the speed of light

\[ I_\nu = \rho_\nu c/n = N\hbar \nu c/n \quad (1.21) \]

where \( c \) is the speed of light in vacuo and \( n \) is the refractive index of the medium. Now the change in the photon density between the boundaries \( x \) and \( x + \Delta x \) can be written as

\[ -dN(x) = -\frac{dI(x)}{dx} \cdot \frac{\Delta x n}{\hbar \nu c} \quad (1.22) \]

given that \( \Delta x \) is sufficiently small. Thus the rate of photon density decay equals

\[ \frac{dN}{dt} = \frac{dI(x)}{dx} \cdot \frac{1}{\hbar \nu} \quad (1.23) \]

Comparing eqs. (1.20) and (1.23) one finds for the absorption coefficient \( \alpha \) (using eqs. (1.5), (1.18) and (1.21))

\[ \alpha(\nu) = \left( \frac{g_2}{g_1} N_1 - N_2 \right) \frac{B_{21}\hbar \nu m}{c} g(\nu) \quad (1.24) \]

From this equation one sees that the absorption depends on the difference in the populations of the energy levels \( E_1 \) and \( E_2 \). If, however, a situation can be created where \( N_2 > (g_2/g_1)N_1 \) (this situation is called population inversion) then the absorption coefficient becomes negative. Consequently, the change in the irradiance (1.18) becomes positive and the irradiance grows as it passes through the medium according to

\[ I = I_0 e^{\alpha x} \quad (1.25) \]

where \( \alpha \) is now referred to as the gain coefficient and it is given by

\[ \alpha(\nu) = \left( N_2 - \frac{g_2}{g_1} N_1 \right) \frac{B_{21}\hbar \nu m}{c} g(\nu) \quad (1.26) \]

1.4 Optical feedback

However, only population inversion is not enough for the stimulated emission process to be the main process occurring in the medium. As can be seen from (1.2) the stimulated emission also depends on the photon density \( \rho_\nu \), so a high photon field is necessary. This field has to be generated somewhere. In a laser this is done similarly as in electronic amplifiers: with positive feedback. The noise on the input is amplified and then fed back to the entrance, amplified again, fed back to the entrance and so on until it reaches a stable output signal.

In lasers this positive feedback is performed by a set of mirrors, one at either side of the medium. Part of the spontaneous emission is reflected back into the medium by one of the mirrors, is amplified while passing through the medium, is reflected at the
other mirror, is amplified again and so on. This way a photon field is built up between the two mirrors. Due to saturation of the medium, gain saturation (see section 1.5), inside the resonator a stable photon field builds up. By making one of the mirrors partially transmitting an optical beam can be coupled out of this resonator and a laser has been realised.

Many different types of resonators are possible. In chapter 4 of this thesis a few resonator configurations that are applicable to long pulse, low gain XeCl* excimer lasers will be treated.

1.5 Gain saturation

For stimulated emission at a photon density \( \rho_\nu \) we can write (compare (1.20))

\[
\frac{d \rho_\nu}{dt} = -\frac{c}{n} \frac{d \rho_\nu}{dx} = \hbar \nu \rho_\nu B_{21} \Delta N g(\nu) \tag{1.27}
\]

where the inversion \( \Delta N \) equals

\[
\Delta N = \left( N_2 - \frac{g_2}{g_1} N_1 \right) \tag{1.28}
\]

Assuming a homogeneous interaction of the inversion over the whole line, i.e. homogeneous broadening, we may write for the inversion

\[
\frac{d}{dt} \Delta N = -2 \rho_\nu B_{21} g(\nu) \Delta N - \frac{\Delta N}{\tau} + P \tag{1.29}
\]

where \( -\Delta N/\tau \) is the loss of inversion due to spontaneous emission, with \( \tau \) the life time of the upper laser level, and \( P \) the pump term, which is the production of inversion per unit of volume. If the decay of the lower level is very fast so that \( \Delta N \) is only the density of the upper laser level, as in a bound-free transition, the factor 2 has to be dropped. For a stationary state one finds from (1.27) and (1.29)

\[
\frac{d \rho_\nu}{\rho_\nu dx} = \frac{n \sigma P}{1 + \frac{2c \rho_\nu \sigma \tau}{\hbar \nu}} \tag{1.30}
\]

where

\[
\sigma = \frac{\hbar \nu B_{21} g(\nu)}{c} = \frac{A_{21} \lambda^2}{8\pi} g(\nu) \tag{1.31}
\]

is the cross-section for stimulated emission. The radiation intensity \( I_\nu \) is equal to \( c \rho_\nu / n \) and the saturable gain \( \alpha_s \) is equal to \( dI_\nu / I_\nu dx \) so that one finds that

\[
\alpha_s = \frac{n \sigma P}{1 + \frac{I_\nu}{I_s}} \tag{1.32}
\]

where \( I_s = \hbar \nu / 2 \sigma n \) is the saturation intensity (for very fast decay of the lower laser level the factor 2 has to be dropped again). From (1.32) one can see that the gain drops if the radiation intensity becomes too high. This process is called gain saturation.
1.6 Threshold condition

As explained before, a steady state photon field is reached when the amplification rate equals the loss rate. This is the case for continuous wave (c.w.) lasers; for pulsed lasers it is slightly different. Thus, while population inversion is a necessary condition for laser action, it is not a sufficient one because there must be enough gain to overcome the losses and to sustain the photon field.

The total loss in the system is due to a number of processes:

1. Transmission at the mirrors
2. Absorption and scattering at the mirrors.
3. Diffraction losses at the mirrors.
4. Absorption in the laser medium due to transitions other than the desired transitions.
5. Scattering at optical inhomogeneities in the laser medium.

As a simplification all losses except for the mirror transmission losses are included in one single loss factor $\gamma$. The effective gain coefficient now becomes $\alpha - \gamma$. If the beam travels a distance $L$ through the medium (with $L$ the length of the medium) the beam irradiance will increase from $I_0$ to $I$, where

$$I = I_0 e^{(\alpha - \gamma)L}$$

(1.33)

Reflection at one of the mirrors leads to a decrease from $I$ to $RI$, where $R$ is the (effective) reflectance of the mirror. For a complete round trip through the resonator one finds that the gain $G$ equals

$$G = \frac{I_{\text{final}}}{I_{\text{initial}}} = R_1 R_2 e^{2(\alpha - \gamma)L}$$

(1.34)

If $G$ is larger than unity the photon field will grow; if $G$ is less than unity the field will die out. Thus the threshold condition for laser action equals

$$R_1 R_2 e^{2(\alpha_{\text{th}} - \gamma)L} = 1$$

(1.35)

It is important to realise that the steady state gain is equal to this threshold gain, due to gain saturation. At threshold the gain $\alpha_{\text{th}}$ equals

$$\alpha_{\text{th}} = \gamma + \frac{1}{2L} \ln \left( \frac{1}{R_1 R_2} \right)$$

(1.36)

1.7 Laser modes

Inside the cavity two different types of modes exist: the axial or longitudinal modes and the transverse modes.
1.7.1 Axial modes

The two mirrors of the laser form a resonant cavity and within this resonant cavity standing wave patterns build up. For an empty resonator these standing waves satisfy the equation

\[ p \frac{\lambda}{2} = L \]  

(1.37)

where \( p \) is an integer, \( \lambda \) the wavelength and \( L \) the optical cavity length. The index of refraction of the medium has been taken equal to 1, which is a good approximation for a gaseous medium.

From (1.37) it is easily found that all possible cavity frequencies are given by

\[ \nu = \frac{pc}{2L} \]  

(1.38)

with \( c \) the speed of light. From (1.38) one finds for the frequency separation of the cavity

\[ \delta \nu = \frac{c}{2L} \]  

(1.39)

These axial frequencies are all possible oscillation frequencies of the resonator. However, only those which lie within the gain curve and come above threshold will actually oscillate: the axial modes of the laser. In chapter 5 a more detailed discussion is given about the linewidth of the laser and of the frequencies found in the laser light. Also techniques to reduce the number of frequencies, i.e. axial modes, will be treated.

1.7.2 Transverse modes

Axial modes are formed by plane waves travelling axially along the the optical axis of the cavity. For any real laser cavity there will probably also be plane waves travelling just off-axis that are able to replicate themselves after covering a closed path. These will also give rise to resonant modes, but because they have components of their electromagnetic fields transverse to the direction of propagation they are called transverse electromagnetic modes.

The analysis of the transverse modes is quite complicated and falls outside the scope of this introduction. In chapter 4 these transverse modes will be treated in more detail in combination with the resonators in which they occur.

1.8 Linewidth of the output

Because of the various broadening mechanisms mentioned in section 1.2 a group of atoms can not be treated as if they all radiate at the same frequency. Instead a small frequency spread must be considered. It might then be expected that the laser output would have the same frequency distribution, this is however not the case. This difference is accounted for by two factors: the effects of the optical resonator and the effects of the amplification process.
1.8.1 The optical resonator

The change in the photon field in an empty cavity, i.e. a cavity without an amplifying or absorbing medium, can be described by

$$-2L \frac{d \rho_\nu}{dt} = c \rho_\nu f$$  \hspace{1cm} (1.40)

where $L$ is the cavity length, $c$ the speed of light and $f$ the loss factor of the cavity. The solution to this differential equation is

$$\rho_\nu = \rho_{\nu,0} e^{-t/\tau_{\text{cav}}}$$  \hspace{1cm} (1.41)

where $\tau_{\text{cav}} = 2L/fc$ is the cavity decay time. From the spectrum of the cavity, which is found from the Fourier transform of the electrical field in the cavity, one finds for the bandwidth of the resonator

$$\Delta \nu_{\text{cav}} = \frac{1}{2\pi \tau_{\text{cav}}}$$  \hspace{1cm} (1.42)

1.8.2 The amplification process

As can be seen from equation (1.26) the small signal gain coefficient $\alpha(\nu)$ depends on the lineshape function $g(\nu)$. The irradiance of the light travelling through an amplifying medium varies as

$$I(\nu, x) = I(\nu, 0) e^{\alpha(\nu)x}$$  \hspace{1cm} (1.43)

so the irradiance is related exponentially to $g(\nu)$. Thus the amplification is much higher in the center than in the ‘tails’ of the atomic lineshape. This effect is called spectral narrowing.

1.9 Hermite-Gaussian beams

Any physical optical beam has a finite transverse cross-section. Beams of finite cross-section may be described in terms of a superposition of plane waves. The paraxial approximation simplifies the analysis of the propagation of these beams. In this paraxial approximation Hermite-Gauss functions are solutions of the wave equation (in cartesian coordinates). In cylindrical coordinates similar functions can be derived: The Laguerre-Gaussian functions (see e.g. [8]).

1.9.1 Paraxial approximation

In free space the optical field obeys the scalar wave equation [8]

$$\nabla^2 \psi + k^2 \psi = 0$$  \hspace{1cm} (1.44)
A general plane-wave solution of this scalar wave equation in Cartesian coordinates is of the form

$$e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

(1.45)

with

$$k_x^2 + k_y^2 + k_z^2 = k^2 = \frac{\omega^2}{c^2} = \left( \frac{2\pi}{\lambda} \right)^2$$

(1.46)

If the propagation vector \( \mathbf{k} \) is inclined by a small angle with respect to the \( z \) axis, then the wave vector is paraxial, and

$$k_z = \sqrt{k^2 - k_x^2 - k_y^2} \approx k - \frac{k_x^2 + k_y^2}{2k}$$

(1.47)

This suggests that \( \psi(x, y, z) \) can be written as

$$\psi(x, y, z) = u(x, y, z)e^{-jkz}$$

(1.48)

where the \( x \), \( y \) and \( z \) dependences of \( u \) are much less rapid than that of the factor \( \exp(-jkz) \). When introducing this in the wave equation and the term \( \partial^2 u / \partial z^2 \) is neglected the paraxial wave equation is obtained

$$\nabla_T^2 u - 2jk \frac{\partial u}{\partial z} = 0$$

(1.49)

where

$$\nabla_T \equiv \hat{x} \frac{\partial}{\partial x} + \hat{y} \frac{\partial}{\partial y}$$

(1.50)

### 1.9.2 Gaussian beams

A solution of the paraxial wave equation is a wave travelling in the \(+z\) direction with a Gaussian amplitude profile (Gaussian beam) and with curved phase fronts of radius \( R(z) \) [8]

$$u_{00}(x, y, z) = \frac{\sqrt{2}}{\sqrt{\pi w}} e^{i\phi} e^{-\frac{x^2 + y^2}{\omega^2}} e^{-\frac{i}{2\pi} (x^2 + y^2)}$$

(1.51)

where

$$w^2 = w_0^2 \left[ 1 + \left( \frac{z}{\pi w_0^2 / \lambda} \right)^2 \right]$$

(1.52)

$$\frac{1}{R} = \frac{z}{z^2 + (\pi w_0^2 / \lambda)^2}$$

(1.53)

$$\tan \phi = \frac{z}{\pi w_0^2 / \lambda}$$

(1.54)

Here \( w_0 \) is the minimum radius of the Gaussian beam. The factor \( \frac{\pi w_0^2}{\lambda} \) is called the Rayleigh range. This is the distance between the focus of the beam and the point where the width of the beam is equal to \( \sqrt{2} w_0 \).
1.9.3 Higher order modes

For the representation of a beam with an arbitrary amplitude distribution at the
input plane, an infinite set of (orthogonal) solutions of the paraxial wave equation
is required. Such a set of functions exists: functions formed of products of Hermit-
Gaussians. A Hermite-Gaussian of order \( m \) of the independent variable \( \xi \) is defined as
the product of a Hermite polynomial of order \( m \), \( H_m(\xi) \), and the Gaussian \( \exp(-\xi^2/2) \).
The lowest-order Hermite polynomials are

\[
H_0(\xi) = 1, \quad H_1(\xi) = 2\xi, \quad H_2(\xi) = 4\xi^2 - 2
\]

The higher-order Hermite polynomials can be obtained with the recursive formula

\[
H_{n+1}(\xi) - 2\xi H_n(\xi) + 2nH_{n-1}(\xi) = 0 \tag{1.55}
\]

If the abbreviated symbol \( \psi_m(\xi) \) is used for the Hermite-Gaussian of \( m \)th order, where

\[
\psi_m(\xi) \equiv H_m(\xi)e^{-\xi^2/2} \tag{1.56}
\]

then the (two-dimensional) Hermite-Gaussian forward travelling wave of the mode
of order \( m, n \) at \( z = 0 \), \( u_{mn}(x_0, y_0) \), can be defined by

\[
u_{mn}(x_0, y_0) = C_{mn}\psi_m\left(\frac{\sqrt{2}x_0}{w_0}\right)\psi_n\left(\frac{\sqrt{2}y_0}{w_0}\right) \tag{1.57}
\]

And the forward travelling wave of the mode \( u_{mn} \) evaluated at \( z \) via the Fresnel
diffraction integral is

\[
u_{mn}(x, y, z) = \frac{C_{mn}}{\sqrt{1 + \left(\frac{x}{\lambda w(z)}\right)^2}}\psi_m\left(\frac{\sqrt{2}x}{w}\right)\psi_n\left(\frac{\sqrt{2}y}{w}\right)e^{-j\frac{\lambda}{2w}(x^2+y^2)}e^{j(m+n+1)\phi} \tag{1.58}
\]

where \( w, R \) and \( \phi \), functions of \( z \), were defined by (1.52) through (1.54) for the
Gaussian beam solution.

1.9.4 Gaussian \( q \) parameter

A Gaussian beam is completely described by the complex parameter \( q \) \([8]\), where \( q \) is
defined as

\[
\frac{1}{q(z)} \equiv \frac{1}{R(z)} - j\frac{\lambda}{\pi w(z)^2} \tag{1.59}
\]

The real part of \( 1/q \) gives the radius of curvature of the beam at position \( z \) and the
imaginary part the spotsize. Thus, if we know how \( q \) transforms, we know how to transform Gaussian beams. Because all higher order Hermite-Gaussian modes are
described by the same \( R \) and \( w \), also the same \( q \) parameter, their transformation is governed by the same law, except in so far as the phase change \( (m + n + 1)\phi \) is concerned.
The transformation of the $q$ parameter may be expressed as a bilinear transformation

$$q' = \frac{Aq + B}{Cq + D} \quad (1.60)$$

with the transformation matrix

$$M = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \quad (1.61)$$

The ABCD-matrices for transformation of the $q$ parameter are the same matrices as those found for pure geometrical ray optics (see e.g. [8,9]).

References


Chapter 2

The XeCl* excimer laser

There are different types of lasers: gas lasers, solid state lasers, free electron lasers, semiconductor lasers etc. Among these lasers the gas laser is (still) the most promising laser for obtaining very high output powers. One special type of gas laser is the excimer laser. This is a class of lasers emitting mainly in the ultraviolet wavelength range of the spectrum. The word excimer is a contraction of excited dimer and is the name for a number of molecules that only exist in an excited state. In the ground level these excimers are in principle unstable. Therefore, inversion can be easily established. Originally the term excimer was used for rare gas molecules (e.g. XeF₂). First laser action for the first excimer molecule (XeF₂) was shown in 1971 by Basov et al. [1]. This laser operated at a wavelength of 172 nm.

Later new families of excimer molecules, which were called exciplexes at first (from excited complex), were found, one of which is the family of rare gas halogens. These rare gas halogens are excimer molecules consisting of a rare gas atom and a halogen atom. The first rare gas halogen laser was the XeBr laser from Searles and Hart in 1975 [2]. Since then different rare gas halogens showed lasing transitions. Table 2.1 shows the wavelengths of these lasing transitions. One of these rare gas halogen excimer molecules is XeCl*. The work described in this thesis is based on experiments performed with an electrically optimised XeCl* excimer laser system [4]. However the obtained results may also be applicable to other excimer lasers which operate in the same low gain, long pulse regime for example the ArF laser [5] which operates at a wavelength of 193 nm and the F₂ laser operating at 157 nm [6]. In this chapter a

<table>
<thead>
<tr>
<th></th>
<th>He</th>
<th>Ne</th>
<th>Ar</th>
<th>Kr</th>
<th>Xe</th>
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<td>-</td>
<td>-</td>
<td>206f</td>
<td>282</td>
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<tr>
<td>I</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>253f</td>
</tr>
</tbody>
</table>
short description of the XeCl* excimer laser and its modus operandi will be given.

2.1 The XeCl* molecule

XeCl* is one of the rare gas halogen molecules. It only exists in the excited state. In the ground state the atoms are repulsive, hence the molecule is unstable in the ground state. Figure 2.1 shows the energy levels of the XeCl* molecule. XeCl* shows two main upper levels, the B and the C state, which show a rather strong bond. There are two possible lower states, the A state and the X state, of which the A state is strongly repulsive. The X state is actually loosely bound. However the energy needed to break this bond is so small that the thermal energy available in the form of vibrations is enough to break it.

According to the Franck-Condon principle the internuclear distance does not change during the transition: there is no time for the nuclei to move [8]. So the energy of the photon emitted at the transition is determined by the energy difference between the upper and lower lever for the same internuclear distance (the vertical arrows in figure 2.1). In XeCl* two lasing transitions are possible: the C→A transition which emits at \( \lambda = 345 \) nm and the B→X transition which emits at 308 nm. The B→X transition is the strongest transition; hence the operating wavelength of 308 nm.

The A state is strongly repulsive. This leads to a broad line: the linewidth \( \Delta \lambda \) is 20 nm. Small variations in the internuclear distance in the XeCl* molecule result in large differences in photon energy at the transition to the lower level due to the large slope of the lower level energy function. The X state is loosely bound and results therefore in a smaller linewidth: \( \Delta \lambda = 0.5 \) nm for the B→X transition [9].
Sec. 2.2 Discharge versus e-beam

The excited (B) state of XeCl* has vibrational and rotational levels. In the formation process of XeCl* the inversion is spread over these levels as there is no thermal equilibrium. It depends on the relaxation time of the levels compared to the life time of the levels whether the different levels will be found in the laser spectrum. If the relaxation of the higher vibrational levels to the lowest vibrational level in the exited state is much faster than the (stimulated) emission process, the vibrational levels will barely be seen in the laser spectrum. The loosely bound lower (X) state of XeCl* shows also some vibrational levels. The transition can therefore take place from a few different upper levels to a few different lower levels.

Table 2.2 shows the main, vibrationally different, B→X transitions of XeCl*. Of these the 0→1 and the 0→2 transitions are found to be the strongest in a non selective cavity (see e.g. [11,12] and chapter 5). This is in agreement with the Franck-Condon Factors (FCF’s) for the different transitions, which are the largest for these two transitions (see table 2.2).

The Franck-Condon Factors give the probability for each transition to occur from a certain upper level. Of the transitions, the most intense are those in which the internuclear distance is a highly probable one for both the upper and the lower state [8].

The linewidth of the two main transitions is estimated to be in the order of 60 pm [11–13]. To select one of the lines dispersive elements have to be introduced in the resonator. This will be treated in more detail in chapter 5. Furthermore, line narrowing techniques will be described in that chapter.

2.2 Discharge versus e-beam

The first excimer lasers were mainly electron beam pumped lasers. For the XeCl* excimer laser this was also the first pumping scheme tried [14]. Ischenko et al. constructed the first discharge pumped XeCl* excimer laser in 1977 [15]. A discharge laser has many advantages compared to an electron beam pumped laser [4]:

- In an electron beam pumped laser the electrons, generated in a vacuum chamber, have to enter the laser gas mixture through a thin foil. This foil causes not only losses with respect to the number of electrons reaching the gas, but also is a

<table>
<thead>
<tr>
<th>Transition</th>
<th>λ [nm]</th>
<th>FCF</th>
<th>Transition</th>
<th>λ [nm]</th>
<th>FCF</th>
</tr>
</thead>
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<td>1→5</td>
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<td>0.114</td>
<td>0→1</td>
<td>307.95</td>
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<td>0→2</td>
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<td>0.125</td>
<td>0→3</td>
<td>308.45</td>
<td>0.190</td>
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<tr>
<td>0→0</td>
<td>307.70</td>
<td>0.104</td>
<td>0→4</td>
<td>308.69</td>
<td>0.112</td>
</tr>
</tbody>
</table>

Table 2.2: The main B→X transitions in XeCl*, their wavelength and their Franck-Condon Factors. From [9,10].
very weak component in the system, especially when a high pressure laser gas mixture is used and the laser has to be operated at high repetition rates. In a gas discharge solid electrodes can be used which means a less fragile construction. However, if x-ray preionisation is used, as in our system, one of the electrodes has to be made of a material that shows a good x-ray transparency, which usually also means that as little material as possible should be used.

- The construction of the laser is relatively simple. No vacuum chamber and electron source are needed, but only two electrodes. The voltages are generally lower for a gas discharge pumped system, but the electrical circuit is more complicated.

- A high power discharge (100 MW) in a gas mixture at high pressure (5 bar) containing halogens is always unstable, hence it is impossible to obtain continuous operation. With a gas discharge high average powers can be obtained because a gas discharge can be operated at high repetition rates (up to at least 1 kHz) and with an electron beam pumped system this is almost impossible (typical repetition rate 1 shot per minute).

A disadvantage of a discharge pumped system is that the average electron energy is rather low. The high energy electrons of an electron beam are more efficient in producing the XeCl* molecule resulting in an efficiency of 8 % [16]. The highest reported efficiency with a gas discharge pumped XeCl* excimer laser is 5.5 % [17].

In principle the XeCl* excimer laser has a high quantum efficiency because the radiative decay is directly to the ground level. For the most important formation channel at least one Xe+ ion is needed to create a XeCl* molecule. The energy needed for the ionisation of Xe is 12 eV. With a photon energy of 4 eV this results in a theoretical maximum obtainable efficiency of 33 %. In practice this value is never reached as in the discharge also several unwanted side-processes occur and, of course the shortest, and most efficient, route is not always possible. The highest reported optical efficiency from a XeCl* discharge is 12 %, but this was a dc glow discharge in a mixture of Xe and Cl₂ [18]. Furthermore, this was an excilamp and not a laser. Thus the emitted radiation was incoherent.

### 2.3 Discharge properties

The laser chamber used in the experiments described in this thesis is shown in figure 2.2. The electrode distance in the system is 2.5 cm. The width and length of the discharge are determined by the preionisation. As the window for the x-ray preionisation to enter the discharge mixture, which is in the anode, is 2 cm x 60 cm, this results in a 2 cm wide and 60 cm long discharge. The total discharge volume is 0.3 l.

In this section a short treatment of the processes connected with the discharge is given. A more extensive survey can be found in [4].
Sec. 2.3 Discharge properties

2.3.1 Gas mixture

A typical gas mixture used in the experiments contains 0.70 mbar hydrogen chloride (HCl) and 8 mbar xenon (Xe). As a buffer gas neon (Ne) is used. The total pressure of the gas mixture is 5 bar.

In the past helium has also been tried as a buffer gas, but Ne showed better performance [4]. As halogen donor several gases were tested like BCl₃ [15], CCl₄, CH₂Cl₂, CCl₂F₂, CFCl₃ and CHCl₃ [19], however the best results were obtained with HCl [19].

In our system the partial pressures of HCl and Xe are kept constant by a cold trap gas purifier. By controlling the temperature of the purifier the vapour pressures of Xe and HCl are controlled. The purifier also acts as a buffer for HCl and Xe as the system is filled with an extra amount of these gases. The purifier controls the vapour pressures and the surplus is frozen in the cold trap. This buffer is particularly useful for the HCl as HCl is consumed during operation. During the discharge HCl dissociates and some chlorine ions attach to the electrodes and walls of the laser chamber. The remaining H ions eventually form H₂ molecules. Because of the HCl buffer in the purifier the HCl concentration can be kept constant and the system can be operated for a relatively long time before the gas mixture needs to be replaced. The purifier also has another purpose: it freezes in contaminations like water and CO₂ released from the walls. A typical temperature for the purifier is 122 K.

2.3.2 Discharge kinetics

To obtain laser operation the upper laser level needs to be filled. In the case of the XeCl* excimer laser this means that the XeCl* molecule needs to be formed. Sorkina [20] gives four formation reactions for the XeCl* molecule:

\[
\begin{align*}
\text{Xe}^+ + \text{Cl}^- + \text{Ne} & \rightarrow \text{XeCl}^* + \text{Ne} \\
\text{Xe}_2^+ + \text{Cl}^- & \rightarrow \text{XeCl}^* + \text{Xe} \\
\text{Xe}^+ + \text{HCl}_{\nu=1,2} & \rightarrow \text{XeCl}^* + \text{H} \\
\text{Xe}^{**} + \text{HCl} & \rightarrow \text{XeCl}^* + \text{H}
\end{align*}
\]

(2.1a) (2.1b) (2.1c) (2.1d)

of which the first one is the most important one. In a discharge system the electron energy is quite low (only a few electron volts) so the ionisation of Xe occurs via the
excited states Xe* and Xe** [20].

\[ \begin{align*}
    \text{Xe} + e & \rightarrow \text{Xe}^* + e & (2.2a) \\
    \text{Xe}^* + e & \rightarrow \text{Xe}^{**} + e & (2.2b) \\
    \text{Xe}^{**} + e & \rightarrow \text{Xe}^+ + 2e & (2.2c)
\end{align*} \]

The Cl⁻ ion is formed by dissociative attachment of electrons to HCl:

\[ \text{HCl} + e^- \rightarrow \text{Cl}^- + \text{H} \quad (2.3) \]

From the reaction equation it can be seen that the electron density is decreased by this formation channel for the Cl⁻. This reaction also shows the reason why the HCl partial pressure should not be too high. If there is too much HCl present in the gas mixture the electrons will get attached to HCl, resulting in discharge instabilities. On the other hand, the formation reaction for HCl from H and Cl is very slow. This means that the HCl is consumed during the discharge. If the gas mixture runs out of HCl during the discharge then the formation of XeCl* is reduced, resulting in loss of gain. Therefore, the partial pressure of HCl is very critical.

During the discharge many other reactions take place, but this falls outside the scope of this thesis. For more detailed information see for example [20–26].

2.3.3 The discharge

During a stable period of the discharge a glow discharge is formed. A general property of a glow discharge is its constant voltage across the electrodes almost independent of the current flowing through the discharge. At this voltage there is an equilibrium between the production and the loss of electrons. If the discharge becomes unstable during the current pulse, i.e. if this equilibrium is disturbed locally, streamers and eventually arcs will start to grow. Arcs shorten the life time of the electrodes, so to prevent electrode wear the system has been optimised to prevent arcs [4].

The steady state voltage during the discharge is determined by the gas mixture. It was found that at a fixed pressure of the buffer gas Neon the HCl content strongly influences the steady state voltage [4]. This means that if the HCl partial pressure is changed by changing the purifier temperature the steady state voltage changes.

During the steady state phase of the discharge, the electrical circuit can be described with a simple LC-circuit as shown in figure 2.3 [4,27]. This simple circuit can only be used for the first half cycle of the current. The voltage across the electrodes is kept constant at the steady state voltage \( V_{ss} \). The differential equation of this circuit is

\[ \frac{i}{C_{pfn}} + L \frac{d^2i}{dt^2} = 0 \quad (2.4) \]

with the initial conditions

\[ i(0) = 0 \quad i'(0) = \frac{V_{pfn} - V_{ss}}{L} \quad (2.5) \]
Sec. 2.3 Discharge properties

![Diagram of an LC model for the discharge circuit during steady state.](image)

Figure 2.3: Simple LC model for the discharge circuit during steady state.

The solution of this differential equation and its starting conditions is

$$i(t) = \sqrt{\frac{C_{\text{pfn}}}{L}} (V_{pfn} - V_{ss}) \sin \left( \frac{t}{\sqrt{LC_{\text{pfn}}}} \right)$$  \hspace{1cm} (2.6)

In these equations $V_{pfn}$ is the charging voltage, $C_{\text{pfn}}$ and $L$ are the capacitance of the capacitor bank and inductance of the circuit respectively.

It can easily be seen that if the charging voltage $V_{pfn}$ is chosen to be twice the steady state voltage $V_{ss}$, no voltage is left in the capacitor bank after the first half cycle of the current. Thus no reverse current will occur. This is the so-called matched discharge situation. This is important for our system as a second current flow would lead to an unstable discharge i.e. arcs, resulting in electrode wear, which is not preferable (especially not for a high repetition rate system).

Also, the efficiency for our system is optimal if the system is operated under matched discharge conditions. The electrical efficiency of the system is, according to this simple model, equal to

$$\eta = \frac{\pi \sqrt{LC}}{\int_{0}^{\pi} V_{ss}i(t)dt} = \frac{4V_{ss}(V_{pfn} - V_{ss})}{V_{pfn}^2}$$  \hspace{1cm} (2.7)

It can easily be shown that the efficiency has its maximum at $V_{pfn} = 2V_{ss}$.

2.3.4 Preionisation

To get a homogeneous discharge the gas mixture has to be preionised [28–30]. Without preionisation the discharge will show instabilities from the beginning, evolving into arcs instead of a stable discharge.

The process that creates these instabilities is shown in figure 2.4 in a simplified way. The electron density needed for operation of a XeCl* laser can be obtained by applying a high voltage, much larger than the steady state voltage, across the electrodes. The individual electrons present in the gas will be accelerated and will collide with Ne and Xe atoms resulting in ionisation of these atoms. Thus extra electrons are created, which will be accelerated too and collide with Ne and Xe and so on. The electrons move towards the positive electrode and leave the slow positive ions behind. Thus each
electron contributes to the formation of a cone of positive space charges as visualised in figure 2.4a. This positive space charge attracts nearby electrons, thus creating new cones of positive space charges (figure 2.4b). When finally the anode and cathode get connected through such a, usually narrow, channel a streamer is formed. These channels are narrow because of the low diffusion rate of electrons due to the high gas pressure. It is obvious that streamers do not lead to a good quality discharge as the current and electron density are too high in the streamers: the XeCl⁺ molecules are quenched by electron collisions before they can radiate. An inhomogeneous discharge will strongly decrease the optical quality of the beam because of variations in refractive index of the medium.

One way to diminish this problem is preionisation. Preionisation creates a sufficient electron density to ensure overlap of the avalanche heads after the high voltage is applied, as is shown in figure 2.5. The required preionisation electron density depends on the gas mixture. An electron density of about $10^5 - 10^6$ cm$^{-3}$ should provide enough overlap [29].

There are different preionisation techniques available: preionisation with UV, with x-rays, α particles or electron beams. In our system an x-ray source is used for preionisation. X-ray preionisation has a few advantages over the other techniques [4]:

- X-rays have a large penetration depth into the gas mixture, thus resulting in a
homogeneous electron density.

- X-rays are generated outside the laser chamber. Thus the source does not contaminate the gas mixture as is the case with UV preionisation by bare sparks, which is the most common used method of UV preionisation. UV preionisation through a window is in principle clean too, but windows for the deep UV (150-200 nm) are difficult to obtain. Also, this UV wavelength range is absorbed by the oxygen in the air, so the system has to be put in a oxygen free environment.

- The x-rays can be ‘collimated’ by letting them penetrate through a well defined window so that only the intended discharge volume is irradiated. The discharge volume is then determined by the preionisation rather than by the shape of the electrical field, which is determined by the electrode profiles. As a consequence, flat electrodes can be used, making the construction of the electrodes a lot easier.

The x-ray source used in our system is shown in figure 2.6 [31]. A tantalum foil is used as anode and two corona plasma tubes are used as cathode. The cathodes are made of thin tungsten wires wound round a quartz tube. When a fast high voltage pulse is applied to the trigger pen inside the quartz tubes, a corona discharge on the quartz surface around the grounded tungsten wires is formed. From this plasma electrons are extracted and accelerated towards the high voltage anode. There the electrons collide with the tantalum where they are decelerated. A small part of the electron energy is emitted as x-rays (Bremsstrahlung). These x-rays are emitted in all directions. Only the x-rays in the direction of the aluminium exit window between the two cathodes can leave the source and can enter the laser chamber.

The electrical circuit is optimised in such way that the x-ray output pulse is short (50 ns) and has a short rise time (< 10 ns). The short rise time of the x-ray pulse is necessary for our system, as there is already a voltage across the laser electrodes at the moment of x-ray injection. As the electron avalanche starts immediately after the injection of the first x-rays (which create the first free electrons), the x-ray pulse should be fast enough and strong enough to create the minimum number of electrons needed
for a good avalanche on a time scale shorter than the time scale of the avalanche. Hence the short duration, but more important low rise time x-ray pulse.

The anode voltage is about 120 kV so the maximum x-ray photon energy is about 120 keV. The aluminium window(s) block the lower energy x-rays. The result is an energy distribution which has its maximum at approximately 60 keV.

2.3.5 Discharge instabilities

In the previous section the preionisation has been introduced as a means against the problem of initial discharge instabilities. The initial discharge instabilities can be prevented by sufficient and properly timed preionisation. The initial electron density just after preionisation, however, is not enough for a stable discharge, as for a stable discharge roughly $10^{14} - 10^{15}$ electrons/cm$^3$ are needed [28]. The electron density is increased by the avalanche that starts after a high voltage is applied to the electrodes. The risetime of this high voltage should be as short as possible to obtain a homogeneous avalanche [4]. A short rise time induces a high breakdown voltage. This is very important because a high voltage results in large avalanche cones, which will show sufficient overlap to lead to a good and homogeneous discharge. A low breakdown voltage induces instabilities because the avalanche cones remain small and do not overlap sufficiently.

Even with perfect preionisation and a good avalanche the discharge will eventually transform from a homogeneous glow discharge into an arc. This is due to a process called halogen depletion [32]. In a stable discharge the amount of electrons is in balance. During the discharge HCl is consumed as the reaction which re-forms HCl is too slow on the time scale of the discharge. This HCl consumption follows reaction (2.3) and is thus electron density dependent. If the electron concentration is now slightly larger than average for a certain position, then the HCl consumption on that position is also slightly larger. This results in a local decrease of HCl concentration, which causes a locally lower electron loss resulting in an even larger electron density. This can be the start of a streamer, thus an instability.

At the cathode hot spots are formed which emit electrons better than the surrounding cold material. These hot spots create electron density variations which grow via the halogen depletion process. Furthermore, a rough surface of the electrodes causes field variations which initiate instabilities. Hence electrode material and surface quality can determine whether a laser operates properly or not. In our system we used a solid brass cathode and an aluminium x-ray window covered by a thin nickel foil as the anode.

2.4 The electrical circuit

For the laser to operate properly the electrical circuit has been designed to meet the requirements of the gas discharge. It has been found that the XeCl* excimer laser can be best operated in the following three step scheme:
1. First the gas is preionised with the x-ray source described in section 2.3.4.

2. Subsequently the electron avalanche has to be started to begin a homogeneous discharge. This can be done best with a short high voltage pulse considerably exceeding the dc breakdown voltage.

3. When the conducting plasma is formed the main current should be allowed to flow to sustain the discharge. The voltage across the electrodes will then be the steady state voltage. As mentioned before, the voltage on the capacitance bank should be twice the steady state voltage to obtain optimum efficiency.

Different circuits have been tested in the past. With an x-ray triggered system 0.8 % efficiency has been reached, but this system resulted in a bad discharge quality [33]. The same system showed better performance after making the electrical circuit a bit more complicated. With improved x-ray triggering the efficiency became 1.8 % [33].

These two methods were based on one single circuit to perform both the avalanche and the discharge sustaining step. In a double discharge system, also called a spiker-sustainer system or a prepulse mainpulse system, these two steps are separated by making use of two different electrical circuits instead of one. A fast, high voltage, but low energy circuit for the avalanche (the prepulse circuit) and a slower, high energy, but relatively low voltage circuit to sustain the discharge (the mainline circuit). To prevent the prepulse from disappearing into the mainpulse circuit a switch has to be used to separate the two circuits. The inductance of this switch should be as low as possible in the closed (conducting) state in order to get a fast rising mainpulse current.

In 1983 Long et al. used a double discharge circuit for the first time for a XeCl* laser [34]. In his system, Long et al. used a railgap to separate the two electrical circuits. In our system we use magnetic switches because these are more suitable for high repetition rate operation. These magnetic switches have a high inductance state and a low inductance state and they can be switched by applying a voltage across them. The switching time is inversely proportional to the applied voltage. A more extensive survey on magnetic switching can be found in [35] A disadvantage of magnetic switches is that the mainpulse voltage is present on the electrodes from the beginning. This mainpulse voltage is larger than the dc breakdown voltage. The x-ray preionisation pulse should therefore have a short rise time, as already explained in section 2.3.4. The x-ray pulse should be applied just before the prepulse.

The best efficiency reported using a prepulse mainpulse circuit is 5.5 % [17].

Commercially available excimer lasers, for example the XeCl lasers from Lambda Physik, are all based on the single circuit laser, as these are relatively simple to manufacture and, more important, less vulnerable. The preionisation in these systems is performed by small arcs close to the discharge region generating UV radiation, so-called UV-preionisation. Both the simple circuit and the not optimal preionisation lead to a rather poor performance of these lasers. Disadvantages of this kind of systems are:
Figure 2.7: The basic prepulse mainpulse circuit

- Only short optical pulses possible (in the order of 30 ns), as the voltage on the capacitors is too high for a stable discharge formation for a long period.

- Very high output power density, due to the high peak current, which is caused by the high charging voltage needed to breakdown the gas. This high output power density is a disadvantage because damage to the optics is very likely.

- Low efficiency, as the charging voltage is not matched to the discharge.

- Ringing current, again because the charging voltage is not matched to the discharge. The overvoltage leads to ringing of the current, as all energy has to be dissipated in the gas mixture eventually. During the ringing of the current no stable discharge is possible, hence arcs will occur, which lead to electrode wear. This is not preferable for a high repetition rate system.

- Low beam quality of the output, as there is no time for the laser to build up a proper beam. In chapter 4 this will be explained in more detail.

2.4.1 Prepulse mainpulse circuits

Figure 2.7 shows a basic prepulse mainpulse circuit. The prepulse (pp) starts the electron avalanche, then the switch has to be closed so that the mainpulse (mp) can sustain the discharge. The energy of the mainpulse is stored in the main capacitor $C_{pfn}$ and the peaking capacitor $C_p$ is used by the prepulse to supply energy during the avalanche.

The advantages of a prepulse mainpulse circuit are [4]

- The breakdown voltage and the sustaining voltage can be chosen independently.

- The mainpulse circuit can be charged to twice the steady state voltage to obtain maximum efficiency.

- The energy in the prepulse can be low.

- Before the breakdown of the gas the impedance of the discharge is high, so the prepulse can have a fast rise time and hence a high overvoltage, resulting in a homogeneous start of the discharge.
Sec. 2.4 The electrical circuit

Depending on the polarity and the magnitude of the applied voltages a prepulse mainpulse circuit can be operated in different modes (see e.g. [36]).

Diode mode

If the prepulse and the mainpulse have the same polarity the laser is operated in diode mode. The voltage across the electrodes has now always the same polarity and current is flowing only in one direction (hence the name diode mode). A disadvantage of the diode mode is the relatively long delay between the breakdown and the onset of the current as the magnetic switch separating the prepulse and mainpulse circuits needs to be saturated in the opposite direction.

Switch mode

If the prepulse and the mainpulse have different polarities the laser can be operated in switch mode or resonant overshoot mode. In switch mode the avalanche current flows in the opposite direction compared to the main current. The current through the discharge has to switch direction, hence the name. The current from the mainpulse circuit, however, leaves the magnetic switch in a low inductance saturated state, eliminating the delay between breakdown and onset of the mainpulse current.

Resonant overshoot mode

In the resonant overshoot mode the polarities of prepulse and mainpulse are opposite, as in the switch mode, but the prepulse voltage is lower [37]. Because the laser gas does not break down on the prepulse, a second high voltage spike on the peaking capacitors is caused by a resonant overshoot of the voltage which comes from the main capacitor. If the laser gas breaks down in the rising edge of the second voltage spike the main current can almost immediately begin to flow. Thus the delay between breakdown and the onset of the main current is very small, resulting in a more homogeneous discharge. As the prepulse voltage is lower for the resonant overshoot mode than for the switch mode, the resonant overshoot mode is more efficient.

Charge mode

Using a circuit with a pulse compressor in the prepulse circuit a fourth operation mode is possible: the charge mode [38]. The charging voltages are equal to the resonant overshoot mode. The difference is the moment of firing the prepulse. In the charge mode the prepulse is fired when the charging current to the main capacitor bank is still flowing. This results in a higher peak voltage at which breakdown occurs. The laser output is found to be the same in the charge mode as in the resonant overshoot mode. However, the higher peak voltage results in a wider and more stable discharge. A disadvantage of the charge mode is a larger delay between breakdown and onset of the main current. Because of this delay it is possible to get a central dip in the output [39]. By tuning this delay a flat-top output beam can be created [39].
2.4.2 The electrical circuit used for the XeCl* excimer laser

Parametric studies on different electrical circuits resulted in the electrical circuit for the XeCl* laser shown in figure 2.8 [4,27]. The values for the capacitors in the circuit have been optimised for the highest quality discharge and the highest optical output.

The main energy is stored in a pulse forming network (PFN). In the system the inductance of the pulse forming network is small compared to the inductance of the laserhead and the saturable inductor, so the pulse forming network acts as a lumped capacitor ($C_{fhn}$). This capacitor is charged in 2-3 $\mu$s from a DC-charged storage capacitor ($C_{ml}$) via a thyatron switch.

Close to the laserhead small peaking capacitors ($C_p$) are used, where a high voltage pulse can be applied to get breakdown of the gas mixture. This high voltage pulse is generated in the prepulse circuit where $C_{pp}$ is the DC-charged storage capacitor. The prepulse and mainpulse circuit are separated by a saturable inductor $L_{rt}$, also called the ‘race-track’. This race-track saturable inductor is also used as a pulse transformer for the prepulse to provide the opposite polarity of the prepulse with respect to the mainpulse, so we can use grounded cathode thyratrons for both the mainpulse and the prepulse circuit. The circuit includes a pulse compressor ($C_{pc}$ and $L_{pc}$) in the prepulse circuit to sharpen the prepulse.

The system is operated in the resonant overshoot mode as this mode shows the best results (highest output and highest efficiency) [37,35,40].

2.5 The laser system

In the previous sections parts of the laser system used in the experiments described in this thesis have been treated. Figure 2.9 shows a cross-section of the complete discharge unit used as gain medium for the laser. The laser chamber with the two electrodes is shown in the centre. Below the chamber the x-ray source is placed for the preionisation. The x-rays enter the laser chamber through a window in the anode. On top of the laser chamber the electrical circuit is build, which ignites and sustains
Figure 2.9: The complete discharge unit.

Figure 2.10: Photograph of the experimental setup.

the discharge. In figure 2.10 a photograph of the experimental setup is given.

In figure 2.11 typical waveforms for the x-ray preionisation pulse, voltage across the discharge and current through the discharge are shown. The x-ray pulse is measured with a fast plastic scintillator NE 102A from Nuclear Enterprises which emits an optical signal if x-rays are impinging on the material. The optical pulse from
Figure 2.11: Typical waveforms of the x-ray preionisation pulse, voltage across the discharge, current through the discharge and the optical signal.

This scintillator is transported by a fiber to a fast Hamamatsu photomultiplier (type H3167W/R2027). The voltage across the electrodes is measured with a resistive voltage divider and the discharge current is monitored by measuring the voltage across a small resistor in the PFN circuit. All signals are monitored with a digital oscilloscope, type DSA 602 from Tektronix.

The prepulse and resonant overshoot on which the gas breaks down are clearly seen. The short delay between breakdown and onset of the current is caused by the ferrites in the racetrack inductor. These have to get saturated again after the high spike of the resonant overshoot. The ferrites are reset by the charging current of the PFN.

Also a typical optical pulse from a stable resonator is shown. It can be seen clearly that the optical pulse does not start immediately after the start of the discharge. A certain build-up time is needed before output is generated. This build-up time has two reasons. In the first place some time is needed for the discharge to create XeCl* in order to build up population inversion. Secondly, some time is needed for the optical field to build up starting from noise (spontaneous emission). The build-up time for the optical field is unique for low gain, long pulse lasers. Commercially available lasers (Lambda Physik, Lumonics) all have very high gain and short pulses because of the simple, single thyatron circuits used in these lasers.

These commercial lasers are characterised by discharges with a high pumping power density and hence a short duration (≤ 50 ns) in order to prevent the onset of discharge instabilities during the laser pulse. This means a high gain and only a few cavity round trips for the optical pulse. The optical output from these short pulse lasers is given by the superposition of outputs from each cavity round trip [41] and the output therefore has a low spatial coherency. The divergence of the optical pulse can be decreased considerably if an active medium with a lower gain and longer stability period is used instead, so that more round trips are used to reach saturation. With such a system the mode can be established before saturation of the gain medium occurs, resulting in a beam with a lower divergence. The system used for the experiments described in
2.6 Performance of the system

In this section some important characteristics of the system used in the experiments will be given.

2.6.1 Efficiency

To determine the optimal operation point for the laser with respect to the charging voltage on the pulse forming network ($C_{Pfn}$ in figure 2.8) the efficiency is chosen as the main parameter. Figure 2.12 shows the efficiency of the system as a function of the PFN voltage for two different definitions of the efficiency. The filled squares show the value of the ratio of the electrical output energy and the energy stored on the PFN. The highest efficiency according to this definition is somewhere between 6 and 8 kV.

If the energy stored in the prepulse circuit is added, the efficiency is reduced as shown by the filled circles in figure 2.12. This definition of the efficiency is more realistic as the energy stored in the prepulse circuit is about 5 J, which is not negligible compared to the ~12.5 J stored on the pulse forming network (at a PFN voltage of 8 kV). Now the efficiency shows a maximum at approximately 8 kV. To minimise current reversal, which causes electrode erosion because no discharge is formed during the following current pulses but only arcs, the PFN charging voltage of 8.2 kV is used. Which is twice the steady state voltage, as can be seen from figure 2.11.

The obtained efficiency is lower than the reported maximal efficiency of 5.5 %. Hence it can be concluded that the electrical system is not optimal, especially the prepulse circuit is found to be very inefficient. A further optimisation falls outside the scope
of the research described in this thesis. Furthermore, the efficiency of the total laser system is not an important parameter for the optical behaviour of the laser.

2.6.2 Discharge width

The prepulse is used to create an avalanche in the gas mixture after the first electrons are produced by the x-ray preionisation pulse. A higher prepulse leads to a better (more homogeneous) discharge. The x-ray preionisation pulse enters the laser chamber through a 20 mm wide window. The discharge should therefore be confined to this 20 mm. If the prepulse voltage is not high enough, the discharge will become narrower because then only in the middle of the discharge the electron density is high enough to get a stable discharge [39].

This can be shown experimentally by looking at the width of the optical beam from a plano-plano cavity. In such a resonator the width of the beam is determined by the width of the discharge. If the discharge is narrower this indicates that the discharge quality is not optimal. The width of the beam is determined by measuring the width of the burn-spot on thermofax paper. Figure 2.13 shows the width of the discharge as a function of the prepulse voltage. It is clearly seen that the discharge width increases with increasing prepulse voltage (as expected). The discharge width stabilises if the prepulse voltage is increased above 45 kV. The electrical setup of the prepulse circuit allowed single shots with voltages up to 56 kV but could not withstand these high voltages for a long time. In the experiments a prepulse voltage of 45 kV has been used.

From the thermofax paper burn spots of the beam the quality of the discharge could also be seen. The streamers in unstable discharges could easily be discerned in the burn spots.
Figure 2.14: Experimental setup used for the gain measurement experiments.

Figure 2.15: The small signal gain as a function of the amplifier PFN voltage with the HCl concentration as a parameter.

2.6.3 Gain measurements

For resonator design knowledge about the gain is important. A high gain laser demands another approach than a low gain laser. Because we had two nearly identical XeCl* systems available the gain could be measured by using one of them as a probe laser for the other.

In a single pass setup there are only two main variables with which the gain medium can be influenced, namely the mainline voltage and the HCl concentration. Other variables are of course the total gas pressure and the capacity of the PFN, but these were kept fixed as the system was optimised with respect to these two variables. Figure 2.14 shows the experimental setup used for the gain measurements. Laser 1 is used as a probe laser and laser 2 as the probed laser of which the gain is measured. The optical signal before and after laser 2 are measured with high speed photodiodes (type DET200/M from Thorlabs Inc.). The value of the gain is determined from
single pass energy gain. The pulse energy is given by the integral of the photodiode signal. As the photodiodes were not calibrated the absolute energy is not known. However if the photodiodes are calibrated with respect to each other with an inactive laser 2, the energy increase due to an active laser 2 can be determined from which the gain can be calculated.

According to McKee [42] the saturation intensity for a long pulse XeCl* excimer laser is approximately 250 kW/cm². To ensure measurement of the small signal gain, it was verified that both the intensity of the probe pulse and the intensity of the amplified pulse were lower than this saturation intensity.

Figure 2.15 shows the gain as a function of the PFN charging voltage for different HCl concentrations. The gain is relatively small and varies between 2.5 and 6.0 % cm⁻¹. At our standard operation point (V_{PFN} = 8.2 kV and 0.70 mbar HCl) the gain is about 5 % cm⁻¹.
These measurements were performed with a relatively long injection pulse (70 ns). To measure the time behaviour of the gain, short pulses (5 ns) were sliced out of this long pulse using a Pockels cell. The experimental setup is shown in figure 2.16. Figure 2.17 shows the time resolved behaviour of the gain. The gain grows and diminishes with the current through the discharge.

References


References


Chapter 3

Beam quality parameters

The brightness of the laser output beam is an important parameter in the performance of a laser since it sets a limit to the temperature that can be reached with a focused beam on a target. The brightness is given by the beam power divided by the beam size and the angular spread of the beam. The quantity that describes the beam size and its spread is the beam quality. The smaller this quantity, the better the beam can be focussed. It would be very practical to be able to describe this beam quality with one single parameter.

A few parameters for describing this beam quality have been introduced in literature, of which Siegman’s $M^2$ is probably the best known [1–4]. Others are the power-in-the-bucket (PIB) value [5,6], Strehl’s ratio $S_r$ [7], the recently introduced time-diffraction-limited (TDL) parameter from Bollanti et al. [8] and the $K$ factor (see e.g. [9,10]). The $M^2$ is a parameter based on the propagation of the beam. The PIB value, Strehl’s ratio and the TDL parameter are parameters which make a comparison between the actual beam and a theoretical beam. The $K$ factor is a measure for the near and far field behaviour of the actual beam. In the following sections the definitions of these parameters will be presented.

3.1 The $M^2$ parameter

The $M^2$ parameter can be derived in different ways. The most straightforward way is from geometrical optics.

3.1.1 $M^2$ in geometrical optics

To derive the $M^2$ factor from ray optics, one has to start with the ray matrix (ABCD matrix) formulation for pure geometrical ray propagation in the paraxial approximation

$$x_2 = Ax_1 + B	heta_1$$  \hspace{1cm} (3.1a)

$$
\theta_2 = Cx_1 + D	heta_1$$  \hspace{1cm} (3.1b)
Consider a bundle of rays. The mean values (first moments) of the position and angle of the rays are given by

\[ \langle x \rangle = \frac{1}{N} \sum_{i=1}^{N} x^i \]  
(3.2a)

\[ \langle \theta \rangle = \frac{1}{N} \sum_{i=1}^{N} \theta^i \]  
(3.2b)

and the variances (second moments) are given by

\[ \langle x^2 \rangle = \frac{1}{N} \sum_{i=1}^{N} (x^i)^2 \]  
(3.3a)

\[ \langle \theta^2 \rangle = \frac{1}{N} \sum_{i=1}^{N} (\theta^i)^2 \]  
(3.3b)

For the propagation of the first moments, the same rules hold as for a single ray

\[ \langle x_2 \rangle = A\langle x_1 \rangle + B\langle \theta_1 \rangle \]  
(3.4a)

\[ \langle \theta_2 \rangle = C\langle x_1 \rangle + D\langle \theta_1 \rangle \]  
(3.4b)

Ray matrix formulas yield for the second moments

\[ \langle x_2^2 \rangle = A^2\langle x_1^2 \rangle + 2AB\langle x_1 \theta_1 \rangle + B^2\langle \theta_1^2 \rangle \]  
(3.5a)

\[ \langle x_2 \theta_2 \rangle = AC\langle x_1^2 \rangle + (AD + BC)\langle x_1 \theta_1 \rangle + BD\langle \theta_1^2 \rangle \]  
(3.5b)

\[ \langle \theta_2^2 \rangle = C^2\langle x_1^2 \rangle + 2CD\langle x_1 \theta_1 \rangle + D^2\langle \theta_1^2 \rangle \]  
(3.5c)

These equations lead to an invariant parameter (for any ray bundle and any ABCD system)

\[ \langle x_2^2 \rangle\langle \theta_2^2 \rangle - \langle x_2 \theta_2 \rangle^2 = \langle x_1^2 \rangle\langle \theta_1^2 \rangle - \langle x_1 \theta_1 \rangle^2 \]

\[ = (x_0^2)\langle \theta_0^2 \rangle \]

\[ = \text{invariant} \]  
(3.6)

Based on this invariant parameter is the definition of the “\( M^2 \) parameter”:

\[ M^2_2 \equiv \frac{4\pi}{\lambda} \sqrt{\langle x^2 \rangle\langle \theta^2 \rangle - \langle x \theta \rangle^2} \]  
(3.7)

where \( M^2_2 \geq 1 \) for any real optical beam.

The ray matrix for free space propagation from \( z_1 \) to \( z \) is given by

\[ \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & z - z_1 \\ 0 & 1 \end{bmatrix} \]  
(3.8)

So the second moment has a quadratic dependency on \( z \):

\[ \langle x^2 \rangle(z) = \langle x_1^2 \rangle + 2\langle x_1 \theta_1 \rangle(z - z_1) + \langle \theta_1^2 \rangle(z - z_1)^2 \]  
(3.9)
Using the waist parameters
\[
\begin{align*}
    z_{0,x} &= z_1 - \langle x_1 \theta_1 \rangle / \langle \theta_1^2 \rangle \\
    \langle x_0^2 \rangle &= \langle x_1^2 \rangle - \langle x_1 \theta_1 \rangle^2 / \langle \theta_1^2 \rangle \\
    \langle \theta_0^2 \rangle &= \langle \theta_1^2 \rangle
\end{align*}
\] (3.10a, b, c)

one finds that every real beam shows a beam size in the form
\[
\langle x^2 \rangle(z) = \langle x_0^2 \rangle + \langle \theta_0^2 \rangle (z - z_{0,x})^2
\] (3.11)

Using the definitions
\[
M_x^2 \equiv \frac{4 \pi}{\lambda} \sqrt{\langle x_0^2 \rangle / \langle \theta_0^2 \rangle}
\] (3.12)

and
\[
W_x^2(z) \equiv 4 \langle x^2 \rangle
\] (3.13)

this converts to
\[
W_x^2(z) = W_{0,x}^2 + \left( \frac{M_x^2 \lambda}{\pi W_{0,x}} \right)^2 (z - z_{0,x})^2
\] (3.14)

When this equation is compared with the equation for free space expansion to the far field of a Gaussian beam, which is diffraction limited (see section 1.9.2)
\[
w^2(z) = w_0^2 + \left( \frac{\lambda}{\pi w_0} \right)^2 (z - z_0)^2
\] (3.15)

it is seen that the real beam propagation behaviour is similar to the propagation of a Gaussian beam. The difference between (3.14) and (3.15) is a scaling factor $M^2$, which causes a larger divergence of the real beam.

From (3.15) it can be seen that the free space propagation of a Gaussian beam is fully characterised by its waist spot size $w_0$ and its waist location $z_0$ (and of course the wavelength $\lambda$), with the far field angular spread being given by $\theta(z) = \lambda / \pi w_0$. The free space propagation of any real laser beam however, cannot solely be described with the waist spot size $W_{0,x}$ and the waist location $z_{0,x}$. It is seen that the beam spread of the real beam is $M_x^2$ times that of the diffraction limited beam. This $M^2$ can be used as a measure for the beam quality. It indicates, a shown above, the deviation from the Gaussian beam.

It can easily be derived that the $M^2$ factor is a measure of the amount of unwanted higher order transverse mode oscillations that may be present in the laser cavity. If a stable laser cavity oscillates in multiple lowest and higher order Hermite-Gaussian modes $u_{mn}(x,y)$ with relative amplitudes $\hat{C}_{mn}$, with the eigenmodes $\hat{u}_{mn}(x,y)$ normalized so that the total power $\sum_{m,n} |\hat{C}_{mn}|^2 = 1$, the beam quality factors of the multimode output beam are given by
\[
\begin{align*}
    M_x^2 &= \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} (2m + 1) |\hat{C}_{mn}|^2 \\
    M_y^2 &= \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} (2n + 1) |\hat{C}_{mn}|^2
\end{align*}
\] (3.16a, b)
3.1.2 Physical interpretation

In a physical interpretation the beam quality factor $M^2$ is a measure of the near field times far field “space-beamwidth product” of the arbitrary real beam normalised to the space-beamwidth product for an ideal TEM$_{00}$ Gaussian. The definition of the beam quality factor $M^2$ can be found by the determination of the space-beamwidth product for an optical beam, where the spatial and angular widths (or near field and far field beamwidths) for the laser beam are defined as the variances of the beam intensity profile in the spatial and spatial-frequency domains.

The variances of a real beam with a time average intensity profile $I(x, y, z)$ and a spatial-frequency distribution $\tilde{I}(s_x, s_y)$ are given by

$$\sigma_x^2(z) = \frac{\iint (x - \bar{x})^2 I(x, y, z)dx\,dy}{\iint I(x, y, z)dx\,dy}$$

(3.17)

and

$$\sigma_{s_x}^2(z) = \frac{\iint (s_x - \bar{s}_x)^2 \tilde{I}(s_x, s_y)ds_x\,ds_y}{\iint \tilde{I}(s_x, s_y)ds_x\,ds_y}$$

(3.18)

Using the definitions

$$W_x(z) = 2\sigma_x(z)$$

(3.19a)

$$W_{0,x} = 2\sigma_{0,x}$$

(3.19b)

$$M_x^2 = 4\pi\sigma_{0,x}\sigma_{s_x}$$

(3.19c)

one can show that for any arbitrary real laser beam the spatial variance $\sigma_x^2(z)$ obeys the free space propagation rule (3.14) [1]

$$\sigma_x^2(z) = \sigma_{0,x}^2 + \lambda^2\sigma_{s_x}^2(z - z_{0,x})^2$$

(3.20)

The beam profile $I(x, y, z)$ may change shape with propagation distance $z$ in a complex way due to diffraction or interference effects, but the variance $\sigma_x^2(z)$ nonetheless will have a minimum value $\sigma_{0,x}$ at some position $z_{0,x}$, and $\sigma_x^2(z)$ will vary quadratically with $z$ on either side of this waist, just as in the ideal Gaussian case. This rule can also be proved rigorously [1,11] starting with (3.17) and (3.18), so the assumption (3.19c) proves to be correct.

In the ideal case of a Gaussian beam the transverse intensity profile in one transverse coordinate is given by

$$I(x, z) = \text{const} \times e^{-2x^2/w_x^2(z)}$$

(3.21)

with the Gaussian spot size parameter $w(z)$ having a free space variation given by

$$w_x^2(z) = w_{0,x}^2 + \frac{\lambda^2}{\pi^2 w_{0,x}^2} (z - z_{0,x})^2$$

(3.22)

This ideal TEM$_{00}$ beam will also have a Gaussian distribution $\tilde{I}(s_x)$ in the spatial frequency or $s_x$ domain which is given by

$$\tilde{I}(s_x) = \text{const} \times e^{-2\pi^2 w_{s_x}^2 s_x^2}$$

(3.23)
Sec. 3.1 The $M^2$ parameter

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.1}
\caption{Determination of the $M^2$ value of a Gaussian beam. The fit parameters are $w_0 = 20 \ \mu m$, $z_0 = 1.00 \ m$ and $M^2 = 1$.}
\end{figure}

where the spatial frequency $s_x$ is related to the propagation angle $\theta$ for any given plane wave component of the Gaussian beam by $s_x \equiv \lambda^{-1} \sin \theta \approx \theta/\lambda$. The variances of these distributions $I(x,z)$ and $I(s_x)$ are

\begin{align}
\sigma_x(z) &= \frac{w(z)}{2} \\
\sigma_{0,x} &= \frac{w_{0,x}}{2} \\
\sigma_{s_x} &= \frac{1}{2 \pi w_{0,x}}
\end{align}

so that

$$M_x^2 = 4 \pi \sigma_{0,x} \sigma_{s_x} = 1$$  \hspace{1cm} (3.25)

Thus the $M^2$ parameter gives a value for the beam quality of an arbitrary beam normalized on the ideal case of a Gaussian beam. It follows that for any real laser beam one finds a value for $M_x^2$ equal to or larger than 1.

### 3.1.3 Measuring $M^2$ - standard method

The easiest way to measure $M^2$ is based on the propagation of a real optical beam, see (3.14). $M^2$ can be determined by measuring the second moment $< x^2 >$ (and therewith the width $W_x$) of the beam as a function of position $(z)$ and fitting equation (3.14) through the obtained points. Figure 3.1 shows this procedure for an ideal Gaussian beam. $M^2$ of this beam is (of course) 1. The disadvantage of this method is that measurement of the second moment $< x^2 >$ has to be performed at several locations before an accurate determination of $M^2$ can be made. For a continuous wave laser this is not problematic but for a pulsed laser one prefers to be able to measure $M^2$ in one shot. This can be done with a multiple CCD camera system [12] or with the recently developed micro-lens CCD camera combination [13].
3.1.4 Measuring $M^2$ - alternative method

Offerhaus et al. have recently introduced an alternative way to determine $M^2$ of a beam [14,15]. It is based on the “space-beamwidth” product formulation for $M^2$ introduced in section 3.1.2. The $M^2$ value is calculated using a Fourier transform of the spatial amplitude and phase distribution in the near field.

For this method we measure both the amplitude and the phase of the near field. The measurement thereof can be performed using a self-referencing interferometer as shown in figure 3.2. Part of the beam travels through the interferometer in the clockwise direction and the other part of the beam in the anticlockwise direction. The ring contains a telescope, formed by the two lenses $L_1$ and $L_2$, which compresses the beam travelling in one direction and stretches the beam travelling in the other direction. The two beams are recombined at the beamsplitter. With a proper choice of focal lengths and lens positions the output images are simply scaled copies of the input image. In the enlarged beam, the reference beam, the wavefront curvature is strongly reduced. The spatial variations in phase are therefore reduced. After outcoupling the compressed beam interferes with the reference beam, which has a relatively flat phase front. From this interferogram both the intensity distribution and the phase front can be extracted [16,17]. Together, these two quantities characterise the field of the incoming beam completely.

In general, the field of the incoming beam can be described as

$$u(x, y) = A(x, y)e^{j\phi(x, y)}$$

(3.26)

where $A(x, y)$ is the amplitude distribution and $\phi(x, y)$ the phase front. By Fourier transformation of this field, the (spatial) frequency distribution can be obtained. Using (3.17) and (3.18) the variances of the intensity and spatial frequency distributions can be determined and therewith $M^2$ if (3.19c) is used. (3.19c) shows that this measurement should be performed in the beam waist or else deviations will occur as $\sigma_x$, (the variance of the spatial frequency distribution) remains unaltered but $\sigma_s$ changes as a function of the position $z$ according to (3.23). To be able to determine $M^2$ at an arbitrary position, which is in general not the waist, the measured $u(x, y)$ should
be adapted such that the measurement position becomes a virtual waist. This can be done by removing the quadratic variation from the wavefront, which is caused by the propagation of the beam from the (real) waist to the measurement position. Now $\sigma_{0,x}$ can be determined from the spatial intensity distribution and $\sigma_x$ from the spatial frequency intensity distribution, which can be determined by Fourier transforming the field $u(x,y)$.

Figure 3.3 shows an example for this method. It is taken from a diode pumped Nd:YAG master oscillator power amplifier system. The figure shows the steps in the calculation. The result of the calculation is a value of 1.06 for $M^2$ [14,15].

### 3.2 The power-in-the-bucket (PIB) value

The power-in-the-bucket value is a beam quality parameter which originates from the field of laser applications.

#### 3.2.1 Definition of the PIB value

The power-in-the-bucket ($PIB$) value is defined as the fraction of the total emitted power that falls within a given aperture, or bucket (see e.g. [6]). Usually this aperture is defined as the angle between the first zeros of the far field pattern of an ideal beam with similar dimensions. As the $PIB$ value is a measure of the amount of useful energy in the focus, a low $PIB$ value means a lot of side lobe energy, which is wasted...
energy, and a high PIB value means low side lobe losses. Therefore, a low PIB value means a bad coherence and thus a bad beam quality.

An extension of this method is the comparison of the measured amount of power in the central peak of the far field diffraction pattern, \( PIB_{\text{meas}} \), to that determined by numerically propagating a pupil plane beam with uniform intensity and phase, \( PIB_{\text{theo}} \) [5]. The beam quality \( (BQ) \) is then defined as

\[
BQ = \sqrt{\frac{PIB_{\text{theo}}}{PIB_{\text{meas}}}}
\]

(3.27)

### 3.2.2 Measuring the PIB value

The PIB value can be determined in four steps

1. Measurement of the total beam energy \( E_{\text{tot}} \).
2. Measurement of the near field diameter \( D \).
3. Measurements of the beam energy \( E_{\text{aper}} \) behind the focus of a lens. In the focus of the lens a diaphragm should be positioned, which has a diameter given by the first minimum in the Airy disc [18]

\[
d = \frac{2.44 \lambda f}{D}
\]

(3.28)

4. The PIB value can now be calculated using

\[
PIB = \frac{E_{\text{aper}}}{E_{\text{tot}}}
\]

(3.29)

An example of the determination of the PIB and BQ value is given in figure 3.4. The full curve is the measured focus profile and the dashed vertical lines show the bucket. The value for \( PIB_{\text{meas}} \) is found to be 0.51.
3.2.3 Determination of the beam quality \((BQ)\)

With this measured \(PIB\) value the \(BQ\) can be determined using (3.27) if the theoretical \(PIB\) value is known. This theoretical \(PIB\) value can be determined by numerically propagating a pupil plane beam with uniform intensity and phase to the focus of the lens. The algorithm to perform this numerical propagation is treated in more detail in appendix A. The energy content within the central peak gives the value for \(PIB_{\text{theo}}\).

Also shown in figure 3.4 is the calculated focus profile based on the measured near field. This results in a \(PIB_{\text{theo}}\) of 0.82. Therefore, the value of \(BQ\) for this beam is 1.3.

3.3 Strehl’s ratio

One of the oldest (but not one of the most useful) beam quality parameters is Strehl’s ratio. Strehl’s ratio \(S_r\) compares the maximum intensity in the focus of a real beam to the maximum focus intensity of the same beam without aberrations in the phase front [7].

3.3.1 Definition of Strehl’s ratio

Consider an arbitrary beam passing a plane \(A\) propagating towards a plane \(B\), as illustrated in figure 3.5, plane \(A\) being the exit plane of a lens with a focal length \(L\). Plane \(B\) is the Gaussian focal plane of the beam. The propagation in one dimension of the beam from \(A\) to \(B\) is given by Huygens’ integral [19,20]

\[
u_B(x_B) = e^{-jkL} \sqrt{\frac{j}{L\lambda}} \int_{-\infty}^{\infty} u_A(x_A) \exp \left[ -j \frac{\pi}{L\lambda} (x_A^2 - 2x_Ax_B + x_B^2) \right] dx_A
\]

(3.30)

where \(u_A\) and \(u_B\) are the optical fields in plane \(A\) and plane \(B\) respectively. \(x_A\) and
\( x_B \) are the respective coordinates and \( L \) is the distance from plane A to plane B. The intensity at the Gaussian focus point P is given by

\[
I(P) = |u_B(P)|^2 = \frac{1}{L\lambda} \left| \int_{-\infty}^{\infty} u_A(x_A) e^{-j\frac{x_B^2}{2L}} dx_A \right|^2
\]  

(3.31)

\( L \) also denotes the radius of the Gaussian reference sphere. Let \( \Phi(x_A) \) represent the distance between the wavefront and plane A measured along the ray and \( A(x_A) \) the amplitude, then the optical field at plane A is represented by

\[
u_A(x_A) = A(x_A) e^{-j\Phi(x_A) - \frac{x_B^2}{2L}}
\]  

(3.32)

\( I(P) \) then becomes

\[
I(P) = \frac{1}{L\lambda} \left| \int_{-\infty}^{\infty} A(x_A) e^{-jk\Phi(x_A)} dx_A \right|^2
\]  

(3.33)

If the wave is aberration-free, i.e. \( \Phi = 0 \), then the intensity has a maximum at the Gaussian image point \([7]\), which is given by

\[
I^*(P) = \frac{1}{L\lambda} \left| \int_{-\infty}^{\infty} A(x_A) dx_A \right|^2
\]  

(3.34)

The ratio of these two intensities is known as Strehl’s ratio

\[
S_r = \frac{I(P)}{I^*(P)}
\]  

(3.35)

### 3.3.2 Determination of Strehl’s ratio

Strehl’s ratio can be determined in four steps:

1. Measurement of the maximum intensity in the focus of the real beam \( I_{\text{meas}} \).
2. Measurement of the near field profile.
3. Numerical propagation of the near field to the focus using a flat phasefront in the near field. The numerical method which can perform this is described in appendix A.
4. Strehl’s ratio is now given by the ratio of the measured maximum intensity and the calculated maximum intensity as defined by

\[
S_r = \frac{I_{\text{meas}}}{I_{\text{th}}}
\]  

(3.36)
Sec. 3.3 Strehl’s ratio

Figure 3.6: Determination of Strehl’s ratio (I). Top left: intensity distribution in near field. Top right: phase distribution in near field. Bottom left: intensity distribution in focus. Bottom right: phase distribution in focus.

Figure 3.7: Determination of Strehl’s ratio (II). Top left: intensity distribution in near field. Top right: phase distribution in near field. Bottom left: intensity distribution in focus. Bottom right: phase distribution in focus.
The procedure for determination of Strehl’s ratio is illustrated in figures 3.6 and 3.7. Figure 3.6 shows the intensity and phase distributions of a beam, both in the near field (i.e. plane A in figure 3.5) and in the focus (plane B). The intensity at the Gaussian focus point (point P in figure 3.5) is found to be 10.2. In this case this is also the maximum of the focus intensity distribution. Figure 3.7 shows a beam with the same intensity profile, but with a distorted phase profile. The maximum of the focus intensity distribution of this beam is found to be 9.9. However, this is not in the Gaussian focus point, which is on the optical axis (see figure 3.5), but slightly aside of it, as is shown in the bottom left plot in figure 3.7. The intensity in the Gaussian focus point is equal to 6.9. Therefore, $S_r = 6.9/10.2 = 0.68$.

This example immediately shows the practical problems which will occur when trying to determine $S_r$. As already mentioned, the maximum of the focus intensity of the distorted beam does not necessarily coincide with the Gaussian focus point. In the measurement the Gaussian focus point is hard to recognise. Hence, the maximum intensity in the focus will be used. The example above shows that $S_r$ becomes $9.9/10.2 = 0.98$ in that case, which differs quite lot from the real 0.68. Calculations with other phase front distortions have shown that it is possible to obtain a value for $S_r$ which is larger than 1 if the maximum intensity of the focus field is taken instead of the intensity of the focus field. Thus, $S_r$ is not a very practical parameter.

3.4 The TDL parameter

Recently a new beam parameter has been introduced: the Times Diffraction Limited (TDL) parameter [8]. The TDL parameter has been introduced because the $M^2$ parameter “may not apply to highly diffractive beams such as those produced by unstable resonators or passing through hard-edged apertures” [8,21].

3.4.1 Definition of the TDL parameter

A beam is called diffraction limited if it is spatially coherent, i.e. if it has a constant-phase wavefront. The beam should not necessarily have a Gaussian energy distribution, so the previously defined $M^2$ can be larger than 1. Therefore a reference beam can be defined which has the same near field energy distribution as the actual beam, but which has a constant phase wavefront. Focussing of this reference beam with an aberration-free lens system leads to a diffraction limited far field energy distribution in the focus of the lens system. This (theoretical) diffraction limited far field energy distribution can be described by the parameter $Q$, which is defined as

$$Q = k/\theta^2$$  \hspace{1cm} (3.37)

where $k$ is the ratio between the energy content within the full divergence angle $\theta$ and the total energy. $Q$ is a combination of the power-in-the-bucket value and the divergence angle. This definition is used as $k$ does not give information on how the energy is distributed over the beam size. This can introduce large errors on the estimation of the peak energy density value. $\theta$ gives good information about the width
of the central lobe, however, it does not provide the corresponding energy content. As $Q$ is a combination of these two, it contains all information needed.

When focussing the actual beam, a far field energy distribution is obtained which is wider than the diffraction limited far field energy distribution. Therefore, a good definition of the Times Diffraction Limited ($TDL$) scale factor would be the ratio of these two far field energy distributions. The $TDL$ parameter is defined as

$$TDL = \sqrt{\frac{Q_{\text{ref}}}{Q_{\text{real}}}} = \frac{\theta_{\text{real}}}{\theta_{\text{ref}}} \sqrt{\frac{k_{\text{ref}}}{k_{\text{real}}}}$$

(3.38)

Where $\theta_{\text{real}}$ and $\theta_{\text{ref}}$ are the full divergence angles of the real beam and the reference beam respectively, and $k_{\text{real}}$ and $k_{\text{ref}}$ the respective energy content within that divergence angle.

The $Q$ value can be based on two definitions:

- on the $1/e^2$ width, and
- on the 86.5 % energy content width (which is equal to the $1/e^2$ width for a Gaussian beam).

The $1/e^2$ width is much less dependent on interpolation of the specific data and noise in the data than, for example, the 86.5 % energy content width. Also, the $1/e^2$ width describes the central peak behaviour, which is of main interest in our view, while the 86.5 % width yields more information about large side lobes. Therefore we based the $Q$ value on the $1/e^2$ width.

The $TDL$ parameter compares the measured far field energy distribution with a far field propagated reference beam having the same near field shape, but a constant wavefront. Therefore, the $TDL$ parameter is in fact a kind of generalised Strehl ratio. The fundamental difference between Strehl's ratio and this $TDL$ parameter is that for the $TDL$ parameter the whole spatial distribution is considered, while for Strehl's ratio only the value of the intensity at the Gaussian focus point is considered.

### 3.4.2 Determination of the TDL value

The procedure for determining the $TDL$ parameter has five steps.

1. Measurement of the near field energy distribution.
2. Propagation of the near field energy distribution with a uniform phasefront to the focus of a lens, which can be done using the numerical method described in appendix A.
3. Measurement of the focus field energy distribution.
4. Determination of the $1/e^2$ radius and the energy content within that radius from both focus fields (measured and calculated).
5. With these four values $TDL$ can be determined by using (3.38).

This procedure is illustrated in figure 3.8. The full curve shows the measured focus profile. The $1/e^2$ width of the focus is 26.5 $\mu$rad and the energy content within this radius is 48 %. The dashed curve shows the calculated focus profile based on the measured near field and the assumption that the phasefront is flat. The $1/e^2$ width of the calculated profile is 20.9 $\mu$rad and the energy content 76 %. Hence, $TDL$ is equal to 1.6 for this beam.

3.5 The beam quality number $K$

Another approach is adopted by the International Organization for Standardization (ISO) [9,10,21,22]. They introduced a dimensionless parameter $K$.

$K$, the beam quality number, is defined as the product of the 86.5 % energy content beam waist radius and beam far field divergence

$$K = \left( \frac{\lambda}{\pi} \right) \frac{1}{w_{86.5\%} \theta_{86.5\%}}$$

(3.39)

If the beam is a Gaussian then $K = 1$. For all other fields $K < 1$. It can be easily shown that for Gaussian beams $K = 1/M^2$. This relation is only an approximation and strong deviations may occur for steep limitations in the field [23].

The determination of $K$ is illustrated in figure 3.9. From the near field energy distribution the value for $w_{86.5\%}$ is determined (9.95 mm). The far field energy distribution yields a value of 29.5 $\mu$rad for $\theta_{86.5\%}$. Thus, according to (3.39), $K$ equals 0.33.
Figure 3.9: Determination of the \( K \) factor of a beam. Top left: the near field energy distribution of the beam. Top right: the normalised radial integral of the near field energy distribution. The dashed lines show the borders which determine \( w_{86.5\%} \). Bottom left: the far field energy distribution of the beam. Bottom right: the normalised radial integral of the far field energy distribution. The dashed lines show the borders which determine \( \theta_{86.5\%} \).

### 3.6 Comparison of the methods

Not all aforementioned beam quality parameters are really useful in practice. All parameters have their pros and cons.

**\( M^2 \) Siegman’s beam quality parameter.**

- \( M^2 \) has a solid theoretical base.
- \( M^2 \) compares the beam to a Gaussian beam, therefore mainly interesting for stable resonator beams, as these are a superposition of Gauss modes.
- \( M^2 \) is not very useful for highly diffractive beams. This is illustrated in figure 3.10. Although the hard edge output beam has a flat phase front, i.e. it is fully coherent, the \( M^2 \) value of the beam is 35.9.
- \( M^2 \) is based on second moments and these are hard to measure. Other definitions of the width \( w \) lead to discrepancies.

**PIB/BQ** The power-in-the-bucket value.

- \( PIB \) gives easily measurable, practical information about the energy content within a certain aperture.
Figure 3.10: Determination of the $M^2$ value of a beam from a hard edge unstable resonator. The fit parameters are $w_0 = 230 \, \mu m$, $z_0 = 1.00 \, m$ and $M^2 = 35.9$.

- $PIB$ does not give information about the incoherence of the beam.

$S_r$ Strehl's ratio.

+ $S_r$ gives information about the aberration of the wavefront.
- $S_r$ compares intensities in the Gaussian focus point to describe the whole beam.
- The peak intensity in the focus is quite difficult to measure accurately.
- The intensity in the Gaussian focus point is even harder to determine.

$TDL$ The times diffraction limited parameter.

+ $TDL$ gives information about the incoherence of the wavefront.
+ $TDL$ describes the far field central peak behaviour if the $1/e^2$ width is used.
+ $TDL$ is a combination of $PIB$ and a generalised $S_r$ and combines the good properties of both.

$K$ The $K$ factor.

+ $K$ is a useful and practical parameter.
- $K$ does not give information about the incoherence of the wavefront.

3.7 Conclusions

The beam quality of a laser cannot easily be described with one parameter. Probably the best judgement of the laser beam can be given if both the phase and intensity
distribution are known. However, this makes it very hard to compare beams from different resonators. To be able to compare the beams from different resonators one single parameter is useful.

For resonator research the TDL parameter is the most useful beam quality parameter. All possible resonator configurations can lead to a TDL of 1, if the resonator can be operated in a single transverse mode. This in comparison to the $M^2$ parameter, which results in large values for highly diffractive beams, such as beams from unstable resonators. $M^2$ is rather a beam propagation factor than a beam quality factor. The PIB value, Strehl’s ratio and the K factor give useful information, but as single parameters they don’t give enough information to the researcher. Thus, in chapter 4 the TDL parameter will be used to compare the different resonator configurations.

For laser-users the K factor probably is the best parameter. The K factor is also the planned ISO beam geometry standard [21]. Together with the laser power (or laser energy and pulse duration for a pulsed laser) the brightness of the laser can easily be calculated and the usefulness for the application can be determined.

References


Chapter 4

Resonator studies - I
Spatial quality

As described in chapter 1, a laser basically consists of a gain medium inside a resonator. The gain medium is the optical amplifier and the resonator acts as feedback mechanism. It is the gain medium which gives every laser its specific name and which determines the output energy c.q. power, the wavelength of the radiation and the frequency tuning range. The resonator however, determines the spatial quality and the output linewidth. Therefore, the resonator is a very important part of the laser. In this chapter the spatial quality of the output beam is the important issue. The linewidth of the output beam will be treated in chapter 5.

There are several resonator configurations possible, all with their own applications. In this chapter experiments on resonator configurations for a low gain, long pulse excimer laser are described. The experiments were performed with the XeCl* system, described in chapter 2. However, the resonators treated in this chapter are also applicable to other lasers as long as they have similar specifications with respect to the gain (should be 3-5 % cm⁻¹), the gain length (approximately 60 cm) and the gain duration (should be larger than 250 ns).

4.1 Stable resonators

4.1.1 The plano-plano resonator

The simplest and most commonly used resonator is the plano-plano resonator (see e.g. [1,2]). It consists of two plane mirrors placed at the opposite sides of the gain medium as shown in figure 4.1. In a resonator the conditions for stable resonance can be specified in terms of three parameters which must remain unaltered after one round trip [1]:

1. the phase (distribution) of the optical wavefront,

2. the on-axis amplitude of the wave, and
3. the amplitude distribution transverse to the optical axis.

For the phase to be reproduced after each round trip, the oscillating wavelength of the optical field should be an integer fraction of the resonator round trip length, according to (1.37)

\[ p\lambda = 2L \]  

(4.1)

where \( p \) is an integer, \( \lambda \) the wavelength and \( L \) the optical cavity length. The refractive index of the gain medium is taken to be 1, which is a valid approximation for a gaseous medium. This equation is only valid for the lowest order mode inside a resonator. Higher order modes usually have an extra phase shift, then the equation becomes

\[ p\lambda = 2L + \varphi \]  

(4.2)

where \( \varphi \) depends on the order of the mode.

The optical field within the resonator builds up from the initial low level of spontaneous emission if the net round trip gain is greater than unity. As long as this is the case the optical field grows exponentially until the gain medium saturates. Then the net gain reduces until it becomes unity. The power circulating in the cavity stabilises at a level at which the saturated gain equals the losses. Thus the resonance condition for the amplitude is more or less automatically satisfied for most types of lasers and in most types of resonators.

However, a plano-plano cavity will generally not act as a stable resonator, as it cannot fulfill the third condition for stable resonance. Any beam of finite cross section within such a cavity will diverge due to normal diffraction processes. Consequently the transverse amplitude profile of the optical field will not be reproduced after each round trip.

However, the gain medium does not have infinite dimensions. In a transversely discharge pumped laser the gain medium is confined in one direction by the electrodes and in the other by the edges of the discharge. Therefore small waveguide influences occur which make it possible to use the plano-plano resonator as a (more or less) stable resonator.
4.1.2 Plano-concave resonators

The easiest technique to overcome the divergence problems with the plano-plano resonator is the inclusion of focussing elements inside the cavity, which compensate the diffraction spreading. The simplest solution is exchanging one plane mirror in the plano-plano resonator with a concave one, as shown in figure 4.2.

Stability criterion

Not all plano-concave resonators are stable. A rather simple ABCD-matrix analysis shows conditions for a resonator being stable or not [3]. If one unfolds a basic resonator, it consists of four elements: a free propagation over the resonator length $L$, a reflection at a curved mirror with radius of curvature $R_1$, again a free propagation over the resonator length $L$ and finally a reflection at a curved mirror with a radius of curvature $R_2$. The ABCD matrix for the resonator is therefore given by the multiplication of the ABCD-matrices for these four elements

$$ M_{res} = \begin{bmatrix} 1 & 0 \\ -2/R_2 & 1 \end{bmatrix} \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2/R_1 & 1 \end{bmatrix} \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} \quad (4.3) $$

which is equal to

$$ M_{res} = \begin{bmatrix} 1 - \frac{2L}{R_1} & 2L - \frac{2L^2}{R_1} \\ -\frac{4L}{R_1R_2} - \frac{2}{R_2} & 1 + \frac{4L^2}{R_1R_2} - \frac{2L}{R_2} - \frac{4L}{R_2} \end{bmatrix} \quad (4.4) $$

The propagation of a general beam can be described with the $q$-parameter introduced in section 1.9.4. As already mentioned in that section, the transformation of $q$ can be described as a bilinear transformation using the ABCD matrix as the transformation matrix.

$$ q' = \frac{Aq + B}{Cq + D} \quad (4.5) $$

If a mode is to exist in the resonator, $q$ should be reproduced after one round trip (i.e. $q' = q$). Hence

$$ Cq^2 + (D - A)q - B = 0 \quad (4.6) $$
or, using \( AD - BC = 1 \)

\[
q = \frac{A - D}{2C} \pm \sqrt{\left(\frac{A + D}{2C}\right)^2 - \frac{1}{C^2}} \tag{4.7}
\]

A real beam dimension can only be obtained if the imaginary part of \( q \) is not equal to zero, or

\[
\left(\frac{A + D}{2}\right)^2 < 1 \tag{4.8}
\]

Using (4.4) this leads to the stability criterion

\[
0 \leq \left( 1 - \frac{L}{R_1} \right) \left( 1 - \frac{L}{R_2} \right) \leq 1 \tag{4.9}
\]

As in a plano-concave resonator one of the mirrors has a radius of curvature equal to \( \infty \), this relation reduces to \( R > L \).

**The transverse modes of a plano-concave cavity**

The resonant modes of a plano-concave cavity can be easily derived from the paraxial wave equation [3]. Consider the Gaussian beam solution (1.51)

\[
u_{00}(x, y, z)e^{-jkz} = \frac{\sqrt{2}}{\sqrt{\pi}w}e^{j\phi}e^{-\frac{x^2+y^2}{w^2}}e^{-\frac{jk}{2R}(x^2+y^2)}e^{-jkz} \tag{4.10}
\]

where

\[
w^2 = w_0^2 \left[ 1 + \left( \frac{\lambda z}{\pi w_0^2} \right)^2 \right] \tag{4.11a}
\]

\[
\frac{1}{R} = \frac{z}{z^2 + (\pi w_0^2/\lambda)^2} \tag{4.11b}
\]

\[
\tan \phi = \frac{z}{\pi w_0^2 / \lambda} \tag{4.11c}
\]

This is a wave travelling in the +z direction. The wave in the opposite (-z) direction can be found by replacing \( z \) by \(-z\)

\[
u_{00}(x, y, -z)e^{jkz} = \frac{\sqrt{2}}{\sqrt{\pi}w}e^{j\phi}e^{-\frac{x^2+y^2}{w^2}}e^{-\frac{jk}{2R}(x^2+y^2)}e^{jkz} \tag{4.12}
\]

where

\[
w^2 = w_0^2 \left[ 1 + \left( \frac{\lambda z}{\pi w_0^2} \right)^2 \right] \tag{4.13a}
\]

\[
\frac{1}{R} = \frac{-z}{z^2 + (\pi w_0^2/\lambda)^2} \tag{4.13b}
\]

\[
\tan \phi = \frac{-z}{\pi w_0^2 / \lambda} \tag{4.13c}
\]
Superposition of (4.10) and (4.12) results in standing waves with nodal surfaces of the electric field parallel to the wavefronts, which have radii of curvature $R$ given by

$$\frac{1}{R} = \frac{z}{z^2 + \left( \frac{\pi w_0^2}{\lambda} \right)^2}$$

(4.14)

The plano-concave resonator can only become resonant if the radii of curvature of the wavefronts coincide with the radii of curvature of the two mirrors. This results in a Gaussian beam with its waist at the plane mirror. The size of the waist is given by

$$w_0^2 = \frac{\lambda}{\pi} \sqrt{L(R-L)}$$

(4.15)

A typical waist size for our XeCl* laser is 0.6 mm (using a cavity of 1.5 m length and a concave rear mirror with a radius of curvature of 10 m).

This resonant Gaussian beam is called the fundamental mode of the resonator. In a similar way higher order resonant modes can be found using the Hermite-Gauss modes of section 1.9.3.

**Divergence of the beam from a plano-concave resonator**

The divergence of the beam from a plano-concave resonator can be found from an approximate analysis [2]. The beam can be approximated by a superposition of $N \approx (a/w_0)^2$ Hermite-Gauss modes with a near-field spot size $w_0$, where $a$ is the radius of the beam. Now the far-field angular spread in one plane can be written as an amalgamation of $\sqrt{N}$ modes, each with the characteristic divergence $2\lambda/\pi w_0$. Thus the far-field divergence angle becomes

$$\theta \approx \frac{2\sqrt{N}\lambda}{\pi w_0} \approx \frac{2a\lambda}{\pi w_0^2}$$

(4.16)

The spot size $w_0$ for a Gaussian beam for a long radius stable resonator can be written as

$$w_0^2 \approx \frac{\lambda}{\pi} \sqrt{LR} \text{ for } R \gg L$$

(4.17)

With $D = 2a$ is the dimension of one side of the beam aperture it is found that

$$\theta \approx \frac{D}{\sqrt{LR}}$$

(4.18)

**4.1.3 Experiments**

Different stable resonator configurations were submitted to experiments. There were three main points of interest:

1. the energy extraction,
2. the near field beam profile, and
3. the divergence of the beam.
Figure 4.3: Output energy as a function of the geometrical feedback for three different charging voltages of the pulse forming network. ▲: $V_{pfn} = 5.9$ kV, ■: $V_{pfn} = 8.1$ kV and •: $V_{pfn} = 9.5$ kV.

Plano-plano resonators with different feedback

Good energy extraction from the gain medium can easily be obtained if proper feedback is used. The feedback of a plano-plano resonator can be changed by changing the reflectance of the outcoupling mirror (the other mirror can also be made partially transmitting). The effective feedback of the plano-plano resonator is given by

$$\gamma = R_1 R_2$$

where $R_1$ and $R_2$ are the reflectivities of the two mirrors. Figure 4.3 shows the output energy as a function of the resonator feedback for three different PFN voltages. The large variations in the measurements (even at the same feedback value) are due to the fact that some of the mirrors where of a bad quality and it is found that this influences the output negatively.

It can be seen from figure 4.3 that for a PFN charging voltage above 8 kV the output energy is more or less constant if the feedback is taken between 20 and 70 %. At lower voltages the output increases slightly with increasing feedback.

If the feedback is far below 20 %, the outcoupling losses are larger than the gain, so no measurable output beam can be formed. Above 70 % the intracavity optical field has probably reached its maximum value and, due to the low outcoupling, only a little of this intracavity field is coupled out, resulting in a low energy in the optical output pulse.

If the feedback is lower, it takes more time for the optical pulse to start, because more round trips are needed before saturation. This is shown in figure 4.4. It can be seen that the base-to-base pulse length changes from approximately 300 ns at a feedback of 72 % to approximately 250 ns at a feedback of 14 %. Knowing that the round trip time in a resonator of approximately 1.5 m length is 10 ns, it is found that 5 round trips more were needed for the 14 % feedback resonator before saturation compared...
Figure 4.4: The optical pulse for different resonator feedback values (plano-plano resonators).

Figure 4.5: Current pulse and optical pulse of a plano-concave stable resonator at 8 kV PFN voltage and 50 % feedback.

to the 72 % feedback resonator. It can also be seen that the pulse duration is more or less the same for feedback values larger than 50 %.

When comparing the current pulse and the optical pulse, as is done in figure 4.5, it is found that for a 50 % feedback in a plano-concave resonator (R.C. of the rear mirror = 10 m) approximately 15 round trips are needed before saturation occurs (10 ns round trip time). So that the difference due to variation of the feedback is rather substantial. For stable resonators this is not so important, but for unstable resonators this saturation time determines whether the laser can operate in the lowest-loss mode or that higher order modes also exist. If not enough round trips can be made before saturation, the higher order modes, which are then not dominated by the lowest loss mode, also appear in the output beam.
Beam profile

The near field profile of the output beam is being measured with a gated CCD-camera. This is done by looking with the camera under a small angle at a scintillator. This scintillator is a thin layer of a solution of sodium salicylate in demineralised water. The small angle for the camera is needed, to avoid the direct illumination of the camera by the discharge. The viewing angle is chosen as small as possible to see the beam as if one looks straight into the beam. Although the camera is sensitive for UV, the zoom-lens is not transparent for UV. The beam spot from a plano-plano resonator is shown in figure 4.6.

![Figure 4.6: Beam spot from a plano-plano resonator (left) and from a plano-concave resonator (right).](image)

It can be seen that disturbances occur at the vertical edges of the beam. These edges correspond to the electrodes that determine the width of the discharge. The left side corresponds to the cathode where hot spots are formed. These hot spots disturb the output beam. At the right side reflections at the anode can be seen. The horizontal edges are undisturbed because there is no limitation there. Here the beam edges are determined by the gain medium itself.

These edge effects are not seen in experiments with the plano-concave resonator, as can be seen in figure 4.6. Here the beam is slightly focussed by the concave rear mirror. Hence the beam does not hit the electrodes, but it passes just between them. Therefore, the beam width in this direction is smaller (20.8 mm instead of 22.5 mm). Now the discharge volume is not completely used, but the beam homogeneity has been improved. That the discharge volume is not completely used can be seen from the discharge side view images. Figure 4.7 shows the side view of the discharge when no optical beam is built up. This side view shows fluorescence due to spontaneous emission. Figure 4.8 shows the same side view (with the same camera settings) but now a plano-plano resonator has been installed. The intensity of the fluorescence is reduced as stimulated emission reduces the population inversion. In figure 4.9 the same side view is shown, but now with a plano-concave resonator installed. It can be clearly seen that, with the plano-concave resonator installed, not the whole discharge volume has been used. Close to the anode there is a very thin layer where the beam does not pass. In this layer the fluorescence is not reduced by the stimulated emission in the beam. The central part, i.e. the part where the beam is passing, shows a reduction of the fluorescence. At the cathode a larger region of the discharge is not used by the beam. This asymmetry between the thicknesses of the two not used layers
at the electrodes is deliberate. The layer at the cathode is chosen thicker to avoid disturbances of the beam due to the hot spots occurring at the cathode [4].

The time evolution of the beam can be studied using a gated CCD-camera. The minimum gate time of our camera is approximately 10 ns. Figure 4.10 shows the beam spot at different moments in the pulse. The output profile parallel to the electrodes widens during the first 100 ns (the laser chamber was rotated 90 degrees when these experiments were performed, hence the electrodes are positioned at the left (cathode) and the right side (anode)). It is interesting to observe that the discharge starts
apparently from the centre and widens to its full width as time continues, whereas
the preionisation took place over the full width. This growth of the discharge is caused
by the initial electron and HCl distribution, which is determined by the preionisation
and the delay between breakdown and the onset of the discharge current [5].

In the transverse direction however this effect cannot be seen. The beam is immediately
as wide as is determined by the electrodes.

Later in the pulse the profile becomes flat topped, as expected, and at the end it dies
out homogeneously as can be seen in figure 4.10.

Beam divergence

To investigate the focussability of the beam the output beam has been focussed using
a lens with a focal length of 1 m. The spot size in the focus of the lens was 6.3 mm
both transverse (i.e. from electrode to electrode) and parallel (to the electrodes). The
resonator used consisted of a plane 50% outcoupling mirror and a concave rear mirror (R.C. = 9.5 m). The divergence angle is found to be 6.3 mrad. This spot size was measured from the burn spot on thermofaxpaper. This value corresponds to a value that can be found using (4.18). This approximation predicts a value of 6.4 mrad based on a resonator length $L$ of 1.25 m, a mirror radius of curvature $R$ of 9.5 m and a beam aperture dimension $D$ of 2.2 cm.

Experiments with different radii of curvature for the rear mirror show that the approximation given by (4.18) holds, except for the plano-plano cavity (see figure 4.11). Apparently the wavefront in the plano-plano cavity is not uniform, as one would expect a divergence of $\theta \approx \lambda/D \approx 30 \mu$rad. However, this only holds for an open resonator, which the plano-plano resonator is not. The resonator is limited by mirror edges, but also by the gain edges and the electrodes, which lead to small waveguide influences. This influences the divergence strongly. Hence the large discrepancy between the measurement and the theory.

### 4.1.4 Conclusions

The experiments have shown that stable resonators are straightforward and easy to align cavities, which allow good energy extraction from the gain medium and homogeneous near field energy distributions. To ensure maximum energy extraction a geometrical feedback between 20 and 70% is found to be optimal.

With the plano-plano resonator a better focusability can be reached, but with a bad near field profile. The plano-concave resonator results in a flat top near field profile and a similar far field profile, but the size of this far field profile is large, due to the large number of modes in the output beam, so the focusability of this beam is rather bad.

The divergence of the plano-plano resonator is approximately 3 mrad. The divergence of the plano-concave resonators is larger, depending on the radius of curvature of the

---

*Figure 4.11: Divergence as a function of the radius of curvature of the rear mirror.*
rear (concave) mirror. The divergence for the plano-concave resonator is within the accuracy in accordance with the value expected from an approximate analysis (see (4.18)).

4.2 Unstable resonators

Unstable optical resonators, i.e. resonators that do not comply with the stability criteria mentioned in section 4.1.1, have been found to be very useful as resonant cavities for laser oscillators, particularly whenever any combination of high gain, large mode volume, high energy or high power is present [2,6].

Unstable resonators in general have a few advantages over stable resonators:

- Single transverse mode, near diffraction limited output at high Fresnel numbers.
- High brightness; i.e. high power radiated per unit area of output per unit solid angle.
- Controlable mode-volume.

Especially the first two are important. Single transverse mode operation gives a better beam quality than multimode operation, especially for the XeCl* laser, whose output beam in a stable resonator contains many modes. For comparison: the lowest order (Gauss) mode in the resonator we used (a flat outcoupler and a 10 m radius HR-mirror) has only a radius of 0.6 mm while the output beam is approximately 23 mm x 23 mm. Therefore approximately \( D^2/(4w_0^2) = 360 \) modi are present in the beam from this stable resonator.

For the diffraction limited case it can be shown easily that the expected maximum brightness \( B \) is given by

\[
B \approx 0.3 \times P_L/\lambda^2
\]

(4.20)

where \( P_L \) is the laser power and \( \lambda \) the wavelength. Hence, noting the \( \lambda^2 \) dividing factor, if a large volume laser with proportionally high power output is operated near the diffraction limit, as is possible with unstable resonators, the brightness will be extremely high. With the stable resonators peak powers of 4 MW were obtained (800 mJ in 200 ns), resulting in an expected brightness of \( 1.3 \cdot 10^{15} \text{ W/cm}^2 \cdot \text{sr} \).

4.2.1 Basic concepts

A simple two mirror optical resonator is classified as unstable if (compare (4.9))

\[
g_1 g_2 > 1 \quad \text{or} \quad g_1 g_2 < 0
\]

(4.21)

where \( g_1 = 1 - L/R_1 \) and \( g_2 = 1 - L/R_2 \), \( R_1 \) and \( R_2 \) being the radii of curvature of the mirrors and \( L \) the mirror separation.
Sec. 4.2 Unstable resonators

Due to the finite size of at least one of the mirrors part of the radiation passes aside of this mirror and is so coupled out of the resonator. Given the ABCD matrix for the full round trip around the cavity, one can define the half trace parameter \( m \equiv (A + D)/2 \), with \( |m| > 1 \) for unstable resonators. This leads then to the round-trip geometric magnification \( \mathcal{M} \) given by

\[
\mathcal{M} = \begin{cases} 
  m + \sqrt{m^2 - 1} & \text{positive branch, } m > +1 \\
  -m - \sqrt{m^2 - 1} & \text{negative branch, } m < -1 
\end{cases} \quad (4.22)
\]

In the geometric approximation the output coupling depends only on the magnification factor \( \mathcal{M} \) of the resonator. For two dimensional mirrors the power loss per round trip is found to be \( 1/\mathcal{M}^2 \).

As with stable resonators, there is a constraint placed on the lasing frequencies of the unstable resonator by the requirement that the round trip phase difference should be a multiple of \( 2\pi \)

\[
\varphi + \frac{2\pi L}{\lambda} = 2p\pi \quad (4.23)
\]

where \( p \) is an integer, \( L \) the optical path length of the resonator and \( \varphi \) a (constant) phase shift introduced by the mirrors. The axial modes defined by different \( p \) values are spaced in frequency by \( \Delta \nu = c/2L \), and how many of these modes appear in the output depends on the gain-linewidth of the lasing medium.

### 4.2.2 Canonical analysis for unstable resonators

A simple but very general canonical formulation can be developed with which almost all standard unstable resonator designs of interest can be analysed [2].

**Huygens’ integral**

Assume a reference plane \( z_0 \), located just before the outcoupling aperture/mirror going in the outward direction; and then consider the propagation to a reference plane \( z_2 \) at the same location one round trip later. If it is assumed that the output mirror has a finite width of \( 2a \) in one transverse dimension, one can write the round trip Huygens’ integral for the resonator in that one transverse direction in the form [2]

\[
\tilde{u}_2(x_2) = e^{-jkL} \sqrt{\frac{J}{B\lambda_0}} \int_{-a}^{a} \tilde{\rho}(x_0) \tilde{u}_0(x_0) \exp \left[ -j \frac{\pi}{B\lambda_0} (Ax_0^2 - 2x_2x_0 + Dx_2^2) \right] dx_0 \quad (4.24)
\]

If the output mirror has some form of variable reflection or transmission, rather than being a simple hard edged aperture, this can be taken into account by including this in the transmission function \( \tilde{\rho}(x_0) \) which multiplies the input function \( \tilde{u}_0(x_0) \) inside the integral.
Huygens’ integral can be converted into a general canonical form by a simple transformation which begins by writing the input and the output waves in the forms [2]

\[\tilde{u}_0(x_0) = \tilde{v}_0(x_0) \times \exp\left[ +j\frac{\pi(A - \mathcal{M})x_0^2}{B\lambda_0} \right] \tag{4.25}\]

\[\tilde{u}_2(x_2) = \tilde{v}_2(x_2) \times \exp\left[ -j\frac{\pi(D - 1/\mathcal{M})x_2^2}{B\lambda_0} \right] \tag{4.26}\]

These transformations are physically equivalent to extracting out the spherical curvature of the unstable resonator modes, thereby converting the magnifying wavefronts into collimated wavefronts at both the input and output ends of the cavity. Huygens’ integral now becomes

\[\tilde{v}_2(x_2) = e^{-jkL} \sqrt{\frac{j}{B\lambda_0}} \int_{-\infty}^{\infty} \tilde{v}_0(x_0) \tilde{v}_0(x_0) \exp\left[ -j\frac{\pi}{B\lambda_0} (\mathcal{M}x_0^2 - 2x_2x_0 + x_2^2/\mathcal{M}) \right] dx_0 \]

This form corresponds to a propagation through a simple collimated telescopic system with a ray matrix of the form

\[
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}
\equiv
\begin{bmatrix}
\mathcal{M} & B \\
0 & 1/\mathcal{M}
\end{bmatrix}
\equiv
\begin{bmatrix}
1 & \mathcal{M}B \\
0 & 1
\end{bmatrix}
\times
\begin{bmatrix}
\mathcal{M} & 0 \\
0 & 1/\mathcal{M}
\end{bmatrix}
\tag{4.28}
\]

The overall system, with the spherical curvatures extracted out, can thus be factored into the matrix product of a zero length telescope of magnification \(\mathcal{M}\), plus a free space section of length \(\mathcal{M}B\).

The effective Fresnel number which characterises the effective free space propagation distance in this canonical model is given by

\[N_c = \frac{\mathcal{M}a^2}{B\lambda_0} \equiv \text{collimated Fresnel number} \tag{4.29}\]

This collimated Fresnel number determines the number of Fresnel diffraction ripples that can be seen in the output wave across the output aperture of the unstable resonator.

### Eigenwaves of the unstable resonator

The magnifying geometrical eigenwave solution for an unstable resonator, viewed in the canonical formulation, is clearly just a collimated plane wave that passes through the aperture (or equivalently bounces of the finite output mirror); is expanded transversely by the magnification \(\mathcal{M}\), remaining planar; and then propagates through the distance \(\mathcal{M}B\) back to the same reference plane.

This same canonical system also has, however, a demagnifying geometrical eigenwave, which, in general, is not collimated. The parameters from this demagnifying eigenwave can be obtained quite easily. Consider a spherical wave of radius \(R_0\) viewed at the input reference plane \(z_0\) in the canonical model. When such a spherical wave
is magnified in the transverse direction by a magnification factor $\mathcal{M}$, its radius of curvature is multiplied by $\mathcal{M}^2$, since the phase lag $\Delta \phi(x)$ at the outer edge of the beam should remain the same before and after the magnification. After this magnification the spherical wave propagates through the free space distance $\mathcal{M}B$ back to the reference plane. The radius of curvature should now be the same again (or else it would not be an eigenwave of the resonator), so

$$R_0 = -\frac{\mathcal{M}B}{\mathcal{M}^2 - 1} \quad (4.30)$$

This is evaluated at the reference plane just before the output mirror.

One should keep in mind that these magnifying and demagnifying eigenwaves are, strictly speaking, eigensolutions only for an unbounded or purely geometrical unstable system, not for a hard edged resonator in which finite beam diameters and edge diffraction effects must be included. The dominant mode pattern in an unstable resonator, however, is generally quite similar in its basic properties to the magnifying eigenwave [2].

When this kind of magnifying eigenwave strikes the edges of the output aperture, it can be expected that spherical edge waves will be scattered from the aperture edges in all directions. Some of this edge wave energy will be scattered in a direction which feeds directly into the demagnifying eigensolution. This scattered wave energy will at first demagnify down towards the core of the unstable system, coming closer to the axis of the system after every round trip. After a few round trips this energy will have demagnified down to such a small diameter that diffraction spreading effects become very important. The demagnified energy will be turned back outward and in fact will be converted into the magnifying eigenwave direction. The relative phase angle with which the demagnifying wave is excited by the aperture edges and fed back into the primary magnifying wave is quite significant in determining the mode behaviour of a real unstable resonator [2]. This relative phase angle is expressed in terms of the equivalent Fresnel number

$$N_{eq} = \frac{a^2}{2R_0 \lambda} = \frac{\mathcal{M}^2 - 1}{2 \mathcal{M}} \times \frac{a^2}{2B \lambda_0} = \frac{\mathcal{M}^2 - 1}{2 \mathcal{M}^2} \times N_c \quad (4.31)$$

To find the exact eigenvalues and eigenmodes for an unstable resonator the exact resonator integral equation has to be solved. This can be done either by using a Fox-and-Li type numerical procedure or by one of several complicated analytical methods. A very good method to determine these eigenvalues and eigenmodes is the virtual source method [2].

With the eigenvalues and eigenmodes the mode behaviour of the resonator can be investigated. Figure 4.12 shows the power loss for the four lowest order modes in a hard edge unstable resonator as a function of the equivalent Fresnel number. The figure shows that the equivalent Fresnel number has quite a large influence on the resonator mode behaviour. The cusps in the lowest curve (points with higher power loss) are actually mode crossing points. At these values of the equivalent Fresnel number there are two modes that have exactly the same diffraction losses. However, only the magnitude and not the phase angles of these two modes are the same, so
Figure 4.12: Power losses in a hard edge unstable resonator as a function of the equivalent Fresnel number of the resonator.

Figure 4.13: Intensity (left) and phase profiles (right) of the lowest order (top) and first order (bottom) eigenmode of a hard edge unstable resonator near a mode crossing point ($N_{eq} = 2.9$).

there is no real degeneracy. For higher values of $N_{eq}$ sometimes there is no mode crossing, but only cusping (interaction occurs with the first higher order mode).

Figure 4.13 shows the lowest order and the accompanying first order mode near such a mode crossing point. The feedback for these two modes is nearly equal and the intensity profiles have a similar shape. The phase however is rather different.

Those values of the equivalent Fresnel number halfway between the mode crossings
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Figure 4.14: Intensity (left) and phase profiles (right) of the lowest order (top) and first order (bottom) eigenmode of a hard edge unstable resonator near a optimal mode discrimination point ($N_{eq} = 2.4$).

seem to be the optimum operating points for an unstable resonator, since they combine the largest discrimination between lowest and higher order modes with the lowest diffraction losses for a given magnification $M$. This is illustrated in figure 4.14. The figure shows the two lowest order modes near such an optimum operating point. The feedback is clearly different and there is good mode discrimination therefore.

The question arises how the modes in a resonator will be modified by the effects of a spatially varying gain within the resonator, as well as the effects of gain saturation in the laser medium. This can be studied by approximating the gain medium by a thin saturable gain sheet in the calculations. One finds then, that the general character of the eigenmode changes only very little [2], the primary difference being that some of the higher peaks are pushed down in amplitude by the local gain saturation. This seems to be true in most loaded resonator calculations: transverse gain variations and gain saturation have only minor influence on the mode patterns, however any kind of local transverse phase variations will have large effects on the mode patterns and on the far field beam spread in particular.

4.2.3 Design of an unstable resonator

Different unstable resonator layouts are possible. These layouts can be divided into two classes: the positive branch unstable resonators ($m > 1$) and the negative branch unstable resonators ($m < -1$). The negative branch schemes have the disadvantage of a focal point inside the resonator, which might lead to problems with optical breakdown. This can be overcome by using a Self Filtering Unstable Resonator (SFUR),
which is a frequently used technique for short pulse lasers (see e.g. [7]).

In our long pulse system the energy in the pulse is much higher, so the decision was made to use a positive branch unstable resonator. Now there are a few schemes possible (see e.g. [8,9]). We used the simplest scheme, which is shown in figure 4.15. This scheme consists of one fully HR coated concave mirror with radius of curvature $R_1$ and a convex mirror $M_2$ on which only a partial coating is deposited with a diameter of $2a$, to enable the light to pass past the mirror edges. To provide a collimated output beam, the resonator has to be confocal, i.e. $f_1 + f_2 = L$, which is equal to $R_1 + R_2 = 2L$, where $R_2$ is the radius of curvature of the convex mirror. The convex mirror $M_2$ should have the same radius of curvature (but concave instead of convex) at the backside to ensure a parallel output beam.

The magnification $\mathcal{M}$ can be calculated by using (4.22). For this confocal positive branch unstable resonator we find

$$\mathcal{M} = -\frac{R_1}{R_2}$$  \hspace{1cm} (4.32)

With a magnification $\mathcal{M}$ the geometrical feedback becomes

$$\gamma = \frac{(2a)^2}{(2\mathcal{M}a)^2} = \frac{1}{\mathcal{M}^2}$$  \hspace{1cm} (4.33)

For the reflection profile on the outcoupling mirror two different sorts of profiles were investigated. Hard edge unstable resonators have an outcoupler were the radial reflection profile shows a step in the reflectance. A variable reflectivity unstable resonator (VRUR) has an outcoupling mirror, which has a smoothly varying radial reflection profile. The hard edge unstable resonators will be treated in section 4.4 and the VRURs in section 4.5.

### 4.3 Experimental configuration

To characterise the unstable resonators different measurements were performed on the optical output beam. Apart from the pulse energy and the pulse form, the main points of interest were the near and far field energy distributions.
4.3.1 Near field measurement

For the near field measurements the output beam is imaged on a scintillator, using a relay imaging system of two lenses with focal lengths of 50 cm placed 1 m apart. The used scintillator is a thin film of a solution of sodium salicylate in water between two quartz windows. The beam power is attenuated to $\sim 1\%$ before it reaches the scintillator. In this intensity region the scintillator response is found to be linear with the laser intensity. However, our image intensified CCD camera has a nonlinear response, which is compensated for, by using a correction function.

4.3.2 Far field measurement

The far field pattern is measured by focussing the attenuated beam (attenuation to $\leq 0.05\%$ of the beam) on the scintillator, using a concave mirror with a radius of curvature of 10 m. This attenuation is performed with a 45° 90% reflectivity mirror and a flat HR-coated mirror. The optical beam is unpolarised, hence no effect on the far field pattern, caused by the attenuation, is expected [10]. To record the image the CCD camera has been equipped with a microscope objective.
We decided to use this method of determining the far field pattern as we wanted to study the single shot behaviour of our laser. The laser beam is found to have a rather large angular instability (see section 4.7). The angular instability is influenced by e.g. gas turbulence and discharge inhomogeneities. To diminish these influences the repetition rate of the system should be rather low. Because of this angular instability, methods like the knife-edge method (see e.g. [11]) need large amounts of measurements in order to obtain reasonable values for the beam width. However, as stated by Wright et al. [11], a complete array of beam intensity data, as is obtained with a CCD camera, is also permitted to analyse the beam. A disadvantage of this system is the relatively large noise level. But as we are mainly interested in the central peak behaviour, this is no problem for our application.

4.4 Hard edge unstable resonators

In this section the first type of confocal positive branch unstable resonators will be treated: the hard edge unstable resonators. Hard edge unstable resonators are resonators where the outcoupling mirror shows a step in the reflectivity profile. These mirrors can be made quite easily by putting a mask over the substrate in the layer deposition process.

4.4.1 High reflectivity hard edge unstable resonators

If the central spot on the outcoupling mirror has a high reflectivity (HR) the outcoupled near field is ring-shaped with practically no energy in the center of the beam. Figure 4.18 shows the near field beam profile for a resonator with a geometric magnification \( M \) of 2.4 and a central HR spot of 8 mm diameter. The diffraction pattern at the beam edge is caused by the hard mirror edge which may not be perfect due to the fabrication process of the HR coating.

Figure 4.18: Near field beam profile (left) and focus profile (right) of the beam from a hard edge unstable resonator \( (M = 2.4) \) with a central HR spot of 8 mm diameter focussed by a 10 m radius concave mirror.
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Figure 4.19: Calculated and measured near field energy distributions of a hard edge unstable resonator with $\mathcal{M} = 2.4$ and a central HR spot of 8 mm.

The choice for a magnification $\mathcal{M} = 2.4$ is based on the minimum feedback criterion found in the stable resonator measurements from section 4.1.3. There we found that at least about 20% feedback is needed to get a proper energy extraction. The value of 2.4 for the magnification results in a geometrical feedback of 17% which is just below this value.

As already mentioned in section 4.2.2, an approximation of the eigenmodes of a hard edge resonator can be determined using the virtual source theory. Calculation of the lowest order mode of our resonator results in the near field energy distribution just behind the outcoupling mirror shown in figure 4.19. A measured near field energy distribution is also shown in this figure. The calculation is based on an unloaded resonator, i.e., the gain medium is not taken into account. The gain medium smoothes the profile due to the saturation of the gain [2].

If the gated camera is used to look at the time-behaviour of the near field energy distribution, it is found that the near field energy profile does not vary much during the pulse. The profile is built up before the saturation of the gain and it remains unchanged during the output pulse. The small variations that could be seen were found to be small shot-to-shot variations. The focus energy distribution shows more shot-to-shot variation. This is probably due to the fact that the lowest order mode was not completed before saturation. Thus small phase and amplitude variations may occur in the near field. Especially these phase variations have their effect on the intensity distribution in the far field. However, the focus measurements show, on average, no large differences between the beginning and the end of the pulse.

The measured far field energy distribution of the beam from the HR unstable resonator is shown in figure 4.18. The far field energy distribution shows the ring structure as expected from the diffraction of a cylindrical beam. A cross-section of the focus is shown in figure 4.20. From the above calculated outcoupled field energy distribution the far field energy distribution can be calculated by propagating the beam through a lens to the focal plane of this lens. This propagation of the wavefront is given by
the Huygens integral (4.24). The numerical method used for these calculations is described in appendix A. The result of this calculation is also shown in figure 4.20. It is seen that the measured focus energy distribution is slightly narrower than the calculated focus energy distribution. This is due to the smoother and somewhat wider near field energy distribution. However, if we calculate the focus energy distribution from the measured near field energy distribution shown in figure 4.19 using a uniform phase distribution, a smaller focus is found as can be seen in figure 4.20. The measured focus energy distribution is slightly wider than the calculated distribution thus the output beam is nearly diffraction limited. The FWHM of the measured focus is 16.7 μrad and the FWHM of the calculated focus is 13.1 μrad.

The full divergence angle based on the first minimum in the focus intensity distribution is 32 μrad for the measured beam and 30 μrad for the calculated beam. The measured beam has approximately 60 % of the energy within the central peak and the calculated beam 82 % of the energy. This results in a TDL-parameter, as introduced in chapter 3, of 1.3 for our beam.

The output energy is 339 mJ in a 146 ns pulse under matched discharge conditions. Approximately 60 % of this energy is focussed in the central peak. This results in a brightness of $5.5 \cdot 10^{14}$ W/cm² sr. Here the brightness is defined as

$$B = \frac{kE}{\tau} \left( \frac{1}{\pi \frac{\theta_i \Delta \lambda} {2}} \right)^2$$

(4.34)

If the discharge is pushed to its limits by increasing the PFN voltage the pulse energy can be increased to approximately 850 mJ, leading to a brightness of $1.2 \cdot 10^{15}$ W/cm² sr. In both cases a lot of energy, about 40 %, is lost in the side lobes of the focus intensity profile. This results in a brightness which is lower than the brightness that could be obtained if all energy would be in the central peak.
4.4.2 Partial reflecting hard edge unstable resonators

Calculations predict that the side lobe energy can be reduced by lowering the reflectivity of the central spot on the outcoupling mirror. Figure 4.21 shows the calculated focus energy distributions based on the lowest order eigenmode of a hard edge resonator with a magnification $M$ of 2.4 and an outcoupling mirror radius of 4 mm. It is seen that if the central reflectivity is lowered the side lobe is also reduced. The width of the central peak however increases slightly, thus the beam of a partial reflecting hard edge unstable resonator will show a slightly larger divergence than a high reflectivity hard edge unstable resonator.

Table 4.1 shows the characteristics of different resonators used in the experiments. $R$ in table 4.1 is the reflectivity of the reflecting area and $D_r$ is its diameter. The geometrical feedback $\gamma = R/M^2$, which gives only an indication of the actual feedback, is determined by the magnification $M$ and the reflectivity of the central area $R$ and it is independent of the size of the reflecting area. $\theta_d$ is the full divergence angle based on the first minimum in the focus energy distribution, $E$ is the output energy under matched discharge conditions, $\tau$ is the pulse duration, $k$ is the percentage of energy within the central peak, $TDL$ is the times diffraction limited parameter, $K$ the beam quality number (as defined in chapter 3) and $B$ is the brightness of the output beam.

Figure 4.22 shows the distribution of the energy in the focus for the three resonators with the same magnification $M = 2.4$, mentioned in table 4.1. These resonators are identical (same radii of curvature for both mirrors and same resonator length), except for the reflectivity of the central spot (100, 72 and 45 %). It is seen that the 72 % spot reflectivity shows an improvement of the energy distribution in the focus: the energy in the side lobe is lowered (approximately 70 % of the pulse energy is found in the central peak). However, the width of the focus is slightly larger and the pulse energy slightly lower due to the lower feedback. Therefore the brightness decreases slightly. At 45 % reflectivity the side lobe energy increases to a level above the 100 %
reflection case. This is probably caused by the outcoupler: due to the coating used to obtain the reflectivity profile there will exist a phase difference between the central transmitted part and the edge of the beam. This phase difference results in more energy in the side lobe. Interferometric measurements show that the 70 % mirror has a phase difference of approximately $0.3\pi$ and the 45 % mirror a phase difference of approximately $\pi$ (see appendix D). These values are in agreement with those expected from the coatings on the mirrors. The phase difference introduced by the coating can be reduced by using phase unifying mirrors [12,13]. These will be discussed later in this section and in appendix D.

When the reflectivity of the central part is lowered the geometrical feedback $\gamma$ is also lowered. To keep the geometrical feedback similar for the different reflectance values the resonator magnification has to be decreased for decreasing mirror reflectance. So experiments have been performed using resonators having an equal geometrical feedback. To ensure a proper filling of the gain volume the size of the reflecting spot on the outcoupling mirror was also increased. This, however, has a negative effect on

**Table 4.1:** Characteristics of and experimental results obtained with the laser equipped with different unstable resonator configurations and operated under matched discharge conditions.

<table>
<thead>
<tr>
<th>$\mathcal{M}$</th>
<th>$R$</th>
<th>$D_r$</th>
<th>$\gamma$</th>
<th>$\theta_d$</th>
<th>$E$</th>
<th>$\tau$</th>
<th>$k$</th>
<th>$TDL$</th>
<th>$K$</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[-]</td>
<td>[%]</td>
<td>[mm]</td>
<td>[%]</td>
<td>[rad]</td>
<td>[mJ]</td>
<td>[ns]</td>
<td>[%]</td>
<td>[-]</td>
<td>[-]</td>
<td>[W/cm²·sr]</td>
</tr>
<tr>
<td>2.4</td>
<td>100</td>
<td>8</td>
<td>17.7</td>
<td>32</td>
<td>339</td>
<td>160</td>
<td>60</td>
<td>1.3</td>
<td>0.50</td>
<td>$5.5 \cdot 10^{14}$</td>
</tr>
<tr>
<td>2.4</td>
<td>72</td>
<td>8</td>
<td>12.5</td>
<td>37</td>
<td>334</td>
<td>146</td>
<td>70</td>
<td>1.6</td>
<td>0.67</td>
<td>$4.6 \cdot 10^{14}$</td>
</tr>
<tr>
<td>2.4</td>
<td>45</td>
<td>8</td>
<td>7.8</td>
<td>31</td>
<td>305</td>
<td>146</td>
<td>50</td>
<td>1.6</td>
<td>0.20</td>
<td>$4.1 \cdot 10^{14}$</td>
</tr>
<tr>
<td>2.0</td>
<td>72</td>
<td>10</td>
<td>17.5</td>
<td>36</td>
<td>362</td>
<td>149</td>
<td>70</td>
<td>1.7</td>
<td>0.52</td>
<td>$5.0 \cdot 10^{14}$</td>
</tr>
<tr>
<td>1.6</td>
<td>45</td>
<td>12</td>
<td>17.5</td>
<td>34</td>
<td>359</td>
<td>160</td>
<td>30</td>
<td>2.3</td>
<td>0.16</td>
<td>$2.5 \cdot 10^{14}$</td>
</tr>
</tbody>
</table>

*Figure 4.22: Focus energy distributions from resonators with different central reflectivities (100, 72 and 45 %). The magnification $\mathcal{M} = 2.4$ for all resonators.*
the divergence. The focus from the resonator with a larger mirror size is somewhat larger, hence the brightness will be reduced somewhat. However, if we would have chosen to keep the mirror size the same the brightness would also be reduced due to the smaller outcoupled beam ($M \cdot D_r$ is reduced) and the energy extraction would be reduced because of the worse gain volume usage.

The characteristics of the studied resonators can be found in table 4.1. The obtained results are similar to the results with the resonators having the same magnification mentioned above. A smooth, but slightly wider focus profile (as expected, see above) with practically no side lobes is obtained for the resonator with an outcoupler of 72 % reflectivity and the resonator fitted with an outcoupler having a central reflectivity of 45 % leads to more side lobe energy than the resonator with a 100 % reflecting area. The focus from this last resonator is shown in figure 4.23. The figure shows a clear asymmetry between the horizontal and vertical directions. This effect is most distinct for this specific resonator and the phase unifying resonator of the next paragraph. The focus images from the other resonators treated before showed only a very small asymmetry and look similar to the one depicted in figure 4.18. This small asymmetry in the focus energy distribution probably originates from the small asymmetry which can be found in the near field energy distribution. The latter might be caused by a nonhomogeneous discharge due to a nonuniform deposition of the electrical power into the discharge.

However, figure 4.23 shows a rather large asymmetry, which is not present in the near field energy distribution. We think that this asymmetry might be caused by the electrodes. During the start-up of the system several modes try to oscillate. Due to different losses the respective modes have strong competition. In general the mode with the lowest losses will reach the highest intensity and will suppress the weaker modes. For survival of the strongest mode with the lowest losses several oscillations between the mirrors have to take place before saturation of the gain medium. In the case of a low magnification system there are relatively lower losses for the higher order modes so that the competition process becomes slower and it takes more oscillations for the lowest loss mode to dominate. If the gain medium saturates before the end of the mode competition, as in pulsed systems like ours, the output beam still contains higher order modes. This means that the far field energy distribution has a larger spot size. In the case of the $M = 1.6$ resonator the gain medium is probably saturating too early so that not only the lowest loss mode but also a few other modes are present in the beam. The losses for these higher order modes are different for the horizontal and the vertical direction. In the vertical direction the gain volume is restricted by two hard edges: the electrodes. In the horizontal direction there is no hard restriction as the gain volume ends where the discharge ends: at the border of the preionisation, which is a rather soft transition. Thus in the vertical direction the losses for the higher order modes are larger than in the horizontal direction, resulting in a beam which shows less higher order modes in the vertical direction than in the horizontal direction. This results in an asymmetrical focus having more side lobes in the horizontal direction than in the vertical direction.
Figure 4.23: Focus spots from a hard edge resonator with magnification of 1.6 and a central spot reflectance of 45 % fitted with a standard mirror (left) and fitted with a phase unifying mirror (right).

4.4.3 Phase unifying unstable resonators

If a beam passes a partial reflecting hard edge outcoupling mirror a phase difference between the central part (i.e. the part that passes through the reflecting coating) and the outer part (i.e. the part that passes beside the reflecting coating) can occur. This transmission phase difference between the central spot and the rest of the mirror can be decreased by using so-called phase unifying mirrors [12,13]. As mentioned before the 45 % outcoupling mirror showed a phase difference of nearly $\pi$. Thus experiments were performed with a resonator with a magnification of $M = 1.6$ having a phase unifying outcoupling mirror with a central reflectance of 45 %. Interferometric measurements on this mirror show only a very small phase difference (see appendix D). Calculated from the coating there is a phase difference of only 0.16$\pi$. Figure 4.23 shows typical results. It is clearly seen that the output beam from the phase unifying resonator has a better focus than the non-phase unifying resonator. The non-phase unifying resonator shows a lot more side structure than the phase unifying resonator. The output energy from the phase unifying resonator is a little higher than from the normal hard edge resonators: 399 mJ in a 160 ns pulse instead of 339 mJ in a 160 ns pulse. The diffraction angle of the beam is comparable to the diffraction angle as mentioned in table 4.1 for the $M = 1.6$ resonator with a 45 % outcoupler: 34 $\mu$rad, however approximately 80 % of the energy is found in the central peak instead of less than 30 % as is the case for the standard mirror resonator. This therefore results in a higher brightness: $7.0 \cdot 10^{14}$ W/cm$^2$sr under matched discharge conditions and $1.4 \cdot 10^{15}$ W/cm$^2$sr at maximum energy loading of the discharge. The value for the TDL parameter of the beam from this resonator is 1.4.

4.4.4 Conclusions

It has been demonstrated that with a high reflectivity hard edge unstable resonator with a magnification of 2.4 a nearly diffraction limited beam ($TDL = 1.3$) can be
obtained from our low gain, long pulse XeCl* excimer laser. Under matched discharge conditions the brightness of the pulse is $5.5 \cdot 10^{14}$ W/cm$^2$·sr. The maximum obtained brightness was $1.2 \cdot 10^{15}$ W/cm$^2$·sr.

The side lobe energy in the far field energy distribution can be reduced by using partial reflecting outcoupling mirrors. This leads to a small increase in divergence angle, but if the geometrical feedback is kept constant the brightness of the beam increases due to a higher central peak energy. However, in the case of partial reflecting outcoupling mirrors the coating can cause phase problems as we have seen with the 45% mirrors. It has been shown that these phase problems can be avoided by using phase unifying mirrors. Due to a lower side lobe energy and lower diffraction effects at the beam edge the brightness of the beam from a 45% reflectivity phase unifying resonator with a magnification of 1.6 is $7.0 \cdot 10^{14}$ W/cm$^2$·sr. The maximum obtained brightness with the phase unifying resonator was $1.4 \cdot 10^{15}$ W/cm$^2$·sr.

4.5 Variable reflectivity unstable resonators

The second type of positive branch unstable resonators is the variable reflectivity unstable resonator (VRUR). In hard edge unstable resonators the diffraction at the mirror edges causes disturbances in both the cavity mode profile and the outcoupled mode profile. These disturbances can be overcome by smearing out the edge diffraction effects by tapering the edges of the mirrors. Using a variable-reflectivity mirror (VRM) for the outcoupling the advantages of an unstable resonator, i.e. increased mode volume and good mode discrimination, can be combined with the possibility to control both the mode performance of the unstable resonator and the transverse beam profile of the output beam.

However, the primary problem with both phase and amplitude tapering is finding practical methods for tapering the mirror reflectivity which can be fabricated to adequate optical tolerances at reasonable cost, and which will also stand high output powers if necessary.

In addition, unwanted phase perturbations associated with an amplitude taper may distort the phase front of the lowest order mode, so that this mode, although it may have excellent mode separation and mode discrimination against higher order modes, will have a nonplanar or nonspherical wavefront which cannot easily be converted into a well-collimated output beam. A possible, but very difficult, solution is the use of a phase filter, either intracavity or extracavity, or both.

The technique we developed in cooperation with the optics workshop in our department to produce these special mirrors is explained in appendix E.

4.5.1 Theory

The simplest solution for an outcoupling mirror of a VRUR, at least analytically, is a Gaussian variable reflectivity mirror, because a resonator fitted with such a mirror will have well understood Hermite-Gauss or Laguerre-Gauss modes, which can be
described and analysed more or less exactly using the ABCD-method.

To analyse an unstable resonator with a Gaussian variable reflectivity mirror, the hard edged aperture in the canonical model as described in section 4.2.2 should be replaced by an aperture with the appropriate amplitude transmission coefficient \( \tilde{r}(r) \) (the mirror reflection coefficient \( \tilde{r}(r) \) in the real laser becomes the aperture transmission coefficient in the equivalent lensguide model). For the ideal Gaussian situation it is assumed that this reflection or transmission coefficient is given by

\[
\tilde{r}(r) = e^{-\frac{r^2}{w_a^2}}
\]

where \( w_a \) is the \( 1/e \) radius for the amplitude transmission, or the \( 1/e^2 \) radius for intensity transmission, through the aperture.

The ABCD matrix of a Gaussian aperture is given by [2]

\[
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix} = \begin{bmatrix}
1 & 0 \\
-j\lambda/\pi w_a^2 & 1
\end{bmatrix}
\]

(4.36)

Combining this ABCD matrix for the Gaussian aperture with the ABCD matrix of the unstable resonator (4.28) one obtains as ABCD matrix for the variable reflectivity unstable resonator

\[
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix} = \begin{bmatrix}
\mathcal{M} & -j\lambda B/\pi w_a^2 \\
-j\lambda/\pi w_a^2 \mathcal{M} & 1/\mathcal{M}
\end{bmatrix}
\]

(4.37)

One finds from the eigenvalues and eigenmodes that the lowest order mode is a collimated Gaussian beam with an infinite radius of curvature and a finite spot size (see appendix B)

\[
w^2 \approx (\mathcal{M}^2 - 1) \times w_a^2 \quad \text{for} \quad \frac{\mathcal{M}^2 + 1}{2\mathcal{M}} \gg \frac{B\lambda}{2\pi w_a^2}
\]

(4.38)

The amplitude discrimination between the lowest order and higher order TEM\(_{mn}\) modes will roughly be \((1/\mathcal{M})^{m+n}\) on each round trip.

Although the transverse profile of the lowest eigenmode will be Gaussian \textit{inside} the laser cavity, the output beam will have this Gaussian profile multiplied by the radially varying mirror transmission \( T(r) \equiv 1 - \tilde{r}(r) \) outside the cavity. For a mirror having a reflectivity of the general form

\[
R(r) \equiv |\tilde{r}(r)|^2 = R_0 e^{-\frac{r^2}{w_a^2}}
\]

(4.39)

where \( R_0 \) is the central value of the reflection coefficient, the output beam will become of the form

\[
I_{\text{out}}(r) = I_0 \left[ 1 - R_0 e^{-\frac{2r^2}{w_a^2}} \right] e^{-\frac{2r^2}{w^2}}
\]

(4.40)

The effective feedback into the resonator is found to be

\[
\tilde{R} = \frac{R_0}{\mathcal{M}^2}
\]

(4.41)
The smoothest and most uniform output beam profile is obtained when the centre reflection and the resonator magnification are conform the "maximally flat" condition (see appendix C)

\[ R_0M^2 = 1 \]  \hspace{1cm} (4.42)

### 4.5.2 Experiments

In table 4.2 the characteristics of the variable reflectivity unstable resonators are given.

<table>
<thead>
<tr>
<th>M [%]</th>
<th>R_e [%]</th>
<th>D_e [\mu m]</th>
<th>\gamma [%]</th>
<th>\theta_d [\mu rad]</th>
<th>E [\mu J]</th>
<th>\tau [ns]</th>
<th>k [%]</th>
<th>TDL [%]</th>
<th>K [-]</th>
<th>B [W/cm^2 sr]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.4</td>
<td>65</td>
<td>8</td>
<td>11.3</td>
<td>25</td>
<td>412</td>
<td>144</td>
<td>43</td>
<td>1.8</td>
<td>0.71</td>
<td>8.7 \times 10^{14}</td>
</tr>
<tr>
<td>2.4</td>
<td>45</td>
<td>8</td>
<td>7.8</td>
<td>28</td>
<td>252</td>
<td>120</td>
<td>53</td>
<td>1.8</td>
<td>0.79</td>
<td>6.2 \times 10^{14}</td>
</tr>
<tr>
<td>2.4</td>
<td>17</td>
<td>8</td>
<td>3.0</td>
<td>27</td>
<td>212</td>
<td>115</td>
<td>55</td>
<td>1.7</td>
<td>0.91</td>
<td>5.9 \times 10^{14}</td>
</tr>
<tr>
<td>2.0</td>
<td>65</td>
<td>10</td>
<td>16.3</td>
<td>27</td>
<td>440</td>
<td>137</td>
<td>51</td>
<td>1.7</td>
<td>0.67</td>
<td>9.1 \times 10^{14}</td>
</tr>
<tr>
<td>2.0</td>
<td>45</td>
<td>10</td>
<td>11.3</td>
<td>37</td>
<td>342</td>
<td>129</td>
<td>64</td>
<td>2.2</td>
<td>0.73</td>
<td>5.0 \times 10^{14}</td>
</tr>
<tr>
<td>2.0</td>
<td>25</td>
<td>10</td>
<td>6.3</td>
<td>35</td>
<td>167</td>
<td>100</td>
<td>64</td>
<td>1.8</td>
<td>0.87</td>
<td>3.5 \times 10^{14}</td>
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<tr>
<td>1.6</td>
<td>45</td>
<td>12</td>
<td>17.6</td>
<td>50</td>
<td>\sim 200</td>
<td>\sim 130</td>
<td>n.m.</td>
<td>n.m.</td>
<td>n.m.</td>
<td>n.m.</td>
</tr>
</tbody>
</table>
Figure 4.24: The near field spots of two VRURs. Left: $\mathcal{M} = 2.4$, $R_c = 65 \%$. Right: $\mathcal{M} = 2.0$, $R_c = 25 \%$.

Figure 4.25: Measured and calculated focus energy distributions for two VRURs. Left: $\mathcal{M} = 2.4$, $R_c = 65 \%$. Right: $\mathcal{M} = 2.0$, $R_c = 25 \%$.

To obtain a near field close to a gaussian near field, a low central reflectivity has to be used for the gaussian reflection profile on the outcoupling mirror. Figure 4.24 shows a near field from a VRUR with a low central reflectivity ($\mathcal{M} = 2.0$, $R_c = 25 \%$). This resonator was designed to have a maximally flat near field, fulfilling the condition (4.42). The output energy from this resonator is (of course) rather low, as the feedback into the cavity is quite low. Therefore, the filling of the gain volume with the beam is not very good either. However, the output beam is flat topped, as is expected. Even with this smooth near field without a central hole, a small side lobe is expected theoretically in the far field as is shown in figure 4.25. The experiments show a slightly wider focus with a low but wide shoulder (see figure 4.25). This shoulder appears for nearly all resonators, but it is more clearly to discern for the $\mathcal{M} = 2.0$ resonators than for the $\mathcal{M} = 2.4$ resonators.

We believe that this shoulder is caused by the presence of higher order modes in the cavity, which have not been eliminated in the mode competition process. As the mode competition process is slower in resonators with a lower magnification, the $\mathcal{M} = 2.0$
resonators should suffer more from this effect than the $M = 2.4$ resonators, which is confirmed by our experiments.

The rather wide shoulder results in a lower energy content within the $1/e^2$ diameter of the beam. In table 4.2 the energy content within this $1/e^2$ diameter is given for all resonators. This energy content is checked by creating pinholes in a thin brass foil and by determinating of the energy transmission of these holes.

With the measured energy content and the measured beam divergence angle, the beam can be characterised using the TDL-parameter. In table 4.2 the value of the TDL-parameter for each resonator is given. All resonators result in a beam that is about 2 times diffraction limited. This is due to the relative large shoulder that all foci show.

An interesting effect seen in the experiments with the VRURs is the rather good performance of the low reflectivity resonators. In literature it is stated that for a given active medium an unstable resonator with a certain feedback should result in the same output energy as a stable cavity with the same feedback [2]. This is not confirmed by our experiments. We found that a variable reflectivity unstable resonator with an effective feedback of only 3.0 % results in a beam with reasonable energy while a stable resonator with similar feedback does not result in a measurable formation of an optical beam. In a stable cavity the lower limit for reasonable output is about 17 % feedback. We obtained similar results with the hard edge unstable resonators (described in the previous section). We believe that these resonators behave so well, because the optical field is built up at the optical axis of the resonator. Near the optical axis the feedback is higher, because the reflectivity of the VRM is larger close to the centre of the mirror, which coincides with the optical axis of the resonator. Hence the optical field can build up near the optical axis. As the magnification of the resonator is rather low, the optical field stays close to the optical axis for quite a long time, thus experiencing the higher central reflectivity of the VRM for quite a long time allowing the optical field to increase in strength. The outer part of the resonator merely acts as a (single pass) amplifier.

### 4.5.3 Conclusions

Nearly diffraction limited beams (1.7 times diffraction limited) with a smooth near field energy distribution can be obtained, using variable reflectivity unstable resonators. The experiments have shown that the high central reflectivity is not strongly influencing the beam quality. However, the output energy of these high central reflectivity VRURs is much larger due to the improved gain volume usage. The brightness of these high central reflectivity VRURs is nearly twice as high compared to the brightness of the low central reflectivity VRURs. The best performing variable reflectivity unstable resonator was the VRUR with a magnification of 2.0 and a central reflectivity of 65 %. With this resonator a brightness of $9.1 \cdot 10^{14} \text{ W/cm}^2\text{ sr}$ could be obtained under matched discharge conditions.

If a flat topped near field is requested, the reflectivity and magnification have to be chosen such that the maximally flat condition (4.42) is satisfied. The experiments
have shown that this condition, which has been derived for an empty resonator, holds for this specific laser.

4.6 Comparison of the different resonator configurations

In this section a comparison will be made between the best performing resonators from the previous sections. The comparison will be made with respect to output energy, pulse duration, average power and brightness. The three resonators used in this comparison are:

1. the stable resonator consisting of a flat outcoupling mirror with a reflectivity of 50 % and a concave HR back mirror with a radius of curvature of 4.75 m,
2. the high reflectivity hard edge unstable resonator with a magnification of 2.4 and a central reflectivity if 100 %, and
3. the high reflectivity variable reflectivity unstable resonator with a magnification of 2.0 and a central reflection of 65 %.

4.6.1 Experiments

Figure 4.26 shows the output energy of different resonators as a function of the charging voltage of the main capacitor. It is seen that, except for the higher mainpulse voltages, the highest output can be obtained with a stable resonator. At the highest mainpulse voltage, i.e. when the discharge is pushed to its limits, the output is the highest for the VRUR. At these high mainpulse voltages the gain is much higher than about 6 % cm⁻¹. This means that, to get optimal energy extraction, the reflectivity
of the outcoupler of the stable resonator should probably be lower than the 50 % we used to obtain maximum energy extraction.

The hard edge resonator shows an output energy reduction of about 20 % compared to the stable resonator. This is caused by two factors: (a) the gain volume is not properly filled as we get a circular beam out of an nearly square gain medium, thus the corners of the gain cross section are not used, and (b) there is less outcoupling in the centre of the beam because of the outcoupling mirror reflectivity. With the VRUR with high central reflectivity it is possible to extract a similar amount of energy from the gain medium as with the stable resonator. This is especially the case for the higher mainpulse voltages, because the gain volume is more completely filled and there is more transmission of the beam near the centre of the VRM, as the reflectivity of the VRM is lower than that of the hard edge unstable resonator. However, the pulse duration is much shorter for the VRUR, as can be seen from figure 4.27. This is caused by the longer time needed for the optical pulse to build up in the VRUR. As the feedback of the stable resonators is much larger than the feedback of the unstable resonators, this results in a shorter build-up time and a longer optical pulse for the stable resonator. The pulse duration in the VRURs is even shorter than the pulse duration from the hard edge unstable resonators, probably because of the lower effective feedback in the beginning phase of the discharge. This results in the highest average pulse powers, defined by the energy divided by the pulse duration (= $E/\tau$), for the VRUR with a high central reflectivity, as shown in figure 4.28.

The brightness of the beam is a convenient parameter to compare the output from the different resonators. The brightness has already been defined by (4.34) as

$$B = \frac{kE}{\tau} \left( \frac{1}{\pi \frac{\theta_d}{2} \frac{\Delta D_L}{2}} \right)^2$$

Under matched discharge conditions, $V_{PFN} \approx 8$ kV, we obtained from the hard edge unstable resonator a brightness of $5.5 \times 10^{14}$ W/(cm$^2$·sr). This is an enormous in-
crease compared to the brightness of the beam from the plano-concave resonator: \( B = 1.5 \times 10^{10} \) W/(cm\(^2\)-sr). The best performing variable reflectivity unstable resonator \( (M = 2.0, R_c = 65 \%) \) resulted in a brightness of \( 9.1 \times 10^{14} \) W/(cm\(^2\)-sr) under matched discharge conditions. Comparing this to the other unstable resonators we conclude that the best results under matched discharge conditions are obtained with the variable reflectivity unstable resonator.

### 4.6.2 Conclusions

The best energy extraction has been obtained by using variable reflectivity unstable resonators with a high central reflectivity. The experiments have shown that the high central reflectivity is not strongly influencing the beam quality. With variable reflectivity unstable resonators a better energy extraction and therefore a higher brightness is obtained than with hard edge unstable resonators.

### 4.7 Beam pointing variation

The position of the focus field of the beam from a long pulse XeCl* excimer laser, equipped with a positive branch unstable resonator, shows some small shot-to-shot variations. This is the result of variations of the direction of the optical axis of the output beam. This effect is called the beam pointing variation and a parameter that can be used to describe it is the beam pointing variation parameter (BPV). The beam pointing variation is an important design parameter for optical resonators, especially in applications like micromachining and hole drilling. In literature the pointing variation has been observed, however not explained [14,15]. Based on calculations and measurements, in this section a more thorough explanation of the beam pointing variation is presented.
Sec. 4.7 Beam pointing variation

The laser was equipped with confocal positive branch unstable resonators, as these were found to be the best choice for our system to obtain minimum output beam divergence and maximum energy extraction (see previous sections). The experiments were performed with hard edge unstable resonators and variable reflectivity unstable resonators.

4.7.1 Experimental configuration

The (attenuated) laser beam is focussed by a 10 m concave mirror onto a scintillator. The scintillator image is recorded by a fast image intensified CCD camera equipped with a microscope objective. The position of the maximum of the focus field is determined for 150 shots. The standard deviation of the maximumom of the focus field is taken as a measure for the beam pointing variation (BPV). This definition is slightly different from the beam pointing stability (BPS) definition used in the ISO norm, which is based on the first moment of the intensity distribution. We used BPV instead of BPS for practical reasons as the measured focus spots were rather good in symmetry. Calculations have shown that the theory for the beam pointing variation described in this section gives similar results for the beam pointing stability.

The measurements presented in this section were based on the beam pointing variation in the horizontal direction, i.e. perpendicular to the electric field. The beam pointing variation in the vertical direction showed similar results. The measurements were corrected for slow variations of the average in time. These slow variations were found to be a problem of our experimental setup. The vertical direction showed a larger slow variation. After this correction it was found that the beam pointing variation shows practically no anisotropy.

4.7.2 Theory

As aforementioned, the number of cavity round trips of the optical field in an unstable resonator before saturation of the gain determines whether the beam becomes single mode or stays multimode [2,8,6]. The intracavity field builds up from noise (spontaneous emission). The lowest order mode in general has the lowest losses and it will therefore survive in the mode competition process if the modi have sufficient overlap. If there are enough round trips between the start of the gain and its saturation the intracavity field will only consist of the lowest order resonator eigenmode. If the gain saturates earlier, higher order modes will be present as well, resulting in a decreased beam quality. This mode competition process does not only influence the intensity distribution of the beam, but it influences the phase front as well. Therefore, the phase front also improves during the build-up phase. A distorted wavefront results in a variation of the direction of the optical axis of the beam, i.e. a variation of the position of the focus field.
4.7.3 Calculations

The evolution of the optical field inside a cavity, starting from noise, can be calculated using Huygens’ integral for the resonator (see section 4.2.2). For simplicity we assume that the intracavity wavefront is separable in the transverse plane, i.e. \( \tilde{u}(x, y) = \tilde{u}_x(x)\tilde{u}_y(y) \). For the simple resonator layout used in our experiments this is a valid assumption. The calculations can now be performed one-dimensionally, which results in a reduction of the necessary calculatory effort.

The calculations have been performed in three steps. First, the calculations for a certain number of round trips through the cavity are executed. The gain in the resonator was taken into account by introducing a saturable gain in the cavity calculations. The average signal gain was taken to be 3 \% cm and the saturation intensity 250 kW/cm\(^2\) [8]. Subsequently, the resulting intracavity field is transported through the outcoupling mirror. The last step is the propagation of the beam to the focus of a 10 m mirror. This calculation procedure is illustrated in figure 4.29.

Figure 4.29a shows the starting point of the calculations: randomly generated noise for both intensity and phase. Figure 4.29b shows the intracavity field just after saturation of the gain (i.e. after 7 round trips). The intensity profile shows a flat top, just as is seen in the experiments. The phase profile shows some tilt, so the beam is expected to deviate from the optical axis. The outcoupled field is shown in figure 4.29c and the focus is shown in figure 4.29d. It can clearly be seen that the maximum of the field is along the optical axis due to the tilt of the intracavity phasefront, as is expected.

The calculated beam pointing variation is determined by performing this procedure 500 times for different noise inputs. For each noise input the position of the maximum of the focussed intensity is found. The calculated pointing variation is determined as the standard deviation of the distribution of these 500 focus maxima. In addition there are also mechanical vibrations that contribute to the beam pointing variation. This part of the beam pointing variation will be determined experimentally.

The broken line in figure 4.30 shows the dependence of the pointing variation on the number of intracavity round trips. This calculation is performed for a resonator with \( \mathcal{M} = 2.4 \) and \( R_c = 100\% \). It can be seen that pointing variation decreases with an increasing number of round trips. This means that the beam pointing variation decreases with an increasing number of round trips. As aforementioned, the number of round trips before saturation of the gain determines the mode character of the beam. Similarly it determines the pointing variation of the beam. It turns out that different values for \( R_c \), but the same value for \( \mathcal{M} \) gives practically the same result.

The number of pre-saturation round trips is determined by the gain medium and the resonator feedback. Thus, if resonators with different magnification but equal feedback are used, a fixed saturation moment is obtained if the gain medium remains unaltered. This can be obtained for a constant \( \gamma = R/\mathcal{M}^2 \). If \( \gamma \) is kept constant the beam pointing variation will be solely determined by the resonator magnification \( \mathcal{M} \). The broken line in figure 4.31 shows the influence of the magnification on the pointing variation for \( \gamma = 17.5\% \). In these calculations the beam pointing variation is determined after 7 round trips, which is found to be a good estimate of the number of pre-saturation round trips in our experiments. The figure shows that a larger
Figure 4.29: The calculation of the beam pointing variation. On the left the intensity of the optical field and on the right the phase. (a) Start from noise, (b) calculation of intracavity field, (c) passage through the outcoupler and (d) the propagation of the beam to the focus.

magnification results in a smaller beam pointing variation.

4.7.4 Results

In table 4.3 the measured beam pointing variation is given for different hard edge unstable resonators. For the first group of three resonators the resonator magnification $M$ is kept constant while the reflectivity of the outcoupler $R_c$ is varied. For the second
Figure 4.30: Beam pointing variation as a function of the number of cavity round trips for a fixed magnification ($M = 2.4$) of the hard edge unstable resonator.

Figure 4.31: Beam pointing variation as a function of magnification of the resonator for a fixed number of round trips in a hard edge unstable resonator.

group of three resonators, the magnification is reduced. In order to keep the feedback $\gamma$ equal, the reflectivity of the outcoupler had to be reduced as well.

In table 4.3 the duration of the optical pulse $\tau_{opt}$ (FWHM) and the build-up time of the optical pulse $\tau_{bu}$ are given. The build-up time is determined from the time dependent gain behaviour (see section 2.6.3 for the measurement of this time dependent gain behaviour). The procedure for determining $\tau_{bu}$ is illustrated in figure 4.32. The end of the optical pulse is assumed to coincide with the end of the gain curve. As we use low feedback resonators, this is a reasonable assumption. The threshold is determined by the geometrical feedback of the resonator. The error in the build-up time is $\sim 10$ ns, due to shot-to-shot variations of the optical pulse. The number of pre-saturation round trips $N$ is equal to the build-up time divided by the cavity round
trip time. The beam pointing variation is measured by determination of the standard deviation of the position of the maximum of about 200 focus energy distributions.

It can be seen from the first set of three measurements in table 4.3 that the beam pointing variation that for a constant resonator magnification the beam pointing variation decreases slightly with an increasing number of pre-saturation round trips, as was predicted by the calculations (see figure 4.30). However, the effect is not very clear as the measurements are very close to each other and the measurement errors were relatively large. Nevertheless, the trend is observable. In order to fit the experimental data with the calculation an offset of 6.3 $\mu$rad was introduced to account for the background vibrations of the laser system. This offset value turns out to be constant for all configurations studied.

This set of three resonators shows a rather strange behaviour: a decrease of the reflectivity and therefore a decrease in feedback results in a decrease in build-up time. The decrease is quite small, but distinct. This might be caused by ringing of the

**Table 4.3: Measured beam pointing variation for different hard edge unstable resonators.**

<table>
<thead>
<tr>
<th>$M$</th>
<th>$R_e$</th>
<th>$\gamma$</th>
<th>$\tau_{opt}$</th>
<th>$\tau_{bu}$</th>
<th>$N \pm 1$</th>
<th>$BPS \pm 0.3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[-]</td>
<td>[%]</td>
<td>[%]</td>
<td>[ns]</td>
<td>[ns]</td>
<td>[-]</td>
<td>[\mu rad]</td>
</tr>
<tr>
<td>2.4</td>
<td>100</td>
<td>17.7</td>
<td>139</td>
<td>81</td>
<td>9</td>
<td>8.0</td>
</tr>
<tr>
<td>2.4</td>
<td>72</td>
<td>12.5</td>
<td>132</td>
<td>77</td>
<td>8</td>
<td>8.1</td>
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<tr>
<td>2.4</td>
<td>45</td>
<td>7.8</td>
<td>128</td>
<td>69</td>
<td>8</td>
<td>9.5</td>
</tr>
<tr>
<td>2.4</td>
<td>100</td>
<td>17.7</td>
<td>139</td>
<td>81</td>
<td>9</td>
<td>8.0</td>
</tr>
<tr>
<td>2.0</td>
<td>72</td>
<td>17.5</td>
<td>135</td>
<td>85</td>
<td>9</td>
<td>17.3</td>
</tr>
<tr>
<td>1.6</td>
<td>45</td>
<td>17.5</td>
<td>135</td>
<td>87</td>
<td>9</td>
<td>21.8</td>
</tr>
</tbody>
</table>
power deposition, and therefore the gain, at the beginning of the pulse. From figure 2.11 it can be seen that both the current through and the voltage across the laser ring with a period of about 20 ns. This leads to a ringing power deposition, which in turn results in a ringing gain. If the threshold becomes higher it crosses the gain curve at a different point. If the gain in the low threshold resonator crosses the threshold just before a downward slope and the gain in the slightly higher threshold resonator crosses the threshold just before an upward swing of the gain, then the build-up time in the high threshold resonator can be smaller than the build-up time in the low threshold resonator. The difference has to be smaller than the ringing time of the gain. And that is the case in our system, thus we think that the ringing of the gain causes this rather odd behaviour.

From the second set of measurements it can be seen that for equal build-up time the pointing variation decreases with increasing resonator magnification. This is in good agreement with our theoretical expectations (see figure 4.31). The discrimination between the modes is better in a resonator with a larger magnification. Thus the mode competition process is faster, resulting in an reduced pointing variation for resonators with larger magnification.

The measurements in figure 4.31 show a larger variation than the data in figure 4.30. This may be due to some variation in the build-up time. Therefore the data can not be fitted properly with one line. In figure 4.31 two boundaries were drawn based on the number of cavity round trips. The measurements were found within this boundaries.

Similar measurements were performed using variable reflectivity unstable resonators. The results are shown in table 4.4.

Figures 4.33 and 4.34 show the beam pointing variation as a function of the number of round trips in a variable reflectivity unstable resonator for a fixed magnification ($\mathcal{M} = 2.4$ and $\mathcal{M} = 2.0$ respectively). In the theoretical model the same offset is used for the background vibrations (6.3 $\mu$rad). Both figures show a good agreement between the experimental data and the calculations. We can conclude that the reflection profile on the outcoupling mirror is of no large influence on the beam pointing variation.

<table>
<thead>
<tr>
<th>$\mathcal{M}$</th>
<th>$R_e$</th>
<th>$\gamma$</th>
<th>$\tau_{opt}$</th>
<th>$\tau_{bn}$</th>
<th>$N \pm 1$</th>
<th>$BPS \pm 0.3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.4</td>
<td>65</td>
<td>11.3</td>
<td>100</td>
<td>70</td>
<td>8</td>
<td>9.1</td>
</tr>
<tr>
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<td>45</td>
<td>7.8</td>
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<td>76</td>
<td>8</td>
<td>8.3</td>
</tr>
<tr>
<td>2.4</td>
<td>17</td>
<td>3.0</td>
<td>64</td>
<td>65</td>
<td>7</td>
<td>11.1</td>
</tr>
<tr>
<td>2.0</td>
<td>65</td>
<td>16.3</td>
<td>95</td>
<td>75</td>
<td>8</td>
<td>13.5</td>
</tr>
<tr>
<td>2.0</td>
<td>45</td>
<td>11.3</td>
<td>70</td>
<td>70</td>
<td>7</td>
<td>14.4</td>
</tr>
<tr>
<td>2.0</td>
<td>17</td>
<td>6.3</td>
<td>60</td>
<td>86</td>
<td>9</td>
<td>12.2</td>
</tr>
</tbody>
</table>
Sec. 4.7 Beam pointing variation

Figure 4.33: Beam pointing variation as a function of the number of round trips in a variable reflectivity unstable resonator for a fixed resonator magnification ($\mathcal{M} = 2.4$).

Figure 4.34: Beam pointing variation as a function of the number of round trips in a variable reflectivity unstable resonator for a fixed resonator magnification ($\mathcal{M} = 2.0$).

The mode competition process continues after saturation of the gain. Due to the gain saturation the losses of the different modes change, hence the speed of the competition process changes. Nevertheless it is still expected that the beam pointing variation decreases during the optical pulse. Figure 4.35 shows the variation of the pointing variation during the optical pulse for the $\mathcal{M} = 2.0$, $R_c = 72$% resonator, where $T = 0$ ns coincides with the beginning of the laser pulse. These measurements were performed using the fast gated CCD-camera. No measurable reduction of the beam pointing variation during the pulse was observed. A probable reason for this lack of reduction of the beam pointing variation is the decrease of the discharge stability throughout the pulse [16]. This decrease of the discharge stability probably compensates for the expected reduction of the beam pointing variation. Measurements for other resonators also show this absence of the reduction of the beam pointing variation.
variation during the pulse.

Another effect that has to be taken into account is that part of the beam pointing variation is caused by mechanical vibrations of the laser system as a whole. The system was not mounted completely vibration free. Therefore, the limit of the beam pointing variation will never be zero, as in figures 4.30 and 4.31, but there will always be some pointing instability remaining.

### 4.7.5 Conclusions

The measurements are in good agreement with the calculations; a trend is clearly visible. Therefore, the introduced model seems to explain at least part of the pointing variation. In this model, it is assumed that the beam pointing variation originates from the build-up phase of the optical field in the resonator. If the optical field in the resonator gets more build-up time before the gain saturates the beam pointing variation decreases. It is found that the pointing variation of the entire pulse is determined by the pointing variation at the beginning of the pulse. Growth of discharge instabilities might cause a stabilisation of the pointing variation during the optical pulse.

The pointing variation of the optical beam can be influenced both by the gain (i.e. the build-up time) and by the resonator magnification. The influence of the resonator magnification is especially important, as it provides us with an extra criterion in the design of unstable resonators for pulsed laser systems.

Both calculations and measurements have shown that the reflection profile on the outcoupling mirror has no large influence on the beam pointing variation.
4.8 Conclusions

The experiments have shown that stable resonators are straightforward and easy to align cavities, which allow good energy extraction from the gain medium and homogeneous near field energy distributions. To ensure maximum energy extraction a geometrical feedback between 20 and 70 % is found to be optimal.

With the plano-plano resonator a better focusability can be reached, but with a bad near field profile. The plano-concave resonator results in a flat top near field profile and a similar far field profile, but the size of this far field profile is large due to the large number of modes in the output beam, so the focusability of this beam is rather bad.

The divergence of the plano-plano resonator is approximately 3 mrad. The divergence of the plano-concave resonators is larger, depending on the radius of curvature of the rear (concave) mirror. The divergence for the plano-concave resonator is within the accuracy in accordance with the value expected from an approximate analysis (see (4.18)).

With a high reflectivity hard edge unstable resonator having a magnification of 2.4 a nearly diffraction limited beam (TDL = 1.3) can be obtained from our laser. Under matched discharge conditions the brightness of the pulse is $5.5 \cdot 10^{14}$ W/cm$^2$·sr. The maximum obtained brightness was $1.2 \cdot 10^{15}$ W/cm$^2$·sr.

The side lobe energy in the far field energy distribution can be reduced by using partial reflecting outcoupling mirrors. This leads to a small increase in divergence angle, but if the geometrical feedback is kept constant the brightness of the beam increases due to a higher central peak energy. However, in the case of partial reflecting outcoupling mirrors the coating can cause phase problems, as we have seen with the 45 % mirrors. It has been shown that these phase problems can be avoided by using phase unifying mirrors. Due to a lower side lobe energy and lower diffraction effects at the beam edge the brightness of the beam from a 45 % reflectivity phase unifying resonator with a magnification of 1.6 is $7.0 \cdot 10^{14}$ W/cm$^2$·sr. The maximum obtained brightness with the phase unifying resonator was $1.4 \cdot 10^{15}$ W/cm$^2$·sr.

Further reduction of the side lobe energy can be obtained by using variable reflectivity unstable resonators. Using these resonators nearly diffraction limited beams (1.7 times diffraction limited) with a smooth near field energy distribution can be obtained. The experiments have shown that the high central reflectivity is not strongly influencing the beam quality. However, the output energy of these high central reflectivity VRURs is much larger due to the improved gain volume usage. The brightness of these high central reflectivity VRURs is nearly twice as high compared to the brightness of the low central reflectivity VRURs. The best performing variable reflectivity unstable resonator was the VRUR with a magnification of 2.0 and a central reflectivity of 65 %. With this resonator a brightness of $9.1 \cdot 10^{14}$ W/cm$^2$·sr could be obtained under matched discharge conditions.

If a flat topped near field is requested, the reflectivity and magnification have to be chosen such that the maximally flat condition (4.42) is satisfied. The experiments have shown that this condition, which has been derived for an empty resonator, holds
for this specific laser.

The best energy extraction has been obtained by using variable reflectivity unstable resonators with a high central reflectivity. The experiments have shown that the high central reflectivity is not strongly influencing the beam quality. With variable reflectivity unstable resonators a better energy extraction and therefore a higher brightness is obtained than with hard edge unstable resonators.

The position of the focus field of the beam is found to show a variation: the beam pointing variation. A model has been introduced to explain this beam pointing variation. The measurements are in good agreement with the calculations. In the model, it is assumed that the beam pointing variation originates from the build-up phase of the optical field in the resonator. If the optical field in the resonator gets more build-up time before the gain saturates the beam pointing variation decreases. It is found that the pointing variation of the entire pulse is determined by the pointing variation at the beginning of the pulse. Growth of discharge instabilities might cause a stabilisation of the pointing variation during the optical pulse.

The pointing variation of the optical beam can be influenced both by the gain (i.e. the build-up time) and by the resonator magnification. The influence of the resonator magnification is especially important, as it provides us with an extra criterion in the design of unstable resonators for pulsed laser systems.

Both calculations and measurements have shown that the reflection profile on the outcoupling mirror has no large influence on the beam pointing variation.

References


References


Chapter 5

Resonator studies - II
Linewidth

In the previous chapter the spatial coherence of the laser has been improved by properly designing the laser resonator. The temporal coherence, i.e. the linewidth of the laser, is the subject of this chapter.

The line structure of XeCl* has already been treated in chapter 2. A free running oscillator emits mainly in the $0\rightarrow1$ and $0\rightarrow2$ transitions as these show the highest gain [1,2]. The wavelengths of these transitions are 307.95 nm and 308.20 nm respectively. Figure 5.1 shows an example of the spontaneous and the stimulated emission spectra of XeCl*. The linewidth of the two strongest lines is estimated to be in the order of 60 pm [1–3]. The wavelength of the output beam can be adjusted by introducing dispersive elements into the cavity. The experiments described in this chapter were performed with the XeCl* system described in chapter 2.

Figure 5.1: Spectra of spontaneous and stimulated emission of XeCl*. In the detailed view the two strongest lined of the stimulated emission spectrum is shown. Adapted from: [4].
5.1 Line narrowing

The dispersive linewidth is determined by the expression [4]

$$\Delta \lambda \approx \frac{\Delta \theta}{(\partial \theta/\partial \lambda)}$$  \hspace{1cm} (5.1)

where $\Delta \theta$ is the beam divergence and $(\partial \theta/\partial \lambda)$ is the intracavity dispersion. This simple equation indicates that to reduce the linewidth $\Delta \lambda$, $\Delta \theta$ should be minimised and $(\partial \theta/\partial \lambda)$ should be maximised.

In a stable cavity the beam divergence can be minimised by introducing losses for higher order modes (i.e. a diaphragm), thus forcing the laser to operate in the lowest order Gauss mode (see also chapter 4). A simple resonator shows almost no intracavity dispersion, except that the resonator itself is a Fabry-Perot etalon with a very small Free Spectral Range (FSR). The intracavity dispersion can be increased by adding dispersive elements to the cavity, such as

- prisms,
- gratings, and
- etalons.

The use of prisms becomes more effective at shorter wavelengths, owing to the increasing dispersion [5]. Prisms have a wide tunability, but the linewidth reported in the literature is not as narrow as can be reached with the other methods [1] (for results with other excimer laser types, see [6]). Gratings are most commonly used with lasers which have a medium with a large bandwidth, allowing a tunability over a large range. The Littrow grating technique exhibits a number of discontinuities in performance and tuning due to the necessity to change the grating order [1,7,8]. Also, the linewidth obtained with these methods is such that usually etalons are used for further narrowing, thereby reducing the tunability. Fabry-Perot etalons as spectral narrowing elements have definite advantages in efficiency, simplicity and stability. However the tunability is not so good, but in the XeCl* excimer laser there are only two strong relatively narrow spectral lines in the stimulated emission spectrum (see figure 5.1), so there is not much tunability available anyway. Therefore, we have chosen to use Fabry-Perot etalons to narrow the linewidth of our XeCl* laser.

5.1.1 Fabry-Perot etalon theory

Consider an etalon consisting of two reflecting surfaces with a reflectivity $R$ a distance $d$ apart and let $n$ be the refractive index of the material between the two reflecting surfaces. Then maxima of the transmittance are found if the order of interference $m$, defined by [5]

$$m = \frac{\delta}{2\pi} = \frac{2nd\cos\theta}{\lambda}$$  \hspace{1cm} (5.2)
Sec. 5.1 Line narrowing

![Figure 5.2: A basic laser cavity with a line-narrowing element.](image)

has integer values and minima when $m$ has half-integral values ($\theta$ being the angle of incidence). For small angles of incidence the maxima are $\Delta \lambda_{FSR} = \lambda^2/2nd$ apart. $\Delta \lambda_{FSR}$ is called the Free Spectral Range of the etalon. Usually the free spectral range of an etalon is given in wave numbers, therefore $FSR = 1/2nd$.

The transmitted intensity $I_t$ of a ray which enters the etalon is given by

$$I_t = I_i \frac{1}{1 + F \sin^2 \frac{\theta}{2}}$$

(5.3)

where $I_i$ is the incident intensity and the parameter $F$ is defined by

$$F = \frac{4R}{(1 - R)^2}$$

(5.4)

If the reflectivity of the surfaces is low, only a small variation in intensity is seen in the transmitted light. If the reflectivity approaches 1 the transmitted light from a diverging source consists of narrow bright circular fringes on an almost completely dark background. The sharpness of the fringes is measured by their half-intensity width. The ratio between the separation of the adjacent fringes and their half-width is defined as the finesse $\mathcal{F}$ of the etalon. The finesse $\mathcal{F}$ is found to be determined by the surface reflectivities [5]

$$\mathcal{F} = \frac{\pi \sqrt{F}}{2} = \frac{\pi \sqrt{R}}{1 - R}$$

(5.5)

Thus, the linewidth of the transmitted light is given by

$$\Delta \lambda = \frac{\lambda^2}{2nd\mathcal{F}}$$

(5.6)

5.1.2 Intracavity dispersion

When a dispersive element is placed in the laser cavity, the optical beam passes several times though the element. This results in increased spectral narrowing.

The effect of gain in the laser medium on line narrowing can be described with a simplified analysis [9]. Assume the laser cavity to be as shown in figure 5.2. The
intensity distribution over the various frequencies, corresponding to the electric field inside the cavity, before saturation of the gain, is given by

\[ I_n(\nu) = I_{sp}(e^{\alpha L R})^{2n} f(\nu)^{2n} \]  

(5.7)

where \( n \) is the number of round trips, \( e^{\alpha L} \) is the single pass gain, assuming that the width of the gain function is much larger than the linewidth of the line narrowing element (thus, \( \alpha(\nu) \approx \alpha \), independent of \( \nu \)), \( R \) is the reflectivity of the mirrors and \( f(\nu) \) is the transmission function of the line-narrowing element. \( I_{sp} \) is the intensity of the spontaneous emission in those cavity modes which are able to grow. The intensity distribution passes the line narrowing element twice per cavity round trip. The transmission function shapes the intensity distribution per pass: there are more losses at the frequency edges and less losses at the resonant frequencies. Therefore, the more passes through the cavity before saturation, the narrower the resulting line.

For a Fabry-Perot etalon the transmission function is given by (5.3))

\[ f(\delta) = \frac{1}{1 + F \sin^2 \frac{\delta}{2}} \]  

(5.8)

where \( \delta \) is related to \( \nu \) through (5.2) The halfwidth \( \Delta \nu_\ell \) of the transmission function for a typical etalon \( (F \gg 1) \) can be calculated by approximating \( \arcsin 1/\sqrt{F} \) by \( 1/\sqrt{F} \). Similarly the halfwidth \( \Delta \nu_n \) for the intensity \( I_n(\nu) \) can be calculated. Expressing \( \Delta \nu_n \) as a function of the halfwidth of the etalon, \( \Delta \nu_\ell \) one finds (see appendix F)

\[ \Delta \nu_n = \left( 2^{\frac{1}{2n}} - 1 \right)^\frac{1}{2} \Delta \nu_\ell \]  

(5.9)

This equation shows how the number of round trips in the cavity build-up time affects the line-narrowing.

5.2 Experiments

In the free running mode our XeCl* laser generates an output energy of about 400 mJ in a 22 mm × 22 mm beam. Using a diaphragm, the beam dimensions of the oscillator can be reduced to such a diameter that the beam becomes almost single transversal mode. Doing this we find a Gaussian beam with a radius of 0.6 mm, a pulse energy of 1 mJ and a pulse length of 155 ns. The divergence of this beam is about 0.5 mrad.

As already mentioned, in the free running mode the laser spectrum consists of two lines. Using a double pass Czerny-Turner monochromator (see figure 5.3) with a 1200 lines/mm grating and the exit slit replaced by a CCD camera, the spectrum of the laser beam can be measured. Figure 5.4 shows the camera image at the exit window of the monochromator. The left line is the 308.19 nm line and the right line the 307.90 nm line. The measured linewidth is 140 ± 40 pm for each line. However, the resolution of this measurement is not very large, due to the low resolution of the CCD camera-monochromator combination.
Improved measurement resolution can be obtained using a Fabry-Perot etalon in combination with a Polaroid camera to determine the linewidth. Figure 5.5 shows the experimental setup for this measurement. The camera used in this setup is a Polaroid camera without the lens and the diaphragm. The attenuated UV radiation from the laser is directly projected onto the film. The distance from the etalon to the Polaroid camera can be adjusted to obtain a properly sized image of the ring pattern. Figure 5.6 shows the fringe pattern from the free running laser. The measurement etalon is a 0.1 mm thick etalon with 80% reflective sides. The width of the two lines is again found to be 140 pm, but now with an error of 15 pm instead of 40 pm, which was the error in the monochromator measurement. 22 pm can be attributed to the linewidth of the measurement etalon (see table 5.1), so the linewidth becomes 120 ± 15 pm. This value is quite a lot larger than the literature value mentioned before (60 pm) [1-3]. We believe that this is caused by the higher pressure we use.
in our laser. The linewidth of XeCl* is homogeneously broadened due to collisions in the gas, so-called collision broadening or pressure broadening (see section 1.2). Our system is operated at 5 bar, while the systems described in the literature are operated at lower pressures [1–3]. Therefore, our system is expected to show a larger linewidth than those systems.

5.2.1 Intracavity etalons

Table 5.1 shows the specifications of the etalons used in the experiments.

Figure 5.7 shows the fringe pattern of the laser when the 0.1 mm etalon (see table 5.1) is added in the oscillator cavity. With this intracavity etalon the oscillator can be tuned to emit one single line. The measurement is performed with the 0.7 mm etalon. The output energy of the oscillator is reduced to 0.6 mJ because of the etalon. Furthermore, the pulse length is reduced to 125 ns. Both XeCl* lines result in nearly
Figure 5.7: Photograph of the fringe pattern from the oscillator with an intracavity 0.1 mm Fabry-Perot etalon.

the same energy and pulse length.

From the fringewidth the linewidth can be determined. Based on the central five fringes the linewidth is found to be $6.7 \pm 0.5$ pm. However, this value is the result of the convolution of the linewidth and the etalon transmission function. The width of the etalon transmission function is $3.2$ pm (see table 5.1). Hence, the actual linewidth is slightly smaller: $4 \pm 1$ pm.

When the 0.7 mm etalon is added to the oscillator cavity the losses become too large and no output beam is formed. To reduce the losses, an etalon with a lower reflectivity was added in the oscillator cavity: a 1 mm thick etalon with 50 % reflective sides. Figure 5.8b shows the fringe pattern after insertion of the 1 mm etalon and the 0.1 mm etalon in the oscillator cavity. Again the output energy is reduced (to 0.1 mJ). The pulse length of the optical pulse is 90 ns. However, no improvement of the linewidth is seen, compared to the situation with only the intracavity 0.1 mm etalon (see figure 5.8a), which is probably caused by the reduction of build up time. Measurements with a different measurement etalon, the 3 mm etalon (see table 5.1), also show no measurable improvement, see figure 5.9. The energy reduction is large: from 0.6 mJ to 0.1 mJ when the 1 mm $R = 50$ % etalon is added, while the insertion of the 0.1 mm

<table>
<thead>
<tr>
<th>$d$ [mm]</th>
<th>$FSR$ [cm$^{-1}$]</th>
<th>$\Delta \lambda_{FSR}$ [pm]</th>
<th>$R$ [%]</th>
<th>$\mathcal{F}$</th>
<th>$\Delta \lambda$ [pm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>333</td>
<td>316</td>
<td>80</td>
<td>14</td>
<td>22</td>
</tr>
<tr>
<td>0.7</td>
<td>4.76</td>
<td>45</td>
<td>80</td>
<td>14</td>
<td>3.2</td>
</tr>
<tr>
<td>1.0</td>
<td>3.33</td>
<td>31.6</td>
<td>50</td>
<td>4.4</td>
<td>7.0</td>
</tr>
<tr>
<td>3.0</td>
<td>1.11</td>
<td>10.5</td>
<td>50</td>
<td>4.4</td>
<td>2.4</td>
</tr>
</tbody>
</table>

Table 5.1: Specifications of the etalons used in our experiments.
Figure 5.8: Photograph of the fringe pattern from the oscillator with an intracavity 0.1 mm etalon (a) and (b) with an additional intracavity 1 mm etalon. Measured with the 0.7 mm etalon.

Figure 5.9: Photograph of the fringe pattern from the oscillator with an intracavity 0.1 mm etalon (a) and (b) with an additional intracavity 1 mm etalon. Measured with the 3.0 mm etalon.

R = 80 % etalon only leads to an output energy reduction from 1 mJ to 0.6 mJ.

5.3 Oscillator modelling

In the previous section it was seen that resonators with intracavity etalons resulted in an oscillator output beam with very low output energy or no output energy at all in some cases. We will show that an explanation for this observation can be found in
Sec. 5.3 Oscillator modelling

the build-up of the coherence length of the optical beam. In this section a model is introduced which describes this process.

From the injection locking experiments with an amplifier, which are described later in this chapter, it is found that the laser line is homogeneously broadened (see also section 1.2). For a high pressure system like our laser pressure (or collision) broadening is expected to be the main broadening process.

5.3.1 Coherence length distribution function

As the collisions lead to dephasing events for the emission, the coherence length of the spontaneous emission, \( L_c \), is related to the mean time between collisions, \( \tau_{dp} \), by

\[
L_c = c \tau_{dp}
\]

where \( c \) is the speed of light in vacuo. However, a given atom does not have collisions in evenly spaced intervals of time \( \tau_{dp} \). It can only be said that the probability that a given atom has a collision in a small time interval \( \Delta t \) is given by \( \Delta t \) times the mean number of collisions per unit time, \( 1/\tau_{dp} \). Simple mathematics show that the probability that a given atom has had no collision for a time \( T \) is equal to [10]

\[
P(T) = e^{-T/\tau_{dp}}
\]

Similarly, one can obtain an expression for the probability that a certain coherence length \( L \) occurs

\[
P(L) = e^{-L/L_c}
\]

Thus, a beam has a coherence length distribution in the form of (5.12).

5.3.2 Etalon transmission function

The transmission function of a Fabry-Perot etalon is based on constructive and destructive interference of the various reflections travelling back and forth in the etalon. If two beams travel separate paths, each having a different optical path length, there will be a difference in the phases of the beams at the point of recombination. For a monochromatic source the phase difference is constant in time. Usually the time average of the power density, i.e. the intensity, is measured, which is proportional to the time average of the local optical field (from [11])

\[
\langle U \rangle = \varepsilon \langle \mathbf{E} \cdot \mathbf{E} \rangle
\]

The electric field \( \mathbf{E} \) is the resultant of the vector sum of the electric fields in each of the two beams, \( \mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 \), which leads to

\[
\langle U \rangle = \varepsilon \langle \mathbf{E}_1 \cdot \mathbf{E}_1 + \mathbf{E}_2 \cdot \mathbf{E}_2 + 2 \mathbf{E}_1 \cdot \mathbf{E}_2 \rangle
\]

or

\[
\langle U \rangle = \langle U_1 \rangle + \langle U_2 \rangle + 2 \varepsilon \langle \mathbf{E}_1 \cdot \mathbf{E}_2 \rangle
\]
The first two terms are the average energy densities of the two beams independently. The last term is caused by interference, which may be constructive or destructive depending on whether the sign of the interference term is positive or negative, respectively. When we restrict ourselves to the case where the two beams travel in (nearly) the same direction, which is the case in an etalon, the vector behaviour of the electric field can be neglected. It can be easily shown that one can write for the intensity \[ I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi \] (5.16) where \( \phi \) is the phase difference between the two beams. This equation can be generalised to the multiple beam interference situation of the etalon \[ I = \sum_{n=1}^{N} I_n + \sum_{n=1}^{N} \sum_{m=1, m \neq n}^{N} \sqrt{I_n I_m} \cos(\phi_m - \phi_n) \] (5.17)

This is the well known series of multiple reflections within a Fabry-Perot etalon. In general \( N \) rises to \( \infty \) and this summation becomes a mathematical series which reduces to (5.3).

However, if the light entering the etalon has only a limited coherence length \( L_c \), interference can only occur between those reflections that are coherent. Therefore, the value of \( N \) is determined by the coherence length and \( N \) is given by \( N = L_c / 2nd \). Figure 5.10 shows the transmission of a Fabry-Perot etalon as a function of the wavelength for different coherence lengths. The full line shows the transmission function of the etalon without any interference. In this situation the transmission is independent of the wavelength. The dotted line shows the transmission where the coherence length is just large enough to allow the direct transmission and the first reflection to interfere. The larger the number of reflections to add coherently, the narrower and the higher the transmission peaks, as is shown by the other lines in figure 5.10.
5.3.3 Cavity model

Combining the etalon transmission function (5.17) with the coherence length distribution function (5.12) one can determine the total transmission function of the etalon for a given average coherence length $L_c$ or linewidth $\Delta \lambda$, which is related to the coherence length (using (5.10) and (1.16))

$$L_c = \frac{c}{\pi \Delta \nu_C} = \frac{\lambda^2}{\pi \Delta \lambda}$$

(5.18)

The growth of the coherence length (or the decrease of the linewidth) can now be modelled as a function of the number of round trips in the cavity. A cavity round trip can be divided in a few steps (the plane just before the outcoupling mirror is used as reference plane):

1. reflection at the outcoupling mirror,
2. passage through one or more etalons,
3. passage through the gain medium,
4. reflection at the rear mirror,
5. passage through the gain medium, and
6. passage through the etalon(s).

As the optical field builds up from noise, spontaneous emission has to be added while passing through the gain. In appendix G the practical implementation of the model is treated in more detail.

5.3.4 Results

Figure 5.11 shows the intensity and linewidth evolution for four different resonator configurations. In figure 5.11a the standard situation without an intracavity etalon is shown. The linewidth remains constant during the pulse as there is no line narrowing element in the cavity except for the resonator itself, which is also a Fabry-Perot etalon. This Fabry-Perot etalon narrows the closely packed individual (longitudinal) resonator modes. Figure 5.11b shows the results when using the 0.1 mm etalon in the oscillator cavity. It can be seen that the linewidth starts to reduce when the gain passes threshold and the beam starts to build up. The intensity reduces due to the remaining losses caused by the etalon. When the 0.7 mm etalon is placed in the cavity the output reduces strongly, as can be seen in figure 5.11c. The losses in the cavity become so large that threshold is not reached and the etalons can not become resonant. The linewidth is not reduced and no measurable output can be generated. Lowering the losses of the second etalon by lowering the reflectivity of the etalon results in a reduced linewidth compared to the single etalon case, but also in a reduced output intensity, as shown by figure 5.11d.
Figure 5.11: Calculated intensity and linewidth evolution for four different configurations. (a) No intracavity etalon, (b) 0.1 mm $R = 80\%$ intracavity etalon, (c) 0.1 mm $R = 80\%$ and 0.7 mm $R = 80\%$ intracavity etalons and (d) 0.1 mm $R = 80\%$ and 1.0 mm $R = 50\%$ intracavity etalons. Note the different scales used for the intensity.

These results are comparable to the measured results described in the previous section. Pulse energies and pulse durations are not exactly the same, but this is due to the fact that the gain and resonator losses are only estimates. However, the model explains why certain oscillator configurations do work and why others do not.

5.4 Injection locking

When the narrow linewidth pulse from the master oscillator is injected in a slave oscillator, it is expected that the output of the latter will reproduce the narrow linewidth of the master oscillator and that the output energy will be similar to the energy generated by the free running slave oscillator, i.e. the slave oscillator will lock to the master oscillator. The experimental configuration used for these experiments is shown in figure 5.12. The slave oscillator is equipped with a hard edge unstable resonator with a magnification of 2.5 and a central reflectivity of 100\% (see also chapter 4). The master oscillator beam is injected in the slave oscillator through a hole with a diameter of 1 mm in the concave rear mirror as shown in figure 5.12.

The experiments were performed with the 0.1 mm thick etalon with $R_{side} = 80\%$ in the master oscillator cavity. The master oscillator linewidth is about 4 pm and the energy is about 0.6 mJ. Figure 5.13 shows the fringe patterns of the master oscillator...
and the injection locked slave oscillator. A good correspondence between the two fringe patterns can be seen. The linewidth of the slave oscillator is found to be equal to the linewidth of the master oscillator, 4 ± 1 pm. Figure 5.14 shows the waveforms of the optical pulse for the free running slave oscillator and for the injection locked slave oscillator. The injection locked slave oscillator shows a slightly longer pulse, which is due to the injection pulse. The output energy of the injection locked slave oscillator is also somewhat larger: 325 mJ without injection and 365 mJ with injection.

The experiments show that the spectrum of the XeCl* laser is homogeneously broadened, as the slave oscillator locks to the master oscillator with almost no linebroadening and with complete energy extraction.

The injection pulse has a pulse length of 125 ns. Varying the delay between the master
oscillator and the slave oscillator shows a range of about 280 ns for proper injection locking. Thus, the injection timing is not very critical. The transition areas at the beginning and the end of this timing interval are rather sharp. The injection locking process turns out to develop properly or there is no locking at all. This is probably due to the relatively high injection intensity, which is several orders of magnitude larger than the noise equivalent power (NEP), which is estimated to be 1 W/cm² [3].

When the injection energy is reduced, using attenuators between the master oscillator and the slave oscillator, it is found that the spontaneous build-up of the slave oscillator begins at injection intensities below 100 W/cm². Figure 5.15 shows the fringe pattern from the injection locked slave oscillator at an injection energy of approximately 20 W/cm². It is clearly seen that other wavelengths also appear in the output beam.
5.5 Conclusions

Fabry-Perot etalons have proven to be efficient, easy to align and stable elements to reduce the linewidth of a long pulse XeCl\(^*\) excimer laser. A linewidth of 4 pm can be obtained using one 0.1 mm thick etalon with \(R_{\text{side}} = 80\%\) in the oscillator cavity. The output energy of this pulse is found to be more than enough to injection lock a slave oscillator.

When trying to reduce the linewidth further we ran into the problem of too high cavity losses due to the etalons. A model has been developed to describe the build-up of the coherence length in the cavity. Using this model, it is seen that if the cavity losses are too high the linewidth does not reduce fast enough. Therefore the etalon does not become fully resonant. This means that the cavity losses due to the etalon remain high and no beam can build up. Hence, with our relatively low gain system (3-5 \(\%\) cm\(^{-1}\)) the narrow linewidths as reported in the literature [12-14] can not be reproduced. The linewidths reported there were obtained with high gain systems (10-12 \(\%\) cm\(^{-1}\)).

The injection locking experiments show that narrow bandwidth, high output energy and good beam quality can be obtained for a long pulse XeCl\(^*\) excimer laser in a master-oscillator-slave-oscillator setup. The experiments show that the spectrum of the XeCl\(^*\) laser is homogeneously broadened, as the slave oscillator locks to the oscillator with nearly no line broadening and with complete energy extraction. The injection intensity can be reduced to a value of 100 W/cm\(^2\) before injection locking stops. Long injection pulses allow a broad injection timing interval.

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Appendix A

Solving Huygens’ integral numerically

Propagation of an arbitrary wavefront through a complex optical system, consisting of a hard edge aperture, a mask, a gainsheet and a paraxial optical system specified by its ABCD matrix can be performed by solving Huygens’ integral (4.24) if the problem is separable in the transverse plane, either in cartesian or in cylindrical coordinates [1,2].

In cartesian coordinates Huygens’ integral for one transverse dimension is (4.24)

$$\tilde{u}_2(x_2) = e^{-jL} \sqrt{\frac{j}{B\lambda_0}} \int_{-a}^{a} \tilde{\rho}(x_0) \tilde{u}_0(x_0) \exp \left[ -j \frac{\pi}{B\lambda_0} (Ax_0^2 - 2x_2x_0 + Dx_2^2) \right] dx_0$$

(A.1)

The hard edge aperture is determined by the limits of the integration (aperture size = 2a). \(\tilde{\rho}(x_0)\) is the (arbitrary) mask and/or gainsheet.

In order to use Fast Fourier Transform (FFT) propagation methods, the integral has to be re-written as a convolution. If an arbitrary scaling factor \(M\) is introduced and the spherical curvature of the unstable resonator modes is extracted, thereby converting the magnifying wavefronts into collimated wavefronts at both the input and output ends of the cavity by the transformation

$$\tilde{u}_0(x_0) = \tilde{v}_0(x_0) \times \exp \left[ +j \frac{\pi(A - M)x_0^2}{B\lambda_0} \right]$$

(A.2)

$$\tilde{u}_2(x_2) = \tilde{v}_2(x_2) \times \exp \left[ -j \frac{\pi(D - 1/M)x_2^2}{B\lambda_0} \right]$$

(A.3)

then Huygens’ integral becomes

$$\tilde{v}_2(x_2) = \sqrt{\frac{j}{B\lambda_0}} \int_{-a}^{a} \tilde{\rho}(x_0) \tilde{v}_0(x_0) \exp \left[ -j \frac{\pi}{B\lambda_0} (Mx_0^2 - 2x_2x_0 + x_2^2/M) \right] dx_0$$

(A.4)

Using the change of coordinates \(X_0 = x_0/a\) and \(X_2 = x_2/Ma\) this last integral can
be rewritten. Huygens’ integral now becomes

\[ \tilde{v}_2(X_2) = \sqrt{\frac{j}{B\lambda_0}} \int_{-1}^{1} \tilde{\rho}(X_0) \tilde{v}_0(X_0) \exp \left[ -j \frac{M\pi a^2}{B\lambda_0} (X_0 - X_2)^2 \right] dX_0 \]

(A.5)

This integral is just a convolution of the input function \( \tilde{v}_0(X_0) \) and the kernel \( K(X_0) = \exp \left[ -j \frac{M\pi a^2}{B\lambda_0} X_0^2 \right] \). This convolution can be easily evaluated by Fourier transform techniques, by transforming the two functions, multiplying them and inverse transforming the product.

For cylindrical coordinates a similar procedure can be derived quite easily. The solving technique then involves Hankel transforms instead of Fourier transforms. But for our purpose the one dimensional cartesian solution is sufficient.

References


Appendix B

Approximate derivation of the lowest order eigenmode in a Gaussian unstable resonator

The equation

\[ w^2 \approx (M^2 - 1) \times w_0^2 \]  \hspace{1cm} (B.1)

can be quite easily derived using the \( q \)-parameter

\[ \frac{1}{q} = \frac{1}{R} - j \frac{\lambda}{\pi w^2} \]  \hspace{1cm} (B.2)

which characterizes a Gaussian beam completely. \( R \) in this equation is the radius of curvature of the wavefront, \( w \) the width of the Gaussian beam and \( \lambda \) is the wavelength. The transformation of \( q \) in an optical system can be expressed as a \( \text{bilinear transformation} \) using the ABCD matrix of that optical system

\[ q' = \frac{Aq + B}{Cq + D} \]  \hspace{1cm} (B.3)

For the Gaussian unstable resonator the ABCD matrix equals (see (4.37))

\[ \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} M - j\lambda B / \pi w_0^2 & B \\ -j\lambda / \pi w_0^2 M & 1/M \end{bmatrix} \]  \hspace{1cm} (B.4)

If a Gaussian mode exists in the resonator, \( q \) must repeat after one full round trip: \( q' = q \). We now get the following equation for \( q \)

\[ Cq^2 + (D - A)q - B = 0 \]  \hspace{1cm} (B.5)

This quadratic equation has two solutions for \( q \)

\[ q = \frac{A - D \pm \sqrt{(D - A)^2 + 4BC}}{2C} \]  \hspace{1cm} (B.6)
Using \( AD - BC = 1 \) (the determinant of the ABCD matrix of an optical system always equals 1), this can be rewritten in the form

\[
q = \frac{1}{2c} \left( A - D \pm \sqrt{(D + A)^2 - 4} \right) \tag{B.7}
\]

From the ABCD matrix we find

\[
A - D = \frac{M^2 - 1}{M} - j \frac{\lambda B}{\pi w_a^2} \tag{B.8a}
\]
\[
A + D = \frac{M^2 + 1}{M} - j \frac{\lambda B}{\pi w_a^2} \tag{B.8b}
\]
\[
C = -j \frac{\lambda}{\pi w_a^2 M} \tag{B.8c}
\]

If we now assume

\[
\frac{\lambda B}{\pi w_a^2} \ll \frac{M^2 + 1}{M} \tag{B.9}
\]

The result for \( q \) becomes

\[
q \approx \frac{-M \pi w_a^2}{2j \lambda} \left( \frac{M^2 - 1}{M} \pm \sqrt{\left( \frac{M^2 + 1}{M} \right) - 4} \right) \tag{B.10}
\]

which has two solutions

\[
q_- = 0 \tag{B.11a}
\]
\[
q_+ = \frac{-\pi w_a^2 (M^2 - 1)}{j \lambda} \tag{B.11b}
\]

of which the first has no value in this approximation. If we compare the second solution to the definition of \( q \)

\[
\frac{1}{q} = \frac{1}{R} - j \frac{\lambda}{\pi w_a^2} \tag{B.12}
\]

we find for this approximation a Gaussian mode with a flat wavefront \( R = \infty \) and a width \( w_a = (M^2 - 1) w_a^2 \) as the lowest order eigenmode in this resonator.
Appendix C

Derivation of the maximally flat condition

The maximally flat condition (4.42) can easily be derived from (4.40).

\[ I_{\text{out}}(r) = I_0 \left[ 1 - R_0 e^{-\frac{2r^2}{w_0^2}} \right] e^{-\frac{2r^2}{w_a^2}} \]  \hspace{0.5cm} (C.1)

If \( R_0 = 0 \) this becomes the simple gaussian function

\[ I_{\text{out}}(r) = I_0 e^{-\frac{2r^2}{w_a^2}} \]  \hspace{0.5cm} (C.2)

If \( R_0 = 1 \) the intensity becomes zero at \( r = 0 \) and has a peak at a certain radial position \( r \neq 0 \). So when increasing the value of \( R_0 \) from 0 to 1 the profile changes from a gaussian with only a central peak to a ring shaped profile (in the case of cylindrical symmetry). With increasing \( R_0 \) the central peak is broadened and at a certain moment it is split into two peaks (when looking along a crosssection) with a slight minimum in the centre, which gets deeper until it reaches zero central intensity at \( R_0 = 1 \). The point where the splitting into two peaks occurs is the point where the output beam is maximally flat. It can be found analytically by looking for the derivative of (C.1) to be zero outside \( r = 0 \).

\[ \frac{dI_{\text{out}}(r)}{dr} = -I_0 \frac{4r}{w_0^2} \left[ 1 - R_0 e^{-\frac{2r^2}{w_0^2}} \right] e^{-\frac{2r^2}{w_a^2}} + I_0 \frac{4r}{w_a^2} R_0 e^{-\frac{2r^2}{w_a^2}} e^{-\frac{2r^2}{w_a^2}} \]  \hspace{0.5cm} (C.3)

Setting

\[ \frac{dI_{\text{out}}(r)}{dr} = 0 \]  \hspace{0.5cm} (C.4)

results in the equation

\[ -\frac{4r}{w_0^2} \left[ 1 - R_0 e^{-\frac{2r^2}{w_0^2}} \right] + \frac{4r}{w_a^2} R_0 e^{-\frac{2r^2}{w_a^2}} = 0 \]  \hspace{0.5cm} (C.5)

Using (4.38)

\[ w^2 = w_a^2 (\mathcal{M}^2 - 1) \]  \hspace{0.5cm} (C.6)
this can be rewritten as

\[ 1 - R_0 \mathcal{M}^2 e^{-\frac{2\pi^2}{\mathcal{M}^2}} = 0 \quad \text{for } r \neq 0 \]  

(C.7)

From this equation it can be seen that if $R_0 \mathcal{M}^2 > 1$ a maximum will be found for $r \neq 0$ and for $R_0 \mathcal{M}^2 < 1$ no solution is found. Thus, at $R_0 \mathcal{M}^2 = 1$ the output profile is maximally flat.
Appendix D

Phase unifying mirrors

The outcoupling mirror in a partial reflecting hard edge unstable resonator introduces a phase distortion when the beam passes through the mirror. This is caused by the coating used to obtain the reflectivity profile. In this appendix the background of and the solution to this problem will be given.

D.1 Standard coating

The standard way to obtain the (step-wise) reflectivity profile on the outcoupling mirror of a partial reflecting hard edge unstable resonator is to put a mask over the substrate. Therefore, a coating will be deposited on a part of the substrate. Figure D.1 shows such a coating. This coating was designed to have a reflection of 45%.

The phase distortion of the mirror was measured using a Michelson interferometer, as shown in figure D.2. As a light source a HeNe laser has been used. As the light passes the mirror twice, the beam suffers the phase distortion twice. However, as the wavelength of XeCl* is about half that of HeNe, the measured phase shift using the

![Diagram](image)

Figure D.1: Coating package on the outcoupling mirror of a partial reflecting hard edge unstable resonator.
Figure D.2: Interferometer setup used to determine the phase distortion in the out-coupling mirror.

Figure D.3: Interferogram of the non-phase unifying 45 % mirror.

double pass HeNe is nearly equal to the real phase shift for single pass XeCl*. Figure D.3 shows the interferogram of the 45 % mirror described above. A phase shift of nearly $\pi$ is found from this interferogram.

This result is in good agreement with the expected value of the phase shift based on the coating design. The physical thickness of the coating is 215 nm. The optical thickness of the coating is 350 nm, using an index of refraction of 1.81 for $Y_2O_3$ and 1.46 for $SiO_2$. The pathlength difference is therefore 135 nm which is equal to a phase difference of $0.9\pi$ at a wavelength of 308 nm.

Figure D.4 shows the influence of the phase shift on the focus energy distribution of a homogeneous beam with a uniform phase front. It is clearly seen that the phase shift distorts the focus energy distribution dramatically.
Figure D.4: The focus of a uniform output beam with and without a phase shift in the near field.

Figure D.5: Phase unifying coating package.

D.2 Phase unifying coating

To diminish the effect of the phase distortion, the coating has to be designed in such a way that the phase shift is small. This can be done by using special mirrors: phase unifying mirrors [1]. Figure D.5 shows such a coating design. This coating shows a reflectivity of 45% in the centre and nearly 0% at the mirror edges. The thickness difference is only 24.2 nm, resulting in a phase shift of only $0.16\pi$.

References

Appendix E

Variable reflectivity mirrors
The production technique

The variable reflectivity mirrors used in the experiments described in chapter 4 are not “off the shelf” optics. For 1064 nm variable reflectivity mirrors are commercially available at the moment, but for 308 nm we found that our request for these optics frightened off most optics retailers. Therefore we decided to try to make these mirrors in house; together with the optics group of the FFW (Fijnmechanisch Fysisch Werkgebied) at the University of Twente. This cooperation lead to good mirrors [1]. In this section some of the underlying theory of the production of these mirrors will be treated and some crucial parts of the developed apparatus will be shown.

The specifications of the first variable reflectivity mirror were:

- the reflectivity profile should be gaussian shaped,
- the central reflectivity should be 45 %,
- the reflectivity at the edge should be 0 %, and
- the 1/e radius of the amplitude reflection profile should be 6.5 mm.

The coating design described in this section is based on these specifications.

E.1 Theory

The central reflectivity of the above specified mirror can be obtained with a coating of 5 nearly quarter wave layers of alternately SiO$_2$ and Y$_2$O$_3$. By varying thickness of the middle layer of these five layers the reflection of the coating can vary between the rest reflection of the substrate, when the middle layer has zero thickness, and the maximal reflection, when the layer is a quarter wave layer. By adding two extra layers, which form an anti-reflection coating, underneath these five, right on top of the substrate, the rest reflection at the mirror edge can be reduced [2].
To get a variable reflection mirror the reflection, and therefore the layer thickness of the fifth layer, should vary radially, see figure E.1.

Figure E.2 shows the requested amplitude reflection profile. Figure E.3 shows the reflection of the total coating as a function of the thickness of the variable fifth layer. Taking these two curves together the needed radially varying thickness profile can be obtained, which is shown in figure E.4.

To vary the thickness of the layer a rotating mask is used, which is placed close to the substrate. This mask has to have a special shape to create the right layer thickness variation. From figure E.4 the thickness of the fifth layer as a function of the radius is known. So it is possible to determine which fraction the layer thickness at a certain radius is compared to the maximal thickness at the centre of the mirror. When for example the thickness at a certain radius is 25% of the central thickness, this means that, when the mask rotates over 180°, the mask should be open 25% and closed.
Figure E.3: The reflection of the total coating as a function of the layer thickness of the variable fifth layer.

Figure E.4: Thickness of the fifth layer of the coating as a function of the radius.

75 % of the time at this certain radius. Using this procedure for all radii the mask can be designed. A typical mask is shown in figure E.5. This mask is only to be used for the fifth layer, so it should be possible to move the mask out of the way of the deposition material.

E.2 Apparatus

The thin film group of the FFW has a Varian vapour deposition installation, which is upgraded with an IL 820 process controller from Intellemetrics, which has been made PC-controllable. The mask and the substrate were mounted in a specially designed mask and substrate holder, which is shown in figure E.6. As the mask is needed only
for the fifth layer, the mask holder can be shifted in front of the substrate if needed. This can be done externally, without opening the vacuum chamber.

E.3 Results

Figure E.7 shows the reflection of the variable reflectivity mirror as a function of the radius of the mirror. The x-axis in the figure has been shifted due to the measurement system.
Figure E.7: The result: the radially varying reflection of the variable reflectivity mirror.

The characteristics of the curve, a central reflection of 43.1 % and a 1/e width of 6.4 mm, are in very good agreement with the specifications (45 % and 6.5 mm respectively).

References


Appendix F

Intracavity dispersion

The equation showing how the number of round trips affects the line-narrowing effect of an etalon, (5.9),

\[ \Delta \nu_n = \left(2^{\frac{n}{2}} - 1\right)^{\frac{1}{3}} \Delta \nu_e \]  \hspace{1cm} (F.1)

can be easily derived from (5.7)

\[ I_n(\nu) = I_{sp}(e^{\alpha L} R)^{2n} f(\nu)^{2n} \]  \hspace{1cm} (F.2)

using (5.8)

\[ f(\delta) = \frac{1}{1 + F \sin^2 \frac{\delta}{2}} \]  \hspace{1cm} (F.3)

The half width of the etalon transmission function (F.3) is determined by the values for \( \delta \) where \( f(\delta) \) drops to half its maximum value (i.e. when \( f(\delta_{1/2}) = 1/2 \)). Thus

\[ \frac{1}{1 + \sin^2 \frac{\delta_{1/2}}{2}} = \frac{1}{2} \]  \hspace{1cm} (F.4)

Therefore

\[ \delta_{1/2} = 2 \arcsin \left( \frac{1}{\sqrt{F}} \right) \]  \hspace{1cm} (F.5)

Since \( F \) is rather large, \( \sin(1/\sqrt{F}) \approx 1/\sqrt{F} \), and therefore the half width of the etalon, \( \Delta \delta_e = 2\delta_{1/2} \), becomes

\[ \Delta \delta_e = \frac{4}{\sqrt{F}} \]  \hspace{1cm} (F.6)

2n passes through the etalon, which occurs in n cavity round trips, results, according to (F.2), in a total transmission function

\[ f_n(\delta) = f(\delta)^{2n} = \left( \frac{1}{1 + F \sin^2 \frac{\delta_{1/2}}{2}} \right)^{2n} \]  \hspace{1cm} (F.7)
The half width of this function can be determined in a similar way as before.

\[
\left( \frac{1}{1 + F \sin^2 \frac{\delta_{1/2}}{2}} \right)^{2n} = \frac{1}{2} \quad (F.8)
\]

equals

\[
1 + F \sin^2 \frac{\delta_{1/2}}{2} = 2^{\frac{1}{2n}}
\quad (F.9)
\]

This results in

\[
\delta_{1/2} = 2 \arcsin \left( \frac{\left(2^{\frac{1}{2n}} - 1\right)^{\frac{1}{2}}}{\sqrt{F}} \right) \quad (F.10)
\]

and

\[
\Delta \delta_n = \left(2^{\frac{1}{2n}} - 1\right)^{\frac{1}{2}} \frac{4}{\sqrt{F}} \quad (F.11)
\]

Comparing (F.11) with (F.6) it is seen that

\[
\Delta \delta_n = \left(2^{\frac{1}{2n}} - 1\right)^{\frac{1}{2}} \Delta \delta_e \quad (F.12)
\]

and similarly

\[
\Delta \nu_n = \left(2^{\frac{1}{2n}} - 1\right)^{\frac{1}{2}} \Delta \nu_e \quad (F.13)
\]

using

\[
\nu = \frac{\delta}{2\pi} \frac{c}{2nd \cos \theta} \quad (F.14)
\]
Appendix G

A model for the beam build-up in a resonator with intracavity etalons

In section 5.3 a model for the build-up of the beam in a resonator with intracavity etalons is described. In this appendix the practical implementation of the model will be shown.

First the etalon transmission function has to be determined using (5.17) as a function of wavelength $\lambda$ and number of coherent reflections $N$

$$T(\lambda, N) = \frac{1}{I_{in}} \left[ \sum_{n=0}^{N} I_n + \sum_{n=0}^{N} \sum_{m=0}^{N} \sqrt{I_n I_m} \cos(\phi_m(\lambda) - \phi_n(\lambda)) \right]$$  \hspace{1cm} (G.1)

where $I_{in}$ is the impinging intensity and $I_n$ are the intensities of the transmissions with $I_0$ the direct transmission, $I_1$ the transmission after one double pass through the etalon, $I_2$ after two double passes through the etalon and so on. The phase $\phi$ is a function of the wavelength $\lambda$

$$\phi(\lambda) = \frac{4\pi mnd}{\lambda}$$  \hspace{1cm} (G.2)

where $m$ is the number of double passes through the etalon, $n$ is the refractive index of the etalon substrate and $d$ is the thickness of the etalon.

Using these results the growth of the optical pulse can be calculated as a function of the number of cavity round trips. The plane just before the outcoupling mirror is used as a reference plane. The intensity of the beam is distributed over different wavelengths, $I = I(\lambda)$. Each cavity round trip exists of a number of steps.

1. **Reflection at the outcoupling mirror.** Only part of the intensity is reflected.
   The reflection is not wavelength dependent, so we can simply calculate

$$I(\lambda) = RI(\lambda)$$  \hspace{1cm} (G.3)
2. **Passage through one or more etalons.** As the etalon transmission function depends on the wavelength and the linewidth, the coherence length needs to be determined using (5.18)

\[ L_c = \frac{\lambda^2}{\pi \Delta \lambda} \quad \text{(G.4)} \]

Using the coherence length distribution function (5.12)

\[ P(L) = e^{-L/L_c} \quad \text{(G.5)} \]

and the wavelength and coherence length dependent etalon transmission function \( T(\lambda, N) \) given by (G.1), the total transmission function \( T(\lambda) \) of the etalon can be determined. Finally, the intensity has to be multiplied with this function

\[ I(\lambda) = T(\lambda)I(\lambda) \quad \text{(G.6)} \]

3. **Passage through the gain, reflection at the rear mirror and passage through the gain.** The gain medium also adds spontaneous emission. Therefore, spontaneous emission should be incorporated (it is even necessary for the build up phase!).

The small signal gain has been measured as a function of time in section 2.6.3. Figure G.1 shows the gain curve used in the calculations. It is assumed that the peak gain can change, but that the shape remains the same.

As saturation of the gain is important in this model, it has to be incorporated also. The line is homogeneously broadened. Therefore, the saturation of the gain occurs for all wavelengths. The gain saturates according to (see also section 1.5)

\[ \alpha_s = \frac{\alpha}{1 + \int \frac{I(\lambda)d\lambda}{I_s}} \quad \text{(G.7)} \]
where $\alpha$ is the small signal gain and $I_s$ the saturated gain. The gain is compressed to a thin sheet. To prevent problems with the saturation, we have chosen to divide the gain medium in 10 such sheets.

The spontaneous emission is introduced by adding a noise term before passing the gain elements. This spontaneous emission term has a Lorentz shape and is scaled with the small signal gain

$$I_{\text{s spont}}(\lambda) = \frac{A}{g_{\text{max}}} \frac{1}{2\pi (\lambda - \lambda_0)^2 + (\Delta \lambda/2)^2}$$  \hspace{1cm} (G.8)

where $A$ is a scaling factor used to set the noise intensity. In the calculations a noise intensity of 1 W/cm$^2$ is used (conform [1]).

Thus the intensity is increased with the spontaneous emission

$$I(\lambda) = I(\lambda) + I_{\text{s spont}}(\lambda)$$  \hspace{1cm} (G.9)

and subsequently amplified (in 10 steps)

$$I(\lambda) = e^{\alpha I} I(\lambda)$$  \hspace{1cm} (G.10)

where $l$ is the length of the gain section.

4. Passage through the etalon(s). See step 2.

Figure G.2 shows a typical result from the calculations. The intensity and the linewidth are shown as a function of the number of cavity round trips. The pulse form of the intensity is in rather good agreement with the measurements (see for example figure 2.11).

References

References
Abstract

The properties of the laser beam are mainly influenced by the resonator. In this thesis the results of the research on the spatial and spectral beam quality of a long pulse, low gain XeCl* excimer laser are described. The experimental work described in this thesis can be divided into two parts: the experiments on the spatial beam quality and the experiments on the spectral quality.

It would be very practical to be able to describe the spatial beam quality with one single parameter. However, this is not easy. Probably the best judgement of the laser beam can be given if both the phase and intensity distribution are known. However, this makes it very hard to compare beams from different resonators. To be able to compare the beams from different resonators one single parameter is useful.

For resonator research the TDL (Times Diffraction Limited) parameter is the most useful beam quality parameter. All possible resonator configurations can lead to a TDL of 1, if the resonator can be operated in a single transverse mode. This in comparison to the $M^2$ parameter, which results in large values for highly diffractive beams, such as beams from unstable resonators. $M^2$ is rather a beam propagation factor than a beam quality factor. The PIB value, Strehl's ratio and the $K$ factor give useful information, but as single parameters they don't give enough information to the researcher. Thus, the TDL parameter will be used to compare the different resonator configurations.

For laser-users the $K$ factor probably is the best parameter. The $K$ factor is also the planned ISO beam geometry standard. Together with the laser power (or laser energy and pulse duration for a pulsed laser) the brightness of the laser can easily be calculated and the usefulness of the laser for the application can be determined.

The experiments on the spatial beam quality have shown that stable resonators are straightforward and easy to align cavities, which allow good energy extraction from the gain medium and homogeneous near field energy distributions. To ensure maximum energy extraction a geometrical feedback between 20 and 70 % is found to be optimal.

With the plano-plano resonator a better focusability can be reached, but with a bad near field profile. The plano-concave resonator results in a flat top near field profile and a similar far field profile, but the size of this far field profile is large due to the large number of modes in the output beam, so the focusability of this beam is rather bad.

The divergence of the plano-plano resonator is approximately 3 mrad. The divergence
of the plano-concave resonators is larger, depending on the radius of curvature of the rear (concave) mirror. The divergence for the plano-concave resonator is within the accuracy in accordance with the value expected from an approximate analysis.

With a high reflectivity hard edge unstable resonator having a magnification of 2.4 a nearly diffraction limited beam (TDL = 1.3) can be obtained from our laser. Under matched discharge conditions the brightness of the pulse is $5.5 \times 10^{14}$ W/cm$^2$·sr. The maximum obtained brightness was $1.2 \times 10^{15}$ W/cm$^2$·sr.

The side lobe energy in the far field energy distribution can be reduced by using partial reflecting outcoupling mirrors. This leads to a small increase in divergence angle, but if the geometrical feedback is kept constant the brightness of the beam increases due to a higher central peak energy. However, in the case of partial reflecting outcoupling mirrors the coating can cause phase problems, as we have seen with the 45% mirrors. It has been shown that these phase problems can be avoided by using phase unifying mirrors. Due to a lower side lobe energy and lower diffraction effects at the beam edge the brightness of the beam from a 45% reflectivity phase unifying resonator with a magnification of 1.6 is $7.0 \times 10^{14}$ W/cm$^2$·sr. The maximum obtained brightness with the phase unifying resonator was $1.4 \times 10^{15}$ W/cm$^2$·sr.

Further reduction of the side lobe energy can be obtained by using variable reflectivity unstable resonators (VRURs). Using these resonators nearly diffraction limited beams (1.7 times diffraction limited) with a smooth near field energy distribution can be obtained. The experiments have shown that the high central reflectivity is not strongly influencing the beam quality. However, the output energy of these high central reflectivity VRURs is much larger due to the improved gain volume usage. The brightness of these high central reflectivity VRURs is nearly twice as high compared to the brightness of the low central reflectivity VRURs. The best performing variable reflectivity unstable resonator was the VRUR with a magnification of 2.0 and a central reflectivity of 65%. With this resonator a brightness of $9.1 \times 10^{14}$ W/cm$^2$·sr could be obtained under matched discharge conditions.

If a flat topped near field is requested, the reflectivity and magnification have to be chosen such that the maximally flat condition is satisfied. The experiments have shown that this condition, which has been derived for an empty resonator, holds for this specific laser.

The best energy extraction has been obtained by using variable reflectivity unstable resonators with a high central reflectivity. The experiments have shown that the high central reflectivity is not strongly influencing the beam quality. With variable reflectivity unstable resonators a better energy extraction and therefore a higher brightness is obtained than with hard edge unstable resonators.

The position of the focus field of the beam is found to show a variation: the beam pointing variation. A model has been introduced to explain this beam pointing variation. The measurements are in good agreement with the calculations. In the model, it is assumed that the beam pointing variation originates from the build-up phase of the optical field in the resonator. If the optical field in the resonator gets more build-up time before the gain saturates the beam pointing variation decreases. It is found that the pointing variation of the entire pulse is determined by the pointing
variation at the beginning of the pulse. Growth of discharge instabilities might cause a stabilisation of the pointing variation during the optical pulse.

The pointing variation of the optical beam can be influenced both by the gain (i.e. the build-up time) and by the resonator magnification. The influence of the resonator magnification is especially important, as it provides us with an extra criterion in the design of unstable resonators for pulsed laser systems.

Both calculations and measurements have shown that the reflection profile on the outcoupling mirror has no large influence on the beam pointing variation.

The spectral quality is usually denoted by the linewidth. In the spectral beam quality improvement experiments, the use of Fabry-Perot etalons has proved to be efficient, easy to align and stable elements to reduce the linewidth of a long pulse XeCl* excimer laser. A linewidth of 4 pm can be obtained using one 0.1 mm thick etalon with $R_{side} = 80\%$ in the oscillator cavity. The output energy of this pulse is found to be more than enough to injection lock a slave oscillator.

When trying to reduce the linewidth further, we ran into the problem of too high cavity losses due to the etalons. A model has been developed to describe the build-up of the coherence length in the cavity. Using this model, it is seen that if the cavity losses are too high, the linewidth does not reduce fast enough. Therefore the etalon does not become fully resonant. This means that the cavity losses due to the etalon remain high and no beam can build up. Hence, with our relatively low gain system (3-5 % cm$^{-1}$) the narrow linewidths as reported in the literature can not be reproduced. The linewidths reported there were obtained with high gain systems (10-12 % cm$^{-1}$).

The injection locking experiments show that narrow bandwidth, high output energy and good beam quality can be obtained for a long pulse XeCl* excimer laser in a master-oscillator-slave-oscillator setup. The experiments show that the spectrum of the XeCl* laser is homogeneously broadened, as the slave oscillator locks to the oscillator with nearly no line broadening and with complete energy extraction. The injection intensity can be reduced to a value of 100 W cm$^{-2}$ before injection locking stops. Long injection pulses allow a broad injection timing interval.
Samenvatting

De eigenschappen van een laserbundel worden in sterke mate bepaald door de resonator. In dit proefschrift worden de resultaten beschreven van het onderzoek aan de ruimtelijke en spectrale kwaliteit van een lange puls, lage gain XeCl* excimer laser. Het in dit proefschrift beschreven experimentele werk kan onderscheeld worden in twee delen: de experimenten naar de ruimtelijke bundelkwaliteit en de experimenten aan de spectrale kwaliteit.

Het zou erg praktisch zijn als het mogelijk was om de ruimtelijke bundelkwaliteit met een enkele parameter te beschrijven. Dit is echter niet eenvoudig. De waarschijnlijk beste beoordeling van de laser bundel kan plaats vinden als zowel de fase als intensiteitsverdeling van de bundel bekend zijn. Dit maakt het echter moeilijk om bundels uit verschillende resonatoren te kunnen vergelijken. Om deze vergelijking te kunnen maken is een enkele parameter toch wenselijk.

Voor het resonator onderzoek is de TDL (Times Diffraction Limited) parameter de meest bruikbare bundelkwaliteitsparameter. Iedere resonator configuratie kan leiden tot een TDL van 1 als de resonator in de laagste orde mode bedreven kan worden. In tegenstelling tot de $M^2$ parameter, die een grote waarde krijgt voor sterk diffracterende bundels, zoals bundels uit instabiele resonatoren. $M^2$ is meer een bundelpropagatieparameter dan een bundelkwaliteitsparameter. De PIB-factor, Strehl's ratio en de $K$-factor geven zinvolle informatie, maar als enkele parameter beschrijven ze niet genoeg voor de onderzoeker. Dus om resonatoren te vergelijken is gebruik gemaakt van de TDL parameter.

Voor gebruikers van lasers is de $K$-factor waarschijnlijk de beste parameter. De $K$-factor is dan ook de geplande ISO-bundelgeometrie-standaard. Samen met het vermogen (of puls energie en pulsduur voor een gepulst systeem) is de helderheid van de laser eenvoudig te bepalen en kan de bruikbaarheid van de laser voor een bepaalde toepassing bepaald worden.

De ruimtelijke bundelkwaliteit experimenten hebben laten zien dat stabiele resonatoren simpele en gemakkelijk uit te lijnen resonatoren zijn, die een goede energie extractie uit het medium mogelijk maken en die resulteren in homogene nabije veld bundelprofielen. Om de maximale energie extractie te bereiken is een geometrische terugkoppeling van 20 tot 70 % noodzakelijk bevonden.

Met een vlak-vlak resonator is een betere focussenbaarheid haalbaar, maar dit leidt wel tot een slechter nabije veld bundelprofiel. Vlak-concaaf resonatoren leiden tot een vlak top-hat nabije veld bundelprofiel en een dito verre veld profiel, maar de grootte
van dit verre veld is groter door het grote aantal modi in de output bundel, zodat de focussenbaarheid van deze bundels tamelijk slecht is.

De divergentie van de bundel uit een vlak-vlak resonator is ongeveer 3 mrad. De divergentie van de bundel uit de vlak-concaaf resonator is groter, afhankelijk van de kromtestraal van de concave achterspiegel. De divergentie van de bundel uit de vlak-concaaf resonator is binnen de meetnauwkeurigheid in overeenstemming met de waarde verwacht op basis van een benaderingsanalyse.

Met een hoge reflectie hard edge instabiele resonator met een vergroting van 2.4 is een vrijwel diffractie gelimiteerde bundel (∏DL = 1.3) mogelijk uit onze laser. Onder gemachte ontlaadscondities resulteert dit in een helderheid van de bundel van $5.5 \cdot 10^{14}$ W/cm$^2$·sr. De maximaal behaalde helderheid van de laser met deze resonator is $1.2 \cdot 10^{15}$ W/cm$^2$·sr.

De energie in de zijlobben van de verre veld energie verdeling kan gereduceerd worden door deel doorlatende uitkoppelspiegels te gebruiken in de instabiele resonatoren. Dit leidt tot een kleine toename van de divergentie, maar de geometrische terugkoppeling gelijkgoud gehouden neemt de helderheid toe door de hogere energie in de centrale piek. Echter, in deel doorlatende uitkoppelspiegels kunnen een probleem veroorzaken met betrekking tot de fase, zoals we hebben gezien bij de 45% spiegels. We hebben laten zien dat het mogelijk is om deze problemen te voorkomen door gebruik te maken van zogenoemde phase-unifying spiegels. Door de lagere zijlob energie neemt de helderheid van de bundel uit een instabiele resonator met een 45% reflecterende uitkoppelspiegel met een vergroting van 1.6 toe tot $7.0 \cdot 10^{14}$ W/cm$^2$·sr onder gemachte condities. De maximum helderheid bedroeg $1.4 \cdot 10^{15}$ W/cm$^2$·sr.

Verdere vermindering van de zijlob energie kan verkregen worden door gebruik te gaan maken van variabele reflectie instabiele resonatoren (VRURs). Met deze resonatoren zijn vrijwel diffractie begrenste bundels (1.7 maal diffractie gelimiteerd) met een glad verlopend nabij veld gerealiseerd. De experimenten hebben aangetoond dat een hoge centrale reflectie de bundelkwaliteit nauwelijks nadelig beïnvloed. Echter, de output van deze hoge centrale reflectie VRURs veel groter is door een beter gain volume gebruik. De helderheid van de hoge reflectie VRURs is bijna twee keer zo groot als die van de lage reflectie VRURs. De best presterende VRUR was degene met een vergroting van 2.0 en een centrale reflectie van 65%. Met deze resonator is een helderheid van $9.1 \cdot 10^{14}$ W/cm$^2$·sr gerealiseerd bij een gemachte ontlanding.

Als een nabij veld met een vlakke top wenselijk is moet de reflectie en de vergroting zodanig gekozen worden dat aan de zogenaamde 'maximally flat'-conditie voldaan is. De experimenten hebben aangetoond dat deze conditie, die is afgeleid voor een lege resonator (dus zonder gain), geldig is voor onze laser.

De beste energie extractie van alle resonatoren is behaald met de variabele reflectie instabiele resonatoren met een hoge centrale reflectie. De experimenten hebben aangetoond dat deze hoge centrale reflectie de bundelkwaliteit niet beïnvloed. Met VRURs is een betere energie extractie mogelijk dan met hard edge instabiele resonatoren en dus ook een grotere helderheid van de bundel.

De positie van het focus veld van de bundel vertoont variaties: de 'beam pointing'-variatie. Een model is opgesteld dat deze pointing variatie verklaart. De metingen
zijn in overeenstemming met de berekeningen. In het model is aangenomen dat de beam pointing variatie zijn oorsprong vindt in de opbouwfase van het optische veld in de resonator. Als het optische veld meer tijd krijgt om op te bouwen voordat de gain verzadigt, neemt de pointing variatie af. We hebben gezien dat de pointing variatie van de gehele puls bepaald wordt door de pointing variatie aan het begin van de puls. Groei van ontladingsinstabiliteiten zou een stabilisatie van de pointing variatie gedurende de puls kunnen veroorzaken.

De pointing variatie van de bundel kan beïnvloed worden met zowel de gain (i.e. de opbouwtijd) en met de vergroting van de resonator. De invloed van de resonatorvergroting is vooral van belang, omdat het een extra criterium oplevert voor het ontwerpen van instabiele resonatoren voor gepulste lasersystemen.

Zowel berekeningen als metingen geven aan dat het reflectieprofiel op de uitkoppelspiegel niet van grote invloed is op de beam pointing variatie.

De spectrale kwaliteit van een laserbundel wordt normaal gesproken aangegeven met de lijnbreedte. In de experimenten uitgevoerd om de spectrale kwaliteit van de lange pulss XeCl* laser te verbeteren is gebleken dat Fabry-Perot etalons efficiënte, makkelijk uit te lijnen en stabiele elementen zijn om de lijnbreedte te versmallen. Een lijnbreedte van 4 pm is verkregen door gebruik te maken van een 0.1 mm dik etalon met 80 % reflectie aan iedere kant in de cavity te plaatsen. De output energie van dit systeem met intracavity etalon is hoog genoeg gebleken om een tweede systeem te laten "injection-locken".

Toen we probeerden de lijnbreedte verder te verminderen, kregen we te maken met het probleem van de hoge cavity verliezen ten gevolge van de etalons. Een model is opgesteld om de opbouw van de coherente lengte in de cavity te beschrijven. Uit dit model blijkt dat, als de cavity verliezen te hoog zijn, de lijnbreedte niet snel genoeg afneemt en het etalon daardoor niet volledig resonant wordt. Dit betekent dat de verliezen door de etalons hoog blijven en geen bundel op kan bouwen voor het einde van de gain. Dus kunnen met ons lage gain systeem (3-5 % cm⁻¹) de smalle lijnbreidtes zoals die in de literatuur gemeld zijn niet gereproduceerd worden. Deze gerapporteerde lijnbreidtes zijn gehaald met hoge gain systemen (10-12 % cm⁻¹).

Injection-locking experimenten hebben aangetoond dat een bundel met een smalle lijnbreedte, een hoge output energie en een goede bundelkwaliteit gerealiseerd kunnen worden met een lange pulss XeCl* excimer laser in een master-oscillator-slave-oscillator configuratie. De experimenten tonen aan dat het spectrum van de XeCl* laser homogeen verbreed is, omdat de slave-oscillator locked aan de master-oscillator zonder een duidelijke lijnverbreding en met volledige energie extractie. De injectie energie kan verlaagd worden tot ongeveer 100 W/cm² voordat de injection-locking stopt. Lange injectie pulsen laten een breed injectie interval toe.
Over het optisch gedrag van de lange puls XeCl* excimeer laser

Samenvatting voor de niet-laserfysicus

Lasers kom je tegenwoordig tegen in alle hoeken van onze samenleving. Bijna iedereen heeft, bewust of onbewust, wel een laser(tje) thuis staan. Er is sinds de bouw van de eerste laser in 1953 een grote verscheidenheid aan lasers ontwikkeld, varierend van de kleine, laagvermogen halfgeleider lasers toegepast in CD-spelers en laser-pointers tot de grote, hoogvermogen gas- en vaste stof lasers die in de industrie gebruikt worden. De gepulste XeCl*-excimeer laser is een laser die tot deze laatste groep behoort.


Laserlicht heeft een aantal specifieke eigenschappen die het onderscheiden van gewoon licht. Ten eerste is een laserbundel sterk gericht doordat het licht deels opgesloten zit in de resonator. Het licht kan alleen over de optische as van de resonator lopen. Ten tweede is laser licht monochromatisch omdat slechts een zeer smalle band van golflengtes uitgezonden wordt. Een derde eigenschap van laserlicht is coherentie. Coherentie betekent dat er een relatie bestaat tussen de verschillende golven waar het licht uit bestaat. Het gevolg van deze eigenschappen is dat een laserbundel een zeer hoge intensiteit heeft.

Het laserlicht vertoont deze eigenschappen echter niet vanzelf. Een deel wordt bepaald door het medium, maar met de resonator kan men invloed uitoefenen op deze eigenschappen. In dit proefschrift worden de resultaten beschreven van het onderzoek aan de bundelkwaliteit van een lange puls XeCl* excimeer laser. Het onderzoek is op te delen in twee vrijwel onafhankelijke delen.

Het belangrijkste deel is de ruimtelijke kwaliteit van de laserbundel. Hiermee wordt bedoeld de gerichtheid van de bundel en de ruimtelijke coherentie van de bundel. Een aantal typen resonatoren is onderzocht op de eigenschappen van het uitgezonden licht.
Stabiele resonatoren bestaan uit eenvoudige, standaard optische componenten, zijn eenvoudig uit te lijnen en resulteren in een goede energie extractie uit het medium. Echter, de bundel bestaat uit een groot aantal resonator modi, wat leidt tot een slechte coherentie en een matige gerichtheid van de bundel. De kwaliteit van de bundel kan verbeterd worden door het aantal resonator modi te verminderen tot bij voorkeur 1. Met instabiele resonatoren is dit te bereiken. Instabiele resonatoren zijn vergro- tende resonatoren. De laagste orde resonator mode wordt daardoor opgeblazen tot de gewenste grootte. Er dient een optimum gevonden te worden tussen de vergroting van de resonator en de mogelijkheden die het medium biedt wat versterking betreft.

Er bestaat een tweetal klassen van instabiele resonatoren, de zogenoemde positieve en negatieve tak instabiele resonatoren. De negatieve tak instabiele resonatoren hebben het nadeel dat ze een focus in de resonator hebben. Om problemen met optische breakdown in de resonator te voorkomen is derhalve voor de positieve tak instabiele resonatoren gekozen. Deze zijn weer onder te verdelen in twee soorten: hard edge en variabele reflectie instabiele resonatoren. De begrippen hard edge en variabele reflectie zeggen iets over het reflectie profiel op de uitkoppspiegel van de resonator. Bij een hard edge uitkoppelspiegel zit er een stap in het reflectie profiel, terwijl bij de variabele reflectie uitkoppelspiegel, de naam zegt het al, het reflectie profiel zonder discontinuiteiten verloopt.

Beide soorten resonatoren zijn onderzocht en leiden tot goede resultaten. Met de hard edge instabiele resonatoren zijn bundels te maken die vrijwel door de diffrac- tie begrensd worden. In het focus vertonen de bundels uit deze resonatoren echter ringen rond de centrale spot. Dit is het gevolg van de harde stap in het reflectie profiel van de uitkoppelspiegel. Dit is te verminderen met variabele reflectie uitkoppelspiegels. Hiermee zijn ook vrijwel diffrac- tie gelimiteerde bundels gemaakt.

De bundelkwaliteit uit beide typen instabiele resonatoren is zeer goed. Ook is de energie extractie uit het medium goed. De variabele reflectie instabiele resonatoren presteerden net iets beter. Ze hebben echter het nadeel dat de spiegels die voor deze resonatoren nodig zijn lastig te maken zijn en daardoor relatief kostbaar zijn.

De instabiele resonatoren brengen echter ook een probleem met zich mee. Tijdens de opbouwfasen van de bundel zijn de start condities erg belangrijk. Deze variëren van schot tot schot en zijn niet allen in de hand te houden. De bundel bouwt namelijk op vanuit de ruis en deze is niet te regelen. Dit veroorzaakt kleine verschillen in de bundel voor de verschillende pulsen. Het duidelijkst zichtbare resultaat hiervan is de variatie van de positie van het focus. Dit betekent dat de optische as van de bundel varieert. In dit proefschrift is een model beschreven dat de oorzaak van deze variatie ten dele verklaart. Experimenten bevestigen het model.

Het tweede deel van het beschreven onderzoek behelst het verbeteren van de spectrale kwaliteit van de XeCl* laser. In de XeCl* laser is een aantal frequenties mogelijk omdat een aantal overgangen tussen boven- en onderniveau mogelijk zijn. De lijnbreedte van iedere golflengte is daarnaast verbreed doordat we met een hoge druk gasontla- ing werken als medium. De totale onversmalde lijnbreedte van het XeCl* spectrum is 0.5 nm.

Voor de XeCl* laser blijken Fabry-Perot etalons efficiënte, makkelijk uit te lijnen en
stabiele elementen te zijn om de lijnbreedte te versmallen. Een lijnbreedte van 4 pm is in de experimenten bereikt. Verder versmallen met extra etalons is, tegen de verwachting in, niet gelukt. Om dit te kunnen verifiëren is een model opgesteld dat de opbouw van coherentie lengte in de resonator beschrijft. Dit model laat zien dat als de verliezen in de resonator te groot zijn, de lijnbreedte niet snel genoeg afneemt. De Farby-Perot etalons blijven dan voor grote verliezen zorgen en een bundel kan niet opbouwen voor het einde van de puls.

Door twee vrijwel identieke XeCl* laser systemen te gebruiken is het mogelijk om zowel een goede spectrale als een goede ruimtelijke bundelkwaliteit te halen. Met een master oscillator kan de smalle lijnbreedte gecreeëerd worden. Deze bundel wordt vervolgens geïnjecteerd in een slave oscillator die de goede bundelkwaliteit en de energie van de totale bundel bepaalt.
Over het optisch gedrag van de lange puls XeCl* excimeer laser
Nawoord

Een proefschrift komt uiteraard niet zonder ondersteuning tot stand. Een aantal bedankjes is derhalve op zijn plaats. Bovenal ben ik dank verschuldigd aan prof.dr.ir. W.J. Witteman voor het mogelijk maken van het onderzoek en het aan mij schenken van het vertrouwen om dit toch zoveel mogelijk zelfstandig te doen. Daarnaast ben ik de Stichting voor Technische Wetenschappen (STW) dankbaar voor het financieel mogelijk maken van dit onderzoek.

Voor de dagelijkse ondersteuning en begeleiding was Fred van Goor gelukkig altijd beschikbaar. Ik heb de afgelopen vier jaar dan ook met plezier met hem samengewerkt. Omdat ik ook in de vakgroep Quantumelectronica ben afgestudeerd op het systeem waar het in dit proefschrift beschreven onderzoek op uitgevoerd is, heb ik dankzij mijn toenmalige begeleider John Timmermans een vliegende start gehad. De begintijd van mijn promotie viel samen met het eind van zijn promotie en gedurende die tijd hebben we nog regelmatig samen experimenten uitgevoerd. Het tweede hoofdstuk van dit proefschrift is min of meer een sterk verkorte versie van een deel van John's proefschrift geworden: 't was dus een zeer dankbaar naslagwerk.

In september '96 kwam er een project van Nederlands Centrum voor Laser Research (NCLR) B.V. en een NCLR-collega bij: Martijn Zwegers. Martijn koos uiteindelijk voor een meer commerciële functie bij Coherent, maar heeft in de anderhalf jaar die hij met mij heeft samengewerkt het nodige gedaan. Een groot deel van de metingen zoals die beschreven zijn in hoofdstuk 4 zijn in die tijd gemeten. En toen ik er na zijn vertrek weer alleen voor stond, heb ik goed gemerkt dat je met twee man in een bepaalde tijd veel meer werk kunt verzetten dan alleen in de dubbele tijd.

Ik had geluk toen ik begon: er stond al een goed werkend systeem. Maar toch, dat was niet genoeg. Vandaar de verhuizing naar een ander lab en de bouw van een tweede (vrijwel identiek) systeem. Reproduceeren van een systeem bleek toch minder eenvoudig dan gedacht. Het duurde uiteindelijk meer dan een jaar langer dan gepland voordat het tweede systeem naar behoren werkte. En toen zaten er al heel wat zweetdruppeltjes in van de heren technici: Henk Prins, Gerard Oude Meijers, Jacob Couperus, Roy Lammerink en Henk Tanke.

In de meetkooi was het akelig stil geworden na het vertrek van John naar de Eureka laser. Gelukkig kwam daar al vrij snel verandering in toen Louw Feenstra het lab aan de andere kant ging gebruiken. Vele discussies vooral over electrische circuits en de onwil van zijn systeem volgden. Ik ben hem zeer erkentelijk voor het herhaaldelijk corrigeren van artikelen en dit proefschrift, aangezien zijn Engels duidelijk beter is dan dat van mij.
De maandagavonden (tegenwoordig woensdagavonden) met de etclub (Peter van der Slot, Frans Blok, Serge Gielkens, Sandra Pagano, John en sinds kort ook Monique Timmermans) waren altijd zeer gezellig en ik hoop dat we dat nog even vol kunnen houden. Voor de dames waren de soms technische discussies waarschijnlijk niet te volgen, maar ze wisten zich er toch door heen te slaan. De kookkunsten van een ieder worden zeer op prijs gesteld, maar eerlijk is eerlijk: wat mij betreft spant Peter wat dat betreft de kroon. Daar kunnen sommige koks in restaurants nog wat van leren.

De rest van de vakgroep Quantumelectronica (tegenwoordig leerstoel Laserfysica als ik het goed begrepen heb) en NCLR veeg ik even op een hoop: bedankt voor alle zinvolle, maar vooral ook voor de zinloze discussies die gevoerd zijn, alle taart die in de loop van de jaren naar binnen is gewerkt, etc. etc. Eentje wil ik er toch nog even uitletten: onze secretaresse Mirella van der Beek. Nou ja, onze. Eigenlijk is ze er alleen maar voor het NCLR, maar de vakgroep doet ze er zo even bij. Bedankt.

De speciale optiek die in de loop van het onderzoek is nodig geweest is grotendeels gemaakt bij de optische groep van de FFW (Johan van Hespen, Tom van Druinick, Bernard Meinders). Hen wil ik van af deze positie hartelijk bedanken voor alle werk dat zij in de loop van die vier jaar voor mij verricht hebben. Met name het maken van de variabele reflectie spiegels is geen gemakkelijke klus geweest en die hebben ze toch maar mooi geklaard.

Tenslotte een welgemeend dank-je-wel voor mijn ouders en mijn zus. Ieder weekend was ik (met de was) welkom. Bedankt voor de steun en het geduld dat jullie met mij hebben gehad. Ik kwam soms lichtelijk moe thuis en dan wilde ik nog wel eens uit mijn slof schieten. Sorry.

Curriculum Vitae


Dit lasersysteem vond hij dermate interessant dat hij besloot te blijven toen er in het kader van het STW-project "Hoogvermogen excimier lasers voor industriële toepassingen" een OIO-plaats vrij kwam met als doel het verbeteren van de bundelkwaliteit van deze laser. De resultaten van dit onderzoek zijn in dit proefschrift beschreven.

Ook bij zijn nieuwe werkgever, Nederlands Centrum voor Laser Research (NCLR) B.V., blijft hij nog even hangen in het excimier laser onderzoek. Daar wordt het de bedoeling om de opgedane kennis te gebruiken bij de verdere ontwikkeling van het 1 kHz, 1 kW XeCl* lasersysteem dat bij NCLR in ontwikkeling is en dat op korte termijn op de markt moet gaan komen.