ON THE FRICTION OF

THIN FILM RIGID DISKS

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ON THE FRICTION OF

THIN FILM RIGID DISKS

PROEFSCHRIFT

ter verkrijging van
de graad van doctor aan de Universiteit Twente,
op gezag van de rector magnificus,
prof. dr. F.A. van Vught,
volgens besluit van het College voor Promoties
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Summary

This thesis describes the investigation into the friction between a head and a rigid disk (the so called head-disk interface) of rigid disk drives.

Rigid disk drives are used to store and retrieve vast amounts of data. These drives consist of a stack of rigid disks on a common rotating spindle. Each disk surface has a head that can move across the disk for recording and reproducing data. The head is connected to a spring suspension which presses the head against the disk surface with a light normal load. During normal operation of the drive, each head is separated from its disk by means of aerodynamic lubrication: As a result of the shape of the head and the relative velocity between the head and the disk, a thin pressurised film of air is generated at the head-disk interface. Due to the load carrying capacity of this film, the head flies above its disk surface. During powering on or off of a drive, the relative velocity is too low to create a sufficient load carrying pressure in each air film, and sliding takes place. During sliding, the friction between head and disks is substantially larger than during flying. During the transition from sliding to flying the friction reduces. This change in friction makes that certain lubrication regimes (boundary, mixed, and aerodynamic lubrication) at the head-disk interface can be distinguished. The friction can be presented in a so called generalised Strubeck curve, which gives the coefficient of friction as a function of a lubrication number.

In the rapid developing information technology, there is a continuous demand for disk drives with higher storage capacities. In order to further increase the capacity, the heads should be in virtual contact with the rigid disks whereby friction and wear have to be kept as low as possible. As a result of this, controlling the friction and wear becomes more and more important in rigid disk drives.

The investigation in this thesis has been carried out in order to come to a better understanding of the different lubrication regimes of a head-disk interface, such that the regimes can be predicted as a function of the operational parameters of the interface. The thesis provides an introduction into the problems of head-disk interfaces, an analysis of the head-disk interface, the development and analysis of a friction model, a dynamic analysis of the instrument that has been used for friction experiments, and, finally, the results of those experiments.

The friction model describes the friction force as a function of a lubrication number of the head-disk interface. The model is based on the following assumptions: the surface of the head is nominally flat, and rough, with a large number of asperities; the rigid disk consists of strong bonded thin films on a substrate
and its surface is covered with a thin liquid lubricant layer; the surface of the rigid disk is considered smooth and flat; furthermore, a stiction force can occur between the rough surface and the liquid lubricant layer. The friction force has three components: a component determined by the microcontacts between the solid surfaces of the head and the disk, a component due to the shearing of the thin liquid lubricant layer, and a component determined by the gaseous lubricant film.

The model predicts that the generalised Strubeck curve shifts toward the left and that the stiction force at the head-disk interface decreases with increasing nominal pressure. The model predicts also a rather high stiction force in the liquid lubricant layer, especially when the thickness of the layer is in the order of the surface roughness of the head-disk interface and the applied normal load is low. The stiction force decreases with decreasing nominal area of contact. It also follows from the model that the normal load, in combination with the dimensions of the head, should be carefully chosen, not only for the purpose of aerodynamic lubrication, but also to minimise the friction in the boundary and mixed lubrication regimes. The elasticity of the solid surfaces of an head-disk interface is mainly determined by the elastic modulus of the substrate, the undercoat and the head. However, these moduli do not have a very significant influence on the generalised Strubeck curve.

Before the results of the friction experiments are discussed, a description and a dynamic analysis is given of the instrument that is used for these experiments. From the analysis the forces and displacements at the instrument are estimated. After describing the friction experiments, the experimental results are compared with the results obtained from the friction model. It follows that the friction model predicts qualitatively the shape of the experimentally derived generalised Strubeck curves. The generalised Strubeck curve shifts toward the left with higher applied normal load. This shift is also predicted by the friction model. In the boundary lubrication regime, the coefficient of friction at low loads is higher than that of an unlubricated disk. The take-off velocity increases slightly with the surface roughness of the head-disk interface.

From the results in this thesis, it can be concluded that rigid disk drives should be designed such that the head-disk interfaces are operating at the transition from mixed to aerodynamic lubrication. At this transition the friction force is minimal and the separation is low. This can be done by adjusting the nominal pressure or the relative velocity of the head-disk interfaces. The nominal pressure can be modified by adjusting the dimensions of the head.
Samenvatting

Dit proefschrift beschrijft het onderzoek naar de wrijving tussen een leeskop en een harde schijf (de zgn. ‘head-disk interface’) van ‘rigid disk drives’.

Rigid disk drives worden gebruikt om grote hoeveelheden data op te slaan en te reproduceren. Deze drives hebben een stapel harde schijven die om een gemeenschappelijk roterende as draaien. Elk schijfoppervlak heeft een leeskop die, voor het opslaan en reproduceren van de data, over de schijf kan bewegen. De leeskop is bevestigd aan een veer die de leeskop met een geringe belasting tegen het schijfoppervlak aan drukt. Wanneer de drive gewoon in werking is, wordt de leeskop door middel van aerodynamische smering gescheiden van de schijf: tengevolge van de vorm van de leeskop en de relatieve snelheid tussen de leeskop en de schijf, wordt een dunne luchtfilm gegenereerd tussen de leeskop en het schijfoppervlak. Ten gevolge van de draagkracht van de luchtfilm, ‘zweeft’ de leeskop boven het schijfoppervlak. Tijdens het aan- of uitschakelen van de drive, is de snelheid te laag om een voldoende draagkracht te genereren in de luchtfilm, waardoor ‘glijden’ plaatsvindt. Tijdens glijden, is de wrijving tussen leeskop en schijf substantieel hoger dan tijdens zweven. Tijdens de overgang van glijden naar zweven neemt de wrijving af. Deze overgang in wrijving maakt het mogelijk dat bepaalde smeringsgebieden (grens-, gemengde en aerodynamische smering) bij de head-disk interface kunnen worden onderscheiden. De wrijving kan worden getoond in een zogenaamde algemene Strubeck kromme, die de wrijvingscoëfficiënt weergeeft als functie van een smeringskental.

Door de snel ontwikkelende informatie-technologie is er een continue behoefte aan disk drives met een hogere opslagcapaciteit. Om de capaciteit nog verder te verhogen, zouden de leeskoppen nagenoeg in contact moeten zijn met de harde schijven, waarbij wrijving en slijtage zo laag mogelijk moeten worden gehouden. Dientengevolge wordt de beheersing van wrijving en slijtage steeds belangrijker in rigid disk drives.

Het onderzoek in dit proefschrift is uitgevoerd om tot een beter begrip te komen van de verschillende smeringsgebieden van een head-disk interface, zodat de gebieden kunnen worden voorspeld als functie van de operationele parameters van de interface. Het proefschrift geeft een introductie in de problemen van head-disk interfaces, een analyse van de head-disk interface, de ontwikkeling en analyse van een wrijvingsmodel, een dynamische analyse van het instrument dat gebruikt is voor de wrijvingsexperimenten, en, tenslotte, de resultaten van deze experimenten.
Het wrijvingsmodel beschrijft de wrijvingskracht als een functie van het smeringskental van de head-disk interface. Het model is gebaseerd op de volgende veronderstellingen: het oppervlak van de leeskop is nominaal vlak en ruw, en heeft een groot aantal ruwheidstoppen; de harde schijf bestaat uit dunne films die stevig zijn aangehecht op een substraat en het oppervlak is bedekt met een dun smerend vloeistof laagje; het oppervlak van de harde schijf is glad en vlak verondersteld; verder kan een stichtekracht optreden tussen het ruwe oppervlak en de smerende vloeistof laag. De wrijvingskracht kent drie componenten: een component die bepaald is door de microcontacten tussen de vaste oppervlakken van de leeskop en de schijf, een component ten gevolge van de afschuiving in de dunne smerende vloeistof laag, en een component die bepaald is door de smerende gasfilm.

Het model voorspelt dat de algemene Strubeck kromme naar links verschuift en dat de stichtiekracht aan een head-disk interface vermindert bij een toename in de nominale druk. Het model voorspelt ook een vrij hoge stichtiekracht in de smerende vloeistof laag, vooral wanneer de dikte van de laag in de orde van grootte van de oppervlaktefornestheid van de head-disk interface is en de aangebrachte normaal belasting laag is. De stichtiekracht neemt af bij een afnemend nominaal contactoppervlak. Uit het model volgt eveneens dat de normaalbelasting, in combinatie met de afmetingen van de leeskop, voorzichtig gekozen moeten worden; niet alleen voor de aerodynamische smering, maar ook om de wrijving in het grens- en gemengde smeringsgebied te minimaliseren. De elasticiteit van de vaste oppervlakken van een head-disk interface wordt voornamelijk bepaald door de elasticiteitsmodulus van het substraat, de onderlaag en de leeskop. Echter, deze moduli hebben geen significante invloed op de algemene Strubeck kromme.

Voordat de resultaten van de wrijvingsexperimenten worden bediscussieerd, is een dynamische analyse gegeven van het instrument dat voor de experimenten is gebruikt. Uit deze analyse worden de krachten en verplaatsingen afgeschat. Na de beschrijving van de wrijvingsexperimenten worden de experimentele resultaten vergeleken met de resultaten verkregen uit het wrijvingsmodel. Hieruit blijkt dat het wrijvingsmodel de vorm van de experimenteel afgeleide algemene Strubeck kromme kwalitatief goed voorspelt. De algemene Strubeck kromme schuift naar links met hogere aangebrachte normaalbelasting. Deze verschuiving wordt ook door het model voorspeld. In het grensmeringsgebied is de wrijvingscoëfficiënt bij lage belasting hoger dan dat van een ongesmeerde schijf. De take-off snelheid neemt licht toe met de oppervlaktefornestheid van de head-disk interface.

Uit de resultaten van dit proefschrift kan worden geconcludeerd dat rigid disk drives zo ontworpen moeten worden dat de head-disk interfaces in het overgangspunt van gemengde naar aerodynamische smering werken. Bij deze overgang is de wrijvingskracht minimaal en de separatie laag. Dit kan verwezenlijkt worden door de nominale druk of de relatieve snelheid van de head-disk interface aan te passen. De nominale druk kan veranderd worden door de afmetingen van de leeskop aan te passen.
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Nomenclature

In choosing the various symbols, the following general principles have been followed, whenever possible. Vectors and matrices are in bold-face type. The transpose of a vector \( \mathbf{A} \) is denoted by \( \mathbf{A}^T \). A dot above a letter denotes differentiation with respect to time. A bar above a letter indicates a mean value or otherwise a dimensionless variable. Primes are used to denote quantities that have been subjected to a transformation of some kind. The list below shows all the symbols used in the various chapters of this thesis. Symbols which are used only locally in the appendices are not given here. In case more than one meaning has been given to a symbol, they are divided by a semicolon. Symbols presenting variables have their dimensions in square brackets. Each dimensional unit is divided by a space.

Index of roman symbols

\begin{itemize}
  \item \( a \) \quad - \quad \text{Magnetisation transition length [m]; Distance in } Z \text{ direction between the centre of the cross-sectional area of a beam and the centre of a strain gauge of the friction force sensor [m]}
  \item \( a_x, a_y \) \quad - \quad \text{Minor and major semi axis, respectively, of the contact ellipse of an ellipsoidal asperity in contact with a smooth surface [m]; Magnetisation transition lengths for longitudinal and perpendicular recording, respectively [m]}
  \item \( A \) \quad - \quad \text{Parameter in the equation of motion of the pendulum}
  \item \( \bar{A} \) \quad - \quad \text{Dimensionless parameter in the dimensionless equation of motion of the pendulum; Projected area of contact [m}^2\text{]}
  \item \( A_a \) \quad - \quad \text{Nominal (or apparent) area of contact [m}^2\text{]}
  \item \( A_h \) \quad - \quad \text{Area of a magnetisation region (bit) [m}^2\text{]}
  \item \( A_c \) \quad - \quad \text{Mean cross sectional area of the core of a ring head [m}^2\text{]}
  \item \( A_F \) \quad - \quad \text{Amplification factor of the amplifier bridge circuit of the friction force sensor bridge}
\end{itemize}
\(A_g\)  
- Cross sectional area of the gap of the core of a ring head \([\text{m}^2]\); Real area of contact remaining for the aerodynamic lubrication \([\text{m}^2]\)

\(A_{p,s}\)  
- Plastic area of contact of the microcontacts between two sliding surfaces \([\text{m}^2]\)

\(A_{r,t}\)  
- Total wetted area (real area of contact of those asperities of the solid surface which are in contact with the liquid lubricant layer) \([\text{m}^2]\)

\(A_{r,s}\)  
- Real area of contact of the microcontacts between two sliding solid surfaces \([\text{m}^2]\)

\(A_X \equiv \frac{a_x}{t_m}\)  
- Dimensionless magnetisation transition lengths for longitudinal recording

\(A_Y \equiv \frac{a_y}{t_m}\)  
- Dimensionless magnetisation transition lengths for perpendicular recording

\(A_g \equiv \frac{A_g}{A_a}\)  
- Dimensionless real area of contact remaining for the aerodynamic lubrication

\(A_{r,t} \equiv \frac{A_{r,t}}{A_a}\)  
- Dimensionless real area of contact of those asperities of the solid surface which are in contact with the liquid lubricant layer

\(A_{r,s} \equiv \frac{A_{r,s}}{A_a}\)  
- Dimensionless real area of contact of the microcontacts between two sliding solid surfaces

\(b_r\)  
- Length of a magnetisation region (bit length) \([\text{m}]\)

\(b_p\)  
- Bit rate (speed of recording) \([\text{s}^{-1}]\)

\(B\)  
- Parameter in the equation of motion of the pendulum

\(B\)  
- Dimensionless parameter in the dimensionless equation of motion of the pendulum

\(B_r\)  
- Flux density of the magnetic field of the permanent magnet in the radial direction of the coil \([\text{T}]\)

\(B_s\)  
- Width of a slider \([\text{m}]\)

\(c\)  
- Constant

\(c_i\)  
- Curve fit constants for \(P_g\) \((i = 1, 2, \ldots, 4)\); Curve fit constants for \(\tau_i/\tau_0\) \((i = 1, 2, \ldots, 5)\)

\(C_1, C_2\)  
- Surface correcting coefficients

\(C_p\)  
- Viscous damping constant of the pendulum \([\text{N m s}]\)

\(C_F\)  
- Friction force calibration factor \([\text{N}]\)

\(C_L\)  
- Load calibration factor of the pendulum \([\text{N}]\)

\(C_\sigma\)  
- Constant

\(d_s\)  
- Separation between the reference planes of two rough solid surfaces \([\text{m}]\)

\(d_{g,\text{min}}\)  
- Minimum air film thickness \([\text{m}]\)

\(d_{s,0}\)  
- Starting separation at \(v_- = v_+ = 0\) \([\text{m}]\)

\(D\)  
- Amplitude of the axial displacement of the disk at the probe tip \([\text{m}]\)
\[ \overline{D} \equiv \frac{D}{R_z} \] - Dimensionless amplitude of the axial displacement of the disk at the probe tip

\[ D_a \] - Areal bit density [m\(^{-2}\)]

\[ D_b \] - Linear bit density [m\(^{-1}\)]

\[ D_t \] - Track density [m\(^{-1}\)]

\( e \) - Output voltage between the ends of the coil of a head [V]

\( e_x \) - Output voltage detected by a ring head from a longitudinal recorded signal [V]

\( e_y \) - Output voltage detected by a ring head from a perpendicular recorded signal [V]

\( \overline{E} \{ \} \) - Expected value of the term inside the brackets

\( \overline{E}_e \) - Reduced elastic modulus [Pa]

\[ \overline{E}_c \equiv \frac{E_c}{E_r} \] - Dimensionless reduced elastic modulus

\( E_e \) - Effective elastic modulus [Pa]

\[ \overline{E}_e \equiv \frac{E_e}{E_r} \] - Dimensionless effective elastic modulus

\( E_{i} \) - Elastic modulus of thin film \( i \) (\( i = 1, 2, \ldots, n \)) [Pa]

\( E_{e} \) - Elastic modulus of the slider (rails) [Pa]

\( E_{a} \) - Elastic modulus of the substrate [Pa]

\( E_X \equiv \frac{e_x \langle \overline{F} \rangle}{n \mu_{0} w w M_r} \) - Dimensionless output voltage for a longitudinal recorded signal

\( E_Y \equiv \frac{e_y \langle \overline{F} \rangle}{n \mu_{0} w w M_r} \) - Dimensionless output voltage for a perpendicular recorded signal

\( \overline{E}_X, \overline{E}_Y \) - Dimensionless loss in amplitude of the output voltage for longitudinal and perpendicular recording, respectively

\( F \) - Friction force [N]; Friction force at the probe tip [N]

\( \mathbf{F}, \mathbf{F}' \) - Friction force vectors at the probe tip [N]

\( F_c \) - Load in X direction generated at the coil of the pendulum [N]

\[ \mathbf{F}_c = (-F_c, 0, 0)^T \] - Load vector of the applied load at the coil [N]

\[ \overline{F}_c \equiv \frac{F_c \ell_1}{M_c g R_z} \] - Dimensionless load in X direction generated at the coil of the pendulum

\( F_{c,\text{min}} \) - Minimum initial load at the coil of the pendulum, needed to bring the pendulum beam against the limit stop [N]

\( \overline{F}_{c,\text{min}} \) - Dimensionless minimum initial load at the coil of the pendulum, needed to bring the pendulum beam against the limit stop

\( F_{m,n}(d_n) \) - Integral function (kind of probability distribution function)
\( F_g \) – Component of the friction force due to the shearing of the gaseous lubricant [N]

\( F_l \) – Component of the friction force due to the shearing of the liquid lubricant layer [N]

\( \overline{F}_{m,n}(h) \) – Dimensionless integral function

\( F_s \) – Component of the friction force due to the sliding of microcontacts [N]

\( F_x \) – Friction force in \( X \) direction at the probe tip of the pendulum [N]

\( F_z \) – Friction force in the \( Z \) direction of the friction force sensor [N]; Friction force in \( Z \) direction at the probe tip of the pendulum [N]

\( g \) – Gap length of a head [m]

\( g = 9.807 \) – Gravitational acceleration [m s\(^{-2}\)]

\( g_c \) – Gap between the circular plates of the parallel plate capacitor [m]

\( g_{c,0} \) – Gap between the circular plates of the parallel plate capacitor, when the angular deflection of the pendulum is zero [m]

\( g_{c,\text{max}} \) – Maximum gap between the circular plates of the parallel plate capacitor, when the pendulum rests against the limit stop [m]

\( g_{c,\text{min}} \) – Minimum gap between the circular plates of the parallel plate capacitor, when the bottom of the movable plate \( C_2 \) touches the fixed plate \( C_1 \) [m]

\( G \) – Gauge factor of each strain gauges of the friction force sensor

\( G \equiv \frac{g}{t_m} \) – Dimensionless gap length of a head

\( G_{m,n}(d_n) \) – Integral function (kind of probability distribution function)

\( \overline{G}_{m,n}(h) \) – Dimensionless integral function

\( h \) – Flying height or head-medium separation (HMS) [m]; Height of each beam of the friction force sensor [m]

\( h \equiv \frac{d_n}{R_{a,x,c}} \) – Dimensionless separation between two rough solid surfaces

\( h_m \) – Magnetic separation [m]

\( h_{\text{min}} \) – Minimum flying height or head-medium separation [m]

\( h_r \) – Height of a slider rail [m]

\( h_{\sigma} \) – Dimensionless separation factor

\( H \) – Effective hardness of the interface [Pa]

\( H \equiv \frac{h_m}{t_m} \) – Dimensionless magnetic separation

\( H_c \) – Coercivity of a medium [A m\(^{-1}\)]; Effective hardness [Pa]

\( \overline{H}_c \equiv \frac{H_c}{M_r} \) – Dimensionless coercivity of a medium
\[H_d\quad \text{Demagnetisation field of a medium [A m}^{-1}]\]
\[H_{\min} \equiv \frac{d_{\min}}{\ell_r} \quad \text{Dimensionless minimum air film thickness}\]
\[H_s \quad \text{Height of a slider [m]}\]
\[H_X \equiv A_X + H \quad \text{Dimensionless separation for a longitudinal recorded signal}\]
\[H_Y \equiv A_Y + H \quad \text{Dimensionless separation for a perpendicular recorded signal}\]
\[i \quad \text{Current passing through the coil of a head [A]}\]
\[I \equiv i \sqrt[4]{\frac{JR_c^2}{(MgR_u)^3}} \quad \text{Dimensionless current in the coil of the pendulum}\]
\[I_c \quad \text{Moment of inertia of the pendulum about its axis of rotation [kg m}^2]\]
\[\bar{I}_c \equiv \frac{I_c}{J} \quad \text{Dimensionless moment of inertia of the pendulum about its axis of rotation}\]
\[I_m \quad \text{Moment of inertia of the calibration weight about the axis of rotation of the pendulum [kg m}^2]\]
\[\bar{I}_m \equiv \frac{I_m}{J} \quad \text{Dimensionless moment of inertia of the calibration weight about the axis of rotation of the pendulum}\]
\[I_p \quad \text{Moment of inertia of the probe about the axis of rotation of the pendulum [kg m}^2]\]
\[\bar{I}_p \equiv \frac{I_p}{J} \quad \text{Dimensionless moment of inertia of the probe about the axis of rotation of the pendulum}\]
\[I_w \quad \text{Moment of inertia of the balance weight about the axis of rotation of the pendulum [kg m}^2]\]
\[\bar{I}_w \equiv \frac{I_w}{J} \quad \text{Dimensionless moment of inertia of the balance weight about the axis of rotation of the pendulum}\]
\[J \quad \text{Moment of inertia of the pendulum system about its axis of rotation [kg m}^2]\]
\[K \equiv \frac{\lambda}{t_m} \quad \text{Dimensionless wavenumber of a sinusoidal recorded signal}\]
\[K(m) \quad \text{Complete elliptic integral of the first kind}\]
\[K_p \quad \text{Torsional spring constant, of the elastic hinge at the pivot point of the pendulum [N m]}\]
\[K_n \equiv \frac{\lambda}{d_g} \quad \text{Knudsen number}\]
\[\ell \quad \text{Length of each beam B}_i \ (i = 1, \ldots, 4) \text{ of the friction force sensor [m]; Length of the wire of the coil at the pendulum [m]}\]
\[\ell_c \quad \text{Mean length of the core of a ring head [m]}\]
\[\ell_g \quad \text{Length of each strain gauge of the friction force sensor [m]}\]
\[\ell_i \quad \text{Dimensions of the pendulum (i = 1, 2, \ldots, 12) [m]}\]
\( \ell_r \)  
- Length of a slider rail [m]

\( \ell_t \)  
- Length of a taper of a taper-flat slider [m]

\( L \)  
- Inductance of the coil of the pendulum [T]

\[ L \equiv \frac{\eta_g v_+}{\pi R_{a,x,c}} \]  
- Dimensionless lubrication number

\[ L \equiv \frac{L}{R_c \sqrt{\frac{M_c g R_z}{J}}} \]  
- Dimensionless inductance of the coil of the pendulum

\( L_A \)  
- Dimensionless lubrication number at the transition ML-AL

\( L_B \)  
- Dimensionless lubrication number at the transition BL-ML

\( L_g \)  
- Dimensionless gap loss for a sinusoidal recorded signal

\( L_H \)  
- Dimensionless separation loss for a sinusoidal recorded signal

\( L_i \)  
- Dimensionless dimensions of the pendulum (i = 1, 2, \ldots, 12)

\( L_s \)  
- Length of a slider [m]

\( L_T \)  
- Dimensionless thickness loss for a sinusoidal recorded signal

\( m \)  
- Mass of the calibration weight at the pulley \([Q]\) of the pendulum [kg]

\( m_c \)  
- Mass of the pendulum [kg]

\( m_p \)  
- Mass of the probe at the pendulum [kg]

\( m_w \)  
- Mass of the balance weight of the pendulum [kg]

\( M \equiv \frac{m_c}{M_c} \)  
- Dimensionless mass of the pendulum

\( M_c \equiv m_c + m_p + m_w \)  
- Total mass of the pendulum system (pendulum beam, balance weight and probe) [kg]

\( M_C \)  
- Moment generated at the pivot point of the pendulum due to the inherent damping of the pendulum [N m]

\( M_K \)  
- Moment generated at the pivot point of the pendulum due to the torsional spring constant of the elastic hinge [N m]

\( M_m \equiv \frac{m}{M_c} \)  
- Dimensionless mass of the calibration weight of the pendulum

\( M_p \equiv \frac{m_p}{M_c} \)  
- Dimensionless mass of the probe of the pendulum

\( M_r \)  
- Remanent magnetisation \([\text{A m}^{-1}]\)

\( M_w \equiv \frac{m_w}{M_c} \)  
- Dimensionless mass of the balance weight of the pendulum

\( M_\theta \)  
- Transformation matrix for the pendulum

\( n \)  
- Number of windings of a head coil; Rotational speed of a disk or a bearing shaft \([\text{s}^{-1}]\); Number of windings of the circular coil at the pendulum; Number of elastic dissimilar thin films; Number of generalised coordinates

\( n, n' \)  
- Unit vector, normal to the specimen surface
\[ n_t \]  - Total number of wetted asperities
\[ n_s \]  - Total number of microcontacts
\[ N \equiv n R B_0 t_1 \]  - Dimensionless magnetic flux of the permanent magnet
\[ \frac{1}{\sqrt{R^2 J M_e g R_z}} \]  - Dimensionless total number of wetted asperities
\[ N_t \equiv \frac{n_t R_{a,x,c}}{A_0 \rho} \]  - Dimensionless total number of microcontacts
\[ \bar{p} \]  - Nominal pressure of a contact [Pa]
\[ p_a \]  - Ambient pressure [Pa]; Nominal (or apparent) pressure in the microcontacts [Pa]
\[ p_g \]  - Pressure in the gaseous lubricant [Pa]
\[ \bar{p}_g \]  - Mean pressure in the gaseous lubricant [Pa]
\[ \bar{p}_l \]  - Nominal pressure in the liquid lubricant layer [Pa]
\[ \bar{p}_s \]  - Nominal pressure at the microcontacts between two sliding solid surfaces [Pa]; Mean pressure in the microcontacts [Pa]
\[ \bar{p}_a \equiv \frac{p_a}{\bar{p}} \]  - Dimensionless ambient pressure
\[ \bar{p}_g \equiv \frac{p_g}{\bar{p}} \]  - Dimensionless pressure in the gaseous lubricant
\[ P \]  - Applied normal load [N]; Applied load at the slider [N]; Applied normal load at the probe tip [N]
\[ \mathbf{P}, \mathbf{P'} \]  - Normal load vector at the probe tip [N]
\[ \bar{P} \equiv \frac{P}{M_c g} \]  - Dimensionless load at the probe tip of the pendulum
\[ P_{25}, P_{50} \]  - Dimensionless pulse width at 25% and 50% of the pulse amplitude, respectively
\[ P_g \]  - Load carried by the gaseous lubricant [N]
\[ P_l \]  - Load carried by the liquid lubricant layer [N]
\[ \bar{P}_o \equiv \frac{P_o}{M_c g} \]  - Dimensionless offset load at the probe tip when the tip is in contact with the specimen, prior to the actual measurements
\[ P_{p-p} \]  - Dimensionless pulse width
\[ P_s \]  - Load carried by the microcontacts of two sliding solid rough surfaces [N]
\[ P_x \]  - Load in X direction at the probe tip of the pendulum [N]
\[ P_z \]  - Load in Z direction at the probe tip of the pendulum [N]
\[ \bar{P}_g \equiv \frac{p_g}{\bar{P}} \]  - Dimensionless load carried by the gaseous lubricant
\[ \bar{P}_l \equiv \frac{p_l}{\bar{P}} \]  - Dimensionless load carried by the liquid lubricant layer
\[ \bar{P}_s \equiv \frac{p_s}{\bar{P}} \]  - Dimensionless load carried by the microcontacts
$r$ - Radius at which the slider contacts the rigid disk [m]

$r_i$ - Position vectors of certain locations at the pendulum [m] ($i = 1, 2, \ldots, 9$)

$r'_i$ - Position vectors of certain locations at the pendulum after the pendulum has deflected by an angle $\theta$ [m] ($i = 1, 2, \ldots, 9$)

$r_p$ - Radius of the circular movable plate $C_2$ of the parallel plate capacitor [m]

$R$ - Radius of the circular coil at the pendulum [m]

$\mathbf{R} = (R_x, R_y, R_z)^T$ - Position vector of the centre of mass of the pendulum system (pendulum beam, balance weight and probe) [m]

$R_a$ - Centre-line-average (CLA) roughness parameter [m]

$R_{a,x,c}$ - Combined centre-line-average (CLA) roughness of the surface heights of solid 1 and 2 [m]

$R_{a,x,i}$ - Centre-line-average (CLA) roughness of the surface heights of solid $i$ [m]

$R_c$ - Resistance of the circular coil at the pendulum [V A$^{-1}$]

$S \equiv \frac{v_-}{v_+}$ - Slip factor

$t$ - Time [s]

$t_i$ - Thickness of thin film $i$ ($i = 1, 2, \ldots, n$) [m]

$t_\ell$ - Thickness of the lubricant layer [m]

$\overline{t}_\ell \equiv \frac{t_\ell}{R_{a,x,c}}$ - Dimensionless thickness of the lubricant layer

$t_m$ - Thickness of the magnetic layer [m]

$t_o$ - Thickness of the overcoat [m]

$T$ - Kinetic energy [N m]

$\overline{T}(\theta) \equiv \frac{T}{M_c g R_z}$ - Dimensionless kinetic energy of the pendulum

$U_c$ - Voltage applied to the coil of the pendulum [V]

$\overline{U_c} \equiv U_c^\prime$ - Dimensionless voltage applied to the coil of the pendulum

$\sqrt{\frac{4}{J}} \frac{1}{R_c^2 (M_c g R_z)^3}$ - Output voltage of the friction force sensor bridge circuit [V]

$U_i$ - Supplied voltage to the Wheatstone bridge of the friction force sensor [V]

$U_o$ - Output voltage of the Wheatstone bridge of the friction force sensor [V]

$U_p$ - Output voltage of the capacitance bridge [V]

$v$ - Relative velocity between head and medium [m s$^{-1}$]; Velocity of the specimen [m s$^{-1}$]

$\mathbf{v}, \mathbf{v}'$ - Velocity vector of the specimen surface at the probe tip [m s$^{-1}$]
$v_- \equiv v_1 - v_2$ - Relative velocity between the sliding solid surfaces [m/s$^{-1}$]

$v_+ \equiv v_1 + v_2$ - Sum velocity of the sliding solid surfaces [m/s$^{-1}$]

$v_A$ - Velocity at which the transition ML-AL takes place [m/s$^{-1}$]

$v_B$ - Velocity at which the transition BL-ML takes place [m/s$^{-1}$]

$v_i$ - Velocity of solid surface i [m/s$^{-1}$]

$v_{\text{max}}$ - Maximum relative velocity at the HDI [m/s$^{-1}$]

$v_{\text{min}}$ - Minimum relative velocity at the HDI [m/s$^{-1}$]

$V(\theta) \equiv \frac{V}{M_c g R_z}$ - Dimensionless potential energy of the pendulum

$w$ - Width of a track [m]; Width of each beam of the friction force sensor [m]

$w_r$ - Width of a slider rail [m]

$x, y, z$ - Cartesian coordinate system; Cartesian coordinate system of the pendulum, with its origin at the pivot point of the elastic hinge

$x_B, y_B, z_B$ - Cartesian coordinate system at the beams of the friction force sensor

$x_c, y_c, z_c$ - Coordinates of the centre of mass of a slider [m]; Coordinates of the centre of mass of the pendulum beam [m]

$x_p, y_p, z_p$ - Coordinates of the pivot point of a slider [m]

$X, Y, Z$ - Dimensionless Cartesian coordinates (Chapter 2); Cartesian coordinates (Chapter 5)

$X', Y', Z'$ - Cartesian coordinates (Chapter 5)

$X'', Y'', Z''$ - Cartesian coordinates (Appendix F)

$X''', Y''', Z'''$ - Cartesian coordinates (Appendix F)

$x \equiv vt$ - Cartesian coordinate in x direction

$\bar{X} \equiv \frac{x}{x_m}$ - Dimensionless coordinate

$X_{25}, X_{50}$ - Dimensionless coordinate at 25% and 50% of the pulse amplitude respectively

$X_c \equiv \frac{x_c}{R_z}$ - Dimensionless $X$ coordinate at the centre of mass of the pendulum

$X_p$ - Dimensionless coordinate at the minimum or maximum of the pulse amplitude

$z$ - Surface height of the topography of a solid surface [m]

$z_i$ - Surface height of the topography of solid surface i [m]

$z_B$ - Surface height of a protuberance at the glass substrate rigid disk [m]

$Z_c \equiv \frac{z_c}{R_z}$ - Dimensionless $Z$ coordinate of the centre of mass of the pendulum
Index of greek symbols

\( \alpha \) – Deflection angle of a suspension of an head-arm assembly; Dimensionless minor semi axis of the contact ellipse of a microcontact; Angle between the velocity vector \( \mathbf{v} \) of the specimen and the \( Y \) axis of the pendulum

\( \alpha_t \) – Taper angle of a taper-flat slider

\( \beta \) – Dimensionless major semi axis of the contact ellipse of a microcontact; Position angle of the specimen surface

\( \gamma \) – Skew or yaw angle of an head-arm assembly; Position angle of the specimen surface;

\( \dot{\gamma} \) – Rate of shearing in a lubricant \( [s^{-1}] \)

\( \delta(t) \) – Axial motion of the disk at the probe tip \( [m] \)

\( \delta_t \) – Height of an asperity immersed into the liquid lubricant layer \( [m] \)

\( \delta_s \) – Mutual approach of two contacting asperities of the solid surfaces \( [m] \)

\( \Delta n \) – Speed step \( [s^{-1}] \)

\( \Delta t \) – Time step \( [s] \)

\( \epsilon_d \equiv D/\ell_3 \) – Dimensionless amplitude of the axial displacement of the disk at the probe tip

\( \zeta \equiv \frac{C_p}{2} \) – Dimensionless viscous damping constant of the pendulum

\( \sqrt[.5]{\frac{1}{J M_c g R_z}} \) – Surface density of asperities of a rough surface \( [m^{-2}] \)

\( \eta \equiv \frac{\eta R_{a,z,c}}{\rho} \) – Dimensionless surface density of the summits of a rough surface

\( \eta_t \equiv \frac{1}{1 + \frac{A g \ell_c}{\mu_c \eta_A c_c}} \) – Head efficiency parameter

\( \eta_b \) – Dynamic viscosity of the gaseous lubricant \( [Pa\ s] \)

\( \eta_t \) – Dynamic viscosity of a liquid lubricant \( [Pa\ s] \); Dynamic (shear thinned) viscosity of the liquid lubricant layer \( [Pa\ s] \)

\( \eta_0 \) – ‘Low-shear’ or ‘zero-shear’ viscosity of a liquid lubricant \( [Pa\ s] \)

\( \theta \) – Pitch angle of a slider; Angular deflection of the pendulum

\( \theta_0 \equiv \frac{\xi}{\omega^2} \) – Angular deflection of the mechanically balanced pendulum when the pendulum is at its equilibrium position
\( \theta_c \) – Angular deflection of the pendulum during load calibration when the pendulum is at its equilibrium position

\( \theta_m \) – Angular deflection of the pendulum, when the probe tip is in contact with the specimen surface prior to actual measurements

\( \theta_{\text{min}} \) – Minimum angular deflection of the pendulum, when the pendulum rests against the limit stop

\( \theta_{\text{max}} \) – Maximum angular deflection of the pendulum, when the bottom of the movable capacitor plate \( C_2 \) touches the fixed capacitor plate \( C_1 \)

\( \dot{\theta} \equiv \frac{d\theta}{dt} \) – Angular velocity of the pendulum \([\text{s}^{-1}]\)

\( \ddot{\theta} \equiv \frac{d^2\theta}{dt^2} \) – Angular acceleration of the pendulum \([\text{s}^{-2}]\)

\( \kappa \equiv \frac{a_x}{a_y} \) – Axes ratio of the contact ellipse of a Hertzian contact

\( \lambda \) – Wavelength of a sinusoidal recorded pattern \([\text{m}]\); Molecular free path of the molecules in a gas \([\text{m}]\)

\( \lambda \equiv \frac{\rho_y}{\rho_x} \) – Ratio of principal curvatures

\( \lambda_a \) – Molecular free mean path of the gas molecules in the gaseous lubricant at ambient conditions \([\text{m}]\)

\( \lambda_A \) – Constant in the lubrication transition diagram \([N^{1/2} \text{m}^{-3/2}]\)

\( \lambda_B \) – Constant in the lubrication transition diagram \([N \text{ m}^{-2}]\)

\( A_a \equiv \frac{C_1 \lambda_a}{R_{a,x,c}} \) – Dimensionless molecular free mean path of the gas molecules in the gaseous lubricant at ambient conditions

\( \mu \equiv \frac{F}{P} \) – Dynamic coefficient of friction

\( \mu_A \) – Dynamic coefficient of friction at the transition ML-AL

\( \mu_B \) – Dynamic coefficient of friction at the transition BL-ML

\( \mu_g \equiv \frac{F_g}{P} \) – Dynamic coefficient of friction due to the gaseous lubricant

\( \mu_l \equiv \frac{F_l}{P} \) – Dynamic coefficient of friction due to the liquid lubricant layer

\( \mu_o = 4\pi \cdot 10^{-7} \) – Magnetic permeability of free space \([\text{V s A}^{-1} \text{m}^{-1}]\)

\( \mu_r \) – Relative magnetic permeability

\( \mu_s \) – Dynamic coefficient of friction at the BL regime; Static coefficient of friction at the probe tip, when the probe tip is in contact with the specimen, prior to the actual measurements

\( \nu_e \) – Effective Poisson ratio

\( \nu_i \) – Poisson ratio of thin film \( i \) \((i = 1, 2, \ldots, n)\)

\( \nu_t \) – Poisson ratio of the slider (rails)

\( \nu_s \) – Poisson ratio of the substrate
\[ \xi \equiv \frac{R_x}{R_z} \]  

- Ratio of \( X \) coordinate and \( Z \) coordinate of the centre of mass of the pendulum

\[ \rho \equiv \rho_x + \rho_y \]  

- Main curvature of the asperities [m\(^{-1}\)]

\[ \rho_x \]  

- Minor principal relative curvature of the elliptic asperities [m\(^{-1}\)]

\[ \overline{\rho}_x \equiv \frac{\rho_x}{\rho} \]  

- Dimensionless minor principal relative curvature of the elliptic asperities

\[ \rho_y \]  

- Major principal relative curvature of the elliptic asperities [m\(^{-1}\)]

\[ \overline{\rho}_y \equiv \frac{\rho_y}{\rho} \]  

- Dimensionless major principal relative curvature of the elliptic asperities

\[ \sigma_{s,c} \]  

- Standard deviation of the summit heights of a solid rough surface [m]

\[ \sigma \]  

- Mean of the surface heights of a rough surface [m]

\[ \sigma_m \]  

- Maximum principal tensile stress in the liquid lubricant layer [Pa]; Maximum principal tensile stress in a lubricant [Pa]

\[ \overline{\sigma}_m \equiv \frac{\sigma_m}{p} \]  

- Dimensionless maximum principal tensile stress in a lubricant

\[ \tau \equiv t \sqrt{\frac{M_cgR_z}{J}} \]  

- Dimensionless time in the equation of motion of the pendulum

\[ \tau_c \]  

- ‘Newtonian’ shear stress that corresponds to the onset of the ‘rupture’ regime [Pa]

\[ \tau_g \]  

- Shear stress in the gaseous lubricant [Pa]

\[ \tau_l \]  

- Shear stress in a liquid lubricant layer [Pa]

\[ \tau_o \]  

- ‘Newtonian’ shear stress in a liquid lubricant layer [Pa]

\[ \overline{\tau}_o \equiv \frac{\eta_g \tau_o}{\eta_o \rho} \]  

- Dimensionless ‘Newtonian’ shear stress in a liquid lubricant layer

\[ \tau_s \]  

- Shear stress at the microcontacts of the sliding solid surfaces [Pa]; ‘Newtonian’ shear stress in a liquid lubricant layer at which shear thinning begins [Pa]

\[ \phi \]  

- Roll angle of a slider

\[ \phi(z) \]  

- Probability density function of the distribution of summit heights of a rough surface

\[ \overline{\phi}(s) \]  

- Dimensionless probability density function of the distribution of summit heights of a rough surface (dimensionless Gaussian summit height distribution)

\[ \chi \equiv \frac{\eta^\gamma}{\gamma} \]  

- Dimensionless roughness parameter

\[ \psi \]  

- Plasticity index (according to Greenwood and Williamson [47])

\[ \psi_c \equiv \frac{E_c}{H} \sqrt{\rho R_{a,x,c}} \]  

- Dimensionless elasticity index
\( \omega \equiv \sqrt{\frac{K_p}{M_c g R_z} + 1} \) – Dimensionless spring rate of the elastic hinge at the pivot point of the pendulum

\( \omega_d \) – Angular frequency of the axial motion of the disk at the probe tip [s\(^{-1}\)]

\( \bar{\omega}_d \equiv \omega_d \sqrt{\frac{J}{M_c g R_z}} \) – Dimensionless angular frequency of the axial motion of the disk at the probe tip

\( \Omega \equiv \frac{1}{\alpha \beta} \frac{E_r}{p} \cdot \sqrt{\frac{\rho R_{a,x,c}}{\gamma}} \) – Dimensionless parameter

**Index of used units**

- [A] Ampere
- [C] Coulomb
- [F = C m\(^{-1}\)] Farad
- [kg] Kilogram
- [N] Newton
- [m] Meter
- [Pa = N m\(^{-2}\)] Pascal
- [s] Second
- [T = V s m\(^{-2}\)] Tesla
- [V] Volt

**Index of used units prefixes**

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<td>terra</td>
<td>T</td>
<td>10(^{12})</td>
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<td>giga</td>
<td>G</td>
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<td>mega</td>
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xxii  Nomenclature
## Abbreviations

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<tr>
<td>AC</td>
<td>alternating current</td>
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<tr>
<td>AD</td>
<td>analogue-digital</td>
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<td>AL</td>
<td>aerodynamic lubrication</td>
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<td>bit</td>
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<td>CaTiO₃</td>
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<td>CLA</td>
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<td>COF</td>
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<td>DASD</td>
<td>direct access storage device</td>
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<td>DCF</td>
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<td>FCF</td>
<td>friction force calibration factor</td>
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<td>FDD</td>
<td>flexible disk drive</td>
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<td>HAA</td>
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<td>hydrodynamic lubrication</td>
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<tr>
<td>HMI</td>
<td>head-medium interface</td>
</tr>
<tr>
<td>HMS</td>
<td>head-medium separation</td>
</tr>
<tr>
<td>ID</td>
<td>inner diameter</td>
</tr>
<tr>
<td>kb</td>
<td>kilo byte</td>
</tr>
<tr>
<td>LCF</td>
<td>load calibration factor</td>
</tr>
<tr>
<td>Mb</td>
<td>mega byte</td>
</tr>
<tr>
<td>ML</td>
<td>mixed lubrication</td>
</tr>
<tr>
<td>MR</td>
<td>magnetic recording</td>
</tr>
<tr>
<td>MS</td>
<td>magnetic separation</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Definition</td>
</tr>
<tr>
<td>-------------</td>
<td>-------------------------------------------------</td>
</tr>
<tr>
<td>MSD</td>
<td>magnetic storage device</td>
</tr>
<tr>
<td>NiP</td>
<td>nickel-phosphorus</td>
</tr>
<tr>
<td>NRZI</td>
<td>nonreturn to zero inverted</td>
</tr>
<tr>
<td>OD</td>
<td>outer diameter</td>
</tr>
<tr>
<td>OECD</td>
<td>organisation for economic cooperation and development</td>
</tr>
<tr>
<td>PCB</td>
<td>printed circuit board</td>
</tr>
<tr>
<td>PFPE</td>
<td>perfluoropolyether</td>
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<tr>
<td>PID</td>
<td>digital compensation filter</td>
</tr>
<tr>
<td>RDD</td>
<td>rigid disk drive</td>
</tr>
<tr>
<td>RMS</td>
<td>root-mean-square</td>
</tr>
<tr>
<td>rpm</td>
<td>rotations per minute</td>
</tr>
<tr>
<td>rps</td>
<td>rotations per second</td>
</tr>
<tr>
<td>SiC</td>
<td>silicon-carbon</td>
</tr>
<tr>
<td>SNR</td>
<td>signal-to-noise ratio</td>
</tr>
<tr>
<td>STLE</td>
<td>society of tribologists and lubrication engineers</td>
</tr>
<tr>
<td>TD</td>
<td>tape drive</td>
</tr>
<tr>
<td>TDV</td>
<td>touch-down velocity</td>
</tr>
<tr>
<td>TOV</td>
<td>take-off velocity</td>
</tr>
<tr>
<td>TPC slider</td>
<td>traverse pressurised contour slider</td>
</tr>
<tr>
<td>VCR</td>
<td>virtual contact recording</td>
</tr>
</tbody>
</table>
1

Introduction to Friction of Thin Film Rigid Disks

This introductory chapter outlines the objectives and scope of this thesis. It starts with a brief description of the magnetic storage devices and their main components. A great deal of attention will be given to the head-medium interface and the nature of its problems regarding the magnetic recording and reproducing process. Several terms will be defined that will be used throughout this thesis. Finally, the organisation of this thesis will be given.

1.1 Magnetic Storage Devices

The amount of information that becomes available to us by electronic means, such as computers, audio and video recorders, is immense. In future, it will grow even more rapidly, as it has been estimated that yet only a few percent of all information is stored onto electronic information storage devices and that information networks and electronic databases continue to grow (see Adams [1] and Bhushan [11]). Today, information is not limited by availability but by issues such as finance, politics, etc. In order to deal with this increasing amount of information, high capacity information storage devices are required. The storage capacity — that is the amount of information that can be stored onto a given area and which is usually expressed in terms of storage density — is therefore one of the most important properties of an information storage device. Other important properties of the device are the input and output information rates. If larger storage capacities are required, then the input and output information rates should be increased too, in order to store or reproduce the same amount of information per unit of time. Furthermore, it is desirable that the stored information can be reproduced reliably, even after several years. The need for low cost devices with higher storage capacities and faster input and output information rates, with a reasonable long-term reliability, are the driving forces behind the rapidly growing market of the information storage technology.

Magnetic recording (MR) is one of several methods of storing and retrieving information. The method has proven to be versatile and the cost per storage capacity is low compared to other methods. This makes that MR can be found in all types of information storage devices, for example, in the exceedingly common
flexible and rigid disk drives of computers and in tape drives of audio, video and data recorders. With MR an electric signal, representing the information, is converted into a highly concentrated magnetic field at a very small area of a magnetisable medium. The inverse process is used for the reproduction of the stored information. In this thesis, a device carrying out these processes will be called a magnetic storage device (MSD).

Definition 1.1 A magnetic storage device (MSD) is any system able to store or reproduce information (whether representing sound, image, text, or numerical data) by converting a time-varying electrical signal into a spatially divided magnetic pattern (recording) or vice versa (reproducing) by means of a magnetic field.

Basically, MR is accomplished by moving a transducer (usually called the head) at a constant velocity relatively to and in close proximity of a magnetisable medium (hereafter shortly called the medium). Usually, the medium consists of a flexible or a rigid substrate and a multilayered structure deposited onto this substrate. Hence, media can be distinguished in flexible and rigid media. Flexible media can be found in tape drives (TDs) and flexible disk drives (FDDs), and rigid media can be found in rigid disk drives (RDDs) and drives for cards. Figure 1.1 shows schematically the head-medium configuration of some basic MSDs together with their cross sections. As can be noticed in this figure, the components of the multilayered structure of the media are given different names, for example, ‘thin film’, ‘coating’, or simply ‘layer’. In order to make a consistent use of these terms throughout this thesis, the following definitions are given:

Definition 1.2 The term ‘layer’ will be used in general sense; it may refer to a thickness of any material, such as a deposited magnetic material, a coating, a contaminant, or a lubricant, etc. The term ‘thin film’ will be used exclusively as a stratum of solid material deposited onto a solid substrate or onto another thin film by thin film deposition processes such as physical vapour deposition, chemical vapour deposition, electro deposition, etc. The term ‘coating’ will be used exclusively for indicating a stratum of solid material or a (thin) film on a solid surface for protection against wear and/or corrosion.

One or more layers of the media in Fig. 1.1 are magnetisable, whereas the others function as intermediary adhesion layers or protective coatings. The layers are usually very thin (below 100 nm) compared to the substrate (2 to 40 μm for tapes and flexible disks and 0.25 to 1.0 mm for rigid disks). Recently, also heads may have a thin coating (for example, heads in TDs of video recorders and in RDDs). A medium can be particulate or thin film (or also called continuously). A particulate medium consists of a magnetic layer of little magnetisable particles dispersed in a polymeric matrix which is deposited onto the substrate. A thin film medium is composed of one or more magnetic layers prepared by thin film processes such as electro- or electroless plating or vacuum techniques (sputtering
a) Heads and thin film disks in a rigid disk drive.

b) Heads and particulate disk in a flexible disk drive.

c) Heads and particulate tape in a video tape recorder.

d) Heads and particulate tape in an audio tape recorder.

Fig. 1.1. Schematics of the heads and media in some MSDs.
and evaporation). In contrast to a particulate (conventional) medium, a thin film medium consists of linked crystals of magnetic material.

In an MSD application there may be several media and each medium may have more than one head. To accomplish MR any MSD has an electro-mechanical system for moving the heads and the media relative to each other with constant velocity and an electronic system for delivering and reproducing the signal to be stored.

The arrows in Fig. 1.1 show the direction of motion of the heads and the media. As will be seen in this thesis, the relative velocity between head and medium plays an important role in MR. Relative velocity can be achieved by moving the medium only, for example, in audio TDs (also called the stationary-head applications); but there are also applications, such as RDDs and FDDs and TDs for video recorders, in which both head and medium move (the so-called moving-head applications).

With an RDD (see Fig. 1.1a), a stack of circular disks rotates continuously around a common spindle at high speed\(^1\). Each disk surface has its own head connected to a spring suspension which presses the head against the disk surface with a light nominal pressure of 10 to 30 kPa. The stack of heads and suspensions rotates around a common spindle, to provide direct access of the heads to any desired position on the disk. Therefore, an RDD is also called a **direct access storage device** (DASD). An RDD allows fairly high velocities between the heads and the disks (as high as 40 m/s), so that sufficient input and output information rates can be achieved.

With FDDs (see Fig. 1.1b), a single flexible circular disk (diskette) is enclosed in a jacket. This makes the medium replaceable. When the jacket is inserted in the drive, the disk is clamped at its centre and is accessed by two heads through two slots at either side of the jacket. The heads are mounted on spring suspensions and when recording or reproducing takes place, the disk rotates at relatively low speed (usually at 5 rps), while the heads are pushed together with the disk in between. The heads can move in a radial direction across the disk surface to provide direct access of the heads to any desired position on the disk.

In audio TDs (see Fig. 1.1d), stationary heads are usually applied. Only during recording or reproducing, the tape moves over the heads at a constant velocity of 0.048 m/s. The tension in the tape provides a constant load at the tape and presses the tape against the head surface.

TDs for video recorders (see Fig. 1.1c) require high input and data information rates. These have been achieved by wrapping the tape around a rotary and stationary drum. The heads are mounted on the rotary drum. During recording or reproducing the tape is held stationary, and the heads rotate rapidly, thereby

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\(^1\)In this thesis a clear distinction will be made between the speed and the velocity of a disk. The term **speed** will be used for the rate of rotation, expressed in the number of revolutions per unit time, for example, revolutions per second (rps). The speed depends on the angular acceleration of the disk. The term **velocity** is used for the rate of change of position with respect to time at a point at the disk. Velocity is expressed in unit distance per unit time, for example, metres per second (m/s), and depends on both the angular acceleration and the radius of the point at the disk.
scanning the tape. After the heads have passed, the tape steps to the next position. The tension in the tape provides a constant load between the heads and the tape and presses the tape against the surface of the heads.

With TDs the information is available only sequentially and the finite length of the tape makes that a compromise must be made between tape velocity (sufficient input and output information rates) and the required maximum recording length or time. Furthermore, tapes as well as flexible disks must be exchangeable between drives of different manufacturers. Therefore, the velocities of these types of media have been standardised. RDDs and TDs for video recorders operate at much higher velocities (as high as 40 m/s) than FDDs and TDs for audio recorders (0.21 m/s and 0.048 m/s respectively).

1.2 The Head-Medium Interface

The heart of an MSD is the interface of the head and the medium, or shortly called the head-medium interface (HMI). Here, the information must be recorded and reproduced, whereby the head must remain at extremely close distance from the medium and, simultaneously, the relative velocity between the head and the medium must be overcome. This may, but not necessarily will, give rise to mechanical interaction and thus to dissipation of energy (friction²) and dissipation of material (wear³). In general, knowledge of friction and wear comes mostly from tribology: the science and technology of interacting surfaces in relative motion. Therefore, it follows that the interface of the head and medium is essentially a complex tribological system.

In the definition of friction, the term ‘common boundary’, or shortly, the ‘interface’, is used. The HMI, however, is no interface in the ordinary sense. As with all physical solids, the surfaces of the head and the medium are not perfectly flat nor smooth; their actual topography deviates from their nominal surface, and, even though they appear very smooth, they have all sorts of topographical features, such as valleys and peaks, or asperities, of varying length and width. These deviations and features lead to a certain waviness and roughness of the surfaces. Consequently, when the head and medium are in contact or in close proximity of each other, contact may occur at local spots, or so called microcontacts. Furthermore, in the previous section, it was mentioned that the medium has a thin multilayered structure at its surface. Therefore, the definition of an interface, as the separating boundaries between the head and the medium, is not much use. In this thesis, the definition of a HMI, that will be used throughout is as follows (Godet [42, page 29]):

²Friction is the resistance force tangential to the common boundary between two bodies when, under action of an external force, one body moves or tends to move relative to the surface of the other (OECD [41]).

³Wear is the progressive loss of substance from the surface of a solid body caused by mechanical action, i.e. contact and relative motion of a solid, liquid or gaseous counter body (OECD [41]).
Definition 1.3 The head-medium interface (HMI) is the zone across which the difference in velocity between head and medium is accommodated.

In this definition the word ‘zone’ is used intentionally in order to distinguish it from the several layers of the medium, which do not necessarily all belong to the HMI. Some HMIs are shown schematically at the right hand side of each figure in Fig. 1.1. In a similar way, the head-disk interface (HDI) of an RDD may be defined by Definition 1.3, in which the word medium is replaced by the word disk.

1.3 The Head-Medium Separation Versus Contact Recording

For the last 40 years, the storage capacity of MSDs has been continuously increased, by designing new heads, by improving the magnetic layers and by applying thin film media instead of particulate media. Particularly due to the application of thin film media it has been recognised that in order to continue the increases in storage capacity the thickness of the magnetic layer(s) of the medium, the roughness of head and medium and also the distance between head and magnetic layer, i.e. the magnetic separation (MS), have to be minimised. In order to achieve a minimum MS, the head-medium separation (HMS), i.e. the distance between the surfaces of head and medium, should be critically small and uniform. The HMS is currently considered to be the main obstacle in achieving larger storage capacities. In order to achieve a low HMS, the roughness of head and medium should be as low as possible. Recognition of the importance of minimising the HMS, in order to increase the storage capacity of MSDs, led to a considerable increase of research into tribology of MSDs (see, for example, Bhushan [11]-[16]).

For maximum storage density it is likely that during recording and reproducing the head is in contact with the magnetic layer of the medium. Unfortunately, zero MS cannot be achieved, because the minimum separation is limited by the thickness of the overcoat and the lubricant, the HMS and also by the roughness of the head and the medium surfaces. In the last few years the terms contact recording (CR), virtual contact recording (VCR) or near contact recording are increasingly used in the MR industry to indicate recording at zero HMS or near zero HMS respectively (see, for example, Adams [1], Yeack-Scranton et al. [157]-[156]). However, as the roughness of the head and the medium and the thickness of the layers (thin films) of the medium are on the nanometre scale and continue to decrease, it is difficult to define ‘zero HMS’. It is likely that no particular HMS can be given at which there is contact but merely a region of separations in which the load at the HMI is mainly carried by the interacting solid surfaces. Therefore, contact recording is in this thesis defined as follows:
**Definition 1.4** Contact recording (CR) is magnetic recording or reproducing whereby the load, acting on the head-medium interface, is carried or partly carried by the interacting solid surfaces of head and medium.

### 1.4 The Head-Medium Separation in Rigid Disk Drives

Defined in the above way, CR has been achieved in principle in some MSDs such as FDDs and audio TDs, which operate at relatively low velocities and only when the devices are recording or reproducing. As the difference in velocity between head and medium must be accommodated at the HMI and the load must be carried by the solid surfaces, the head and medium must be protected sufficiently in order to prevent long-term mechanical deterioration. Eventually, the media of these devices do wear out with use, but they can be replaced, and often the heads can be cleaned.

DASDs, such as RDDs, are sealed and after powering on, they operate continuously at relative high velocities, in order to achieve the desired input and output information rates. Consequently, head cleaning and medium replacements are not possible, and CR would quickly lead to excessive heat generation and unacceptable mechanical deterioration of the head and medium surfaces. The solution to these problems has been found by applying aerodynamic lubrication (AL) to the HDIs and thus by allowing an HMS. By arranging the surfaces of each head and disk slightly convergent instead of parallel, a head can use the relative velocity of the disk to generate a thin pressurised film of air between them. Due to the load carrying capacity of this film, the head flies above its disk surface. As a result of the lubricating properties of the air film, the friction can be kept low and wear is eliminated.

However, a certain minimum relative velocity is required to allow the head to take off (the so called *take-off velocity* (TOV)). After powering on an RDD, the disks accelerate from zero to final speed. During (part of) this acceleration period, the relative velocity is too low to create a sufficient load carrying pressure in each air film, and *sliding* takes place. Also, after powering off the RDD, the relative velocity becomes too low to build up sufficient pressure in the air films and by the time the heads touch down (at the so called *touch-down velocity* (TDV)) on their disk surfaces, sliding takes place until the rotation of the disks ceases. The sliding period during the starting and stopping process of the disks usually takes a few seconds. In the literature, the conditions of starting and stopping are frequently referred to as *contact-start-stop* conditions (CSS-conditions).

In case of video TDs, which require high relative velocities during recording and reproducing, the heads and the medium are also aerodynamic lubricated.
1.5 Towards Contact Recording in Rigid Disk Drives

1.5.1 Contact Recording

As a result of the continuous efforts to achieve higher storage capacities of RDDs, the HMS has been considerably reduced. For example, Fig. 1.2 shows the reduction of the HMS over the years. Taking into consideration the logarithmic scale of the figure, a 200-fold decrease in the HMS during the last 40 years can be noticed. The air film thickness of present RDDs is below 100 nm. In the last years different technologies have been studied which utilise CR (see, for example, Adams [1], Bhatia and Menon [8] and Talke [137]). It is predicted that in a few years time the head and medium of RDDs will be in nominal contact.

1.5.2 Miniaturisation

Parallel to the move towards CR there is a trend to miniaturise the HMI, i.e. to reduce the dimensions of MSDs. For example, small TDs for backing up the stored information were required after the introduction of personal computers. The introduction of portable and notebook computers required smaller RDDs. Table 1.1 shows the trends in miniaturisation for the RDD industry (see also Grochowski et al. [48], [49]). For example, the increase in storage capacity of disk drives made smaller disk diameters possible. Consequently, the diameter of large capacity disks has reduced from 355 mm or larger to 203, 133, 89 mm and below.

![Graph](image-url)

**Fig. 1.2.** The minimum head-medium separation in rigid disk drives, as a function of the year of product shipment. Each dot represents the year of the first shipment of a product. Data from Mee and Daniel (Eds.) [91] and Nashua Co., Santa Clara, USA.
The reason for this reduction is that smaller disks improve the input and output information rates. Given a certain storage capacity, a smaller disk diameter allows the average distance travelled by the head to be shorter. Consequently, it takes less time to traverse the head to other information areas on the disk. The improvement in information rates is expected to continue when disk diameters become even smaller (in the range 64 to 25 mm) and disk speed increases (above 60 rps). Another important factor is that smaller disks can be made flatter and smoother and, consequently, smaller HMSs can be achieved, which, in turn, increase the storage capacities. Furthermore, the mass and the applied load will be reduced (below 95 mN, see Table 1.1), in order to minimise the possibility of head crash. The magnetic layers and coatings become thinner (below 50 nm thickness) and smoother (below 5 nm centre-line-average (CLA) roughness). So a strange paradox occurs: as the capacity of RDDs increases the size of the RDDs reduces.

### 1.5.3 Reliability Issues

As a result of the move towards CR and the miniaturisation of RDDs, a series of problems, all relating to reliability issues, will become increasingly important. The relations between the different aspects of these problems are given in Fig. 1.3. The first problem concerns the friction and wear between the head and the disk. It will

| Table 1.1. Trends for rigid disks and drives in the last decennium (After information from Nashua Co., Santa Clara, California, and Applied Magnetics Co., Goleta, California). |
|---|---|
| Substrate | AlMg, 1 to 2 mm |
| Undercoat | NiP, 50 μm |
| Magnetic layer | e.g. CoCrTa, 50 nm |
| Coercivity, $H_c$ | 150 kA/m |
| Overcoat | C, 20 nm |
| Lubricant | PFPE, 2 to 5 nm |
| HMS | $> 100$ nm |
| CLA-roughness medium | $\approx 5$ nm |
| CLA-roughness head | $\approx 2$ nm |
| Disk speed | 60 rps |
| Disk diameter | 64 to 89 mm |
| Storage density | 775 kBits/mm² |
| Nominal contact area head | 1 to 2 mm² |
| Load at head | 68 to 95 mN |
| Near future | Canasite, Glass, SiC |
| | 0.25 mm |
| | e.g. CoPt, 10 to 20 nm |
| | (multilayered) |
| | $> 300$ kA/m |
| | C, 4 to 10 nm |
| | ‘advanced’, 0.5 to 1 nm |
| | 5 to 10 nm (CR) |
| | $\approx 1$ nm |
| | $\approx 1$ nm |
| | $> 160$ rps |
| | 25.4 to 38 mm |
| | $> 62$ MBits/mm² |
| (‘50% to 100% heads’) |
| (‘15% heads’) |
| < 40 mN |
**Fig. 1.3.** Relationships between the different aspects of RDDs (△ = beneficial, ▲ = detrimental). Terms indicated by '1' will be explained in Chapter 2 and beyond. The grey area indicates the field of tribology. Note that a reduction of the HMS has a detrimental effect on the mechanical performance, while it has a beneficial effect on the magnetic performance of an RDD.
be expected that during sliding, the friction between head and disk is substantially larger than during flying. This has been confirmed by friction measurements (see, for example, Jiaa and Eltoukhy [65] and Marchon et al. [88]). During the transition from sliding to flying the friction reduces. This change in friction, makes that certain lubrication regimes\(^4\), can be characterised by their frictional behaviour. With CR high friction would also occur during normal operation. As a result of the miniaturisation of disk drives, the spindle motors of the drives have less power and the construction of the arms on which the heads are mounted become less rigid. In order to reduce the torsion at the spindle and to reduce vibrations of the head arms, the friction between head and disk should be as low as possible. If the friction is too high, the acceleration and deceleration of the disk can be affected and eventually stalling of the motor spindle may occur. High friction between head and disk may also lead to deterioration of the head surface.

Smaller HMSs require also smoother and flatter head and disk surfaces. However, smoother surfaces will increase stiction\(^5\). Because of the less powerful spindle motors and less rigid head arms, stiction could lead to stalling of the motor spindle.

Several attempts have been made to reduce the above friction and stiction problems. For example, by using a dynamic load-unload head-arm mechanism, whereby the head arms are lifted (unloading) or lowered (loading) when the drive is stopped or started up. The loading and unloading can be done by using a piezoelectric actuator (see, for example, Hashimoto and Tagawa [51], Tagawa and Hashimoto [134]–[136]) or by moving the head arms up and down along a ramp (Fu and Bogy [34], [35], Jeong and Bogy [59]–[64] and Yamada and Bogy [154]).

As the HMS will be reduced further, fluctuations of the HMS, for example, induced by disk run-out, surface roughness, mechanical vibrations or air flow, become more significant and will increase the incidence of intermittent contact between the head and disk during normal operation. This may lead to damage of the surfaces and eventually to a so called head-crash. A smaller disk would lead to less vibrations and therefore could improve the reliability of the HDI.

In order to reduce the risk of intermittent contact, so called active heads have been developed which allow the head to fly at a relatively ‘save’ HMS and which lower the head onto the disk only when a recording or reproducing operation must be carried out (see Khanna [69]). An example of these active heads is the programmable air bearing head (Khanna and Hendriks [70], [71]) in which the HMS can be adjusted by changing the curvature of the head with the use of piezoelectric materials. In this way, the head can fly high (when idle) or low (when recording or reproducing) on demand. In a similar approach only the transducer

\(^4\)The lubrication regimes will be discussed in Chapter 4.

\(^5\)The term stiction is usually associated with liquid-mediated solid surfaces (see Bhushan [10]), but it has been used in the literature to indicate different phenomena. Both, the large adhesive and the frictional force between the liquid-mediated solid surfaces are called stiction. The difference between them is the direction of the force. In the former case the force is in the direction of the normal to the interface, so that the term stiction refers to adhesion. In the latter case the force is in the direction tangential to the interface, and occurs usually at relatively low velocities. Therefore, stiction in this case refers in fact to the term static friction.
(the part of the head that actual records or reproduces the information) is adjusted by means of a piezoelectric element and the head itself remains at a save distance to the disk (Khanna et al. [72]). However, with the trends to miniaturise the heads, the active heads will become more difficult to realise, because extra piezoelectric elements have to be implemented into the heads.

1.6 Objectives and Scope of the Thesis

From the previous sections it follows that with the continuing move towards CR and miniaturisation of HDIs, AL becomes less important and research efforts should be focused more on friction, wear and the transitions between the different lubrication regimes. Furthermore, current HDIs are optimised for separation rather than for friction.

An impressive amount of papers and articles on friction and wear studies of HDIs have been produced late. However, the emphasis of these studies is mainly on wear reduction, durability of materials or on AL of the HDI. Only a few studies are about the transitions between the different lubrication regimes (see, for example, Benson and Talke [4, 5], Chiang et al. [26], Marchon et al. [88], Trauner et al. [138], and Tseng and Talke [139]). Transitions between lubrication regimes have been studied by Begelinger and de Gee [3], Landheer et al. [78], Schipper [113] and Schipper et al. [114]-[116] for interfaces with ‘conventional’ operational parameters (such as load, initial roughness, velocity, etc.). In this thesis a similar approach will be developed for HDIs, thus for systems with smooth thin films operating at low loads and high velocities. An HDI is ideal for such an investigation of transitions between the different lubrication regimes.

The objective of this study is to investigate the friction of thin film rigid disks such that this could lead to a better understanding of the different lubrication regimes of head-disk interfaces. The central problem of this thesis is how to measure the friction at HDIs and how to determine the transitions between the lubrication regimes of HDIs under similar conditions as those found in RDDs, and such that the regimes can be predicted as a function of the operational parameters of the HDI. A friction model for HDIs will be derived that will be used in analysing the friction and the transitions between the lubrication regimes of thin film rigid disks. Then it will be explained how friction measurements on HDIs can be carried out and how they were conducted in this thesis. The results of these measurements will be compared with those of the friction model.

The study in this thesis has been confined to thin film rigid disks which are made by thin film deposition processes (sputtering and electroless plating). An RDD usually consists of a stack of HDIs. Nevertheless, in this study only single HDIs have been studied.

\[\text{Many of these papers and articles can be found in special annual publications of the Society of Tribologists and Lubrication Engineers (STLE), entitled: 'Tribology and Mechanics of Magnetic Storage Systems’. Another rich source of information is the series, entitled: 'Advances in Information Storage Systems’ (see, for example, Bhushan [11]-[16]).}\]
1.7 Organisation of the Thesis

The organisation of this thesis is centred on the HDI. This chapter (Chapter 1) explained the objectives and scope of the thesis. In Chapter 2 the basic principle of MR will be briefly explained. By means of this principle it will be shown that smaller MSs are required in order to improve the magnetic performance of HMIs; it follows also from this chapter that a mere introduction of smaller MSs has the apparently paradoxical effect of reducing the mechanical performance of the HMI. In Chapter 3, the HDI will be analysed from a tribological point of view, in order to arrive at a model for the HDI. It provides an overview of the main mechanical components of an RDD. The HDI can be regarded as a complex tribological system, which, when in operation, has several lubrication regimes. These lubrication regimes will be discussed in Chapter 4 where a transition diagram of lubrication regimes will be introduced. This diagram will be extrapolated towards lower pressures as are found in HDIs. The validity of this extrapolation will be considered. It will be discussed how to modify or extend the lubrication transition diagram in order to make it applicable to low load systems such as an HDI. Furthermore, a contact model for thin film rigid disks will be given. This model will be used in the development of a friction model for HDIs. In Chapter 5 the instrument that has been made suitable for the friction experiments will be described and analysed. The operating principles of the instrument will be explained and the dynamics of the instrument will be discussed. It will be shown that with the modified instrument it is possible to measure the friction of HDIs. Chapter 6 discusses the results that are obtained from the friction measurements. The agreement between the measured friction and the friction predicted from the friction model will be discussed. Finally, Chapter 7 summarises the results and gives the final conclusions together with some recommendations for further investigations.

The references given in this thesis are alphabetically listed at the end of Chapter 7. For convenience a list of the mathematical symbols used in this thesis is given at pages ix to xxi and a summary of terms and abbreviations is given at pages xxiii to xxiv.
Magnetic Recording and Tribology

MR is a highly interdisciplinary field in which magnetic recording engineers, tribologists and designers should all take part in order to improve the performance of HMIs. In this chapter it will be shown that the magnetic performance and the tribological behaviour of an HMI are interrelated through parameters, such as the magnetic separation, the relative velocity between the head and medium and the thickness of the layers of the medium. When storage capacities and information rates of MSDs have to be increased further, the contributions in the field of tribology of HMIs becomes even more essential.

This chapter starts with a brief explanation of the principle of MR. By means of a few simplified equations for the recording and reproducing process it will then be shown that it is desirable to maintain the head in close proximity to the medium, and to apply thin magnetic layers. This implies that the magnetic performance of an MSD may be affected by mechanical interaction.

2.1 Principle of Magnetic Recording and Reproducing

Basically, the MR process involves the interaction of a magnetisable medium and a transducer (usually called the head). The medium is usually a multilayered structure deposited onto a flexible or rigid substrate, which moves relative to and in close proximity of the transducer (see Fig. 2.1). Recording takes place by sending an electric current \( i(t) \) through the transducer (see Fig. 2.2). The current induces a magnetic field in the core of the transducer. The transducer intensifies, localises and extends this field near the gap \( g \) into the medium surface, thereby magnetising the medium. If the field is strong enough, i.e. if the field exceeds the coercivity\(^1\) of the medium, then the direction of magnetisation of the medium will be locally modified in such a way that it corresponds to the polarisation of the current \( i(t) \). As a result of the relative velocity \( v \) between the transducer and the medium, a pattern of remanent magnetisation regions will be established along the medium which corresponds to the input current \( i(t) \). In practise, the change in direction of magnetisation between two adjacent magnetised regions is never abrupt, but reverses over a certain distance. This distance is called the magnetisation transition length \( \alpha \). It will be shown in the next section that this

\(^1\)The coercivity is the strength of a magnetic field that is necessary to demagnetise a material that is magnetised to saturation. The coercivity is an extrinsic property of a material.
length plays an important role in the recording and reproducing process. The pattern of magnetisation regions and reversals is called a track (shown in Fig. 2.1). A track is always parallel to the direction of relative motion between transducer and medium. Reproducing takes also place by moving the transducer relative to the medium (see Fig. 2.3). The magnetic field of the passing magnetisation regions of the medium extends into the transducer. This field induces a change in voltage $e(t)$ in the transducer, according to changes in the direction of magnetisation of the medium.

For further information about the fundamentals of the magnetic recording and reproducing process the reader is referred to Bertram [7], Hoagland [56], Jorgensen [66], Mallinson [87], Mee and Daniel [91], O’Grady and Laidler [104], and Sharrock [118].

### 2.1.1 Bits of Information

Information can be stored analogously or digitally. With digital MR, the recorded magnetisation regions represent binary information, i.e. a combination of bits\(^2\). During recording the binary information must be encoded. One encoding method is the so called nonreturn to zero inverted (NRZI). With this method a 1 is represented by a magnetisation reversal in the medium and a 0 by the absence of such a reversal. During reproducing information, the output voltage will be decoded into binary form. With NRZI, an output voltage pulse represents a 1 and no output voltage pulse represents a 0. A bit can therefore also represent a

\(^2\)A bit (a contraction of the words binary and digit) is the smallest unit of information in a binary number system. The value of a bit is usually referred to as a zero or a one.
region of the medium. In this thesis a bit will be defined as a magnetised region between adjacent magnetisation reversals\(^3\) and the bit length \(b_t\) is then defined as the distance between two magnetisation reversals. The bit rate \(b_r\), i.e. the speed of recording measured in bits per second, is given by

\[
b_r = \frac{v}{b_t + a},
\]

where \(v\) is the relative velocity between head and medium at the bit and \(a\) is the magnetisation transition length. The area \(A_b\) of a bit is then

\[
A_b = (b_t + a)w = v \frac{w}{b_r},
\]

in which \(w\) is the width of the track corresponding to the bit. The linear bit density \(D_b\) is defined as the number of bits (magnetisation regions) per unit of length of the track, thus

\[
D_b \overset{\text{def}}{=} \frac{1}{b_t + a} = \frac{b_r}{v}.
\]

The track density \(D_t\) is defined as the number of tracks per unit of length

\[
D_t \overset{\text{def}}{=} \frac{1}{w}.
\]

The areal bit density \(D_a\) of a medium is then the product of linear bit density and track density, which becomes

\[
D_a = D_b D_t = \frac{1}{(b_t + a)w} = \frac{1}{A_b} = \frac{b_r}{vw}.
\]

\(^3\)In contrast to this definition, a bit is sometimes defined in the literature as the region of a magnetisation reversal of the medium.
2.1.2 Recording Modes

Two different recording modes may be used to magnetise a medium. In the *longitudinal* (also called *in-plane* or *horizontal*) recording mode the direction of magnetisation of the medium is parallel to the plane of the medium and parallel to the direction of motion of the medium (see Fig. 2.4). In the *perpendicular* (also called *out-of-plane* or *vertical*) recording mode the direction of magnetisation is normal to the plane of the medium and normal to the direction of motion of the medium (see Fig. 2.5). The type of mode applicable in a medium depends amongst other things on the type of material used for the magnetic layer. Certain materials exhibit a longitudinal magnetisation direction, while others show a perpendicular one. The longitudinal recording mode has been applied successfully for many years. Notwithstanding its potential for high density recording (see, for example, Lodder [84]), the application of the perpendicular recording mode is not yet fully under way. This potential is mainly due to the smaller magnetisation transition lengths which can be achieved with perpendicular recording.

When the direction of magnetisation of a small region in a longitudinal recording medium reverses, the pole on each side of the region becomes similar to the pole of its adjacent region (shown at the top of Fig. 2.4) and a demagnetisation field $H_d$ will be introduced. If this field is larger than the coercivity $H_c$ of the medium then the direction of magnetisation will be reversed and part of the information will be lost. Thus, regions with opposite magnetisation directions tend to demagnetise each other and broaden the magnetisation transition length. This *self-demagnetisation* becomes more pronounced when smaller magnetisation regions (this means smaller bits, thus, according to (2.5), a higher linear bit density) are applied. It will be shown in the following sections that the self-demagnetisation effect can be reduced by increasing the coercivity $H_c$ of the

![Fig. 2.4. Longitudinal (in-plane) recording mode.](image1)

![Fig. 2.5. Perpendicular (out-of-plane) recording mode.](image2)
magnetic layer and by reducing the remanent magnetisation $M_r$ or the thickness $t_m$ of the magnetic layer.

When the direction of magnetisation of a small region in a perpendicular recording medium reverses, the pole on each side of the region becomes opposite to the pole of its adjacent region (shown at the top of Fig. 2.5). As the magnetic fields of the regions reinforce each other, the magnetisation transition length becomes narrower and consequently the magnetisation regions can be placed nearer together and thus a higher linear bit density of the medium can be achieved.

2.2 Means to Increase the Storage Capacity

The storage capacity of a medium is determined by its areal dimensions and the areal bit density, given by (2.5). In principle, the storage capacity of a medium could be increased by simply enlarging the areal dimensions of the medium or by improving the areal bit density, i.e. by increasing the linear bit and/or track density. However, medium enlargement is usually not desirable because with a larger medium it takes more time to move the head (in case of a disk) or medium (in case of a tape) to other information areas on the medium, which may result in reduced input and output information rates. Thus, improvements on input and output information rates will be achieved mainly by reducing the dimensions of the medium and increases in storage capacity are expected to continue by increasing the areal bit density. Fig. 2.6 shows the increase in linear bit density and track density of rigid disk drives over the years. Current linear bit densities are in the order of 5000 bits/mm and track densities are about 250 tracks/mm.

From (2.3) it follows that the achievable linear bit density $D_b$ is inversely proportional to the sum of the length of the magnetisation regions (bit length) $b_l$ and the magnetisation transition length $a$. Thus, it depends on the extent to which the magnetisation transition length $a$ can be narrowed. However, the linear bit density is limited by the width of the resulting output voltage pulse of a single magnetisation reversal. When the linear bit density is too high compared to the pulse width, the pulses may interfere with each other during reproducing. Thus, for high linear bit densities it is essential to have narrow pulse widths.

The track density defined by (2.4) may be improved by creating narrower tracks. However, when reducing the width $w$ of the tracks the amplitude of the output voltage reduces more than the noise level and this may lead to a low signal-to-noise ratio (SNR) and eventually may cause reproducing errors. Thus, when improving the track density the amplitude of the output voltage should be increased.

It becomes clear that larger output voltage amplitudes and narrower pulse widths are necessary in order to improve the storage capacity. In the following sections it will be shown how the output voltage and the magnetisation transition length are affected by parameters such as the magnetic layer thickness, gap length, and magnetic separation. This will be done by considering some simple equations for the recording and reproducing processes for both the longitudinal and perpendicular recording mode. In order to estimate the influence of the pa-
Fig. 2.6. The linear bit density $D_b$ and the track density $D_t$ of rigid disk drives, as a function of the year of product shipment. Each dot represents the year of the first shipment of a product. Data from Mee and Daniel (Eds.) [91] and Nashua Co., Santa Clara, USA.

rameters involved, the equations have been made dimensionless by applying a similarity analysis (according to the analysing method given by Moes [98]).

2.3 Longitudinal Recording

For longitudinal recording it can be shown (see, for example, Middleton [92]) that the dimensionless magnetisation transition length $A_X$ is given by

$$A_X = \frac{1}{4} + \sqrt{\left(\frac{1}{4}\right)^2 + \frac{2}{\pi \sqrt{3}} \frac{H + \frac{1}{2}}{H_c}},$$

(2.6)

in which the dimensionless groups are given by

$$A_X \equiv \frac{a_x}{t_m}; \quad H \equiv \frac{h_m}{t_m}; \quad \overline{H_c} \equiv \frac{H_c}{M_r}.$$  

(2.7)

Here, $a_x$ is the magnetisation transition length parameter for longitudinal recording, $t_m$ is the thickness of the magnetic layer, $h_m$ is the magnetic separation, i.e. the distance between the head and the magnetic layer, $H_c$ is the coercivity of the medium, and $M_r$ is the remanent magnetisation in the medium. This equation shows that in order to achieve small magnetisation transition lengths, and, thus, high linear bit densities, the ratio $M_r/H_c$, the medium thickness $t_m$, and the magnetic separation $h_m$, should be as small as possible. The magnetisation transition length is proportional to the square root of the magnetic separation.
2.4 Perpendicular Recording

It can be shown (see, for example, Middleton [92]) that the dimensionless magnetisation transition length \( A_Y \) for perpendicular recording is given by

\[
A_Y = \frac{c}{\pi} \left( H + \frac{1}{2} \right) - \frac{1}{2},
\]

(2.8)

in which \( H \) is given by (2.7) and \( A_Y \) by

\[
A_Y \equiv \frac{a_y}{t_m},
\]

(2.9)

where \( c \) is a constant depending on the type of head used and \( a_y \) is the magnetisation transition length parameter for perpendicular recording. It follows from (2.8) that the magnetisation transition length \( a_y \) is independent of the remanent magnetisation and coercivity of the medium and that it is mainly determined by the magnetic separation and thickness of the magnetic layer. The magnetisation transition length is directly proportional to the magnetic separation, thus perpendicular recording is much more sensitive to variations in magnetic separation than is longitudinal recording. In general, it can be concluded from (2.6) and (2.8) that reduced magnetic separations would lead to sharper transitions.

2.5 Reproducing a Longitudinally Recorded Signal

Analytic equations for the output voltage \( e_x(\pi) \) (in which \( \pi = vt \)) of a longitudinally recorded signal detected by a ring head, have been derived for example by Middleton and Davies [93], Middleton [92], Potter [110], Speliotis [125], Speiliotis and Morrison [126], Wallace [144] and Westmijze [145]. In this section two equations — one for analogue and one for digital reproducing — will be discussed.

2.5.1 Analogue Reproducing

Assuming a particulate medium at which a signal has been recorded longitudinally by a sinusoidal variation of the magnetisation, Wallace [144] and Westmijze [145] derived an equation for the output voltage \( e_x(\pi) \) of a ring head. Here, this equation is presented in a dimensionless form as follows

\[
E_X(\pi) = -\eta_e \left( \frac{2\pi}{K} \right) L_H L_T L_g \cos \left( \frac{2\pi \pi}{K} \right),
\]

(2.10)

in which the separation loss \( L_H \), the thickness loss \( L_T \), and the gap loss \( L_g \), are given by

\[
L_H = e^{-2\pi H/K}; \quad L_T = \frac{1 - e^{-2\pi/K}}{2\pi/K}; \quad L_g = \frac{\sin(\pi G/K)}{\pi G/K}.
\]

(2.11)
The dimensionless parameters occurring in these equations are given by (2.7) and by

\[
E_X = \frac{e_x(\bar{x})}{n \mu_0 \pi w M_r}; \quad X = \frac{\bar{x}}{t_m};
\]

\[
\eta_e = \frac{1}{1 + \frac{A_c \ell_c}{\mu_r g A_c}}; \quad K = \frac{\lambda}{t_m}; \quad G = \frac{g}{t_m}.
\]

(2.12)

Here, \(e_x(\bar{x})\) is the output voltage at the head coil, \(\lambda\) is the wavelength of the sinusoidally recorded signal, \(n\) is the number of windings of the head coil, \(g\) is the gap length of the head, \(\ell_c\) is the mean length of the core (shown in Fig. 2.1), \(A_c\) is the mean area of a cross section of the core, \(A_g\) is the area of a cross section of the core at the gap, \(\mu_0\) is the magnetic permeability of vacuum, \(\mu_r\) is the relative magnetic permeability of the core material, \(\bar{x} = vt\), \(v\) is the relative velocity between head and medium, \(w\) is the track width, \(M_r\) is the remanent magnetisation in the medium and \(\eta_e\) is the so called head efficiency parameter. In a well designed head, \(A_g \ell_c / (\mu_r A_c) \ll 1\) (see, for example, Walker [143]), and \(\eta_e = 0.70\) to 0.80.

The loss in amplitude of the output voltage, due to the magnetic separation is given by

\[
\overline{E}_X \equiv \frac{E_X(X)}{(E_X(X))_{H=0}} = L_H = e^{-2 \pi H/K}.
\]

(2.13)

![Graph](image)

**Fig. 2.7.** The separation loss \(L_H\) and the thickness loss \(L_T\) as a function of the dimensionless separation \(H/K\) and the dimensionless thickness \(1/K\), respectively, of an analogously reproduced longitudinally recorded signal.
This can be expressed in decibels as follows

\[
20 \log E_X = 20 \log \left( e^{-2\pi H/K} \right) = -54.6 H/K = -54.6 \frac{h_m}{\lambda} \text{dB}.
\]  

(2.14)

A similar equation can be derived for the thickness loss \( L_T \). Figure 2.7 shows the separation loss \( L_H \) and the thickness loss \( L_T \) as a function of the dimensionless separation \( H/K \) and the dimensionless thickness \( 1/K \), respectively. From (2.10) and (2.14) and Fig. 2.7 it follows that the amplitude of the output voltage \( e_\alpha(\bar{r}) \) is directly proportional to the track width \( w \) and that \( M_r \) falls off exponentially with the magnetic separation \( h_m \) and the thickness of the magnetic layer \( t_m \) and increases exponentially with decreasing wavelength \( \lambda \). Thus the amplitude reduces as either the track density \( D \) or the linear bit density \( D_b \) increases. In section 2.1.2 it was mentioned that self-demagnetisation can be reduced by decreasing the thickness \( t_m \), by increasing the coercivity \( H_c \) of the magnetic layer and by reducing the remanent magnetisation \( M_r \). Thus, when the linear bit density and/or the track density are to be increased, the loss in amplitude should be compensated for by decreasing the magnetic separation \( h_m \) and the thickness \( t_m \) of the magnetic layer and by increasing the coercivity \( H_c \) of the magnetic layer.

2.5.2 Digital Reproducing

Middleton and Davies [93], Middleton [92], Potter [110], Speliotis [125] and Speliotis and Morrison [126] derived an analytic equation for the output voltage \( e_\alpha(\bar{r}) \) due to a single magnetisation reversal of a longitudinal recorded signal, detected by a ring head. The coordinates used in this equation are shown in Fig. 2.3. In these studies, the track width \( w \) was considered much larger than the other dimensions of the HMI. Here, the equation is written in dimensionless form as follows

\[
E_X(\bar{X}) = -\frac{1}{\pi G} \left( \frac{H_x}{2} \right) \left( \arctan \frac{\frac{1}{2} G + \bar{X}}{H_x + 1} + \arctan \frac{\frac{1}{2} G - \bar{X}}{H_x + 1} \right)
\]

\[
- 2H_x \left( \arctan \frac{\frac{1}{2} G + \bar{X}}{H_x} + \arctan \frac{\frac{1}{2} G - \bar{X}}{H_x} \right)
\]

\[
+ \left( \frac{1}{2} G + \bar{X} \right) \ln \frac{(H_x + 1)^2 + \left( \frac{1}{2} G + \bar{X} \right)^2}{H_x^2 + \left( \frac{1}{2} G + \bar{X} \right)^2}
\]

\[
+ \left( \frac{1}{2} G - \bar{X} \right) \ln \frac{(H_x + 1)^2 + \left( \frac{1}{2} G - \bar{X} \right)^2}{H_x^2 + \left( \frac{1}{2} G - \bar{X} \right)^2}
\],

(2.15)

in which the dimensionless separation \( H_x \) is given by

\[
H_x \equiv A_X \left( X \right) + H = \frac{a_\alpha + h_m}{t_m},
\]

(2.16)

and with \( A_X \) and \( H \) defined in (2.7) and \( E_X, \bar{X}, \) and \( \eta_k \) given by (2.12). Figure 2.8 shows that the dimensionless output voltage \( E_X(\bar{X}) \) represents a pulse shape.
Fig. 2.8. Dimensionless output voltage $|E_X(\bar{X})|$, due to a single magnetisation reversal of a longitudinally recorded signal ($H_X = 1.2$, $G = 2.0$ and $\eta_e = 0.80$).

Fig. 2.9. Pulse amplitude loss $E_X$ and dimensionless pulse width $P_{50}$ as a function of dimensionless magnetic separation $H_X$ of a digitally reproduced longitudinally recorded signal.
The width of the pulse can be defined for example as the width at 50% or 25% of the pulse amplitude, and it can be obtained by solving $\mathbf{X}_{50}$ and $\mathbf{X}_{25}$ from the equations

$$\frac{1}{2}E_X(0) = E_X(\mathbf{X}_{50}) \quad \text{and} \quad \frac{1}{4}E_X(0) = E_X(\mathbf{X}_{25}),$$

respectively. The pulse widths are then

$$P_{50} = 2|\mathbf{X}_{50}| \quad \text{and} \quad P_{25} = 2|\mathbf{X}_{25}|,$$

respectively. Unfortunately, no analytic solution exists for these equations. For this reason (2.17) has been solved numerically for some values of $H_X$ and $G$. Figure 2.9 shows the loss in amplitude

$$\mathbf{E}_X = \frac{E_X(0)}{(E_X(0))_{H=0}},$$

in decibels of the dimensionless pulse and the dimensionless pulse width $P_{50}$ as a function of the dimensionless separation $H_X$. From (2.15) and Fig. 2.9 it follows that the pulse amplitude is directly proportional to the track width $w$ and to the remanent magnetisation $M_r$. Furthermore, the pulse amplitude decreases and the pulse width broadens exponentially with the dimensionless separation $H_X$. Hence, when the linear bit density and/or the track density are to be increased, the loss in pulse amplitude and the increase in pulse width should be compensated for by decreasing the magnetic separation $h_m$. With current linear bit densities, $D_b \approx 5000$ bits/mm, magnetic layer thicknesses, $t_m \approx 50$ nm, gap lengths, $g \approx 0.1 \mu m$, and magnetic separations $h_m \approx 100$ nm, the dimensionless parameters become: $A_X = 4.0$, $H = 2.0$ and $H_X = 6$. Hence, from Fig. 2.9 it follows that the linear bit densities can be improved mainly by further reduction of the transition length $a_x$. According to (2.6) this can be achieved mainly by reducing the magnetic separation $h_m$.

### 2.6 Reproducing a Perpendicularly Recorded Signal

In a similar way as for longitudinal reproducing, Middleton [92] derived the output voltage $e_y(\mathbf{X})$ for reproducing from a perpendicular medium. Here, the output voltage is written in dimensionless form as follows

$$E_Y(\mathbf{X}) = -\frac{1}{\pi} \eta e \left( H_Y + 1 \right) \ln \left( \frac{(H_Y + 1)^2}{(H_Y + 1)^2 + \left( \frac{1}{2} G + \mathbf{X} \right)^2} \right)$$

$$- H_Y \ln \frac{H_Y^2 + \left( \frac{1}{2} G + \mathbf{X} \right)^2}{H_Y^2 + \left( \frac{1}{2} G - \mathbf{X} \right)^2}$$

$$+ 2 \left( \frac{1}{2} G + \mathbf{X} \right) \left( \arctan \frac{H_Y + 1}{\frac{1}{2} G + \mathbf{X}} - \arctan \frac{H_Y}{\frac{1}{2} G + \mathbf{X}} \right)$$

$$+ 2 \left( \frac{1}{2} G - \mathbf{X} \right) \left( \arctan \frac{H_Y}{\frac{1}{2} G - \mathbf{X}} - \arctan \frac{H_Y + 1}{\frac{1}{2} G - \mathbf{X}} \right).$$
where the dimensionless separation $H_Y$ is defined as

$$H_Y \equiv A_Y + H = \frac{a_y + h_m}{t_m}, \quad (2.21)$$

and in which the dimensionless parameters, $E_Y$, $A_Y$, $\bar{X}$, $H$ and $\eta_e$ are given by (2.7), (2.9), (2.12) and by

$$E_Y \equiv \frac{e_y(\tau)}{n\mu_0 v w M_i}. \quad (2.22)$$

Figure 2.10 shows the shape of the dimensionless output voltage $E_Y(\bar{X})$. As $E_Y(0) \equiv 0$, the pulse width can not simply be obtained as in case of longitudinal recording. However, the distance $P_{p-p}$ between two peaks in the output voltage can be considered as the pulse width, thus

$$P_{p-p} = 2|\bar{X}_p|. \quad (2.23)$$

In Fig. 2.11 the pulse amplitude loss

$$\bar{E}_Y \equiv \frac{E_Y(\bar{X}_p)}{(E_Y(\bar{X}_p))_{H=0}}, \quad (2.24)$$

and the pulse width $P_{p-p}$ are plotted as a function of the dimensionless magnetic separation $H$. The resulting curves appear to be very similar to those of Fig. 2.9 for longitudinal reproducing and similar conclusions can be drawn as for longitudinal recording, i.e. the magnetic separation $h_m$ should be small compared to the thickness $t_m$ of the magnetic layer, in order to compensate for the reduction in pulse amplitude and the increase in pulse width when the linear bit density and/or the track density are increased.

### 2.7 Summary

In this chapter it has been shown that the shortest magnetisation transition length, i.e. the smallest size of the magnetisation reversals which can be detected, depends strongly on the MS, i.e. the distance between the transducer and the magnetic layer. The smaller the MS, the smaller the magnetisation reversals which can be reproduced. Therefore, information regions, with higher linear bit densities, can be achieved by reducing the MS.

Furthermore, it has been shown that the amplitude and the width of the output voltage pulse of longitudinally and perpendicularly recorded signals are very susceptible to the magnetisation transition length and the MS. The pulse amplitude decreases and the pulse width increases exponentially with the MS and the magnetisation transition length. When track densities are to be increased, the pulse amplitude reduces to a greater extent than the noise level and this may lead to a low SNR. This reduction in pulse amplitude may be compensated for by reducing the magnetisation transition lengths and the MS. With current
Fig. 2.10. Dimensionless output voltage $E_Y(X)$, due to a single magnetisation reversal of a perpendicularly recorded signal ($H_Y = 1.2$, $G = 2.0$ and $n_e = 0.80$).

Fig. 2.11. Pulse amplitude loss $E_Y$ and dimensionless pulse width $P_{p-p}$ as a function of dimensionless magnetic separation $H$ of a digitally reproduced perpendicularly recorded signal.
magnetic media, reducing the magnetisation transition lengths will have the most effect on the output voltage.

The dependency of the output voltage and the magnetisation transition length on the MS leads to the importance of tribology in HMs. With regard to the magnetic performance it may become clear from this chapter that it is important to make the MSs as small as possible. With the areal bit densities of today, the MS becomes critical in such a way that the magnetic performance and the tribological behaviour of an MSD is determined by parameters such as the relative velocity between the head and medium, the roughness of the head and medium and the thickness of the layers of the medium. Because of the requirements as to the thinness of the magnetic layer and the smallness of the MS, the medium needs protection against high friction and wear. With thin film media, this can be done by applying an overcoat and a lubricant layer on top of the magnetic layer. The MS then becomes (see Fig. 2.12),

\[ h_m = t_o + t_l + h, \]

in which \( t_o \) is the thickness of the overcoat, \( t_l \) is the thickness of the lubricant layer and \( h \) is the HMS (at the location of the transducer). Due to the required small MS only thin overcoats and lubricant layers can be applied and head and medium should be as smooth as possible.

In the following chapter the HDI will be analysed and a model of the HDI will be presented.

![Diagram](image)

**Fig. 2.12.** The relation between the head-medium separation \( h \) and the magnetic separation \( h_m \) (with \( h_m = t_o + t_l + h \)).
A Model of the Head-Disk Interface

This chapter will be concerned with a model of the HDI of an RDD. The model will be used for analysing the friction at the HDI. To introduce this model, it is first necessary to describe the main mechanical components and features of an RDD. It will then be shown that the HDI can be regarded as a complex tribological system which is subjected to friction. In general, friction is a characteristic of the system rather than of the materials (Czichos [31]). Therefore, the entire HDI system will be considered. The elements and the operational parameters of the HDI (those parameters of the environment acting on the system) will be discussed.

3.1 Rigid Disk Drives

In this section the main mechanical components of an RDD and the motions during recording and reproducing will be discussed together with the trends in the RDD industry. It will be shown that in the move towards higher storage capacities the HDIs become more and more critical components of an RDD.

3.1.1 Main Mechanical Components

Modern RDD designs are based on the so called Winchester technology, first introduced by IBM in 1973 (see, for example, Harker et al. [50]). Fig. 3.1 shows an example of this design. With this design, an RDD consists of a stack of co-rotating rigid disks on a common rotating spindle which is driven by a brushless dc drive motor. Each disk is made of a rigid, smooth substrate on which several layers are deposited. One of the layers is magnetisable and is used for the storage of information, according to the principles described in Chapter 2 (Section 2.1). The information is stored in the form of remanent magnetisation regions and reversals on narrow concentric rings, usually referred to as tracks.

Each disk surface has an electromagnetic transducer for recording and reproducing information at the tracks. According to Chapter 2 the transducers should be in close proximity to their corresponding disk surface in order to accomplish proper recording and reproducing. Therefore, each transducer is embedded into a slider. With conventional RDDs, the sliders have the form of a sledge, consisting of two flat rails. Both rails have a taper at their leading edge (taper-flat sliders).
The actuator is embedded near the trailing edge of a rail. The combination of transducer and slider will be called the head in this thesis. During normal operation of the RDD, the rotation of the stack of disks generate a thin lubricating air film between each slider and disk surface. The pressure in the air film causes each slider to fly above its corresponding disk surface; slider, air film and rotating disk surface form a self-acting air lubricated slider bearing. By means of this slider bearing the separation between the head and its corresponding disk surface is maintained as small and constant as possible during normal operation. Each time the RDD is powered on or off, the heads take off from or touch down on the surfaces of their corresponding disks.

Each head is connected to the end of a miniaturised stainless steel suspension. There are a number of suspension types in use, depending on the applied actuator and stack of head-arm assemblies (to be discussed in the paragraph below). In the experimental work of this thesis, the so called in-line swayable type suspension has been used which is shown in Fig. 3.2. The suspension is attached to a mounting base by means of weld spots. The suspension has a formed area near the mounting base which is flexible in a direction perpendicular to the disk surface but rigid in all directions in the plane of the disk surface. In this way, the head can freely follow the variations of the disk surface. These variations cannot be eliminated and are introduced, for example, by the waviness of the disk, disk flutter, vibrations in the spindle bearing, and vertical run-out. The vertical run-out can be in the order of micrometres. By reducing the deflection angle \( \alpha \) of the suspension, a preload force — resulting from the spring force of the formed area — can be exerted.
onto the head. For conventional heads the preload force is below 100 mN. During normal operation, this preload force should be in equilibrium with the pressure in the air film between the slider and the disk in order to minimise fluctuations in the air film thickness. The suspension is designed in such a way that its resonant frequencies are much lower than that of the slider bearing. Figure 3.3 shows how a flexure has been used for the attachment of the head onto the suspension. In principle, the flexure is a small leaf spring consisting of two parts. Part 1 is spot welded to the suspension. By using an epoxy adhesive the head is glued onto part 2 of the flexure. This part contains a hemispherical pivot knob which rests against the bottom of the suspension and separates the head from the suspension. The knob transmits the preload force to the head. In this way, the freedom of motion of the head in pitch $\theta$ and roll $\phi$ direction, is provided by the flexure and the head can move freely around the pivot point while it can resist forces in the plane of the disk surface. Each suspension is mounted onto an arm. Head and arm form the so called head-arm assembly (HAA).

The stack of HAAs are attached to a common actuator arm which is driven by an actuator. In this way the heads can be positioned at the required tracks and can follow the recorded information along the tracks. In large form factor RDDs, the actuator may be driven by an electromagnetic voice-coil motor (with a structure similar to that of a moving coil loudspeaker). In small form factor RDDs, the actuator is usually driven by a simple stepper motor and a steel band. Actuators can be linearly or rotary. With linear actuators the HAAs move in the plane of the disk surface and in radial direction to the desired track (see Fig. 3.4). With rotary actuators the HAAs rotate around an axis perpendicular

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig32.png}
\caption{In-line oriented, swayable type head-arm assembly.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig33.png}
\caption{The mounting of the head onto the suspension.}
\end{figure}
to the plane of the disk surface (see Fig. 3.5). Rotary actuators have usually less mass and require less space in the drive, making them more useful in small RDDs than linear actuators.

The stack of rotating disks, the actuators and the HAAs are in a sealed enclosure and form, together with the system for contamination control, the so called head-disk assembly (HDA). In small form factor RDDs controlling of the contamination is simply done by an air filter in front of the air inlet and the drive is usually equipped with desiccators to reduce local humidity. In large form factor RDDs a more complex system of contamination control is used and, in addition, the HDA is isolated from the external supporting structure by compliant shock mounts.

### 3.1.2 Head Motions During Starting, Steady Operation and Stopping

A conventional RDD has three different dynamic operational modes, which are: starting (acceleration of the stack of disks), steady (constant speed) and stopping (deceleration of the stack of disks). During these operational modes the motion of the heads can be in the plane of their disks, perpendicular to their disk surfaces and in pitch and roll directions. In this section the different motions of the heads will be discussed.

#### 3.1.2.1 Track-Accessing and Track-Following

In general, two motions are required by the actuator and HAAs during recording and reproducing.

The first motion is a rapid track-accessing (seeking) motion of the actuator and stack of HAAs across the disk surfaces in order to access information from a different track than the present track. During this motion of the stack of HAAs the heads move in the plane of the disks. During track-accessing, by rotary actuators, the orientation of the heads changes continuously with respect to the disk radius.
The angle $\gamma$ between the direction of the tangential velocity of the disk and the longitudinal direction of the head is called the skew or yaw angle. Rotary actuators have a skew angle of maximum 12 to 13 degrees. The velocity of track-accessing is important because it affects the access time. The access time is the time required for the heads to find a track and to begin reproducing and recording information. The access time should be as short as possible in order to acquire fast input and output information rates.

The second motion is a track-following motion for maintaining the steady-state radial position of the heads. Track-accessing involves larger motions of the heads than track-following, depending on the diameters of the disks. The accuracy of track-accessing and track-following is important because it affects the maximum attainable track density. The acceleration during track-accessing may be several hundred metres per second, and the accuracy of accessing is on the order of submicrometres.

In the past, on small RDDs at lower track densities (approximately 10 tracks/mm) track-accessing and track-following could be realised generally without using a servomechanism. However, in practice, the tracks are not perfectly centred, due to mechanical inaccuracy, and can be slightly noncircular, due to differential thermal expansion. In addition, high frequency mechanical vibration of the HAAs, spindle bearing run-out, and turbulent air flow within the drive enclosure generate nonrepeatable disturbances to the actuator and the HAAs. Therefore, on current RDDs at larger densities (above approximately 20 tracks/mm) a servomechanism is needed to realise accurate track-accessing and track-following. This servomechanism determines the position of the actuator, compares the measured position against the desired position and determines the best control signal to reduce the position error to as close to zero as is practical. The positional information of the tracks is usually stored onto the disks.

### 3.1.2.2 Take-off and Touch-down.

In Chapter 1, Section 1.4 it has been explained that during acceleration and deceleration, when the relative velocity between the head and the disk is below the TOV or below the TDV, the head is in sliding contact with the disk. Although the acceleration and deceleration periods of conventional RDDs are only a few seconds, the sliding action and the thinness of the magnetic layers of the disks make that the HDIs need protection against friction. In order to obviate the risk of surface damage, and, consequently, loss of information due to the sliding action, each head is automatically redirected, prior to deceleration, to a certain region on the disk surface, that is free of information — the so called landing zone. In a similar way, each head takes off from the landing zone, during acceleration, and after the final velocity has been reached, the heads are directed to the desired tracks. The landing zone is usually located near the inner (ID) or outer diameter (OD) of a disk. When the disks rotate at constant final speed the heads fly above their disks between the ID and OD. Heads and disks will be separated in this situation. The region of the disk where the head flies above the disk and where the data is stored will be called the data zone. It is a mistake, to think that the HDI does not need protection in
the data zone, because at any moment during normal operation, there is the risk of intermittent contact between the head and the disk, due to fluctuations of the HMS.

3.1.3 Trends in the Rigid Disk Drive Industry

The trends in the RDD industry are to produce more compact RDDs with larger storage capacities and improved input and output information rates (see Chapter 1, Section 1.5). In Chapter 2, Section 2.2, it has been argued that improvements on input and output information rates will be mainly achieved by reducing the diameters of the disks while increases in storage capacities are expected to continue by increasing the areal bit densities (not by increasing the diameter of the disks or the number of disks). Now the main components of an RDD have been discussed, this section elaborates upon these trends (see also Fig. 1.3, at page 10).

Smaller disk diameters may result in shorter access times, because they shorten the average distance travelled by the heads. Shorter access times result in faster input and output information rates. In addition, since the amplitudes of vibrations are proportional to the disk diameter, smaller disk diameters reduce vibrations in the HDA. From the previous section it follows that less vibrations make higher track densities possible, and, according to (2.5) in Chapter 2, enable therefore higher areal bit densities. With rotary actuators, reduced disk diameters result also in smaller skew angles, which, in turn, result in a more uniform air film thickness across the disk surface. If AL between sliders and disks has to be maintained, and, at the same time, if the disks become smaller, then higher disk speeds (see footnote on page 4) are required. Furthermore, higher disk speeds reduce the access time and according to (2.1) the bit rate. Higher speeds are also required in order to maintain an acceptable SNR at smaller disks (see Chapter 2).

Areal bit densities of RDDs have been increased by improvements in the AL of the slider bearings, the suspensions and the magnetic layers as well as the transducers. In Chapter 2, Section 2.2, it was mentioned that it is important to have MSs as small as possible in order to enable high areal bit densities. Therefore, the HMS decreased continuously in the past and will so in future. This has been done by reducing the load bearing capacity of the slider, by improving the geometry of the sliders and by reducing their mass and preload force. Ultimately, RDDs will be build which utilise CR. Besides the potential improvements on areal bit densities, application of CR eliminates also the effects of skew on the nonuniformity of the air film thickness. However, as a consequence of CR, the friction at the HDI becomes an important issue.

3.2 Head-Disk Interface

The focus in this thesis is on friction at HDIs of RDDs. In order to arrive at a model of the HDI for studying the friction at the HDI, the HDI will be analysed in
more detail in this section. The HDI has been defined in Chapter 1, Definition 1.3. The head (slider) and the disk are therefore the main elements of an HDI to be considered. In this section the features and materials of these elements will be described. The discussion will be based on conventional HDIs, i.e. HDIs which do not utilise CR.

3.2.1 Sliders

In Chapter 2 it has been shown that the magnetic separation, and thus the HMS, strongly affects the amplitude and the width of the output voltage pulse. Therefore, the HMS should be as constant as possible in order to obtain uniform pulses. The function of the slider of the HDI is to maintain the transducer at a constant and as small as possible HMS by means of a lubricating air film. This means that a thin air film, a high stiffness, a low load capacity and a low TOV are the most important requirements of the slider bearing.

For the generation of an air film between a slider and a disk it is necessary to have a converging wedge-shaped slider (see, for example, Reynolds [112]) and a sufficient sum velocity between slider and disk. Therefore, with conventional RDDs, the sliders are in the form of a sledge with two or more rails. The wedge-shape is obtained by a taper at the leading edge of each rail (the so called taper-flat sliders). The first taper-flat sliders had three rails and were introduced in 1973. Nowadays, mostly sliders with two rails are used. The length, width and

![Fig. 3.6. Geometry of a standard two rail taper-flat slider made of CaTiO₃.](image-url)
Table 3.1. Dimensions of a standard two rail taper-flat slider made of CaTiO$_3$.

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_s \times B_s \times H_s$</td>
<td>$4.3 \times 3.2 \times 0.84$ mm</td>
</tr>
<tr>
<td>$(x_p, y_p)$</td>
<td>$(2.2,1.5)$ mm</td>
</tr>
<tr>
<td>$(x_c, y_c, z_c)$</td>
<td>$(2.0,1.5,0.75)$ mm</td>
</tr>
<tr>
<td>$\ell_t$</td>
<td>$366$ $\mu$m</td>
</tr>
<tr>
<td>$a_t$</td>
<td>$14$ mrad</td>
</tr>
<tr>
<td>$\ell_r$</td>
<td>$4.126$ mm</td>
</tr>
<tr>
<td>$w_r$</td>
<td>$500$ $\mu$m</td>
</tr>
<tr>
<td>$h_r$</td>
<td>$400$ $\mu$m</td>
</tr>
<tr>
<td>$(\theta, \phi)$</td>
<td>$(&lt;100, &lt;20)$ $\mu$rad</td>
</tr>
<tr>
<td>$h_{\text{min}}$</td>
<td>$&lt;200$ nm</td>
</tr>
</tbody>
</table>

Table 3.2. Physical properties of a standard two rail taper-flat slider made of CaTiO$_3$.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>$3950$ kg/m$^3$</td>
</tr>
<tr>
<td>Elastic modulus, $E_t$</td>
<td>$110$ GPa</td>
</tr>
<tr>
<td>Hardness</td>
<td>$9.3$ GPa</td>
</tr>
<tr>
<td>Poisson ratio, $\nu_t$</td>
<td>$0.28$</td>
</tr>
</tbody>
</table>

height of a standard two rail taper-flat slider are 4.3, 3.2 and 0.84 mm respectively. In order to reduce the air film thickness further, the size of this type of sliders has been reduced to 70 and 50 percent in recent years (see, for example, Kumaran et al. [77]) and, simultaneously, the load at the slider has been reduced from 95 mN to 62 mN and below. Furthermore, in the literature all sorts of geometrical slider shapes have been proposed which differ from the standard two rail taper-flat slider, for example, a dual slider assembly (Wickert et al. [152]), active (programmable) sliders (Khamm [70]), the subambient pressure slider (Cha and Bogy [23]), the shaped rail slider (Peng and Hardie [107]), the zero-load slider (White [146], [147]), the traverse pressurised contour slider (TPC-slider, Cha and Bogy [23], White [150]), a micromachined dual slider (Wickert et al. [152]), and, recently, a tri-pad slider (see, for example, Stenberg et al. [128]) and a new design (the ‘tango’ slider) has been proposed by Hendriks [53]. Figure 3.6 shows the geometry of a standard two rail taper-flat slider, which has been used in the experimental work of this thesis. The dimensions of this type of slider are shown in Table 3.1 and some of its physical properties are given in Table 3.2.

A slider has three degrees of freedom of motion, given by the $z$ coordinate of the pivot point, together with the pitch and roll angles $\theta$ and $\phi$. The flying height $h_{\text{min}}$ is defined as the distance between the transducer and the disk surface at the trailing edge of the rails. The flying height $h_{\text{min}}$ of a standard two rail taper-flat slider is usually below 200 nm. The spacing function $h(x, y, t)$ between disk and slider bearing surface depends upon the geometry of the slider, the flying height $h_{\text{min}}$ and the slider pitch and roll angles, $\theta$ and $\phi$. The combination of steady-state flying height $h_{\text{min}}$, pitch and roll angles $(h_{\text{min}}, \theta, \phi)$ is called the flying attitude of the slider. In order to reduce the possibility of contact between the slider and the disk surface, the pitch angle may not be too small. The pitch angle is usually increased by slightly offsetting the pivot point towards the trailing edge of the rails.
The air pressure between the slider and the disk can be calculated by the time-dependent, compressible, modified Reynolds equation (see, for example, Mitsuya [94], Mitsuya et al. [96], [97] and White [148], [149]). To solve this equation for the air pressure, the spacing function \( h(x, y, t) \), and thus, the flying attitude is needed. The flying attitude of the slider is determined by the equations of motion of the slider, which can be solved for a certain slider and suspension when the air pressure is known. Therefore, the modified Reynolds equations and the equations of motion of the slider should be solved simultaneously in order to find the flying attitude of the slider.

In the literature a large number of studies can be found which analyse and solve these equations numerically (for example, Berg and Buettner [6], Crone et al. [29], [30], Kumaran et al. [77], Smith et al. [122], [123], Wahl et al. [142], and White [148]). From these studies and also from experimental work (see, for example, Jiaa and Eltoukhy [65]) it follows that there are a number of physical parameters which influence the flying attitude of a slider. For example, asymmetries, such as rails of different widths \( w \), or the suspension pivot point \((x_p, y_p)\), offset from centre, will increase the coupling of the roll motion with pitch and flying height motions. Furthermore, the skew angle \( \gamma \) distorts the air pressure distribution in such a way that it decreases the flying height and increases the roll angle (White [148]). Large roll angles \( \phi \) result in less air bearing stiffness and reduced air bearing natural frequencies, making the flying height more sensitive to both static and dynamic effects. Therefore, the roll angle should be kept as close to zero as possible. Some of these parameters have been used to reduce the load bearing capacity of the slider. For example, reducing the slider rail width \( w_r \) is often used to reduce the flying height \( h_{\text{min}} \) of the slider. However, reducing the rail width also lowers the pitch angle \( \theta \). In addition, Jiaa and Eltoukhy [65] found that the geometry of the rails, such as the crown, the camber, and the twist angle, vary statistically from their design values and affects the flying attitude of the slider. Therefore, the flying attitude will also have a statistical distribution.

It is, therefore, important to know the geometry of the rails of the slider. The geometry and the surface topography of the sliders used in this thesis are all measured by an optical phase shift interferometry profiler\(^1\). An example of the shape of a rail of a standard two rail taper-flat slider used in this thesis is given in Fig. 3.7. It has been found that near the leading and trailing edges the rails are not perfectly flat. Figure 3.8 illustrates the surface topography of part of this rail. Figure 3.9 shows that the surface heights of the rails of a slider are Gaussian distributed.

### 3.2.2 Rigid Disks

In Chapter 1 it has been mentioned that media can be particulate or thin film. In Chapter 2 it has been shown that the magnetic layer should be thin and

\(^1\)The optical phase shift interferometry profiler measures the topography of a surface and therefore, all roughness parameters given in this thesis, such as the CLA roughness \( R_a \) are surface statistical parameters and are not based on a single profile.
Fig. 3.7. The shape of a rail of a standard two rail taper-flat slider, measured by an optical phase shift interferometry profiler along the dashed line. The $x$ axis represents the position at the rail along this line and the $z$ axis represents the surface height. The surface topography at the location indicated by A is given in Fig. 3.8.

Fig. 3.8. The surface topography of a rail of a standard two rail taper-flat slider, measured by an optical phase shift interferometry profiler at point A in Fig. 3.7. (Measurement area: 807×62 μm, $R_a = 1.1$ nm.)
should possess a high permanent magnetisation and coercivity in order to realise a small transition length and therefore a high linear bit density. These requirements can be fulfilled very well by using thin films instead of particulates. Nowadays, therefore, thin film rigid disks are widely employed. The rigid disks, used in the experimental work of this thesis, are all thin film rigid disks and, therefore, particulate rigid disks will not be discussed further.

An HDI is liable to frictional processes, therefore, the magnetic layer needs protection against mechanical (and also chemical) deterioration. In order to reduce friction and wear at the HDI, the magnetic layer of thin film rigid disks is protected by a thin overcoat and the HDI is usually lubricated by a liquid lubricant layer at the disk surface. An undercoat is usually deposited between the rigid substrate and the magnetic layer. Figure 3.10 shows a cross section of a thin film rigid disk used in the experimental work of this thesis. In the remaining of this section, the substrate, the layers, the lubricant and the surface topography of rigid disks will be briefly discussed.

**Substrates.** During starting up, the acceleration of the disks should be as high as possible. Therefore the mass and inertia of rigid disks are kept as low as possible by choosing light and thin substrates, for example an aluminium-magnesium (AlMg) alloy. Recently, also chemically strengthened glass and ceramic substrates are used, mainly because of their smoothness, hardness and stiffness, see, for example, Fujii et al. [36], Horikawa et al. [57], Kogure et al. [74], Li et al. [83], and Suzuki et al. [133]. A harder and stiffer substrate requires less
thickness. The experiments in this thesis are mainly done on rigid disks with an A1Mg alloy based substrate and with an OD of 63.5 mm.

**Undercoats.** Substrates of A1Mg alloys have inadequate hardness. Furthermore, the layers in thin film disks are usually very thin (less than 100 nm thickness) and consequently their contribution to the substrate hardness is inadequate. Furthermore, their adherence directly onto the substrate is usually insufficient. Therefore, substrates of A1Mg alloys are provided with a relative thick undercoat to improve the adherence of the thin films and increase the hardness. The relative thick undercoats are usually polished away to a certain thickness and then textured before the magnetic layer(s) and overcoat are applied, because otherwise the topography of the substrate may be replicated during subsequent depositions of the upper layers. The texturing is carried out in order to reduce stiction between the slider and the disk. Polishing may also mask away substrate defects such as depressions or pits and individual high asperities which could otherwise affect the output voltage signal and the AL of the slider bearing. The rigid disks used in this thesis have an electroless plated nickel-phosphorus (NiP) undercoat with a thickness of 1 to 2.5 μm.

**Magnetic Layers and Overcoats.** Thin film rigid disks have usually thin magnetic layers of metal or oxide (50 to 150 nm thickness). The magnetic layers of the rigid disks used in this thesis consist of cobalt-nickel-chromium (CoNiCr) and cobalt-chromium-platinum (CoCrPt). Metallic magnetic layers have relative low hardness and are susceptible to corrosion. Therefore they are provided with a solid overcoat of about 20 to 80 nm thick, to improve wear and corrosion resistance. Also oxide magnetic layers are usually provided with an overcoat in order to reduce friction and wear. From Chapter 2 it follows that overcoats should be as thin as possible. Furthermore, they should be hard and fracture resistant, and they should be continuous in order to protect the magnetic layer. Moreover, overcoats should be chemically compatible with lubricants, they should provide a surface for lubricant bonding and they must be continuous to prevent interaction between the lubricant and metal ions from the magnetic layer. A good review about overcoats is given by Chandrasekar and Bhushan [24], [25] and Bhushan and Gupta [17, Chapter 14]. Many studies, for example those of Khan et al. [68], Kovac and Novotny [76], Lauer and DuPlessis [79], Lempert et al. [80],
Matsuura et al. [89], Nakatsuka et al. [100], Novotny and Kao [103], Schulz and Viswanathan [117], Yamashita et al. [155], Yeh et al. [159], [160] and Zeira et al. [161], show that the microstructure and therefore the tribological performance of deposited overcoats, strongly depend on the type of deposition process applied and also on the deposition process conditions and, therefore, indirectly affects the friction and wear behaviour of the overcoat of an HDI. Overcoats for thin film rigid disks are usually made of carbon, zirconia or silica thin films. Of all the overcoats, carbon and spin-coated silica are most widely used. The thin film rigid disks used in this thesis (see Fig. 3.10) have 20 to 40 nm thick amorphous carbon overcoats which have been deposited by sputtering.

Lubricant Layers. In order to reduce the friction at the interface during sliding of the head, the HDI is lubricated by a liquid. Solid lubricants cannot be used at an HDI, because they tend to wear away in tracks under the slider and the wear debris may cause catastrophic failure of the HDI. An HDI is therefore internally or topically lubricated by a liquid lubricant layer at the disk surface. Internal lubrication is a method of lubrication in which a lubricant is included in (one of) the sliding surface(s). For example, particulate rigid disks are internally lubricated. Topical lubrication is a system of lubrication in which a strong bonding lubricant is added on top of the sliding surface(s). Thin film rigid disks cannot be internally lubricated because the lubricant cannot be included during the deposition process of the thin films which are usually continuous. Therefore, they are topically lubricated with a 3 nm thick perfluoropolyether (PFPE) layer. The thin film rigid disks studied in this thesis also have a PFPE lubricant layer. The thickness of the lubricant layer is usually less than the CLA roughness of the disk surface (the ratio of the lubricant layer thickness to the CLA roughness is about 0.5). This implies that the lubricant behaves like a boundary lubricant layer. Furthermore, the maximum rate of shearing at an HDI can be as high as $10^8$ s$^{-1}$. Bhushan [10], Streater [129] and Streater et al. [131] showed that at these high rates PFPE lubricant layers may rupture when the slider touches down on the disk, indicating that full hydrodynamic action certainly will not occur.

Surface Topography. Nowadays, the HMSs in HDIs are below 60 nm and as they continue to decrease the effect of surface roughness becomes more important. Very smooth disk surfaces are desirable for the recording and reproducing process, but when the surfaces are too smooth, the slider tends to stick to the disk surface (stiction). To reduce the problem of stiction, thin film rigid disks are usually mechanically textured. Figure 3.11 shows the surface topography of a thin film rigid disk used in this thesis. The disks are circumferentially textured and the CLA roughness is usually 3 to 6 nm. The roughness in tangential direction (along the tracks) is lower than that in radial direction. Figure 3.12 shows that the surface height of the disk is Gaussian distributed.

3.3 A Model of the Head-Disk Interface

In the previous sections it has been shown that an HDI is a very complex tribological system, which has the following features:
Fig. 3.11. The surface topography of a thin film rigid disk used in the experimental work of this thesis, measured by an optical phase shift interferometry profiler. (Measurement area: 1.84×1.41 mm, \( R_a = 3.1 \) nm.)

Fig. 3.12. The distribution of surface heights of a thin film rigid disk used in the experimental work of this thesis, measured by an optical phase shift interferometry profiler. (The total number of data points is 305 920.)
1. The rigid disk is a multilayered structure, made of a substrate and very thin films. The lubricant layer, the overcoat and the magnetic layer have thicknesses of about 3 nm, 20 nm and 50 nm, respectively.

2. The normal load applied to the slider is low (below 100 mN) and is transmitted to the disk surface by a rather small nominal contact area (smaller than 2.5 mm$^2$). Consequently, a very low nominal pressure exists at the interface (lower than 40 kPa).

3. The surfaces of the slider and the disk are rather smooth and have a CLA roughness below 6 nm. This means that the average separation between the slider and the disk surface during acceleration and deceleration is of the same order as the roughness. Consequently, relatively large parts of the interface are virtually in contact. Furthermore, the surface heights of the slider and disks are Gaussian distributed.

4. The relative velocity between the slider and the disk during normal operation is very high (about 12 m/s for a disk with a diameter of 63.5 mm).

Figure 3.13 gives an illustration of a cross section of the model of the HDI to be analysed in the next chapter. It will be assumed that the surfaces are not contaminated, and that the asperities of the slider do not break through the magnetic layer. Furthermore, the layers of the disk are considered to be homogeneous and continuous, but their thickness may not necessarily be uniform. Given these assumptions, shear can take place in the layers and at the interfaces, as is given

![Image of a thin film HDI model](image_url)

**Fig. 3.13.** Model of a thin film HDI. The roughness profiles of the head and disk are measured by an optical phase shift interferometry profiler. The thickness of the layers are not on the proper scale.
3. A Model of the Head-Disk Interface

**Fig. 3.14.** Possible microcontacts at a thin film HDI.

<table>
<thead>
<tr>
<th>Shear in layers</th>
<th>Shear at interfaces</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air film</td>
<td>Slider-Lubricant</td>
</tr>
<tr>
<td>Lubricant</td>
<td>Slider-Overcoat</td>
</tr>
<tr>
<td>Overcoat</td>
<td>Slider-Magnetic layer</td>
</tr>
<tr>
<td>Magnetic layer</td>
<td></td>
</tr>
<tr>
<td>Slider</td>
<td></td>
</tr>
</tbody>
</table>

by Table 3.3. In this model, microcontacts may appear between the slider and the overcoat and between the slider and the magnetic layer (see Fig. 3.14). In the following chapter the friction between the slider and the rigid disk will be analysed by means of this model.
A Friction Model for Head-Disk Interfaces

In the previous chapter, it has been shown that the HDI can be considered as a lubricated and multilayered tribological system which is subjected to sliding, and that its performance is affected by the minimum friction and the minimum HMS at the velocities required for recording and reproducing.

It is generally known, that friction at an interface can be associated with different lubrication regimes which may occur at the interface. Lubrication regimes are also closely linked to the HMS of the HDI. It is therefore important to identify the possible lubrication regimes and to understand how an HDI moves from one regime into another. These aspects will be the subject of this chapter.

The chapter starts with a short introducional summary of the different lubrication regimes, which, in general, may occur at lubricated sliding interfaces. The transitions from one regime to another can be characterised by a so called generalised Stribeck curve. It will be shown how this curve can be applied to the HDI in consideration. Furthermore, a friction model for HDIs will be developed from which the friction and the transitions can be computed. With the help of this model, the effects of some important parameters on the friction at an HDI and on the transitions will be discussed.

4.1 Lubrication Regimes

In a fluid (liquid or gaseous) lubricated interface of two rough, solid surfaces sliding against each other, and subjected to a normal load, the following lubrication regimes can be distinguished:

- **Full film lubrication**: In this regime the applied load is entirely carried by the fluid lubricant under pressure. The lubricant can be pressurised, for example, by supplying it from an external source, or, in case the two solid surfaces are slightly convergent, by the motion at the interface. The pressure in the lubricant causes a complete separation of the solid surfaces (see Fig. 4.1). The difference in relative velocity between the solid surfaces is accommodated by shear in the lubricant. The viscosity\(^1\) of the lubricant is therefore an important parameter in this regime. In case of a liquid lubricant

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\(^1\)The viscosity of a fluid is a physical property which refers to the resistance to flow. If the viscosity is independent of the rate of shearing, the fluid is usually called a Newtonian fluid, whereas, if the viscosity varies with the rate of shearing, the fluid is called non-Newtonian.
Fig. 4.1. Hydrodynamic lubrication regime (the solid surfaces are completely separated by the lubricant).

Fig. 4.2. Boundary lubrication regime (interaction takes place between boundary layers).

this regime is called hydrodynamic lubrication (HL). When air is used as lubricant, this regime is called aerodynamic lubrication (AL)\(^2\). If the pressure in the lubricant is low, the elastic deformation of the solid surfaces can be neglected. In that case, the separation between the solid surfaces is fully determined by the geometry of the solid surfaces. If the elastic deformation of the solid surfaces is significant with respect to the separation, the separation between the solid surfaces is determined by the geometry as well as the deformation of the solid surfaces. This may occur when the pressure in the lubricant is high or when the elastic moduli of the solid surfaces are low. In that case the regime is called elasto-hydrodynamic lubrication (EHL). In case of an HDI, the nominal pressure at the interface is very low (see also Chapter 3) and the elastic moduli of the surfaces involved are moderate. Consequently, EHL does not occur at HDIs. During normal operation of an HDI, the slider and disk form a self-acting air lubricated slider bearing.

- **Boundary lubrication** (BL): This regime is illustrated in Fig. 4.2; it occurs when the pressure in the lubricant cannot provide a complete separation between the moving solid surfaces. The surfaces are so close together that interaction between boundary layers (such as mono- or multimolecular films of lubricant, moisture, etc.) and the solid surfaces dominate the contact. In this case the applied load is entirely carried by the solid surfaces. Due to the roughness of both solid surfaces, contact between them is discontinuous. Contact occurs at local spots, or so called microcontacts, where asperities of one surface come into contact with those of the opposite surface. The real area of contact is the sum of the areas of the microcontacts. The deformation of the surface at the microcontacts creates stresses which oppose the applied load. As a result, the load will be carried by the microcontacts. Shear takes place in the boundary layers or at the interface of these boundary layers. Therefore, the physical and chemical interaction of the boundary layers with the solid surfaces becomes important rather than the viscosity

\(^2\)In the literature about magnetic recording the term hydrodynamic lubrication has also been erroneously used, see, for example, Mee and Daniel [91].
of the lubricant. In general, gases have no boundary lubricating properties, nevertheless, they may form certain reaction layers at the interface, such as oxides, which may provide some boundary lubrication. Therefore, an HDI is provided with a thin liquid lubricant layer at the disk, which acts as a boundary lubricant (see also Chapter 3).

- **Mixed lubrication** (ML): This is an intermediate regime between the (E)HL or AL regime and the BL regime (see Fig. 4.3). The applied load is partly carried by the microcontacts of the solid surfaces and partly by the pressurised lubricant.

If there are no lubricant and contaminants between the solid surfaces, **dry friction** (DF) occurs (see Fig. 4.4). In this case, the applied load is also carried by the microcontacts and the difference in relative velocity between the surfaces is accommodated in, or at the interface of the interacting asperities of the solid surfaces, causing shear in or at these asperities, respectively. In lubricated contacts it is also possible that a combination of BL and DF occurs. This combination is illustrated in Fig. 4.5.

From experiments (see, for example, Schipper et al. [113]-[115]) it follows that each lubrication regime can be associated with its frictional behaviour, which can be presented in a so called generalised Stiubeck curve, as is shown in Fig. 4.6. The vertical axis in this figure represents the value of the coefficient of friction\(^3\) (COF) \(\mu\). At the horizontal axis a dimensionless lubrication number\(^4\) \(L\) is given, which is defined as

\[
L \overset{\text{def}}{=} \frac{\eta_{\ell}v_+}{pR_{a,z,c}},
\]

\(^3\)The coefficient of friction is defined by the OECD [41] as the ratio of the tangential force resisting motion between two bodies by the normal force pressing these bodies together.

\(^4\)In an original Striebeck curve, the coefficient of friction is plotted against \(n\), which is the speed of the bearing shaft (see Striebeck [132]). Hersch [54] plotted the coefficient of friction against the dimensionless number \(\eta_{\ell}n/p\), where \(\eta_{\ell}\) is the dynamic viscosity of the lubricant, and \(p\) is the projected pressure in the contact. In a generalised Striebeck curve the coefficient of friction is plotted as a function of the dimensionless lubrication number \(L\).
where \( \eta \) is the dynamic viscosity of the lubricant at the inlet or the leading edge of the contact and \( R_{a,z,c} \) is the combined CLA roughness of the surface heights of the solid surfaces, given by

\[
R_{a,z,c} \equiv \sqrt{R_{a,z,1}^2 + R_{a,z,2}^2},
\]

in which \( R_{a,z,i} \) is the CLA roughness of the surface heights of solid surface \( i \) \((i = 1, 2)\) of the interface. Furthermore, \( v_+ \) is the sum velocity of the sliding surfaces, defined as

\[
v_+ \equiv v_1 + v_2,
\]

in which \( v_i \) is the velocity of solid surface \( i \) \((i = 1, 2)\). Finally, \( \overline{p} \) is the nominal (or apparent) pressure in the contact, given by

\[
\overline{p} \equiv \frac{P}{A_a},
\]

in which \( P \) is the applied normal load and \( A_a \) is the nominal (or apparent) area of contact. From Fig. 4.6 it follows that, amongst other things, the lubrication regimes are determined by tribological parameters such as the sum velocity, the dynamic viscosity (which depends on the pressure and the temperature) and the nominal pressure (which depends on the load and the nominal contact area).

Also given in Fig. 4.6 is the separation \( d_a \) between the solid surfaces as a function of the dimensionless lubrication number \( L \). The separation \( d_a \) is illustrated in Fig. 4.7, and is defined as the distance between the mean planes through the summit heights of the rough solid surfaces. It follows that the separation depends on the lubrication regime occurring between the solid surfaces. In the (E)HL or AL regime the separation is largest and the COF is lowest, whereas in the BL regime the separation is smallest and the COF is highest. In Chapter 3 it has been mentioned that an HDI operates in the AL regime, where the friction is low and the separation is large. The HMS of an HDI should be as small as possible in order to achieve high storage capacities, so an optimum solution to achieve this requirement would be to operate at the proper velocity required for recording and reproducing, and at the transition ML to AL \((L_A \) in Fig. 4.6), as in this

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**Fig. 4.5.** Combination of boundary lubrication and dry friction.
Fig. 4.6. Generalised Stribeck curve in which the coefficient of friction $\mu$ between two rough solid surfaces of a fluid lubricated interface is plotted as a function of the dimensionless lubrication number $L$. The curve shows that the different lubrication regimes can be characterised by means of the coefficient of friction and the operational conditions.

Fig. 4.7. Definition of the separations $d_s$ and $d_g$ between two rough solid surfaces ($A$ = Rough surface of solid 1, $B$ = Mean plane of the summits of solid 1, $C$ = Mean plane of the heights of solid 1, $D$ = Rough surface of solid 2, $E$ = Mean plane of the summits of solid 2, $F$ = Mean plane of the heights of solid 2).
4.2 Transitions of Lubrication Regimes

With lubricated contacts, the transitions from BL to ML and from ML to (E)HL or AL take place at certain values of the lubrication numbers, which are indicated in this thesis by \( L_B \) and \( L_A \), respectively. The COF at these lubrication numbers are \( \mu_B \) and \( \mu_A \), respectively. These lubrication numbers can be determined, for example, by taking the tangents of the curve in the BL and ML regime, which are indicated in Fig. 4.6 by the lines \( \overline{AB} \) and \( \overline{BC} \), respectively. The \( L_A \) value is the lubrication number at which the tangent of the ML regime intersects the curve (at point \( C \)). Similarly, the \( L_B \) value is the lubrication number at which the tangent of the BL regime meets the tangent of the ML regime (at point \( B \)). Schipper [113, page 6.3] found that for liquid lubricated concentrated contacts, in the pressure range \( 0.1 \text{ GPa} < \bar{p} < 1.5 \text{ GPa} \) and in the roughness range \( 25 \text{ nm} < R_{a,x,c} < 1000 \text{ nm} \), the transitions can be described by the equations

\[
L_A \approx \lambda_A \sqrt{\frac{R_{a,x,c}}{\bar{p}}} \quad \text{and} \quad L_B \approx \lambda_B \frac{1}{\bar{p}},
\]

(4.5)

in which \( \lambda_A \) and \( \lambda_B \) are constants, given by \( \lambda_A \approx 3.10 \cdot 10^4 \text{ N}^{1/2}/\text{m}^{3/2} \), and \( \lambda_B \approx 1.25 \cdot 10^4 \text{ N/m}^2 \). The transitions can be properly represented in a so-called lubrication transition diagram in which the lubrication numbers \( L_A \) and \( L_B \) are given as a function of the nominal pressure \( \bar{p} \) (see Fig. 4.8). The transition line representing \( L_A \) in this diagram has a slope of approximately \(-1/2\), while the line representing \( L_B \) has a slope of approximately \(-1\). Schipper found that the transitions at \( L_A \) and \( L_B \) are not affected by the type of liquid lubricant, but only by the viscosity \( \eta \) of the lubricant at the inlet or the leading edge of the contact. However, the value of the COF depends on the type of lubricant. From (4.5) it follows that the transition at \( L_B \) is independent of the nominal pressure \( \bar{p} \) in the contact, because both sides of the equation contain \( \bar{p} \) in their denominator. This follows also from the work of Landheer et al. [78]. An important consequence of this is that, if an interface operates in the BL regime at constant product \( \eta v_+ / R_{a,x,c} \), the interface can never enter the ML regime, regardless of the pressure \( \bar{p} \). In other words, in order to move a lubricated interface from the BL regime into the ML regime (i.e. to create more well lubricated microcontacts) the product \( \eta v_+ / R_{a,x,c} \) has to be increased (for example, by means of increasing the velocity, or smoothening the surfaces).
4.3 Transitions of Lubrication Regimes at the Head-Disk Interface

A lubrication transition diagram, such as described in the previous section, is useful for HDIs, because it gives the conditions required for a particular lubrication regime, and hence, the conditions required for CR. However, the two transition lines $L_A$ and $L_B$ of the lubrication transition diagram in the previous section have been derived from experimental work carried out on liquid lubricated concentrated contacts, in the pressure range of 0.1 GPa to 1.5 GPa and with a combined CLA roughness in the range of 25 nm to 1000 nm. According to Table 1.1 of Chapter 1, page 9 the load at a slider is below 95 mN. Furthermore, the nominal area of contact of a rail of a standard two rail taper-flat slider is approximately 1 mm². This means that the nominal pressure $\overline{p}$ at an HDI is about 20 to 40 kPa, thus far below the pressures for which the lubrication transition diagram is valid. Furthermore, the HDI consists of multilayers and two lubricants (a liquid lubricant layer and a gaseous film), and the combined CLA roughness of a disk and a slider of an HDI is below 6 nm, which is also outside the range for which the lubrication transition diagram is valid. Because of these entirely different parameters and structure of the contact, it is necessary to establish to what extent the lubrication transition diagram remains applicable to HDIs. This will be studied in the remainder of this section.
Schipper [113] derived lubrication numbers for the experimental results of other workers (see, for example, Gartner [40] and Hata et al. [52]) and plotted them in a lubrication transition diagram. It was found that at lower pressures the slope of the transition line $L_B$ remains constant over the entire pressure range, and that the slope of line $L_A$ approaches $-1$ (see Fig. 4.9). In this case, the transition at $L_A$ becomes pressure independent just like the transition at $L_B$, and the slope of the transition line $L_B$ seems to be valid throughout the whole pressure range.

Also plotted in Fig. 4.9 are some lubrication numbers derived from results of experiments carried out on HDIs. Those experiments which have been carried out in the AL regime are indicated by open symbols, those experiments in which the transition from ML to AL was measured are indicated by ‘TOV’. All other experiments have been carried out in the BL or ML regime or in the DF regime (filled symbols). It seems that there is a clear distinction between the BL regime and the AL regime and that the slope of the transition line $L_B$ remains indeed constant, over the entire pressure range, but it is not clear yet what the slope of the transition lines $L_A$ and $L_B$ is. It should be noted, however, that for some of the plotted data of Fig. 4.9 the roughness of the slider and/or disk was not given, but has been assumed to be 5 and 10 nm, respectively, which are reasonable values for HDIs.

![Transition line $L_B$](image)

**Fig. 4.9.** Lubrication numbers $L$ derived from published experimental studies (Experiments carried out in the AL regime are indicated by open symbols and experiments carried out in the BL or ML regime are indicated by the filled symbols).
In this thesis the transition lines for HDIs are determined from computations of a friction model of the HDI and from experiments carried out on HDIs. In the next sections this friction model will be derived.

4.4 A Contact Model for the Head-Disk Interface

In Chapter 3, Section 3.3 a contact model of the HDI has been introduced, in which the possible microcontacts at the interface are given. In this section, this model will be used to derive the friction force between a slider and a thin film rigid disk of an HDI. Figure 4.10 shows schematically the type of contact envisaged. The contact model is based on the following assumptions:

1. The surface of the slider is considered to be nominally flat\(^5\) and rough with a large number of asperities. In order to reduce the mathematical complexity of the contact model, the surface of the rigid disk is considered smooth and flat. If the slopes of the surface roughness are small, such as is the case in HDIs, it can be shown that the contact of two rough surfaces is equivalent to the contact of a rough surface and a smooth surface (see Greenwood and Tripp [46]). The rigid disk consists of strong bonded thin films, with thicknesses \(t_i\), elastic moduli \(E_i\), and Poisson ratio’s \(\nu_i\) (\(i = 1, \ldots, n\) in which \(n\) is the number of thin films) and the elastic modulus and the Poisson ratio of the substrate are \(E_s\) and \(\nu_s\), respectively. The elastic modulus and the Poisson ratio of the rough surface of the slider are \(E_r\) and \(\nu_r\), respectively. Furthermore, the smooth surface of the rigid disk is covered with a thin liquid lubricant layer of thickness \(t_l\).

2. All asperities at the rough surface of the slider are identical and have — at least near their summits — a semi ellipsoidal shape.

3. The roughness of the surface of the slider is anisotropic, i.e. the curvatures of the asperities may be different in the two main directions.

4. The height of the asperities at the rough surface of the slider vary randomly according to a Gaussian distribution. In Chapter 3 it has been illustrated that this is true to a high degree.

5. Deformation at the surface does not take place, except for the asperities which may deform elastically during contact. The nominal pressure at a HDI is very low, thus this assumption is certainly valid. Furthermore, the asperities cannot penetrate through the thin films of the rigid disk, otherwise plastic deformation would occur.

6. Sliding at the microcontacts takes place in the BL and ML regime.

\(^5\) A nominally flat surface may be defined as that in which the area of contact is large so that the individual microcontacts are dispersed and the deformation of each asperity is independent of its neighbours.
Fig. 4.10. Contact model of a rough solid surface in contact with a smooth and nominal flat, multilayered, solid surface covered with a thin, liquid, lubricant layer. The load is supported by those asperities whose heights are greater than the separation $d_s$ between the reference planes $A$ and $C$. Those asperities immersed in or touching the liquid lubricant layer have a meniscus around them ($A$=Reference plane of the smooth surface, $B$=Reference plane of the liquid lubricant layer, $C$=Mean plane of the summits of the rough surface, $D$=Mean plane of the heights of the rough surface).

7. If an asperity at the rough surface is touched by or immersed into the liquid lubricant layer a concave meniscus is formed around it. The presence of menisci causes an attractive (adhesive) force between the rough surface and the lubricant layer, which is usually referred to as the *meniscus force* or *stiction force*. This force is ascribed to the lower pressure inside as compared to the pressure outside the menisci, as a result of the surface tension of the liquid.

With the statistical contact model described above, there is a probability that a particular asperity at the surface of the slider is in contact with the rigid disk surface (microcontact), or is immersed in the liquid lubricant layer (wetted asperity) or is in contact with the gaseous lubricant (air). The elastic deformation and the wetting of the asperities will be described below.

\footnote{See also the footnote on stiction in Chapter 1 on page 11.}
4.4.1 The Elastic Deformation of the Asperities

In Appendix A, Section A.1 the elasticity theory of Hertz [55] has been applied to the elastic deformation of a single asperity. The elasticity of the several thin films of the rigid disk has been incorporated (see Section A.2). By a statistical analysis the real area of the microcontacts and the load at the microcontacts have been determined in a similar way as Greenwood et al. [45]–[47] have done (see Appendix A, Section A.3). The equations resulting from this analysis give the number of microcontacts, the total real area of contact and the total load carried by the microcontacts in terms of the separation of their mean planes of a tribological multilayered thin film system.

The total number of microcontacts is,

$$n_s = \eta A_a F_{0,0}(d_s),$$  \hspace{1cm} (4.6)

in which $\eta$ is the density of summits at the rough surface of the slider, $A_a$ is the nominal area of contact and $F_{m,n}(d_s)$ is a function defined as

$$F_{m,n}(d_s) \overset{\text{def}}{=} \int_{d_s}^{\infty} E_c^{m}(z - d_s)(z - d_s)^n \phi(z) \, dz.$$  \hspace{1cm} (4.7)

Here, $d_s$ is the separation between the reference planes $[A]$ and $[C]$ in Fig. 4.10, $\phi(z)$ is the probability density function of the summit height distribution of the rough surface and $E_c(z - d_s)$ is the reduced elastic modulus given by (A.20), which depends on the indentation $z - d_s$. The real area of the microcontacts is,

$$A_{r,s}(d_s) = 2\pi\eta A_a \left( \frac{\alpha \beta}{\gamma \rho} \right) F_{0,1}(d_s),$$  \hspace{1cm} (4.8)

in which $\alpha$ and $\beta$ are the dimensionless minor and major radii of the contact ellipse of a microcontact, $\gamma$ is the dimensionless indentation, $\rho$ is the main curvature of the asperities and $F_{0,1}(d_s)$ is given by (4.7). The plastic area of contact of the microcontacts is given by

$$A_{p,s}(d_s) = 2\pi\eta A_a \left( \frac{\alpha \beta}{\gamma \rho} \right) \left( F_{0,1}(d_s + w_p) + w_p F_{0,0}(d_s + w_p) \right),$$  \hspace{1cm} (4.9)

in which $w_p$ is given by

$$w_p = \frac{1}{\rho} \left( \frac{H}{E_c} \right)^2,$$  \hspace{1cm} (4.10)

with $H$ the effective hardness of the interface. The load carried by the microcontacts is given by,

$$P_s(d_s) = \frac{4}{3} \sqrt{2\eta A_a} \frac{1}{\gamma \beta / \rho^{1/2}} F_{1,3/2}(d_s).$$  \hspace{1cm} (4.11)
4.4.2 The Wetting of the Asperities

In Appendix A, Section A.6 a statistical model is used for determining the number of wetted asperities, the total wetted area of contact and the load carried by the wetted asperities. The number of wetted asperities is,

\[ n_e = \eta A_n G_{0,0}(d_s), \]  

in which \( G_{m,n}(d_s) \) is a function defined as

\[ G_{m,n}(d_s) \equiv \int_{d_s-t_e}^{d_s} \tau^m_\ell(\eta_0, v_-, d_s - z)(z - d_s + t_e)\phi(z)dz, \]  

in which \( t_e \) is the thickness of the liquid lubricant layer, \( \tau^m_\ell(\eta_0, v_-, d_s - z) \) is the shear stress given by (A.63) and \( \eta_0 \) is the ‘low-shear’ or ‘zero-shear’ dynamic viscosity of the liquid lubricant layer. The total wetted area of contact is,

\[ A_{r,\ell}(d_s) = 2\pi \eta A_n \sqrt{\frac{1}{\rho_x \rho_y}} G_{0,1}(d_s), \]  

in which \( \rho_x \) and \( \rho_y \) are the main curvatures of the elliptic asperities, and \( G_{m,n}(d_s) \) is defined by (4.13). The load carried by the lubricant layer is given by

\[ P_{\ell}(d_s) = F_{\ell}(d_s) - \sigma_{m} A_{r,\ell}(d_s), \]  

in which \( \sigma_{m} \) is the tensile strength of the liquid lubricant layer, and \( F_{\ell}(d_s) \) is the friction force due to the shearing of the lubricant layer. This friction force is given by

\[ F_{\ell}(d_s) = 2\pi \eta A_n \sqrt{\frac{1}{\rho_x \rho_y}} G_{1,1}(d_s). \]  

4.5 Friction at the Head-Disk Interface

It is thought that the friction force \( F \) at the HDI is made up of a component \( F_s \), determined by the microcontacts between the solid surfaces of the slider and the disk, a component \( F_{\ell} \), due to the shearing of the thin liquid lubricant layer, and a component \( F_g \), determined by the gaseous lubricant film, i.e.

\[ F = F_s + F_{\ell} + F_g = \tau_s A_{r,s} + \tau_{\ell} A_{r,\ell} + \tau_g A_g, \]  

where \( \tau_s, \tau_{\ell}, \text{and } \tau_g \) are the shear stress at the microcontacts, the shear stress in the liquid lubricant layer, and the shear stress in the gaseous lubricant, respectively. \( A_{r,s} \) is the real area of contact of the microcontacts (given by (4.8)), \( A_{r,\ell} \) is the real area of the asperities in contact with the liquid lubricant layer (see (4.14)) and \( A_g \) is the area remaining for the AL of the gaseous lubricant. The sum of these areas must be equal to the nominal area of contact, i.e.
\[ A_a = A_{r,s} + A_{r,t} + A_g. \]  

(4.18)

The nominal area of contact \( A_a \) at an HDI is given by

\[ A_a = 2w_r \ell_r, \]  

(4.19)

in which \( \ell_r \) and \( w_r \) are the length and width of a rail of the slider, respectively. The mean real contact pressure \( \overline{p}_s \) at the microcontacts is

\[ \overline{p}_s = \frac{P_s}{A_{r,s}}, \]  

(4.20)

in which \( P_s \) is the load carried by the contacting asperities, given by (4.11). Thus, the real area of contact can be written as

\[ A_{r,s} = \frac{P_s}{\overline{p}_s}. \]  

(4.21)

Similarly, the area that remains available for the gaseous lubricant can be written as

\[ A_g = \frac{P_g}{\overline{p}_g}, \]  

(4.22)

in which \( P_g \) is the load carried by the gaseous lubricant and \( \overline{p}_g \) is the mean pressure in the gaseous lubricant. Substituting (4.21) and (4.22) into (4.17) gives

\[ F = \frac{\tau_s}{\overline{p}_s} P_s + F_t + \frac{\tau_s}{\overline{p}_g} P_g. \]  

(4.23)

The ratio \( \tau_s/\overline{p}_s \) is generally known as the COF \( \mu_s \) at the BL regime, thus

\[ \mu_s = \frac{\tau_s}{\overline{p}_s}. \]  

(4.24)

In general, the magnitude of \( \mu_s \) cannot be computed and has to be found experimentally. The component \( F_t \) in (4.23) is already given in (4.16). In Appendix B an expression has been derived for the shear stress \( \tau_g \) in the gaseous lubricant for a first, one and half and second order slip flow model,

\[ \tau_g = \frac{\eta_g v_-}{d_g + 2C_1 \lambda_a p_a / p_g}, \]  

(4.25)

in which \( d_g \) is the separation between the reference planes \([A]\) and \([D]\) in Fig. 4.10, \( \eta_g \) is the viscosity of the gaseous lubricant, \( v_- \) is the relative velocity between the slider and the disk surface, \( p_a \) is the ambient pressure, \( \lambda_a \) is the molecular free mean path of the gas molecules at ambient conditions, \( C_1 \) is the surface correction coefficient and \( p_g \) is the pressure in the gaseous lubricant. For the pressure \( p_g \) the mean pressure \( \overline{p}_g \) in the gaseous lubricant has been chosen. From (4.23) the ratio \( \tau_g/\overline{p}_g \) may be written as

\[ \mu_g = \frac{\tau_g}{\overline{p}_g} = \frac{\eta_g v_-}{\overline{p}_g d_g + 2C_1 \lambda_a p_a}. \]  

(4.26)
At any time, the sum of the loads carried by the microcontacts, the liquid lubricant layer, and the gaseous lubricant must equal the total applied normal load, i.e.,

\[ P = P_s + P_t + P_g. \]  

(4.27)

From this equation, the load carried by the gaseous lubricant becomes,

\[ P_g = P - P_s - P_t, \]  

(4.28)

and the mean pressure \( \overline{P}_g \) in the gaseous lubricant may be written as

\[ \overline{P}_g = \frac{P_g}{A_g} = \frac{P - P_s - P_t}{A_a - A_{r,s} - A_{r,t}}. \]  

(4.29)

From (4.23), (4.24), (4.26) and (4.28) the friction force at the HDI is given by

\[ F = \mu_s P_s + F_t + \mu_g P_g. \]  

(4.30)

This equation gives the friction force \( F \) as a function of separation \( d_g \). However, the friction force cannot be obtained directly from (4.30), because the separation is constrained by the loads \( P_s, P_t \) and the load \( P_g \) by means of equation (4.27). Equation (4.30) should therefore be solved numerically to obtain the separation \( d_g \). The sequence of computing a Strubeck curve, based on the equations derived above, is given in Fig. 4.11.

In general, the load \( P_g \) carried by the gaseous lubricant is a function of the sum velocity \( v_+ \) and the separation \( d_g \), thus

\[ P_g = f(d_g, v_+). \]  

(4.31)

If an expression for \( f(d_g, v_+) \) can be found, then (4.27) and (4.30) completely determine the friction force \( F \) of the HDI, as a function of several parameters (surface topographical parameters, lubricant parameters, and operational parameters).

Mathematically it is possible that \( A_{r,s} + A_{r,t} = A_a \), i.e. \( A_g = 0 \), which result in an infinite friction force. However, from the contact model given in the previous section and the mechanical properties of thin film rigid disks and sliders it follows that the real area of contact \( A_{r,s} \ll A_a \) and \( A_{r,t} \ll A_a \). The largest \( A_{r,s} \) occurs when the surfaces rest against each other at zero velocity, because then the applied load has to be carried entirely by the microcontacts. As a result of AI the load will be partly carried by the microcontacts and partly by the liquid lubricant layer and gaseous lubricant. Consequently, the real area of contact \( A_{r,s} \) decreases with velocity.

From (4.15) it follows that the load carried by the liquid lubricant becomes negative if

\[ F_t(d_g) \leq \sigma_m A_{r,t}. \]  

(4.32)

In that case the liquid lubricant layer introduces an adhesive force (stiction force) which contributes to the applied normal load.

Finally, for a Gaussian summit height distribution,
4.5 Friction at the Head-Disk Interface

Fig. 4.11. Flow chart which gives the sequence for computing the friction force $F$ at an head-disk interface.

\[ \phi(z) = \frac{1}{\sigma_{s,c} \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{z}{\sigma_{s,c}} \right)^2}, \quad (4.33) \]

where $\sigma_{s,c}$ is the standard deviation of the summit height distribution. From the surface topography measurements in Appendix I of this thesis, on thin film rigid disks and standard sliders, it follows that a linear relationship exists between the standard deviation of the summit height distribution $\sigma_{s,c}$ and the combined CLA roughness $R_{a,z,c}$ of the surface height distribution,

\[ \sigma_{s,c} = C_{\sigma} R_{a,z,c}, \quad \text{with} \quad C_{\sigma} = 1.19, \quad (4.34) \]

and where $R_{a,z,c}$ is given by (4.2). From these measurements it also follows that a linear relationship exists between the two separations $d_g$ and $d_s$, according to

\[ d_g = \sigma + d_s, \quad (4.35) \]
in which $\bar{\sigma}$ is a constant. Furthermore, the nominal (or apparent) mean pressure in the contact is given by (4.4), i.e.

$$\bar{p} = \frac{P}{A_a}. \quad (4.36)$$

Note that $\sigma_{s,c}$ is absent in all previous equations but (4.33). Therefore, the roughness only determines the summit height distribution.

### 4.5.1 Air Film Thickness Equation

For the computation of the separation $d_g$ a film thickness equation for the gaseous lubricant is required, of the form

$$P_g = f(d_g, v_+). \quad (4.37)$$

Until now, no such equation exists, and therefore, one has been derived from an extrapolation of numerical data obtained from computations of the modified Reynolds equation of the HDI (see, for example, Van der Stegen [127]). The minimum film thickness and pitch angle of a standard slider, such as described in Chapter 3 has been calculated for loads ranging from 5 to 250 mN, and at velocities ranging from 1 to 30 m/s. Figure 4.12 shows the minimum film thickness $d_{g, \text{min}}$ as a function of the velocity $v_+$. The circled region in this figure, is the area of interest for the film thickness equation to be derived. In Fig. 4.13 the parameter $P_g/v_+$ is plotted as a function of $d_{g, \text{min}}$. It follows that there exist a single film thickness equation. By curve fitting the data, and noticing that for any load the minimum film thickness must be zero at zero velocity, the following equation has been obtained for the film thickness

$$\ln \left( \frac{P_g}{A_a} \right) = \left( \frac{\ln \left( \frac{d_g^{-1}}{c_1} - c_2 v_+^{c_3} \right)}{c_4} \right)^{1/c_4}, \quad (4.38)$$

in which $d_g$, $v_+$, $P_g$ and $A_a$ are in m, m/s, N and m$^2$, respectively, and $c_1$, $c_2$, $c_3$ and $c_4$ are constants given by

$$c_1 = -12.039; \quad c_2 = +0.247;$$

$$c_3 = -0.250; \quad c_4 = -1.443. \quad (4.39)$$

The equations in this section have been put into a computer program in order to easily compute the friction at an HDI.

### 4.6 The Equations in Dimensionless Form

By applying a similarity analysis (according to Moes [98]) to the set of equations (4.27) and (4.30), the following set of dimensionless parameters has been obtained for the situation of a Gaussian summit height distribution,
Fig. 4.12. Minimum air film thickness $d_{g,\text{min}}$ as a function of the velocity $v_+$ of a standard two rail taper-flat slider, with one and a half order slip flow model. The parameter $P_g$ indicates the load at the slider (without the weight of the slider).

Fig. 4.13. $P_g/v_+$ as a function of $d_{g,\text{min}}$ of a standard two rail taper-flat slider, with one and a half order slip flow model.
\[
\mu = \frac{F}{p}; \quad \mu_{\ell} = \frac{F_{\ell}}{p}; \quad \mathcal{P}_s = \frac{P_s}{p}; \quad \mathcal{P}_{\ell} = \frac{P_{\ell}}{p}; \quad \mathcal{P}_g = \frac{P_g}{p}; \quad (4.40)
\]

\[
h = \frac{d_s}{R_{a,x,c}}; \quad h_\sigma = \frac{\sigma}{R_{a,x,c}}; \quad \mathcal{A}_{r,s} = \frac{A_{r,s}}{A_a}; \quad \mathcal{A}_{r,\ell} = \frac{A_{r,\ell}}{A_a}; \quad \mathcal{A}_g = \frac{A_g}{A_a}; \quad (4.41)
\]

\[
S = \frac{v}{v_+}; \quad L = \frac{n_R}{\rho R_{a,x,c}}; \quad \bar{\rho}_s = \frac{\rho_s}{\rho}; \quad \bar{\rho}_y = \frac{\rho_y}{\rho}; \quad \mathcal{A}_a = \frac{C_1 \lambda_a}{R_{a,x,c}}; \quad (4.42)
\]

\[
N_s = \frac{n_s R_{a,x,c}}{A_a \rho}; \quad N_{\ell} = \frac{n_{\ell} R_{a,x,c}}{A_a \rho}; \quad \bar{\eta} = \frac{\eta R_{a,x,c}}{\rho}; \quad \bar{t}_{\ell} = \frac{t_{\ell}}{R_{a,x,c}}; \quad (4.43)
\]

\[
\bar{\rho}_a = \frac{\rho_a}{\bar{p}}; \quad \bar{\rho}_g = \frac{\rho_g}{\bar{p}}; \quad \sigma_m = \frac{\sigma_m}{\bar{p}}; \quad \tau_0 = \frac{\eta \tau_0}{\eta_0 \bar{p}}; \quad \mathcal{E}_c = \frac{E_c}{E_r}; \quad \mathcal{E}_e = \frac{E_e}{E_r}; \quad (4.44)
\]

\[
\chi = \frac{\eta \alpha \beta}{\gamma}; \quad \Omega = \frac{1}{\alpha \beta \bar{p}} \sqrt{\frac{\rho R_{a,x,c}}{\gamma}}; \quad \psi_c = \frac{E_c}{H_c} \sqrt{\rho R_{a,x,c}}. \quad (4.45)
\]

These dimensionless parameters include the COF \( \mu \) and the lubrication number \( L \), defined by Schipper [113] (see also (4.1)). Parameter \( \psi_c \) is of similar shape as the plasticity index \( \psi \), introduced by Greenwood and Williamson [47],

\[
\psi = \frac{E_c}{H_c} \sqrt{\sigma \rho}. \quad (4.46)
\]

Furthermore, \( \bar{\eta} \) is similar to the constant \( \eta \sigma \beta \) in the theory of Greenwood and Williamson [47]. By means of the set of dimensionless parameters (4.40) to (4.45), the equations (4.27) and (4.30) can be written as follows,

\[
\mu = \mu_s \mathcal{P}_s + \mu_{\ell} \mathcal{P}_{\ell} + \mu_g \mathcal{P}_g, \quad (4.47)
\]

in which the dimensionless separation \( h \) is constrained by

\[
\mathcal{P}_s + \mathcal{P}_{\ell} + \mathcal{P}_g = 1, \quad (4.48)
\]

and where the dimensionless loads carried by the microcontacts and the liquid lubricant layer are given by

\[
\mathcal{P}_s = \frac{4}{3} \sqrt{2} \chi \Omega \mathcal{F}_{1,1}(h), \quad (4.49)
\]

and

\[
\mathcal{P}_{\ell} = \mathcal{F}_{\ell} - \sigma_m A_{r,\ell}, \quad (4.50)
\]

respectively. The dimensionless friction force due to the liquid lubricant layer is

\[
\mu_{\ell} = 2 \pi \bar{\eta} \sqrt{\frac{1}{\rho_s \rho_y} \mathcal{G}_{1,1}(h)}. \quad (4.51)
\]

In these equations, \( \mathcal{F}_{m,n}(h) \) and \( \mathcal{G}_{m,n}(h) \) are given by
\[ F_{m,n}(h) = \int_{h}^{\infty} e^{-s} \left( s \right)^{n} \phi(s) ds, \] (4.52)

and
\[ G_{m,n}(h) = \int_{h-\bar{r}}^{h} e^{-s} \left( s \right)^{n} \left( s - (h + \bar{r}) \right)^{n} \phi(s) ds, \] (4.53)

in which \( \phi \) is the dimensionless Gaussian summit height distribution,
\[ \phi(s) = \frac{1}{\sqrt{2\pi} C_{\sigma}} e^{-\frac{1}{2} \left( \frac{s}{C_{\sigma}} \right)^{2}}. \] (4.54)

Furthermore, the COF \( \mu_{g} \) due to the gaseous lubricant is given by
\[ \mu_{g} = \frac{SL}{\bar{p}_{g}(h+h_{\sigma}) + 2\Lambda_{a}\bar{p}_{a}}. \] (4.55)

The dimensionless real area of the microcontacts, the dimensionless plastic area of the microcontacts and the dimensionless wetted area of the asperities are
\[ A_{r,s}(h) = 2\pi \chi F_{0,1}(h), \] (4.56)
\[ A_{p,s}(h) = 2\pi \chi \left( F_{0,1}(h + \psi_{c}^{-2}) + \psi_{c}^{-2} F_{0,0}(h + \psi_{c}^{-2}) \right), \] (4.57)

and
\[ A_{r,\ell} = 2\pi \bar{\eta} \sqrt{\frac{1}{\bar{p}_{s}\bar{p}_{c}}} G_{0,1}(h), \] (4.58)

respectively. Furthermore,
\[ A_{r,s} + A_{r,\ell} + A_{g} = 1. \] (4.59)

Notice, that in these equations,
\[ 0 \leq A_{r,s}(h) \leq 1; \quad 0 \leq A_{r,\ell}(h) \leq 1; \quad 0 \leq A_{g}(h) \leq 1. \] (4.60)

The dimensionless number of microcontacts is given by
\[ N_{s} = \bar{\eta} F_{0,0}(h), \] (4.61)
and the number of wetted asperities is
\[ N_{\ell} = \bar{\eta} G_{0,0}(h). \] (4.62)

Finally, the dimensionless reduced elastic modulus is
\[ \frac{1}{E_{c}} = 1 - \nu_{e}^{2} + \frac{1 - \nu_{e}^{2}}{E_{e}}, \] (4.63)
and
\[ \bar{p}_x + \bar{p}_y = 1. \] 

(4.64)

From (4.47) and (4.55) it follows that for \( P_s \to 0 \) and \( \bar{t}_e \to 0 \) (i.e. \( A_{r,s} \to 0 \), \( A_{r,e} \to 0 \), and \( A_g \to 1 \)),
\[ \mu \approx \frac{S L}{\bar{p}_g (h + h_\sigma) + 2 \Lambda_a \bar{p}_a P_g}. \] 

(4.65)

Thus, for interfaces with \( \Lambda_a = 0 \),
\[ \mu \approx \frac{S L}{\bar{p}_g (h + h_\sigma)} P_g, \] 

(4.66)

or
\[ \mu \approx \frac{\eta_k v_-}{\bar{p}_g d_g}, \] 

(4.67)

which is the familiar COF formula for shear stress.

**4.7 Predictions of the Friction Model**

The friction model presented in the previous section provides a theoretical basis for studying certain parameters of the HDI. In this section the effects of these parameters on the generalised Strubeck curve will be discussed. In each case one or two parameters of a reference HDI will be varied and the resulting generalised Strubeck curves will be compared with the generalised Strubeck curve of the original reference HDI. Table 4.1 gives the operational parameters, the topographical parameters of the surfaces, and the properties and dimensions of the disk and the slider of the reference HDI. The topographical parameters are based on real measurements on sliders and disks. The reference HDI has a standard two rail taper-flat slider made of CaTiO3, which has been discussed in Chapter 3. The nominal (or apparent) pressure \( \bar{p} \) at the reference HDI is 26.8 kPa and the nominal (or apparent) contact area \( A_a \) is 4.13 mm².

**4.7.1 The Reference Head-Disk Interface**

The reduced elastic modulus \( E_c \) and the effective elastic modulus \( E_a \) of the reference HDI are shown in Fig. 4.14 as a function of the deformation depth \( d_a \). The reduced elastic modulus is affected by the elastic moduli of the overcoat and the magnetic layer only at very shallow depth. Figure 4.15 shows the computed generalised Strubeck curve and the separation \( d_s \) for the reference HDI. The lubrication numbers \( L_A \) and \( L_B \) that were derived from these curves are shown in Table 4.2 and are 0.347 and 0.121, respectively. Figure 4.16 shows the variation of the loads \( P_s, P_l \) and \( P_g \) carried by the microcontacts, the liquid lubricant layer and the gaseous lubricant, respectively, with the lubrication number \( L \). In the BL regime the load \( P_s \) is larger than the applied normal load \( P \) (i.e. \( P_s > 1 \)).
### Table 4.1. The parameters for the reference head-disk interface.

<table>
<thead>
<tr>
<th>Operational parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal load, $P$</td>
<td>95 mN (mass of slider excluded)</td>
</tr>
<tr>
<td>Velocity range, $v_-$ = $v_+$</td>
<td>0–200 m/s</td>
</tr>
<tr>
<td>COF BL regime, $\mu_s$</td>
<td>0.250</td>
</tr>
<tr>
<td>Gravitational constant, $g$</td>
<td>$9.807 \text{ m/s}^2$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Topographical parameters of the surfaces</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Curvature, $\rho_x$</td>
<td>200 $1/\text{m}$</td>
</tr>
<tr>
<td>Mean, $\overline{\sigma}$</td>
<td>5 nm</td>
</tr>
<tr>
<td>Curvature, $\rho_y$</td>
<td>200 $1/\text{m}$</td>
</tr>
<tr>
<td>Standard deviation, $\sigma_{s,c}$</td>
<td>5 nm</td>
</tr>
<tr>
<td>Curvature, $\rho_{x,y}$</td>
<td>0 $1/\text{m}$</td>
</tr>
<tr>
<td>CLA roughness, $R_{a,x,y}$</td>
<td>$\sigma_{a,c}/C_\sigma$</td>
</tr>
<tr>
<td>Constant, $\eta\sigma_{s,c}/\rho$</td>
<td>0.017</td>
</tr>
<tr>
<td>Summit height distribution</td>
<td>Gaussian</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Properties of the rigid disk and the slider</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Slider (i = r)</td>
<td>Material</td>
</tr>
<tr>
<td>CaoTiO$_3$</td>
<td>110</td>
</tr>
<tr>
<td>Overcoat (i = 1)</td>
<td>C</td>
</tr>
<tr>
<td>Magnetic layer (i = 2)</td>
<td>CoNiCr</td>
</tr>
<tr>
<td>Undercoat (i = 3)</td>
<td>NiP</td>
</tr>
<tr>
<td>Substrate (i = s)</td>
<td>AlMg</td>
</tr>
<tr>
<td>Effective hardness, $H$</td>
<td>5 (GPa)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Properties of the gaseous lubricant</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of gas</td>
<td>Air</td>
</tr>
<tr>
<td>Molecular mean free path, $\lambda_a$</td>
<td>64 nm</td>
</tr>
<tr>
<td>Ambient temperature</td>
<td>20 °C</td>
</tr>
<tr>
<td>Dynamic viscosity, $\eta_g$</td>
<td>171 mPas</td>
</tr>
<tr>
<td>Ambient pressure, $p_a$</td>
<td>$10^5$ Pa</td>
</tr>
<tr>
<td>Surface correcting coefficient, $C_1$</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Properties of the liquid lubricant</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of liquid</td>
<td>PFPE</td>
</tr>
<tr>
<td>‘Zero shear’ dynamic viscosity, $\eta_0$</td>
<td>195 mPas</td>
</tr>
<tr>
<td>Thickness, $t_l$</td>
<td>3 nm</td>
</tr>
<tr>
<td>Tensile strength, $\sigma_m$</td>
<td>$3 \cdot 10^5$ Pa</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Geometry of the slider</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimension</td>
<td>Height (mm)</td>
</tr>
<tr>
<td>Overall</td>
<td>$H_s = 0.800$</td>
</tr>
<tr>
<td>Rails</td>
<td>$h_r = 0.400$</td>
</tr>
<tr>
<td>Tapers</td>
<td>—</td>
</tr>
</tbody>
</table>
Fig. 4.14. The reduced elastic modulus $E_c$ and the effective elastic modulus $E_e$ as a function of the deformation depth $d_s$ for the reference head-disk interface.

Fig. 4.15. The generalised Striebeck curve and the separation $d_s$ for the reference head-disk interface.
**Table 4.2.** The lubrication numbers $L_A$ and $L_B$ for the reference head-disk interface and each varied parameter.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Lubricated Rigid Disk</th>
<th>Unlubricated Rigid Disk</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$L_A$</td>
<td>$L_B$</td>
</tr>
<tr>
<td>Nothing</td>
<td>0.347</td>
<td>0.121</td>
</tr>
<tr>
<td>$t_t$</td>
<td>$0.354 \pm 0.025$</td>
<td>$0.130 \pm 0.009$</td>
</tr>
<tr>
<td>$E_s$</td>
<td>$0.349 \pm 0.011$</td>
<td>$0.121 \pm 0.006$</td>
</tr>
<tr>
<td>$E_3$</td>
<td>$0.348 \pm 0.005$</td>
<td>$0.121 \pm 0.003$</td>
</tr>
<tr>
<td>$E_2$</td>
<td>$0.347 \pm 0.001$</td>
<td>$0.121 \pm 0.000$</td>
</tr>
<tr>
<td>$E_1$</td>
<td>$0.347 \pm 0.000$</td>
<td>$0.121 \pm 0.000$</td>
</tr>
<tr>
<td>$E_r$</td>
<td>$0.340 \pm 0.019$</td>
<td>$0.117 \pm 0.010$</td>
</tr>
<tr>
<td>$\rho_s = \rho_3$</td>
<td>$0.330 \pm 0.021$</td>
<td>$0.107 \pm 0.017$</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>$0.376 \pm 0.018$</td>
<td>$0.136 \pm 0.009$</td>
</tr>
<tr>
<td>$\sigma_{s,c}$</td>
<td>$0.337 \pm 0.016$</td>
<td>$0.116 \pm 0.006$</td>
</tr>
<tr>
<td>$P$</td>
<td>$0.079 - 1.051$</td>
<td>$0.024 - 0.365$</td>
</tr>
<tr>
<td>$A_{sh}$</td>
<td>$0.070 - 0.642$</td>
<td>$0.020 - 0.221$</td>
</tr>
<tr>
<td>Type of gas</td>
<td>$0.439 \pm 0.138$</td>
<td>$0.153 \pm 0.048$</td>
</tr>
</tbody>
</table>

**Fig. 4.16.** The loads $P_s$, $P_t$, and $P_g$ carried by the microcontacts, the liquid lubricant layer and the gaseous lubricant, respectively, as a function of the lubrication number $L$ for the reference head-disk interface.
The additional load arises from the adhesive or stiction force $P_\ell$ of the liquid lubricant layer, which attracts the slider towards the rigid disk. In the BL regime, the relative velocity $v_-$ between the slider and the rigid disk is too low to create a sufficient load capacity in the air film. In order that (4.48) be satisfied, the microcontacts have to carry both the applied normal load and the stiction force. Consequently, $\mu_\ell > \mu_b$ in the BL regime (see Fig. 4.16). Figure 4.17 shows the real area of contact $A_{r,s}$ of the microcontacts and the number of microcontacts $n_b$ as a function of the load $P_b$ carried by the microcontacts as it changes throughout the generalised Striebeck curve. The real area of contact and the number of microcontacts are proportional to the load carried by the microcontacts, which was also found by Greenwood and Williamson [47] (the real area of contact is also independent of the nominal area of contact). Thus the real mean contact pressure $\bar{p}_b$ in the microcontacts, which is defined in (4.20), is constant throughout the BL and ML regimes. Furthermore, the real area of contact is very small compared to the nominal area of contact (i.e. $A_{r,s} < 0.01$), and no plastic deformation occurs at the microcontacts (i.e. $A_{p,s} = 0$). Figure 4.18 shows the wetted area $A_{r,\ell}$ and the number of wetted asperities $n_\ell$ as a function of the load $P_\ell$ carried by the liquid lubricant layer. The wetted area and the number of wetted asperities are proportional to the load carried by the liquid lubricant layer. Hence, the real mean pressure $\bar{p}_\ell = P_\ell/A_\ell$ below the wetted asperities is constant throughout the BL and the ML regime. Although not necessarily, the number of microcontacts in the case of the reference HDI is less than the number of wetted asperities, throughout the BL and ML regime.

In the following sections the effects of the thickness of the liquid lubricant layer, the elastic moduli of the rigid disk and the slider, the shape of the asperities, the surface roughness, the applied normal load, the nominal area of contact and the type of gas applied, on the generalised Striebeck curve of the reference HDI, will be investigated.

### 4.7.2 Thickness of the Liquid Lubricant Layer

The effect of the amount of liquid lubricant at the rigid disk on the generalised Striebeck curve has been studied by varying the thickness $t_\ell$ of the liquid lubricant layer. Five generalised Striebeck curves have been computed each with a layer thickness $t_\ell$ of 0, 1.5, 3, 4.5 and 6 nm, respectively. The curves and the separation $d_b$ are shown in Fig. 4.19. In the BL and ML regime the COF increases with liquid lubricant layer thickness and if the rigid disk is un lubricated ($t_\ell = 0$) then $\mu_\ell = \mu_b$.

Figure 4.20 shows the effect of the thickness $t_\ell$ of the liquid lubricant layer thickness on the loads $P_s$, $P_\ell$ and $P_b$, carried by the microcontacts, the liquid lubricant layer and the gaseous lubricant, respectively. A peak appears in the load $P_b$ near the transition ML to AL, which height increases with the thickness of the liquid lubricant layer. The peak occurs when the load $P_s$ becomes smaller than the stiction force $-P_\ell$. In that case the microcontacts are unable to carry both the applied normal load $P$ and the stiction force $P_\ell$. As a result the gaseous
**Fig. 4.17.** The variation of the real area of contact $A_{r,s}$ and the number of microcontacts $n_s$ with the load $P_s$ carried by the microcontacts for the reference head-disk interface.

**Fig. 4.18.** The variation of the wetted area $A_{r,\ell}$ and the number of wetted asperities $n_\ell$ with the load $P_\ell$ carried by the liquid lubricant layer for the reference head-disk interface.
lubricant has to carry part of the load too. Beyond the transition ML to AL, the load \( P_s \) and the stiction force \( P_t \) are zero and the gaseous lubricant has to carry the applied normal load by itself.

Figure 4.21 shows the variation of the real area of contact \( A_{r,s} \) and the number of microcontacts \( n_s \) with the load \( P_s \) carried by the microcontacts. It follows from this figure that the thickness of the liquid lubricant layer does not affect the real area of contact \( A_{r,s} \) and the number of microcontacts (compare Figs. 4.17 and 4.21). Furthermore, the real mean contact pressure \( \overline{p}_s \) is constant throughout the BL and ML regime and independent of the thickness \( t_s \) of the liquid lubricant layer. The variation of the wetted area \( A_{r,t} \) and the number of wetted asperities \( n_t \) with the stiction force \( P_t \) is shown in Fig. 4.22, from which it follows that the thickness of the liquid lubricant layer does not affect the wetted area and the mean pressure \( \overline{p}_t \) below the wetted asperities, but only the number of wetted asperities. Hence, the increase of the COF in the BL regime with the thickness of the liquid lubricant layer must be due to an increase in the number of wetted asperities.

In all cases of this Section 4.7, the friction force \( F_f \) is negligible. Consequently, the liquid lubricant layer is basically free of shear. Apparently, (4.32) is valid throughout the BL regime. The large COF in the BL regime is due to the stiction force instead of the shear stress in the liquid lubricant layer. With a thicker liquid lubricant layer, more asperities are immersed in this layer, resulting in a higher stiction force. The microcontacts have to carry this extra load, causing a COF in the BL regime that increases with the thickness of the liquid lubricant layer. The lubrication numbers \( L_A \) and \( L_B \) that were derived from the generalised Strubeck curves are shown in Table 4.2. It can be seen that the transitions are somewhat affected by the liquid lubricant layer thickness.

### 4.7.3 Elastic Moduli

The effect of the reduced elastic modulus \( E_c \) on the generalised Strubeck curve has been studied by varying the elastic moduli of the substrate, the undercoat, the magnetic layer, the overcoat and the slider, respectively.

The elastic modulus \( E_s \) of the substrate has been varied from 10, 50, 200 to 800 GPa. In Fig. 4.23 the reduced elastic modulus \( E_c \) and the effective elastic modulus \( E_a \) are shown as a function of the deformation depth \( d_s \). Figure 4.24 shows the effect of the elastic modulus of the substrate on the generalised Strubeck curve and the separation \( d_s \) for the reference HDI. In the BL regime the COF decreases with the elastic modulus of the substrate. Figure 4.25 shows the effect of the elastic modulus of the substrate on the generalised Strubeck curve and the separation \( d_s \) for the reference HDI with an un lubricated rigid disk. The elastic modulus \( E_s \) has been varied considerably, but the generalised Strubeck curve only shifts slightly to the right. The real area of contact \( A_{r,s} \) and the number of microcontacts \( n_s \) are again proportional to the load \( P_s \) carried by the microcontacts (see Fig. 4.26). The real area of contact decreases with increasing elastic modulus of the substrate. In other words, although the real mean contact pressure \( \overline{p}_s \) at the
**Fig. 4.19.** The effect of the thickness $t_L$ of the liquid lubricant layer on the generalised Strubeck curve and the separation $d_s$ for the reference head-disk interface. The numbers at the curves indicate the thickness of the layer in nm.

**Fig. 4.20.** The effect of the thickness $t_L$ of the liquid lubricant layer on the loads $P_s$, $P_f$ and $P_g$, carried by the microcontacts, the liquid lubricant layer and the gaseous lubricant, respectively, of the reference head-disk interface. The numbers at the curves indicate the thickness of the layer in nm.
4. A Friction Model for Head-Disk Interfaces

![Graph](image)

**Fig. 4.21.** The effect of the thickness $t_l$ of the liquid lubricant layer on the real area of contact $A_{r,s}$ and the number of microcontacts $n_s$ for the reference head-disk interface. The numbers at the curves indicate the thickness of the layer in nm.

Microcontacts is constant throughout the entire BL and ML regime, it rises with the elastic modulus $E_s$ of the substrate. The elastic modulus of the substrate does not affect the wetted area and the number of wetted asperities (compare Figs. 4.18 and 4.27). Hence, the mean pressure $\bar{p}_l$ below a wetted asperity is also constant throughout the entire BL and ML regime and is independent of the elastic modulus of the substrate. With a liquid lubricant layer present, the stiction force reduces with increasing elastic modulus $E_s$ of the substrate. With a higher elastic modulus the microcontacts can carry more load before a certain amount of deformation occurs. Consequently, less microcontacts are required for carrying the load and thus the separation $d_s$ increases. With a larger separation, less asperities are immersed in the liquid lubricant layer, thus the stiction force will be lower.

It should be noted however, that the effect of the elastic modulus $E_s$ of the substrate only has a significant effect on the generalised Strubeck curve if its value is much lower than that of most materials. Furthermore, the elastic modulus of the substrate only affects the value of the COF in the BL and ML regimes but the lubrication transitions $L_A$ and $L_B$ remain unchanged (see Table 4.2).

A similar but less pronounced effect has been obtained by varying the elastic modulus $E_3$ of the undercoat. In Fig. 4.28 the effect of the elastic modulus $E_3$ on the reduced elastic modulus $E_c$ and the effective elastic modulus $E_e$ is shown. The elastic modulus $E_3$ has been varied from 10, 50, 200, to 800 GPa, respectively.
Figure 4.29 shows the effect of the elastic modulus $E_3$ on the generalised Striebeck curve and the separation $d_s$ for the reference HDI and Fig. 4.30 shows its effect for the reference HDI and an unhelubricated rigid disk. Figure 4.31 shows the real area of contact $A_{r,s}$ and the number of microcontacts $n_s$ as a function of the load $P_s$ carried by the microcontacts. The wetted area $A_{r,t}$ and the number of wetted asperities are again independent of the elastic modulus $E_3$ and are therefore not shown here. The lubrication numbers $L_A$ and $L_B$ are shown in Table 4.2.

Hardly any variation in the generalised Striebeck curve of the reference HDI could be found when the elastic modulus $E_2$ or $E_1$ of the magnetic layer and the overcoat, respectively, were changed. These layers are too thin to have a significant effect on the reduced elastic modulus, and thus, they have no influence on the generalised Striebeck curve. Therefore, no figures have been given here for these cases. The lubrication numbers $L_A$ and $L_B$ however are given in Table 4.2.

In Fig. 4.32 the effect of the elastic modulus $E_r$ of the slider on the reduced elastic modulus $E_c$ is shown. Figure 4.33 shows the generalised Striebeck curves and the separation $d_s$ of the reference HDI, in case the elastic modulus $E_r$ of the slider has been varied from 10, 50, 200 to 800 GPa. Figure 4.34 shows the generalised Striebeck curves and the separation $d_s$ in case of the reference HDI with an unhelibrated rigid disk. Figure 4.35 shows the effect of the elastic modulus $E_r$ of the slider on the real area of contact $A_{r,s}$ and the number of microcontacts $n_s$. The wetted area $A_{r,t}$ and the number of wetted asperities $n_s$ are independent of

![Graph](image)

**Fig. 4.22.** The effect of the thickness $t_l$ of the liquid lubricant layer on the wetted area $A_{r,t}$ and the number of wetted asperities $n_t$ for the reference head-disk interface. The numbers at the curves indicate the thickness of the layer in nm.
Fig. 4.23. The reduced elastic modulus $E_c$ and the effective elastic modulus $E_e$ as a function of the deformation depth $d_s$ for various values of the elastic modulus $E_s$ of the substrate. The numbers at the curves indicate the elastic modulus of the substrate in GPa.

Fig. 4.24. The effect of the elastic modulus $E_s$ of the substrate on the generalised Strubeck curve and the separation $d_s$ for the reference head-disk interface. The numbers at the curves indicate the elastic modulus of the substrate in GPa.
Fig. 4.25. The effect of the elastic modulus $E_s$ of the substrate on the generalised Strubeck curve and the separation $d_s$ for the reference head-disk interface with an un lubricated rigid disk. The numbers at the curves indicate the elastic modulus of the substrate in GPa.

Fig. 4.26. The effect of the elastic modulus $E_s$ of the substrate on the real area of contact $A_{r,s}$ and the number of microcontacts $n_s$ for the reference head-disk interface. The numbers at the curves indicate the elastic modulus of the substrate in GPa.
the elastic modulus $E_s$ and are therefore not shown here. The effect of the elastic modulus of the slider on the generalised Striebeck curve is the same as that of the elastic modulus of the substrate. However, it is more pronounced, since in case of the elastic modulus of the substrate, the effect is obscured by the undercoat. A peak occurs in the load $P_g$ carried by the gaseous lubricant layer, where $P_s < P_t$ (see Fig. 4.36). The lubrication numbers $L_A$ and $L_B$ are given in Table 4.2; they are slightly affected by the elastic modulus of the slider.

4.7.4 Topography of the Surfaces

The effect of the topography of the surfaces on the generalised Striebeck curve of the reference HDI has been studied in two ways. Firstly, by varying the curvatures of the asperities and secondly by varying the standard deviation of the summit heights.

The curvatures $\rho_x$ and $\rho_y$ of the asperities have been varied from 100, 500, 2000 to 8000 $\text{1/m}$, which correspond to radii of 10, 2, 1/2 and 1/8 mm, respectively. With these variations the product of $\eta \sigma_{s,c}/\rho$ varied from 0.0390, 0.0068, 0.0017, to 0.0004, respectively. The curvatures $\rho_x$ and $\rho_y$ have been varied equally (i.e. $\rho_x = \rho_y$), such that the area of each microcontact becomes circular. Figure 4.37 shows the generalised Striebeck curves and the separation $d_s$ for the reference HDI with varied curvature $\rho_x = \rho_y$. Figure 4.38 shows the generalised

![Image](https://example.com/image.png)

**Fig. 4.27.** The effect of the elastic modulus $E_s$ of the substrate on the wetted area $A_{r,t}$ and the number of wetted asperities $n_t$ for the reference head-disk interface. The numbers at the curves indicate the elastic modulus of the substrate in GPa.
Fig. 4.28. The reduced elastic modulus $E_c$ and the effective elastic modulus $E_v$ as a function of the deformation depth $d_a$ for various values of the elastic modulus $E_3$ of the undercoat. The numbers at the curves indicate the elastic modulus of the undercoat in GPa.

Fig. 4.29. The effect of the elastic modulus $E_3$ of the undercoat on the generalised Strubeck curve and the separation $d_s$ for the reference head-disk interface. The numbers at the curves indicate the elastic modulus of the undercoat in GPa.
Fig. 4.30. The effect of the elastic modulus $E_3$ of the undercoat on the generalised Strubeck curve and the separation $d_s$ for the reference head-disk interface with an un lubricated rigid disk. The numbers at the curves indicate the elastic modulus of the undercoat in GPa.

Fig. 4.31. The effect of the elastic modulus $E_3$ of the undercoat on the real area of contact $A_{r,s}$ and the number of microcontacts $n_s$ for the reference head-disk interface. The numbers at the curves indicate the elastic modulus of the undercoat in GPa.
Fig. 4.32. The reduced elastic modulus $E_c$ and the effective elastic modulus $E_o$ as a function of the deformation depth $d_h$ for various values of the elastic modulus $E_r$ of the slider. The numbers at the curves indicate the elastic modulus of the slider in GPa.

Fig. 4.33. The effect of the elastic modulus $E_r$ of the slider on the generalised Strubeck curve and the separation $d_h$ for the reference head-disk interface. The numbers at the curves indicate the elastic modulus of the slider in GPa.
Fig. 4.34. The effect of the elastic modulus $E_t$ of the slider on the generalised Strubeck curve and the separation $d_s$ for the reference head-disk interface with an un lubricated rigid disk. The numbers at the curves indicate the elastic modulus of the slider in GPa.

Fig. 4.35. The effect of the elastic modulus $E_t$ of the slider on the real area of contact $A_{rs}$ and the number of microcontacts $n_s$ for the reference head-disk interface. The numbers at the curves indicate the elastic modulus of the slider in GPa.
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Fig. 4.36. The effect of the elastic modulus $E_s$ of the slider on the loads $P_s$, $P_l$ and $P_g$, carried by the microcontacts, the liquid lubricant layer and the gaseous lubricant, respectively, of the reference head-disk interface. The numbers at the curves indicate the elastic modulus of the slider in GPa.

Striebeck curves and the separation $d_s$ for the reference HDI with an unlubricated rigid disk and varied curvature $\rho_x = \rho_y$. Figure 4.39 shows the real area of contact $A_{r,s}$ and the number of microcontacts $n_s$ as a function of the load $P_s$ carried by the microcontacts. Figure 4.40 shows the wetted area $A_{r,l}$ and the number of wetted asperities $n_l$ as a function of the load $P_l$ carried by the liquid lubricant layer. According to (4.8) and (4.11) the real area of contact $A_{r,s}$ changes in inverse proportion to the curvature $\rho$, while the load $P_s$ changes in inverse proportion to the square of the curvature $\rho$. The result is that the real mean contact pressure $\overline{p}_s$ at the microcontacts increases with the curvatures $\rho_x = \rho_y$. Consequently, the asperities at the microcontacts deform more and the separation $d_s$ increases as the curvatures $\rho_x = \rho_y$ increase. As a result the number of microcontacts $n_s$ increases with the curvatures $\rho_x = \rho_y$. Also the wetted area, the friction force and the stiction force decrease with the curvatures $\rho_x = \rho_y$, but each in the same proportion (see (4.14), (4.15) and (4.16)). Consequently, the mean pressure $\overline{p}_l$ at the wetted asperities is constant. Furthermore, the wetted area $A_{r,l}$ is proportional to the load $P_l$. A smaller separation leads to more asperities immersed into the liquid lubricant layer, but according to (4.15) and (4.16) the friction force and the stiction force reduce with the curvatures $\rho_x = \rho_y$. Thus, although the larger number of wetted asperities, the stiction force reduces, and, consequently, the COF in the BL and ML regime reduces with increasing curvatures.
Fig. 4.37. The effect of the curvatures $\rho_x$ and $\rho_y$ of the asperities on the generalised Stroubeck curve and the separation $d_s$ for the reference head-disk interface. The numbers at the curves indicate the curvatures $\rho_x = \rho_y$ of the asperities in 1/m.

Fig. 4.38. The effect of the curvatures $\rho_x$ and $\rho_y$ of the asperities on the generalised Stroubeck curve and the separation $d_s$ for the reference head-disk interface with an un lubricated rigid disk. The numbers at the curves indicate the curvatures $\rho_x = \rho_y$ of the asperities in 1/m.
Fig. 4.39. The effect of the curvatures $\rho_x$ and $\rho_y$ of the asperities on the real area of contact $A_{rs}$ and the number of microcontacts $n_s$ for the reference head-disk interface. The numbers at the curves indicate the curvatures $\rho_x = \rho_y$ of the asperities in 1/m.

The lubrication numbers $L_A$ and $L_B$ at which the transitions ML to BL and ML to AL occur are listed in Table 4.2; they are not affected by the curvatures of the asperities.

The effect of the surface topography has been studied further by varying the curvature of the asperities in one direction only. In this case the area of each microcontact is elliptical. The curvature $\rho_x$ of the asperities has been kept constant at 200 1/m (i.e. a radius of 5 mm) while the curvature $\rho_y$ of the asperities has been varied from 10, 500, 2000, to 8000 1/m, corresponding to asperity radii of 10, 2, 1/2, and 1/8 mm, respectively. Again, the product of $\eta\sigma_{s,e}/\rho$ varied from 0.0226, 0.0097, 0.0031 to 0.0008. Figure 4.41 shows the generalised Strubeck curves and the separation $d_s$ for the reference HDI and Fig. 4.42 shows the generalised Strubeck curves and the separation $d_s$ for the reference HDI with an un lubricated rigid disk. Figure 4.43 shows the real area of contact $A_{rs}$ and the number of microcontacts $n_s$ as a function of the load $P_s$ carried by the microcontacts.

Again, a larger curvature $\rho_y$ reduces the real area of contact, but, unlike the previous case, it gives a larger separation in the BL regime. This results in a smaller stiction force and thus a lower load to be carried by the microcontacts. As the separation $d_s$ increases with the curvature $\rho_y$, the number of microcontacts $n_s$ and the number of wetted asperities $n_t$ decrease. Consequently, the stiction force and the COF in the BL regime will be lower. In case of the reference HDI
**Fig. 4.40.** The effect of the curvatures $\rho_x$ and $\rho_y$ of the asperities on the wetted area $A_{r,t}$ and the number of wetted asperities $n_t$ for the reference head-disk interface. The numbers at the curves indicate the curvatures $\rho_x = \rho_y$ of the asperities in $1/m$.

**Fig. 4.41.** The effect of the curvature $\rho_y$ of the asperities on the generalised Stribeck curve and the separation $d_0$ for the reference head-disk interface. The numbers at the curves indicate the curvature of the asperities in $1/m$. 
**Fig. 4.42.** The effect of the curvature $\rho_z$ of the asperities on the generalised Strubeck curve and the separation $d_s$ for the reference head-disk interface with un lubricated rigid disk. The numbers at the curves indicate the curvature of the asperities in $1/m$.

**Fig. 4.43.** The effect of the curvature $\rho_y$ of the asperities on the real area of contact $A_{r,s}$ and the number of microcontacts $n_s$ for the reference head-disk interface. The numbers at the curves indicate the curvature of the asperities in $1/m$. 
Fig. 4.44. The effect of the curvature \( \rho_y \) of the asperities on the wetted area \( A_{r,t} \) and the number of wetted asperities \( n_t \) for the reference head-disk interface. The numbers at the curves indicate the curvature of the asperities in 1/m.

Fig. 4.45. The effect of the standard deviation \( \sigma_{s,c} \) of the asperities on the generalised Stribeck curve and the separation \( d_s \) for the reference head-disk interface. The numbers at the curves indicate the standard deviation of the asperities in m.
Fig. 4.46. The effect of the standard deviation $\sigma_{s,c}$ of the asperities on the generalised Strubeck curve and the separation $d_s$ for the reference head-disk interface with an un lubricated rigid disk. The numbers at the curves indicate the standard deviation of the asperities in m.

Fig. 4.47. The effect of the standard deviation $\sigma_{s,c}$ on the real area of contact $A_{r,s}$ and the number of microcontacts $n_s$ for the reference head-disk interface. The numbers at the curves indicate the standard deviation of the asperities in m.
with an un lubricated rigid disk, a larger curvature \( \rho_y \) results in a shift of the generalised Stribeck curves towards the left.

The effect of the surface topography on the generalised Stribeck curves has also been studied by varying the standard deviation \( \sigma_{s,c} \) of the Gaussian height distribution of the surface topography from 4, 6, 10, 16, 25 to 40 nm, while the mean value \( \bar{\sigma} \) of the heights has been kept to \( 1.0\sigma_{s,c} \) and the combined CLA roughness to \( R_{a,x,c} = \sigma_{s,c}/C_\tau \). In these cases the product of \( \eta\sigma_{s,c}/\rho \) varied from 0.0136, 0.0203, 0.0339, 0.0542, 0.0848 to 0.1356. Figure 4.45 shows the generalised Stribeck curves and the separation \( d_s \) for the reference HDI in case the standard deviation has been varied. Figure 4.46 shows the generalised Stribeck curves and the separation \( d_s \) for the reference HDI with an un lubricated rigid disk. Figure 4.47 shows the real area of contact \( A_{r,s} \) and the number of micro contacts \( n_o \) as a function of the load \( P_s \) carried by the micro contacts. The real mean contact pressure \( p_o \) at the micro contacts decreases and the separation in the BL regime increases with increasing standard deviation. This means that with a higher standard deviation less asperities are in contact with the liquid lubricant layer. Figure 4.48 shows the wetted area \( A_{r,t} \) and the number of wetted asperities \( n_t \) as a function of the load \( P_t \) carried by the liquid lubricant layer. The wetted area and the number of wetted asperities are independent of the standard deviation \( \sigma_{s,c} \) (compare Figs. 4.18 and 4.48). The stiction force decreases with increasing separation. Consequently, the micro contacts have to carry less load.

Fig. 4.48. The effect of the standard deviation \( \sigma_{s,c} \) on the wetted area \( A_{r,t} \) and the number of wetted asperities \( n_t \) for the reference head-disk interface. The numbers at the curves indicate the standard deviation of the asperities in \( \text{m} \).
Therefore, the COF in the BL regime reduces with increasing standard deviation (or a higher surface roughness $R_{a,z,c}$).

### 4.7.5 Applied Normal Load

The effect of the applied normal load on the generalised Striebeck curve has been studied by varying the applied normal load $P$ from 5 to 795 mN in case of the reference HDI and from 5 to 1995 mN in case of the reference HDI with an un lubricated rigid disk. Figures 4.49 and 4.50 show some of the computed generalised Striebeck curves and the separation $d_s$ for the reference HDI and the reference HDI with an un lubricated rigid disk, respectively. In Fig. 4.51 the real area of contact $A_{r,s}$ and the number of microcontacts $n_s$ are shown as a function of the load $P_s$ carried by the microcontacts. Figure 4.52 shows the wetted area $A_{t,t}$ and the number of wetted asperities $n_t$ as a function of the load $P_t$ carried by the liquid lubricant layer. It follows that the real area of contact and the number of microcontacts are proportional to the load carried by the microcontacts and the real mean contact pressure $\bar{p}_s$ is independent of the load $P$. If the dimensional real area $A_{r,s}$ were plotted against the dimensional load $P_s$, the curves would overlap each other. The same is true for the number of microcontacts $n_s$, plotted against the dimensional load $P_s$. In order to carry the higher load, the microcontacts deform more and the separation $d_s$ reduces. This means that the number of microcontacts increases with load and the generalised Striebeck curve shifts towards the left. This occurs with lubricated as well as with un lubricated rigid disks. With the reference HDI the stiction force and the separation decrease with increasing normal load. A lower stiction force results in a lower load to be carried by the microcontacts, and, consequently, in a lower COF in the BL regime. Also the lubrication number $L_A$ decreases from 1.1 to 0.1 when the normal load increases from 5 mN to 794 mN.

From the generalised Striebeck curves shown in Figs. 4.49 and 4.50, the transitions at $L_A$ and $L_B$ have been determined. In Figs. 4.53 and 4.54 the lubrication transition diagrams are shown for the reference HDI and the reference HDI with an un lubricated rigid disk, respectively. It follows that the transitions at $L_A$ and $L_B$ both depend on the nominal pressure $\bar{p}$ in the contact, and that they can be written as

\[
L_A = \lambda_{A,1} \bar{p}^{\lambda_{A,2}}, \tag{4.68}
\]

and

\[
L_B = \lambda_{B,1} \bar{p}^{\lambda_{B,2}}, \tag{4.69}
\]

in which $\lambda_{A,1}$, $\lambda_{A,2}$, $\lambda_{B,1}$ and $\lambda_{B,2}$ are constants, which, for the reference HDI, are given by

\[
\lambda_{A,1} \approx 36.6; \quad \lambda_{A,2} \approx -0.5; \quad \lambda_{B,1} \approx 14.8, \quad \text{and} \quad \lambda_{B,2} \approx -0.5. \tag{4.70}
\]
Fig. 4.49. The effect of the applied normal load $P$ at the slider on the generalised Striebeck curve and the separation $d_s$ for the reference head-disk interface. The numbers at the curves indicate the applied normal load in mN.

Fig. 4.50. The effect of the applied normal load $P$ at the slider on the generalised Striebeck curve and the separation $d_s$ for the reference head-disk interface with an un lubricated rigid disk. The numbers at the curves indicate the applied normal load in mN.
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Fig. 4.51. The effect of the applied normal load $P$ on the real area of contact $A_{r,s}$ and the number of microcontacts $n_s$ for the reference head-disk interface. The numbers at the curves indicate the applied normal load in mN.

### 4.7.6 Nominal Area of Contact

The effect of the nominal area of contact $A_n$ on the generalised Striebeck curve has been studied by varying all the dimensions of the slider of the reference HDI with a certain factor. The following factors have been applied: 1/4, 1/2, 1 and 2. This resulted in a variation in nominal contact area $A_n$ of 0.22, 0.89, 3.55 and 14.19 mm$^2$, respectively. It must be noted, however, that (4.38) has been used for the load carried by the gaseous film, which was determined for the standard two rail taper-flat slider only. With this warning, the generalised Striebeck curves and the separation $d_n$ for the reference HDI are shown in Fig. 4.55 and those for the reference HDI with an un lubricated rigid disk in Fig. 4.56. Figure 4.57 shows the real area of contact $A_{r,s}$ and the number of microcontacts $n_s$ as a function of the load $P_s$ carried by the microcontacts. It follows that the real area of contact is independent of the nominal area of contact, which was already pointed out by Greenwood and Williamson [47]. If the dimensional real area of contact $A_{r,n}$ were plotted against the dimensional load $P_n$, the curves would overlap each other. This is also true for the number of microcontacts $n_s$ plotted against the dimensional load $P_s$. In both cases (lubricated and un lubricated rigid disks) the generalised Striebeck curve shifts towards the right when the nominal area and the separation increases. In addition, the COF increases in the BL and ML regime in case of a lubricated rigid disk.
Fig. 4.52. The effect of the applied normal load $P$ on the wetted area $A_{r,t}$ and the number of wetted asperities $n_{t}$ for the reference head-disk interface. The numbers at the curves indicate the applied normal load in mN.

Fig. 4.53. Lubrication transition diagram for the reference head-disk interface, in case the applied normal load $P$ has been varied.
**Fig. 4.54.** Lubrication transition diagram for the reference head-disk interface with an unlubricated rigid disk, in case the applied normal load $P$ has been varied.

**Fig. 4.55.** The effect of the nominal area of contact $A_n$ on the generalised Strubeck curve and the separation $d_s$ for the reference head-disk interface. The numbers at the curves indicate the factor with which the slider dimensions have been multiplied.
Fig. 4.56. The effect of the nominal area of contact $A_\mathrm{n}$ on the generalised Strubeck curve and the separation $d_s$ for the reference head-disk interface with an un lubricated rigid disk. The numbers at the curves indicate the factor with which the slider dimensions have been multiplied.

Fig. 4.57. The effect of the nominal area of contact $A_\mathrm{n}$ on the real area of contact $A_\mathrm{rs}$ and the number of microcontacts $n_s$ for the reference head-disk interface. The numbers at the curves indicate the factor with which the slider dimensions have been multiplied.
4.7.7 Type of Gas Applied

The values for the molecular mean free path $\lambda$ and the dynamic viscosity $\eta_g$ at ambient conditions are in general different for each gas. In order to study the effect of the type of gas on the generalised Strubeck curves, three types of gas have been chosen: Air, Helium, and Neon (Helium, has been proposed as a gaseous lubricant for HDIs, see, for example, Bouchard and Talke [18], Ohkubo et al. [105] and Shueh [119]). The molecular mean free path for air, Helium and Neon at ambient conditions is 64, 187 and 131 nm respectively. The dynamic viscosity $\eta_g$ at ambient conditions is 171, 186 and 297 mPas, respectively. Figures 4.61 and 4.62 show the generalised Strubeck curves and the separation $d_s$ for the reference HDI and the reference HDI with an un lubricated rigid disk, respectively. Obviously, the type of gas has no effect on the COF in the BL regime, but only on the COF in the AL regime. Note, that the fitted air film thickness equation is based on the viscosity of air only!

The type of gas already starts to affect the generalised Strubeck curve in the ML regime. The separation $d_s$ is remarkably the same for air and Neon. In the BL and ML regime the generalised Strubeck curve is mainly determined by the asperities. Therefore, the real area of contact is not affected by the type of gas. The deviant behaviour of Neon is due to the different dynamic viscosity of Neon compared to that of air and Helium.

![Diagram](image)

**Fig. 4.58.** The effect of the nominal area of contact $A_a$ on the wetted area $A_{r,t}$ and the number of wetted asperities $n_t$ for the reference head-disk interface. The numbers at the curves indicate the factor with which the slider dimensions have been multiplied.
Fig. 4.59. Lubrication transition diagram for the reference head-disk interface, in case the nominal area of contact $A_a$ has been varied.

Fig. 4.60. Lubrication transition diagram for the reference head-disk interface with an un lubricated rigid disk, in case the nominal contact area $A_a$ has been varied.
**Fig. 4.61.** The effect of the type of gas on the generalised Stißeck curve and the separation $d_s$ for the reference head-disk interface.

**Fig. 4.62.** The effect of the type of gas on the generalised Stißeck curve and the separation $d_s$ for the reference head-disk interface with an un lubricated rigid disk.
4.7.8 Experimental Results from the Literature

Applying the friction model to experiments found in the literature, it follows that the computed generalised Strubeck curves follow the experimental curves reasonably well, see Fig. 4.63. The experiments usually show a somewhat steeper slope in the ML regime than the computed Strubeck curves.

4.8 Conclusions

In this chapter a friction model for HDIs has been developed for prediction of the effects of certain parameters on the generalised Strubeck curve of HDIs. The following conclusions can be drawn:

- The generalised Strubeck curve shifts towards the left and the stiction force at an HDI decreases with increasing nominal pressure $\bar{p}$.

- The slopes of the transition lines $L_A$ and $L_B$ in the lubrication transition diagram of an HDI are less than the slopes of the transition lines in the lubrication transition diagram of Schipper [113].

- As a result of shear thinning in the liquid lubricant layer of an HDI, the friction force in this layer may be neglected. However, a very high stiction force can be generated in the liquid lubricant layer, especially when the thickness of the layer is in the order of the surface roughness of the HDI and the applied normal load is low. This could give serious problems in
the future, when the applied normal load, the mass and the dimensions of the slider will be reduced, in an attempt to move to CR. Fortunately, the results in this chapter indicate that the stiction force decreases with decreasing nominal area of contact. In order to reduce the contribution of the stiction force to the friction force, future HDIs should have a thinner liquid lubricant layer, and they should be applied with a higher nominal pressure. With a higher nominal pressure, the transition from ML to AL takes place at a higher velocity (e.g. a higher take-off velocity). The nominal pressure can be increased by reducing the dimensions of the slider and/or by increasing the applied normal load. However, the total friction force increases with increasing normal load, and, therefore, an increase in normal load is not recommended. In fact, the normal load, in combination with the dimensions of the slider, should be carefully chosen, not only for the purpose of AL but also to minimise the friction in the BL and ML regimes.

- The stiction force at an HDI can be reduced by enlarging the curvature and the standard deviation of the asperities. In other words, a higher surface roughness of the HDI can reduce the stiction force and thus the COF in the BL regime. However, the magnetic recording process requires that the surfaces of an HDI are as smooth as possible (see Chapter 2).

- The elasticity of the solid surfaces of an HDI is mainly determined by the elastic modulus of the substrate, the undercoat and the slider. If one or more of these moduli increase the stiction force reduces and in case of an unlubricated rigid disk the generalised Strubeck curve shifts towards the left. However, the effect of the elastic modulus on the generalised Strubeck curve is only significant if its value is much lower than that of most materials. The elastic moduli of the overcoat and the magnetic layer do not have an effect on the generalised Strubeck curve of an HDI, due to their thinness.

- Applying the friction model to experiments found in the literature, it follows that the computed generalised Strubeck curves follow the experimental curves reasonably well.

To verify the friction model friction experiments have been carried out. The instrument that has been used for these experiments will be discussed in the next chapter.
The friction experiments have been carried out on a modified instrument, the so called NanoTest 550. The NanoTest 550 (see Smith [121]) is an instrument for measuring the mechanical properties (such as the hardness and the elastic modulus) of surface layers and thin films. The instrument is also capable of measuring the surface topography and the 'critical load' in scratch tests. The principle of the instrument is based on a pendulum. With a probe (an indenter or stylus) mounted onto this pendulum, the instrument measures simultaneously the load and the movement of the probe, while the probe is in contact with the specimen surface. In order to use the instrument as a pin-on-disk type apparatus, for the measurement of the friction force at HDIs, the NanoTest 550 has been modified. The main modifications are: a rebuild pendulum, an additional rotating motor attachment and a miniaturised piezo-resistive sensor for the measurement of the friction force.

In this chapter, the instrument and its modifications will be described. Firstly, the operating principle of the instrument will be explained. Then, some of the sensors and actuators and the electronics of the instrument will be described. The resolutions of the signals of the sensors are very high (sub nanometre and micro Newton range). Therefore, the operating environment, the forces acting on the pendulum and the compliance of the pendulum are of crucial importance. In order to assess these forces, a mechanical model of the pendulum will be presented and the dynamics of the pendulum will be analysed.

5.1 Operating Principle

Figures 5.1 and 5.2 show schematically the modified NanoTest 550 instrument and Fig. 5.3 illustrates its operating principle. The instrument consists basically of a pendulum beam on which a probe \[ \text{I} \] is mounted that can be brought into contact with a specimen \[ \text{S} \], attached to a specimen holder. The specimen holder is mounted on an XYZ positioning table. The table can translate in all three directions \( X, Y \) and \( Z \) by means of three linear DC motors which are operating independently of each other. The pendulum beam pivots on a frictionless elastic hinge \[ \text{P} \] and is partly made of a ceramic material to give a light and rigid structure. A circular coil \[ \text{L} \] is fitted at the top of the beam; when a current is applied to its wire, the coil will be attracted towards a large permanent magnet.
Fig. 5.1. Schematic of the NanoTest 550 (side view).
Fig. 5.2. Schematic of the NanoTest 550 (front view).
Fig. 5.3. Operating principle of the NanoTest 550 (E = Circular permanent magnet for load application, L = Circular coil, O = Limit stop, P = Frictionless elastic hinge, W = Balance weight, Q = Pulley, C₁, C₂ = Parallel plate capacitor for displacement measurements, I = Friction force sensor, probe holder and probe, S = Specimen, xyz, XYZ = Coordinate systems).
Table 5.1. The geometrical and physical parameters of the pendulum.

<table>
<thead>
<tr>
<th>Lengths (mm)</th>
<th>Masses (g)</th>
<th>Capacitor plates</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ell_1$</td>
<td>$m_c$</td>
<td>45.186 $r_p$ (mm) 13.5</td>
</tr>
<tr>
<td>$\ell_2$</td>
<td>$m_w$</td>
<td>3.590</td>
</tr>
<tr>
<td>$\ell_3$</td>
<td>$m_p$ (indenter)</td>
<td>0.072</td>
</tr>
<tr>
<td>$\ell_4$</td>
<td>$m_p$ (slider+holder)</td>
<td>0.656</td>
</tr>
<tr>
<td>$\ell_5$</td>
<td>1.35</td>
<td></td>
</tr>
<tr>
<td>$\ell_6$</td>
<td>24.63</td>
<td>Inertia (g-mm$^2$)</td>
</tr>
<tr>
<td>$\ell_7$</td>
<td>23.70</td>
<td>45 000 $R_c$ ($\Omega$) 21.8</td>
</tr>
<tr>
<td>$\ell_8$ (min)</td>
<td>20.23</td>
<td>1 585 $R$ (mm) 18.0</td>
</tr>
<tr>
<td>$\ell_8$ (max)</td>
<td>29.08</td>
<td>3 220 $n$ 200</td>
</tr>
<tr>
<td>$\ell_9$</td>
<td>32.80</td>
<td>210</td>
</tr>
<tr>
<td>$\ell_{10}$</td>
<td>0.00</td>
<td>950</td>
</tr>
<tr>
<td>$\ell_{11}$</td>
<td>27.70</td>
<td></td>
</tr>
<tr>
<td>$\ell_{12}$</td>
<td>28.20</td>
<td>$K_p$ (mN-mm/rad) &lt; 6 000</td>
</tr>
<tr>
<td>$x_c$</td>
<td>2.30</td>
<td></td>
</tr>
<tr>
<td>$z_c$</td>
<td>7.90</td>
<td></td>
</tr>
</tbody>
</table>

which causes the beam to rotate around pivot point $P$. The rotation results in a motion of the probe $\mathbf{I}$ towards the specimen $\mathbf{S}$ and, if required, into the specimen surface. The displacement of the probe is measured by means of a capacitive sensor. This sensor consists of two parallel circular plates which are concentric with the axis of the indenter holder. One plate $\mathbf{C}_2$ is attached to the probe holder and the other $\mathbf{C}_1$ is attached to the frame (plate $\mathbf{C}_1$ is earthed). As a result of the displacement of the probe, the gap between the plates changes and this induces a change in capacitance, which is measured via a capacitance bridge unit. This unit is arranged near the capacitor sensor in order to minimise stray capacitance effects. The maximum outward movement of the probe is defined by an adjustable limit stop (represented by $\mathbf{C}$ in Fig. 5.3). This limit stop defines also the orientation of the pendulum beam when a load is applied. Its position is manually adjusted with a micrometer (see Fig. 5.2). The equilibrium position of the unloaded pendulum beam (i.e. the position with zero coil load current) is adjusted with a balance weight $\mathbf{W}$, which is movable along the horizontal axis. The probe is easily exchangeable (for example, different hardness indenters, styli, frictional probes or head sliders may be used) and can be loaded instantaneously or slowly by applying a block shaped or a ramp shaped load-time pattern, respectively. The maximum load can be adjusted between 0 and 300 mN. The load and depth resolutions are 20 µN and better than 1 nm, respectively.

The specimen is mounted on an $XYZ$ table that can traverse in the three coordinate directions $X$, $Y$ and $Z$. The table is traversed by means of three independently operating linear DC motors, each driving a stage mounted on the
table. For friction measurements at a disk, a rotating motor attachment has been build on the XYZ table. The rotating DC motor is controlled in the same way as the linear DC motors. The attachment consists of a base plate on which a rotating motor and a disk are clamped (see Fig. 5.4). The plate is mounted on the Z stage of the XYZ table. The disk spindle runs in two small ball bearings, which are preloaded and fitted into a small bearing house, in order to eliminate clearances of the spindle. The disk is driven by the rotating motor through a belt. By using a driving belt no additional vibrations are induced to the disk by the motor. The motor can rotate accurately within 1/60 rps. The rotating speed of the disks is in the range of 0 to 55 rps. Disks with different diameters may be clamped onto to the attachment. Figure 5.5 shows a typical motor calibration curve of the rotating motor.

The frictional force between the probe and the specimen is measured by means of a light miniaturised friction force sensor, mounted onto the pendulum beam. This sensor has been specially build for the experiments in this thesis and is based on the principle of piezo-resistivity (see Burger et al. [20]). Figures 5.6 and 5.7 show the friction force sensor. The sensor \(B\) consists of a silicon table and a frame, interconnected by four beams (\(B_1, B_2, B_3\) and \(B_4\)). The frame of the sensor is glued onto a printed circuit board (PCB) \(A\), such that the table suspends freely on the beams. The PCB is glued onto the probe holder \(E\). The probe holder is mounted onto the pendulum beam. The probe (stylus or slider) can be screwed onto the silicon table. Silicon table, frame and beams are one part and are micro machined out of a silicon wafer and therefore, have equal thickness. The stiffness in the \(X\) direction has been realised by choosing a sufficient thickness of the wafer. The length and the width of the beams are such that the beams can bend in the \(Z\) direction and compress or stretch in the \(Y\) direction, but are stiff in the \(X\) direction. Two of the four beams are provided with rectangular piezo-resistive polycrystalline silicon strain gauges; two on each beam (\(S_1, S_2, S_3\) and \(S_4\) in Fig. C.3). Figures C.2 and C.3 of Appendix C show the beams and the strain gauges in more detail. These strain gauges have equal resistance \(R_o\) and form the four elements of a Wheatstone bridge. In general, the friction force and the load acting on the probe may introduce forces and moments in the \(X\), the \(Y\) and the \(Z\) direction of the table. However, the four strain gauges are arranged in the Wheatstone bridge in such a way that the output voltage \(U_o\) of the bridge is only a function of the friction force in the \(Z\) direction. In Appendix C it has been shown that the friction force \(F_z\) in the \(Z\) direction, is given as

\[
F_z = C_F U_F \quad \text{with} \quad C_F = \frac{1}{A_F G U_i} \frac{2 E w h^3}{3 a (\ell - \ell_g)},
\]

in which \(U_F\) is the output voltage of the friction force sensor bridge circuit, \(U_i\) is the supplied voltage across the bridge, \(C_F\) is the friction force calibration factor, \(G\) is the gauge factor of the strain gauges, \(a\) is the distance in \(Z\) direction between the centre of the cross-sectional area of a beam and the centre of the strain gauges, \(\ell, w\) and \(h\) are the length, width and height of the beams respectively; \(\ell_g\) is the length of the strain gauges, \(E\) is the elastic modulus of the beams, and \(A_F\) is the
Fig. 5.4. Schematic of the rotating motor attachment.
**Fig. 5.5.** Motor calibration curve of the rotating motor. The speed of the motor is controlled by issuing velocity counts to its control electronics (Calibration factor = 1344.5 Counts/rps).

**Fig. 5.6.** Schematic of the micro machined friction force sensor and the probe holder (A=PCB, B=Micro machined friction force sensor, C=Slider arm, D=Slider, E=Probe holder).
amplification factor of the amplifier of the friction force sensor bridge. Obviously, the sensor should be placed with its Z axis in the direction of the relative velocity between probe and specimen. In case a slider is used as a probe, part of the arm of the slider is screwed onto the table. In this case the freedom of motion of the slider (as discussed in Chapter 3) is still provided by (part of) the flexure. The calibration of the friction force sensor is done by hanging calibration weights on the probe tip and measuring the output voltage of the bridge for each weight. Figure 5.8 shows a typical friction force calibration curve.

Figure 5.9 shows the electronic block diagram of the NanoTest 550. The instrument is automatically controlled by a computer. An electronic control unit provides the communication between the transducers and the computer via an IEEE-488 bus. The electronic control unit receives the AC or DC output voltage signals from the load coil, the capacitance bridge unit and the friction force sensor. These are then amplified, rectified, digitised and transferred via the IEEE-488 bus to the computer. In addition, analogue and digital control signals can be send by the computer to the electronic control unit. These signals control for example the ramp generator, which provides the current through the coil, as well as the gain and amplification of the signals from the capacitance bridge unit and the friction force sensor. The control electronics for each motor is shown in Fig. 5.9 and consists of a servo controller which contains a velocity-profile generator and a digital compensation filter (PID). The generator calculates the required position and the velocity-profile of the corresponding stage. The digital velocity-profile is converted into an analogue signal by a DA convertor. The signal is then amplified and applied to the corresponding stage. An incremental encoder determines the actual position of the stage, which is feed back to the servo controller. The servo controller subtracts the actual position from the required position, set by the
generator, and the resulting position error is processed by the digital filter to move the stage to the required position. The resolution of the specimen positioning is below 20 nm.

The high load and depth resolution of the NanoTest 550 requires that the instrument operates in a proper environment. In order to eliminate undesired excitation of the instrument by its environment, the instrument is placed on a baseplate which rests inside a cabinet. The cabinet rests on a Micro-g© vibration isolation table which comprises four pneumatic isolation legs with three height control valves. The cabinet is thermostatically controlled within ±0.1 °C. The cabinet suppresses also acoustic vibrations and eliminates draughts. Table and cabinet are situated in a temperature controlled laboratory room with a constant temperature of 21 °C.

With the rotating motor attachment, the miniaturised friction force sensor, and the rebuild pendulum it is made possible to use the NanoTest 550 as a very sensitive pin-on-disk machine and to measure at high speed a frictional force in the milli Newton range and displacement in the nanometre range. The friction force is measured very closely to the surfaces in contact. Hence, it is possible to study very accurately the friction at HDIs. The advantage of using the NanoTest 550 for this purpose is that the load is precisely controlled and that load, frictional force and displacement can be measured simultaneously. Such measurements have not been reported before in the literature.
Fig. 5.9. Electronic block diagram of the NanoTest 550.
5.2 The Pendulum Model

Having introduced the basic principle of the NanoTest 550 instrument, now a model for the pendulum will be given. This model will then be used in the next section to analyse the dynamics of the pendulum.

The pendulum model and the geometrical and physical parameters used in this section are given in Fig. 5.3 and Table 5.1, respectively. Let \(xyz\) be a fixed Cartesian coordinate system of the pendulum, with its origin at the pivot point \(P\) of the elastic hinge (see Fig. 5.3). All parts of the pendulum move in planes parallel to the \(yz\) plane, thus, the pendulum executes plane motion, giving a fixed axis rotation about the pivot point \(P\). Therefore, the pendulum has one degree of freedom and its position is determined by the angular deflection \(\theta\). The pendulum will be considered rigid, i.e. it cannot deform by the forces applied to it. The centre of mass of the pendulum (balance weight \(W\) excluded) is at point 3 in Fig. 5.3, and is given by the position vector \(\mathbf{r}_3\). The centre of mass of the balance weight \(W\), the pulley \(Q\), the probe \(I\), and the centres of the capacitor plates \(C_2\) and \(C_1\) are given by the position vectors \(\mathbf{r}_2, \mathbf{r}_4, \mathbf{r}_7, \mathbf{r}_8\) and \(\mathbf{r}_9\), respectively. In the \(xyz\) coordinate system, these and other points are given by

\[
\mathbf{r}_1 = \begin{pmatrix} -\ell_5 \\ 0 \\ \ell_1 \end{pmatrix}; \quad \mathbf{r}_2 = \begin{pmatrix} \ell_8 \\ 0 \\ -\ell_2 \end{pmatrix}; \quad \mathbf{r}_3 = \begin{pmatrix} -x_c \\ 0 \\ z_c \end{pmatrix}; \quad \mathbf{r}_4 = \begin{pmatrix} 0 \\ 0 \\ -\ell_2 \end{pmatrix};
\]

\[
\mathbf{r}_5 = \begin{pmatrix} -\ell_7 \\ 0 \\ \ell_4 \end{pmatrix}; \quad \mathbf{r}_6 = \begin{pmatrix} -\ell_6 \\ 0 \\ \ell_4 \end{pmatrix}; \quad \mathbf{r}_7 = \begin{pmatrix} -\ell_{10} \\ 0 \\ -\ell_3 \end{pmatrix}; \quad \mathbf{r}_8 = \begin{pmatrix} 0 \\ 0 \\ -\ell_3 \end{pmatrix}; \quad \mathbf{r}_9 = \begin{pmatrix} -\ell_{12} \\ 0 \\ -\ell_3 \end{pmatrix}.
\]

After an angular deflection \(\theta\), the position vector \(\mathbf{r}_i\) of any point \(i\) of the pendulum, becomes position vector \(\mathbf{r}'_i\), given by

\[
\mathbf{r}'_i = M_\theta \mathbf{r}_i, \quad (5.3)
\]

where \(M_\theta\) is the transformation matrix given by

\[
M_\theta = \begin{pmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{pmatrix} \quad \text{and} \quad \mathbf{r}'_i = \begin{pmatrix} r_{i,x}' \\ r_{i,y}' \\ r_{i,z}' \end{pmatrix}; \quad \mathbf{r}_i = \begin{pmatrix} r_{i,x} \\ r_{i,y} \\ r_{i,z} \end{pmatrix}. \quad (5.4)
\]

It will be assumed that the elastic hinge possesses a torsional spring with constant \(K_p\). This spring introduces a moment \(M_K\) at the pivot point given by

\[
M_K \overset{\text{def}}{=} K_p \theta. \quad (5.5)
\]

Furthermore, the pendulum will be assumed to possess some inherent damping, given by the damping coefficient \(C_p\). This damping exerts a moment \(M_C\) given by
Fig. 5.10. Free-body diagram of the pendulum (\(\mathbf{L}\)=Circular coil, \(\mathbf{P}\)=Frictionless elastic hinge, \(\mathbf{W}\)=Balance weight, \(\mathbf{Q}\)=Pulley, \(\mathbf{C}_2\)=Capacitor plate, \(\mathbf{I}\)=Friction force sensor, probe holder and probe).
\[ M_C \stackrel{\text{def}}{=} C_p \dot{\theta}, \quad (5.6) \]

in which \( \dot{\theta} \stackrel{\text{def}}{=} d\theta/dt \). The friction force \( F \) along the specimen surface is defined as
\[ F \stackrel{\text{def}}{=} \mu P, \quad (5.7) \]

where \( \mu \) is the COF and \( P \) is the normal force at the specimen surface. The friction force acts on the probe in the direction of the relative movement of the specimen surface. The attractive force between the capacitor plates will be neglected, according to the analysis given in Appendix D.

### 5.2.1 Centre of Mass of the Pendulum

As pendulum, balance weight \( [W] \) and probe form one rigid body, their masses can be taken together, making the equations of motion to be derived much simpler. The centre of mass \( \mathbf{R} \) of the pendulum, the balance weight \( [W] \), and the probe is at point \( \ominus \) and is defined as
\[ \mathbf{R} \stackrel{\text{def}}{=} \frac{m_c \mathbf{r}_3 + m_p \mathbf{r}_7 + m_w \mathbf{r}_2}{M_c}, \quad (5.8) \]

in which
\[ M_c = m_c + m_p + m_w, \quad (5.9) \]

and where \( m_c \), \( m_w \) and \( m_p \) are the mass of the pendulum, the balance weight and the probe, respectively. Substituting (5.2) into (5.8) gives for the centre of mass of the pendulum,
\[ \mathbf{R} = \begin{pmatrix} -(m_c x_c + m_p \ell_{30} - m_w \ell_8)/M_c \\ 0 \\ (m_c z_c - m_p \ell_3 - m_w \ell_2)/M_c \end{pmatrix}. \quad (5.10) \]

From (5.10) it follows that the centre of mass can be modified in \( x \) direction only, by adjusting length \( \ell_8 \) (in the \( z \) direction the centre of mass remains unaltered, as all other parameters are constant).

### 5.2.2 Misalignment of the Specimen

In order to assess the effects of misalignment of the specimen surface on the dynamics of the pendulum, during contact between probe and specimen surface, it will be assumed here that the specimen surface is flat and that the normal \( \mathbf{n} \) to the surface is not perpendicular to the \( yz \) plane but at the angles \( \beta \) and \( \gamma \), see Fig. 5.11. Furthermore, the contact between probe and specimen surface will be at a distance \(-\ell_{30}\) away from the \( yz \) plane (see Fig. 5.3). It will be assumed that the nominal contact area between probe and specimen surface is small, such that the velocity across the contact is constant. The specimen is allowed to translate in the plane of its surface or may rotate around an axis perpendicular to its surface. A second Cartesian coordinate system \( XYZ \) has its axes in the same direction of
that of the $xyz$ coordinate system and its origin at the point of contact between
the probe and the specimen surface. Let $\alpha$ be the angle between the velocity
vector $\mathbf{v}$ and the $Y$ axis.

It has been shown in Appendix F, that the velocity $\mathbf{v}$ of the specimen, the
friction force $\mathbf{F}$, the normal load $\mathbf{P}$, at the probe tip and the applied load $\mathbf{F}_c$, at
the coil, with respect to the $XYZ$ coordinate system, become

$$
\mathbf{v} = \begin{pmatrix} 
v (\gamma \sin \alpha - \beta \cos \alpha) \\
v (\cos \alpha + \beta \gamma \sin \alpha) \\
v \sin \alpha \end{pmatrix} ; \quad \mathbf{F} = \begin{pmatrix} 
F (\gamma \sin \alpha - \beta \cos \alpha) \\
F (\cos \alpha + \beta \gamma \sin \alpha) \\
F \sin \alpha \end{pmatrix}, \quad (5.11)
$$

and

$$
\mathbf{P} = \begin{pmatrix} 
-P \\
-P \beta \\
P \gamma \end{pmatrix} ; \quad \mathbf{F}_c = \begin{pmatrix} 
-F_c \\
0 \\
0 \end{pmatrix}. \quad (5.12)
$$

5.3 Dynamics of the Pendulum

In this section the dynamics of the pendulum model, described in the previous
section, will be analysed. The equations to be derived here will be presented in
dimensionless form, in order to estimate whether certain terms in the equations
can be neglected to simplify the equations. Three cases will be distinguished in

![Fig. 5.11. The coordinates used in the free-body diagram of the pendulum.](image)
5.3.1 Motion of the Mechanically Balanced Unloaded Pendulum

The pendulum is said to be unloaded, in case the load at the coil is zero, the calibration weight is removed, and the probe is not in contact with the specimen. The unloaded pendulum can be mechanically balanced by adjusting the balance weight \( W \) in Fig. 5.3 along a horizontal axis. When the unloaded pendulum is mechanically balanced, the sum of the (anti)clockwise moments at the pivot point \( P \) is per definition zero. The moments acting on the unloaded pendulum are introduced by the masses \( m_c, m_w \) and \( m_p \) of the pendulum, the balance weight, and the probe, respectively, and the torsional spring of the elastic hinge. In Appendix H, Section H.2 the equation of motion of the unloaded pendulum is derived. Let \( \theta_0 \) be the angular deflection of the mechanically balanced unloaded pendulum in case the pendulum is at its equilibrium position. This position is stable if a small disturbance of the pendulum from equilibrium results in a small bounded motion about the angular deflection \( \theta_0 \). The equilibrium position is unstable if an infinitesimal disturbance eventually produces unbounded motion. If the potential energy \( V(\theta) \) of the pendulum is a minimum at the angular deflection \( \theta_0 \) any deviation from this deflection will produce an increase in \( V(\theta) \). Because the energy must be conserved, the rotational velocity must then decrease and eventually come to zero, indicating bound motion. In Appendix H, Section H.2, (H.30) the dimensionless potential energy of the unloaded pendulum is derived as

\[
\overline{V}(\theta) = \cos \theta - \xi \sin \theta + \frac{1}{2}(\omega^2 + 1)\theta^2 ,
\]

in which \( \omega, \xi \) and \( \overline{V} \) are defined by

\[
\omega \equiv \sqrt{\frac{K_p}{M_c g R_z}} - 1 ; \quad \xi \equiv \frac{R_x}{R_z} ; \quad \overline{V} \equiv \frac{V}{M_c g R_z}. \tag{5.14}
\]

When the unloaded pendulum is in its equilibrium position then it follows from the potential energy of the pendulum, that

\[
\overline{F} = - \frac{\partial \overline{V}}{\partial \theta} \bigg|_{\theta_0} = 0 ,
\]

in which \( \overline{F} \) is the moment at the pendulum. With the potential energy given by (5.13) this gives

\[
\sin \theta_0 + \xi \cos \theta_0 - (\omega^2 + 1)\theta_0 = 0 . \tag{5.16}
\]

If small angular deflections \( \theta_0 \) are assumed then the approximations \( \cos \theta_0 \approx 1 \) and \( \sin \theta_0 \approx \theta_0 \) may be used, and (5.16) becomes

\[
\theta_0 = \frac{\xi}{\omega^2}, \tag{5.17}
\]
which is identical to (H.36). The same equation arises also from the summation of the angular moments about the pivot point \( P \). With \( R_x \) and \( R_z \) given in (5.10) and \( \xi \) and \( \omega \) from (5.14) it follows that

\[
\theta_0 = -\frac{(m_c x_c + m_p \ell_{10} - m_w \ell_8)}{K_p/g - m_{cz_c} + m_p \ell_3 + m_w \ell_2}. \tag{5.18}
\]

Hence, the angular deflection \( \theta_0 \) of the unloaded pendulum, and the direction in which the pendulum falls into equilibrium, depend on the masses and dimensions of the pendulum, the balance weight, and the spring constant of the elastic hinge. For a proper operation of the pendulum, the angular deflection \( \theta_0 \) of the equilibrium position must be greater than zero and the minimum output voltage of the capacitance bridge must occur when the pendulum is close to the equilibrium position. In this case it will be assured that the output voltage of the capacitance bridge goes through its minimum when the pendulum is brought against the limit stop (see Fig. 5.12). The equilibrium position of the unloaded pendulum can be adjusted by changing the position of the balance weight \( W \) along its horizontal axis, by changing \( \ell_8 \). Since the numerator in (5.18) is normally \( > 0 \), it follows by inspection that \( \theta_0 > 0 \) if

\[
m_c x_c + m_p \ell_{10} - m_w \ell_8 < 0, \tag{5.19}
\]

or

*Fig. 5.12.* The output voltage \( U_p \) of the capacitance bridge and the gap \( g_c \) between the capacitor plates as a function of the angular deflection \( \theta \). Note that the direction of the \( F_c \) axis is opposite to the direction of the \( \theta \) axis.
\[
\ell_8 > \frac{m_c}{m_w} x_c \left( 1 + \frac{m_p \ell_{10}}{m_c x_c} \right). \tag{5.20}
\]

Normally, the length \( \ell_{10} \) is very small or zero and the mass \( m_p \) of the probe is such that \( m_p \ll m_c \). Only when the mass of the probe is in the order of the mass of the pendulum, the probe can influence the equilibrium position of the unloaded pendulum. Equation (5.18) also shows that, when the probe has been changed (other mass \( m_p \) and thus other values for \( M_c g R_x \) and \( M_c g R_z \)), the load of the instrument should be recalibrated strictly. From (5.18) it follows that the angular deflection \( \theta_0 \) will become infinite when

\[
K_p / g - m_c z_c + m_p \ell_3 + m_w \ell_2 = 0, \tag{5.21}
\]

and with the data given in Table 5.1 it follows that this situation cannot occur.

In general, the angular deflection of the pendulum during load calibration and contact with the specimen surface will be different from the angular deflection \( \theta_0 \) of the unloaded pendulum. These two cases will therefore be discussed in the next sections.

### 5.3.2 Motion of the Pendulum During Load Calibration

The load at the probe is calibrated by hanging different calibration weights, with mass \( m \), at the pulley \( \mathbb{Q} \) (see Fig. 5.3). For each weight a voltage \( U_c \) will be applied to the ends of the coil wire, such that a load \( F_c \) at the coil will be generated which pulls the pendulum against the limit stop. Then the applied voltage \( U_c \) at the coil will be gradually reduced until the pendulum just falls away from the limit stop, i.e. until the clockwise moment induced by the calibration weight is mechanically balanced by the anticlockwise moment induced by the load \( F_c \) applied to the coil. The falling back is monitored by the computer by measuring the output voltage \( U_p \) of the capacitance bridge and when the computer detects the moment the bridge goes through zero, the DC voltage \( U_c \) at the coil will be read. This procedure is repeated by the computer for all weights.

In Appendix H, Section H.3 the equation of motion of the pendulum during load calibration is derived. Equation (H.45) in this appendix, gives the dimensionless potential energy \( \overline{V}(\theta) \) of the pendulum during load calibration,

\[
\overline{V}(\theta) = (1 - M_m L_2) \cos \theta - (\xi + M_m L_9) \sin \theta + \frac{1}{2} (\omega^2 + 1) \theta^2, \tag{5.22}
\]

in which \( \xi, \omega, M_m, L_2 \) and \( L_9 \) are given by (5.14) and by

\[
L_2 \equiv \frac{\ell_2}{R_z}; \quad L_9 \equiv \frac{\ell_9}{R_z}; \quad M_m \equiv \frac{m}{M_c}. \tag{5.23}
\]

When the pendulum is at its equilibrium position \( \theta_c \) it follows from the potential energy that

\[
\mathcal{F} = -\frac{\partial \overline{V}}{\partial \theta} \bigg|_{\theta_c}, \tag{5.24}
\]
in which $\overline{F}$ is the moment at the pendulum, generated by the applied load $\overline{F}_c$ at the coil. And with the potential energy given by (5.22) this gives

$$(1 - M_m L_2) \sin \theta_c + (\xi + M_m L_9) \cos \theta_c - (\omega^2 + 1) \theta_c$$

$$= \overline{F}_c(L_5 \theta_c + 1), \quad (5.25)$$

in which $\overline{F}_c$ and $L_5$ are defined by

$$L_5 = \frac{\ell_5}{\ell_1} \quad \text{and} \quad \overline{F}_c = \frac{F_c \ell_1}{M_c g R_z}. \quad (5.26)$$

If small angular deflections are assumed, then the approximations $\cos \theta_c \approx 1$ and $\sin \theta_c \approx \theta_c$ may be used and the dimensionless load $\overline{F}_c$ at the coil becomes

$$\overline{F}_c = -\frac{(\omega^2 + M_m L_2) \theta_c + \xi + M_m L_9}{L_5 \theta_c + 1}.$$

After substitution of (5.17) this becomes

$$\overline{F}_c = -\frac{\omega^2 (\theta_c - \theta_0) + M_m (L_2 \theta_c - L_9)}{L_5 \theta_c + 1}. \quad (5.28)$$

During the automatic load calibration the load will be gradually reduced in small steps, such that the falling back of the pendulum beam occurs near the angular deflection $\theta = \theta_0$; the angular deflection at the equilibrium position of the unloaded pendulum. Furthermore, in Appendix E it is shown that the angular deflection $\theta$ is in the range $-25 \text{ mrad} \leq \theta \leq 10 \text{ mrad}$. Therefore, $\theta_c - \theta_0 \approx 0$ and $L_2 \theta_c \ll L_9$, and because $L_5 \theta_c \ll 1$, (5.28) can be written as

$$\overline{F}_c = M_m L_9. \quad (5.29)$$

In Appendix G, equation (G.19) has been derived for the dimensionless load $\overline{F}_c(\tau)$ at the coil as a function of the dimensionless input voltage $\overline{U}_c(\tau)$ applied to the ends of the coil wire and the angular velocity $\dot{\theta}(\tau)$ of the pendulum,

$$\overline{F}_c = 2\pi N \left( \overline{U}_c(\tau) - 2\pi N \dot{\theta}(\tau) \right), \quad (5.30)$$

in which $\overline{U}_c$ and $N$ are given by

$$\overline{U}_c \equiv U_c \sqrt{\frac{J}{R_z^2 (M_c g R_z)^3}} \quad \text{and} \quad N \equiv n R B_0 \ell_1 \sqrt{\frac{1}{R_z^2 J M_c g R_z}}. \quad (5.31)$$

When the pendulum is mechanically balanced, then $\dot{\theta} = 0$ and (5.30) reduces to

$$\overline{F}_c = 2\pi N \overline{U}_c(\tau). \quad (5.32)$$

Combining (5.29) and (5.32) gives

$$\overline{U}_c(\tau) = \frac{L_9}{2\pi N} M_m. \quad (5.33)$$
Hence, if the pendulum is mechanically balanced, the input voltage $U_c$ and the load $F_c$ at the coil are both directly proportional to the mass $M_m$ of the calibration weight. In dimensional form, (5.33) becomes

$$U_c = \frac{g \ell_0}{C_L \ell_3} m, \quad \text{with} \quad C_L = \frac{2\pi n RB_c \ell_1}{R_c \ell_3},$$

in which $C_L$ is the so-called load calibration factor. In the automatic load calibration procedure, the load calibration factor $C_L$ will be calculated from a linear curve fit through the data points $(U_c, m)$. The obtained load calibration factor will then be used in the measurements to calculate the real load $P$ at the probe (see the next section). Figure 5.13 shows a typical load calibration curve.

### 5.3.3 Motion of the Pendulum During Contact Between Probe and Specimen

Before any measurements can be carried out with the NanoTest 550 instrument, the specimen surface must be brought into contact with the probe. This is done by bringing the specimen surface in close range of the probe tip by means of the XYZ table, such that the measuring location of the specimen surface is at its proper Y and Z coordinates, and the specimen surface is at a distance of about 50 μm from the probe tip. Then the pendulum is pulled against the limit stop by applying an initial load to the coil. In Fig. 5.12 it is shown that in this case the output voltage of the capacitance bridge is $|U_p|_{\text{max}}$. Then the specimen

![Graph](image)

**Fig. 5.13.** Load calibration curve. The NanoTest 550 instrument has a low and a high load range (Low load: $C_L = 1.482 \, \mu N/\text{bit}$, high load: $C_L = 9.793 \, \mu N/\text{bit}$).
is moved further in negative X direction until contact is made with the probe. The contact between the specimen and the probe is monitored by the computer which measures the output voltage $U_p$ of the capacitance bridge, and when the computer detects the moment the output voltage deviates from the initial value $|U_{p,max}|$, the X stage ceases. The probe is then moved away from the specimen surface by reducing the initial load at the coil to zero. Next, the specimen surface is moved further towards the pendulum (in the negative X direction, thus, the angular deflection of the pendulum will increase, see Fig. 5.12) until the output voltage $U_p$ of the capacitance bridge is $\frac{1}{2}|U_{p,max}|$, i.e. half the initial output when the pendulum is against the limit stop. In that case the angular deflection of the pendulum is at the measuring position $\theta_{min}$. Finally, the probe will be brought back into contact with the specimen by applying an offset load to the coil. In this way it will be assured that the output voltage of the capacitance bridge is always linear with the gap $g_c$ and the load will be virtual zero at the probe. At the end of the loading procedure, the final load will be applied, whereby the offset load will be subtracted from the final load.

During measurement the probe is in contact with the specimen and the moments acting on the pendulum are generated by the masses $m_c$, $m_p$ and $m_w$ of the pendulum, the probe, and the balance weight $W$, respectively, the torsional spring of the elastic hinge, the load $F_c$ at the coil, the load $P$ and the friction force $\mathbf{F}$ at the probe tip. Note that the friction force acts in the opposite direction of the motion of the probe tip. In Appendix H, Section H.4 the equation of motion (H.46) of the pendulum during contact between the probe and the specimen has been derived. This equation can be put in the form

$$\mathbf{F} = \frac{\ddot{\theta} + 2\ddot{\theta} + (F_c L_5 + \omega^2) \theta + F_c - \xi}{A \theta + B}, \tag{5.35}$$

in which $\xi$, $\omega$, $F_c$ and $L_5$ are defined in (5.14) and (5.26) and in which $A$ and $B$ are defined by

$$A = (\mu \sin \alpha + \gamma) L_3 + (\mu (\gamma \sin \alpha - \beta \cos \alpha) - 1) L_{10},$$

$$B = (\mu \sin \alpha + \gamma) L_{10} - (\mu (\gamma \sin \alpha - \beta \cos \alpha) - 1) L_3 \tag{5.36}.$$

Furthermore, $\zeta$, $L_3$, $L_{10}$ and $\mathbf{F}$ are given by

$$\zeta \equiv \frac{C_p}{2} \frac{1}{J M_c g R_z}; \quad L_3 \equiv \frac{\ell_3}{R_z}; \quad L_{10} \equiv \frac{\ell_{10}}{R_z}; \quad \mathbf{F} \equiv \frac{P}{M_c g}. \tag{5.37}$$

From (5.35) some estimates can readily be made.

**Minimum Initial Load.** Firstly, an estimate can be made of the minimum initial load $\mathbf{F}^{c_{min}}$ at the coil, needed to bring the pendulum beam against the limit stop. In that case, the specimen is not in contact with the probe ($\mathbf{F} = 0$ and $\mu = 0$), and the angular deflection will be $\theta_{min}$. Then, it follows from (5.35) that

$$\left( F^{c_{min}} L_5 + \omega^2 \right) \theta_{min} + F^{c_{min}} - \xi = 0, \tag{5.38}$$
and with (5.17) this gives
\[ F_{c,\text{min}} = \frac{\omega^2(\theta_0 - \theta_{\text{min}})}{L_5 \theta_{\text{min}} + 1} \approx \omega^2(\theta_0 - \theta_{\text{min}}), \]  
(5.39)
because \( L_5 \theta_{\text{min}} \ll 1 \). In dimensional form this becomes
\[ F_{c,\text{min}} \approx \frac{K_p - M_c g R_e}{\ell_1} (\theta_0 - \theta_{\text{min}}). \]  
(5.40)

From this equation it follows that \( F_{c,\text{min}} \) depends mainly on the spring constant, the mass and the position of the centre of mass of the pendulum. With the data given in Table 5.1, and with \( \theta_{\text{min}} = -25 \) mrad (\(-1.5\) degrees) and \( \ell_8 = 25.8 \) mm, the minimum initial load \( F_{c,\text{min}} \) is about 0.5 mN. In the software of the NanoTest 550 this initial load can be setup prior to the loading procedure and the actual measurements.

**Offset Load at the Specimen.** A second estimate concerns the offset load at the specimen prior to the actual measurements. Assume that the angular deflection \( \theta \) has reduced to \( \theta_m \). Then, the friction force is the static friction force and \( \alpha = 90 \) degrees (because the pendulum moves away from the limit stop), thus \( \sin \alpha = 1 \) and \( \cos \alpha = 0 \). The COF is virtually the static COF between probe and specimen surface (\( \mu = \mu_s \)). In that case, the offset load \( \overline{F}_o \) at the specimen follows from (5.35) as
\[ \overline{F}_o = \frac{(\overline{F}_c L_5 + \omega^2) \theta_m + \overline{F}_c - \xi}{A \theta_m + B}. \]  
(5.41)
Substitution of (5.17) into this equation gives
\[ \overline{F}_o = \frac{\omega^2(\theta_m - \theta_0) + \overline{F}_c (L_5 \theta_m + 1)}{A \theta_m + B}. \]  
(5.42)

Assuming that \( \theta_m \approx 0 \) and \( \theta_0 \approx 0 \), and with \( A \) and \( B \) given in (5.36) the offset load \( \overline{F}_o \) becomes
\[ \overline{F}_o \approx \frac{\overline{F}_c}{(\mu_n + \gamma) L_{10} - (\gamma \mu_n - 1) L_3}. \]  
(5.43)

This equation shows how important it is that the length \( L_{10} \) should be zero, i.e. *the pivot point should be in the same plane as that of the probe tip and specimen surface!* In the original design of the pendulum this was not the case (the length \( \ell_{10} \) was considerably: \( \ell_{10} \approx 20 \) mm, see, for example, Smith [121]). According to (5.43) the length \( L_{10} \) introduces an extra moment at the pendulum due to the friction between the probe and the specimen surface and the misalignment of the specimen surface. This extra moment causes a reduction in the real normal load at the specimen.

From (5.41) it follows that \( P_o \rightarrow \infty \) if
\[ \theta_m = -\frac{\overline{F}_o}{A} = \frac{\gamma L_{10} + L_3 + \mu_n (L_{10} - \gamma L_3)}{L_{10} - \gamma L_3 - \mu_n (\gamma L_{10} + L_3)}. \]  
(5.44)
If \( L_{10} = 0 \), then (5.44) becomes

\[
\theta_m = \frac{\gamma \mu_n - 1}{\mu_\delta + \gamma} \approx \frac{1}{\mu_\delta},
\]

(5.45)

because \( \gamma \ll \mu_n \). This is a value of \( \theta_m \) which cannot occur in practice. However, for \( L_{10} \neq 0 \) and \( \gamma = 0 \) (5.44) becomes

\[
\theta_m = \frac{L_3 + \mu_n L_{10}}{L_{10} - \mu_\delta L_3}.
\]

(5.46)

Because, \( \theta_{\text{min}} \leq \theta_m \), it follows that in that case

\[
\theta_{\text{min}} \leq \frac{L_3 + \mu_n L_{10}}{L_{10} - \mu_\delta L_3},
\]

(5.47)

or

\[
L_{10} \geq \frac{1 + \theta_{\text{min}} \mu_n}{\theta_{\text{min}} - \mu_\delta} L_3 \approx \frac{L_3}{\mu_\delta}.
\]

(5.48)

Thus, for \( L_{10} \neq 0 \), the normal load at the probe may well become infinite! This again, shows that it is necessary to make the length \( L_{10} \) equal to zero.

**Load at the Specimen During Quasi Static Measurements.** It follows from (5.35) that the friction force, the masses of the pendulum, the coil, the probe and the balance weight \([W]\), the torsional spring of the elastic hinge, and the friction force, give additional moments at the pivot point and reduce the load. With (5.17), (5.35) can be written as

\[
\overline{P} = \frac{\dot{\theta} + 2\zeta \dot{\theta} + \omega^2 (\theta - \theta_0) + \overline{F}_c (L_3 \theta + 1)}{A \theta + \overline{B}}.
\]

(5.49)

This equation was derived by assuming that \( \theta \) is very small. It becomes immediately clear that for small \( \theta \)'s the products of \( \theta \) and \( \theta_0 \) in (5.49) may be neglected compared to the other products. Then it follows from (5.49) that the load \( \overline{P} \) at the specimen during static contact becomes

\[
\overline{P} = \frac{\overline{F}_c}{\overline{B}},
\]

(5.50)

and with \( \overline{B} \) given in (5.36) this can be written as

\[
\overline{P} \approx \frac{\overline{F}_c}{(\mu \sin \alpha + \gamma) L_{10} - (\mu (\gamma \sin \alpha - \beta \cos \alpha) - 1)L_3}.
\]

(5.51)

From this equation it follows again that the influence of the friction between the probe tip and the specimen surface can be large when the length \( L_{10} \neq 0 \). Therefore, the original pendulum has been redesigned with length \( L_{10} = 0 \). In that case (5.51) becomes

\[
\overline{P} \approx \frac{\overline{F}_c}{(1 - \mu (\gamma \sin \alpha - \beta \cos \alpha))L_3}.
\]

(5.52)
Usually, $\beta$ and $\gamma$ are small (< 5 degrees) and $\mu < 1$. Therefore, the load (5.52) can be written as

$$P \approx \frac{F_c}{L_3},$$  \hspace{1cm} (5.53)

or in dimensional form

$$P \approx \frac{F_c \ell_1}{L_3 \ell_3}.$$  \hspace{1cm} (5.54)

And with (5.32) the load $\overline{P}$ becomes

$$\overline{P} = \frac{2\pi N}{L_3 U_c},$$  \hspace{1cm} (5.55)

or in dimensional form

$$P = C_L U_c,$$  \hspace{1cm} (5.56)

in which $C_L$ is the load calibration factor defined in (5.34). Thus, the load at the specimen surface is proportional to the coil voltage in the quasi static situation. This equation is used for the measurement of the applied load at the specimen. 

*Note that this equation is only valid when $L_{10} = 0$ and $\theta$ is small.* From (5.7) and (5.54) it follows that the COF $\mu$ is

$$\mu = \frac{F}{P} \approx \frac{F \ell_3}{F_c \ell_1}.$$  \hspace{1cm} (5.57)

**Load at the Specimen During Pin-On-Disk Measurements.** A fourth estimate regards the load at the probe tip during pin-on-disk measurements. In this case the motion of the pendulum is restricted by the motion of the disk in axial direction. This motion may be due to the disk run-out or waviness of the disk surface, for example. It will be assumed here that the axial motion of the disk is given by

$$\delta(t) = D \sin(\omega_d t),$$  \hspace{1cm} (5.58)

in which $D$ is the amplitude of the displacement in axial direction and $\omega_d$ is the angular frequency of the axial motion of the disk. Furthermore, it will be assumed that during the rotation, the probe remains in contact with the disk surface. Then

$$r_{r,s}^t - r_{r,s} = \delta(t).$$  \hspace{1cm} (5.59)

Substitution of (5.2), (5.3) and (5.58) into this equation gives

$$L_{10}(1 - \cos \theta) - L_3 \sin \theta = D \sin(\omega_d t).$$  \hspace{1cm} (5.60)

This equation can be made dimensionless as follows

$$L_{10}(1 - \cos \theta) - L_3 \sin \theta = \overline{D} \sin(\overline{\omega}_d \tau),$$  \hspace{1cm} (5.61)

in which $\overline{D}$ is the dimensionless amplitude of the displacement in axial direction and $\overline{\omega}_d$ is the dimensionless angular frequency of the axial motion of the disk, defined by
\[ \overline{D} \equiv \frac{D}{R_x} \quad \text{and} \quad \overline{\omega}_d \equiv \omega_d \sqrt{\frac{J}{M_c g R_x}}, \quad (5.62) \]

respectively. Equation (5.61) does not have an analytic solution for \( \theta \). However, for small angular deflections, the approximations \( \cos \theta \approx 1 \) and \( \sin \theta \approx \theta \) may be used and \( \theta \) becomes

\[ \theta \approx -\epsilon_d \sin(\overline{\omega}_d \tau), \quad (5.63) \]

in which \( \epsilon_d \) is the dimensionless amplitude of the displacement of the disk in axial direction, which is defined as

\[ \epsilon_d \equiv \frac{D}{\ell_3}. \quad (5.64) \]

The angular velocity \( \dot{\theta} \) and the acceleration \( \ddot{\theta} \) follow from (5.63) by differentiating to \( \tau \),

\[ \dot{\theta} = -\epsilon_d \overline{\omega}_d \cos(\overline{\omega}_d \tau) \quad \text{and} \quad \ddot{\theta} = \epsilon_d \overline{\omega}_d^2 \sin(\overline{\omega}_d \tau). \quad (5.65) \]

Substituting (5.63) and (5.65) into (5.35) gives

\[ \overline{P}(\tau) = \frac{\epsilon_d \overline{\omega}_d^2 \sin(\overline{\omega}_d \tau) - 2\zeta \epsilon_d \overline{\omega}_d \cos(\overline{\omega}_d \tau) - (F_c L_5 + \omega^2)\epsilon_d \sin(\overline{\omega}_d \tau) + F_c - \xi}{-A \epsilon_d \sin(\overline{\omega}_d \tau) + B}. \quad (5.66) \]

Usually, \( D \leq 10 \mu m \) and with \( \ell_3 \) given in Table 5.1 it follows that \( \epsilon_d \leq 2.5 \cdot 10^{-4} \). If there is no axial motion of the disk then \( \epsilon_d = 0 \) and (5.66) becomes

\[ \overline{P} = \frac{F_c - \xi}{B} \approx \frac{F_c}{B} = \frac{2\pi N}{B} \overline{U}_c, \quad (5.67) \]

because, \( \xi \ll F_c \). This equation is similar to that of (5.50) in the quasi static situation. In case \( \beta = \gamma = 0 \), then

\[ \overline{A} = -L_{10} + \mu L_3 \sin \alpha \quad \text{and} \quad \overline{B} = L_3 + \mu L_{10} \sin \alpha, \quad (5.68) \]

and the load at the probe becomes

\[ \overline{P} = \frac{\epsilon_d \overline{\omega}_d^2 \sin(\overline{\omega}_d \tau) - 2\zeta \epsilon_d \overline{\omega}_d \cos(\overline{\omega}_d \tau) - (F_c L_5 + \omega^2)\epsilon_d \sin(\overline{\omega}_d \tau) + F_c - \xi}{L_3 + \mu L_{10} \sin \alpha - (\mu L_3 \sin \alpha - L_{10})\epsilon_d \sin(\overline{\omega}_d \tau)}. \quad (5.69) \]

And if \( \epsilon_d = 0 \) then

\[ \overline{P} = \frac{F_c - \xi}{L_3 + \mu L_{10} \sin \alpha}, \quad (5.70) \]

which, again shows the need for having \( L_{10} = 0 \). If \( L_{10} = 0 \) then

\[ \overline{P} = \frac{F_c - \xi}{L_3} \approx \frac{F_c}{L_3}, \quad (5.71) \]

which is identical to (5.53) in case of quasi static measurements and which can be written in dimensionless form as

\[ P = C_L U_c, \quad (5.72) \]

see (5.56). Equation (5.69) shows also that the disk run-out may not be too high in order to have a constant load at the probe tip.
5.4 Summary

In this chapter the pendulum of the NanoTest 550 instrument has been modelled and the dynamics of the pendulum has been analysed. The model considers the masses of the pendulum, the probe, and the balance weight, as well as the load at the coil, the load and the friction force at the specimen, and the torsional spring of the elastic hinge.

Although, the principle of the NanoTest 550 instrument is simple, the detailed analysis of the pendulum beam brings about that the load should be carefully applied in order to measure the displacement and the friction properly. The pivot point of the pendulum beam of the original NanoTest 550 did not lie in the same plane as that of the probe tip and specimen surface. It follows from the analysis in this chapter, that therefore, this pendulum is not suitable for friction measurements, because the friction force between the probe and the specimen introduces an additional moment at the pivot point, such that the actual load at the probe will be reduced. As a consequence, with the original pendulum beam the actual load at the specimen will be different from the load at the specimen, as derived from the measured load at the coil. For this reason, the pendulum has been redesigned in order to measure the friction properly. With the redesigned pendulum the pivot point is in the same plane as that of probe tip and specimen surface.

From the analysis given in this chapter it can also be concluded that the angular deflections of the pendulum beam around its pivot point are extremely small (between $-25$ mrad and $10$ mrad). Therefore, the influence of the masses at the pendulum and the torsional spring of the elastic hinge may be neglected in the calculations of the load at the specimen. Furthermore, a miniaturised friction force sensor has been developed in order to measure the friction force between a slider and a disk. Also, a rotating motor attachment has been built for the pin-on-disk type friction measurements. With the rotating motor attachment, the miniaturised friction force sensor and the redesigned pendulum, it has become possible to use the NanoTest 550 as a sensitive pin-on-disk apparatus.
Head-Disk Interface Friction Measurements on The NanoTest 550 Instrument

This chapter describes and discusses friction experiments performed on HDIs and carried out at the NanoTest 550 instrument. Firstly, the properties of the tested HDIs will be described, and the method of experimenting will be explained. Then the experimental results will be given and discussed; generalised Strubeck curves will be presented, which have been derived from the experimental data. These curves will be compared to computed generalised Strubeck curves which have been derived from the friction model, developed in Chapter 4. The effects of several parameters on the experimental data will be discussed. Finally, conclusions will be drawn from the experimental results.

6.1 The Properties of the Head-Disk Interfaces

The friction experiments consisted of measuring the friction force between the slider and the rigid disk of each HDI. Commercially available rigid disks and experimental rigid disks were tested. Table 6.1 shows the properties of some of the rigid disks used in the friction experiments. All rigid disks had an OD of 65 mm (2.5 inch). Their mechanical properties (for example, the elastic moduli and Poisson ratio of the substrate and the layers) had been given by the suppliers or were taken from the literature.

A cross section of rigid disk 2 is shown in Fig. 3.10 on page 40. These disks were circumferentially textured and had a substrate thickness of 0.80 mm. Figure 3.11 on page 42 illustrates the surface topography of a disk with an AlMg substrate. Rigid disks 3 to 7 were experimental disks and consisted of a glass substrate only. The thickness of the substrate was 0.25 mm and they had an artificially roughened surface; a geometrical pattern consisting of laser textured protuberances. Figures 6.1 and 6.2 show an example of the surface topography of these disks.

The sliders used in the friction experiment were standard two rail taper-flat sliders made of CaTiO$_3$, an example of which is shown in Fig. 3.6 and Table 3.1.
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<th>Substrate</th>
<th>Magnetic layer(s)</th>
<th>Overcoat</th>
<th>Lubricant</th>
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<td>NiP</td>
<td>C</td>
<td>PFPE</td>
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<td>$E_3 = 70.0$ GPa</td>
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<td>NiP</td>
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<td>$E_3 = 130.0$ GPa</td>
<td>$E_1 = 0.22$</td>
</tr>
<tr>
<td></td>
<td>$t_1 = 0.25$</td>
<td>$\nu_3 = \nu_2 = 0.25$</td>
<td>$E_1 = 0.20$</td>
<td></td>
</tr>
</tbody>
</table>
Fig. 6.1. The surface topography of rigid disk no. 3 used in the experimental work of this thesis, measured by an optical phase shift interferometry profiler. $z_B$ indicates the height of a protuberance (Measurement area: $464 \times 356 \mu m$, $R_a = 1.1 \text{ nm}$).

Fig. 6.2. Contour plot of the surface topography of rigid disk no. 3 used in the experimental work of this thesis, measured by an optical phase shift interferometry profiler (Measurement area: $464 \times 356 \mu m$, $R_a = 1.1 \text{ nm}$).
6.2 The Surface Topography of the Head-Disk Interfaces

The experimental results, to be presented in this chapter, will be compared with those obtained from the friction model. In the friction model certain roughness parameters are required, such as the combined CLA roughness of the surfaces, the curvatures of the summits, the summit density, etc. Thus, in order to compare the experimental and the theoretical results, the surface topography of the sliders and rigid disks has been measured before and after each experiment by means of an optical phase shift interferometry profiler. These measurements are described in Appendix I, and their results are summarised in Tables I.1 to I.5.

With the given mechanical properties, the roughness parameters and the operational parameters of each experimental HDI, the COF has been computed for each friction experiment carried out.

6.3 Experimental Method

The friction between the slider and the rigid disk of each HDI was measured on the NanoTest 550 instrument. A typical experiment ran as follows. Firstly, the slider and the disk were mounted on the friction force sensor and the XYZ table, respectively, and the sensor was clamped on the pendulum (see Figs. 5.6 and 5.7). Then the instrument was calibrated (i.e. the applied load, the friction force, the speed of the disk, the slider displacement and the disk centre position were calibrated, according to the procedures described in Chapter 5). After the calibration, the disk surface was brought into contact with the slider by means of the procedure described in Chapter 5, Section 5.3.3 on page 120. After making contact with the slider at a certain disk radius \( r \), the final (constant) load was applied and the disk was started up to a constant speed\(^1\) after which the readings of the normal load, the friction force, the number of disk revolutions and the slider displacement were taken by the computer. The program running the experiment was so contrived that the speed of the disk could be changed at will. The moment the computer was ordered to change speed, first the mean values and the standard deviations of the signals, read during the current speed, were calculated and then the speed was changed, after which a new series of readings were taken. The mean values and standard deviations of the signals and the number of revolutions of the disk at each particular speed level was stored in the memory of the computer. In this way, a certain number of cycles was made, in which the disk speed was increased stepwise from zero to a certain maximum speed and then decreased stepwise from maximum speed to zero again, while before each change in speed, the mean values and standard deviations of the signals, read during the previous disk speed, were calculated. Figure 6.3 shows an example of the variation of the disk speed during a friction experiment. To ensure short acceleration or deceleration periods between the steps, each step corresponded to an increase or decrease in disk speed of just 0.25 rps and the acceleration or deceleration was

---

\(^1\)See also footnote on speed in Chapter 1 on page 4.
Fig. 6.3. The variation of the disk speed $n$ as a function of time $t$. The disk speed could be changed at pleasure (Two cycles are shown).

20 rps/s. The sampling rate of each signal during a particular disk speed was about 0.25 s. The time past between the setting of a speed level was about 0.46 s. The number of speed steps taken was 30 to 40, thus each speed level acquired at least $30 \cdot 0.25 + 0.46 \approx 8$ s. This corresponds to 2 and 160 revolutions at the lowest (0.25 rps) and highest speed level ($\approx 20$ rps), respectively. After the disk rotation ceased, the cycle of stepwise increasing and decreasing disk speed was repeated. After a certain amount of cycles, the disk was retracted from the slider and the data were decoded. All data were stored on the RDD of the computer for analysing purposes. The velocity at each speed level $k$ was calculated from

$$v_+ = v_{+,k} = 2\pi n_k r,$$  \hspace{1cm} (6.1)$$

in which $n_k$ is the speed of the disk at level $k$ and $r$ is the radius at which the centre of the slider is at the disk. The mean COF at a particular speed level $k$ was calculated, according to

$$\mu = \mu_k = \frac{\sum_{i=1}^{n} F_{z,k,i}}{\sum_{i=1}^{n} P_{k,i}},$$  \hspace{1cm} (6.2)$$

in which $n$ is the number of readings of the signals at that particular level, and $F_{z,k,i}$ and $P_{k,i}$ are the measured friction force and the applied normal load, given by (5.1) and (5.72) on pages 106 and 125, respectively.
6.3.1 Experimental Parameters

In Chapter 4, it has been found that the applied normal load, the surface topography of the sliders and the rigid disk and the thickness of the liquid lubricant layer at the disk, are parameters of importance in the characterisation of the lubrication transitions of an HDI. Therefore, the parameters that have been varied in the experiments are:

1. The applied normal load $P$;
2. The type of disk (its roughness, the type of substrate and the type of layers, see Table 6.1);
3. The presence of a liquid lubricant layer on the rigid disk.

The temperature inside the temperature controlled laboratory room was 20.8$\pm$0.5 °C and the temperature inside the cabinet of the NanoTest 550 instrument was kept at 24.9 $\pm$ 0.1 °C. The relative humidity in the laboratory room, during the experiments was 52$\pm$6%. The load calibration factor (LCF), the depth calibration factor (DCF) and the friction force calibration factor (FCF) of the NanoTest 550 instrument were 9.5$\pm$0.1 $\mu$N/bit, 0.26$\pm$0.02 nm/bit and 8$\pm$1 $\mu$N/bit, respectively.

6.4 Discussion of the Experimental Results

A large number of experiments have been carried out on different HDIs, some of which will be discussed here.

6.4.1 Repitition Experiments

Figures 6.4 and 6.5 show the results of some repitition experiments carried out at constant load $P$, constant contact radius $r$ and on the same type of disk. Each symbol represents the mean COF of the experimental data points at the corresponding velocity or lubrication number. The solid line gives the computed COF based on Cycle 4. Each dashed line gives a curve fit through the experimental data. It has been found that with each cycle, the COF in the BL regime increases, until, after a number of cycles, it reaches a more or less constant value. For this reason, only the last cycle in each series of cycles will be considered from now on.

6.4.2 Varying Normal Load

In order to study the effect of the normal load on the lubrication transitions, a series of experiments has been carried out, in which the normal load has been varied, while the other parameters have been kept constant. Figures 6.6 and 6.7 show the results of some of these experiments. It can be seen that the generalised
Fig. 6.4. The variation of the coefficient of friction $\mu$ with the velocity $v_+$ of a repetition experiment (Disk no. 1, Slider no. 1, $P = 70.2 \pm 0.5$ mN).
Fig. 6.5. Computed and experimental generalised Strubeck curves of a repetition experiment (Disk no. 1, Slider no. 1, $P = 70.2 \pm 0.5$ mN).
Fig. 6.6. The variation of the coefficient of friction $\mu$ with the velocity $v_+$ for different normal loads $P$ (Disk no. 1, Slider no. 2).
Fig. 6.7. Computed and experimental generalised Striebeck curves for different normal loads $P$ (Disk no. 1, Slider no. 2).
Striebeck curve shifts significantly towards the left with increasing normal load. The transitions from BL to ML and from ML to AL take place at the lubrication numbers \( L_B \) and \( L_A \), respectively. These numbers have been determined from the experimental and computed generalised Striebeck curves, in a similar way as described in Chapter 4, Section 4.2 on page 50. The lubrication numbers \( L_A \) and \( L_B \) of the experimental and the computed generalised Striebeck curves can be plotted in a lubrication transition diagram (see Fig. 6.8). In case of the experiments, the slopes of the transition lines \( L_A \) and \( L_B \) are \(-0.4\) and \(-0.7\), respectively, while for the computed Striebeck curves, these slopes are both \(-0.5\). The transitions at \( L_A \) and \( L_B \) for the experimental Striebeck curves can be written as

\[
L_A = \lambda_{A,1} p^{\lambda_{A,2}},
\]

and

\[
L_B = \lambda_{B,1} p^{\lambda_{B,2}},
\]

in which \( \lambda_{A,1} \), \( \lambda_{A,2} \), \( \lambda_{B,1} \) and \( \lambda_{B,2} \) are constants given by

\[
\lambda_{A,1} \approx 22.4; \quad \lambda_{A,2} \approx -0.4; \quad \lambda_{B,1} \approx 75.9, \quad \text{and} \quad \lambda_{B,2} \approx -0.7. \quad (6.5)
\]

In Fig. 6.9 the COF in the BL regime at \( L = 10^{-2} \) is shown as a function of the nominal pressure \( p \). It can be seen that at lower nominal pressures the COF is higher than at higher nominal pressures, as already is indicated in Chapter 4.

Fig. 6.8. Lubrication transition diagram derived from the experimental and computed generalised Striebeck curves shown in Fig. 6.7.
6.4.3 Roughness

The effect of the surface topography on the lubrication transitions of the HDI has been studied by means of rigid disks numbers 3 to 7. These disks consisted of a glass substrate only. The normal load in these experiments was kept constant. Figures 6.10 and 6.11 show the results of these experiments. In Table I.5 of Appendix I it can be seen that the height of the protuberances on the disks varied from 10 to 35 nm, while the CLA roughness $R_{a, s, c}$ of the surface heights varied much less. The reason for this is that the CLA roughness represents an average roughness value, while the roughness parameter $z_B$ is a maximum roughness value. In Fig. 6.12 the velocity $v_A$ is plotted as a function of the height $z_B$ of the protuberances. It follows that the transition $L_A$ increases slightly with the roughness parameter $z_B$. This has also been found in the friction model.

6.4.4 The Liquid Lubricant Layer

The effect of the liquid lubricant layer on the friction of an HDI has been studied by carrying out experiments on lubricated and un lubricated disks. Comparing Figs. 6.6 and 6.7 with Figs. 6.10 and 6.11 it follows already that rigid disks with a liquid lubricant layer show a higher COF in the BL regime than rigid disks without such a layer. This can also be seen in Figs. 6.13 and 6.14, which show the results of experiments with and without a liquid lubricant layer. In Fig. 6.9 the COF at $L = 10^{-2}$ is given as a function of the applied normal load for HDIs.
Fig. 6.10. The variation of the coefficient of friction $\mu$ with the velocity $v_+$ for different heights $z_B$ of the protuberances (Disk no. 3 to 7, Slider no. 3, $P = 95$ mN).
Fig. 6.11. Computed and experimental generalised Stribeck curves for different heights $z_B$ of the protuberances (Disk no. 3 to 7, Slider no. 3, $P = 95$ mN).
Fig. 6.12. The variation of the velocity $v_A$ as a function of the height $z_B$ of the protuberances.

with lubricated disks and for HDIs with unlubricated disks. The higher COF with lubricated disks is due to the stiction effect, as already is shown in Chapter 4. As the combined CLA roughness of the HDIs only affects the stiction force and the transition lines $L_A$ and $L_B$, it must be concluded that the higher COF in the BL regime of the lubricated disks compared to the unlubricated disks is due to the stiction in the liquid lubricant layer. If the load at the HDIs will be reduced in future RDDs, it can be expected that the problems with high stiction forces will increase.

6.5 Conclusions

This section summarises the conclusions derived from the experimental results in this chapter.

1. Qualitatively, the shape of the experimental derived generalised Strubeck curve is predicted by the friction model. Also the prediction of the influence of certain parameters, such as the applied normal load and the roughness of the surfaces, on the generalised Strubeck curve are in agreement with the experimental results.

2. If the applied normal load at an HDI is increased (i.e. a higher nominal pressure), while other parameters are kept constant, the generalised Strubeck curve shifts towards the left. This shift is also predicted by the
Fig. 6.13. The variation of the coefficient of friction $\mu$ with the velocity $v_+$ for different loads and for lubricated and un lubricated disks.
Fig. 6.14. Computed and experimental generalised Stribeck curves for different loads and for lubricated and unlubricated disks.
friction model. The transition from BL to ML, obtained from the experimental data, are at lower lubrication numbers \( L_B \) than those obtained from a computation. The friction model predicts a somewhat different slope of the transition line \( L_B \) than that based on the experimental Striebeck curves. The slopes of the transition lines \( L_A \) and \( L_B \) are less than those found by Schipper [113] for more 'conventional' tribological systems.

3. In the BL regime, the presence of a thin liquid lubricant layer at the disk result in a higher COF, compared to an unlubricated disk. This is attributed to the stiction force at low velocities. The stiction force increases with decreasing applied normal load, as a result, in the BL regime, the COF at low loads is higher than that of an unlubricated disk.

4. The TOV increases slightly with the surface roughness of the HDI. The reason of using laser textured rigid disks is to reduce the real area of contact and the stiction force. However, also the TOV will increase with roughness.
Conclusions and Recommendations

In this thesis a friction model has been developed that predicts the friction at HDIs. The model contains equations for the contact between a rough slider and a nominal flat and multilayered rigid disk. It also includes equations for the shear thinning of the liquid lubricant layer at the rigid disk and an equation for the thickness of the gaseous lubricant. A study of the model showed the influence of the operational parameters, the properties and the surface topography of the slider and the rigid disk on the frictional behaviour of the HDI. In order to validate the model, friction experiments have been carried out on HDIs. In this chapter, the conclusions of this thesis will be given per chapter. Finally, recommendations for further research will be given.

7.1 Conclusions

Chapter 1
Due to the growing need for higher storage capacities, there is a continuing move towards CR and miniaturisation of HDIs. Although current HDIs are aerodynamically lubricated, AL in RDDs becomes less important. Unfortunately, current HDIs are optimised for separation rather than for friction. Research efforts should be focussed more on friction, and the transitions between the different lubrication regimes of an HDI. This thesis is a study about the friction and the transitions between the different lubrication regimes of an HDI.

Chapter 2
From a simple analysis of the recording and reproducing processes it follows that a higher storage capacity can be achieved by higher linear bit densities. Higher bit densities can be achieved by reducing the MS and thus the HMS. The reproducing process requires a low HMS in order to improve the amplitude and the width of the output voltage pulses of the signal. It is through the MS that an important link between the storage capacity, the magnetic recording and reproducing processes and the tribological performance of an HDI is made. Because of the thinness of the magnetic layer and the smallness of the magnetic separation, the medium needs protection against high friction and wear. Due to the required small MS only thin overcoats and lubricant layers can be applied and the head and media should be as smooth as possible.
Chapter 3
From an examination of an HDI, it follows that an HDI is a very small but complex tribological system, i.e. it is very low loaded (below 100 mN), and, in order to achieve high input and output information rates, it operates at a high relative velocity (about 10 m/s). The load at the head is transmitted to the disk surface at a very low nominal pressure (lower than 40 kPa). The rigid disk of an HDI consists of a substrate on which several thin films have been deposited. On top of the upper thin film a thin liquid lubricant is applied. The surfaces of the head and the slider are rather smooth (the combined CLA roughness is below 6 nm), and the surface heights and summit heights of the head and the disk are Gaussian distributed to a very high degree. During acceleration and deceleration of an HDI the relative velocity between the head and the disk is below the TOV or TDV, and the head is in contact with the disk. Thus, when an HDI starts, the friction changes from a high value (BL regime), to a low value (AL regime) while the separation changes from a low value (contact) to a high value (flying). From the above considerations, it follows that a friction model for the HDI should be tested against these operational parameters and properties.

Chapter 4
A friction model for the HDI has been developed that predicts the effects of certain parameters on the generalised Striebeck curve of an HDI. The model includes the mechanical and geometrical properties, and the surface topography of the disk and the slider, and the operational parameters of the HDI (such as the applied normal load, the relative velocity, and the viscosity of the liquid lubricant layer and the gas film). The model predicts that the friction force in the liquid lubricant layer may be neglected. However, a very high stiction force, relative to the applied normal load, can be generated in this layer, especially when the thickness of the layer is in the order of the surface roughness of the HDI and the applied normal load is low. The stiction force and thus the friction force in the BL regime can be reduced by a higher surface roughness of the HDI. However, the magnetic recording and reproducing processes require that the surfaces of an HDI are as smooth as possible. In order to reduce the stiction force, future HDIs should have a thinner liquid lubricant layer, and they should be applied with a higher nominal pressure. The model also predicts that the elasticity of the solid surfaces of an HDI is mainly determined by the elastic modulus of the substrate, the undercoat and the slider. However, the friction force is not much affected by a change in elastic moduli. Due to their thinness, the elastic modulus of the overcoat and the magnetic layer(s) do not have an effect on the friction force at an HDI. The transitions from BL to ML and from ML to AL are indicated in the lubrication transition diagram by lines $L_B$ and $L_A$, respectively. The friction model predicts that the slopes of the transition lines $L_B$ and $L_A$ of an HDI are slightly less than those of more 'conventional' tribological systems and that they are less dependent on the combined CLA roughness of the HDI.

Chapter 5
The NanoTest 550 measuring instrument has been used for the friction experiments on HDIs. A dynamic model of the pendulum of this instrument has been developed. From the results of this model it follows that the original straight pendulum is not suitable for the measurements of the friction force between a probe and a specimen. Therefore, a new pendulum and a friction force sensor have been developed. With the modifications, the NanoTest 550 instrument can be used as a sensitive pin-on-disk apparatus, for accurately measuring extremely low friction forces at very low loads and high velocities.

Chapter 6
The friction model predicts correctly the shape of the generalised Strubeck curve as well as the influence of certain parameters on the friction (such as the applied normal load, the surface roughness, etc.). The generalised Strubeck curves of an HDI shift somewhat to the left with higher loads, but the friction model predicts a different transition lubrication diagram than that based on the experimental Strubeck curves. The transitions from BL to ML obtained from the experimental data occur at lower lubrication numbers than those obtained from the friction model. The protuberances on the glass substrate rigid disks result in a lower COF in the BL regime, but they do not have a significant effect on the lubrication transitions. The absence of the liquid lubricant layer, reduces the COF in the BL regime.

7.2 Recommendations

This section gives recommendations for a further experimental study and for improvements on the friction model.

Recommendations for further experiments

1. Probably, future HDIs will still need a thin liquid lubricant layer to reduce friction and wear at the interface. However, much is unknown yet, of the behaviour of extremely thin liquid lubricant layers at smooth solid surfaces operating at high shear rates. The viscosity and the shear thinning behaviour of these layers could be studied experimentally, in order to improve the stiction and shear thinning effects in the friction model.

2. The pendulum beam of the NanoTest 550 instrument has some parts made of strengthened aluminium while other parts are made of ceramic (Al₂O₃). In order to improve its dynamic behaviour, the mass and stiffness of the pendulum (see, for example, Table 5.1) could be further improved by making it out of one piece of a rigid ceramic material (for example, Al₂O₃).

3. The friction experiments could be carried out in an improved environment by placing the NanoTest 550 instrument together with its entire cabinet in a clean room. In this way, besides the humidity and temperature, also the air cleanliness could be better controlled.
4. The friction experiments could be carried out on other instruments or friction testers, but with at least the same or even higher load and friction force resolutions, and which could measure the load and friction force at different sum and relative velocity values. In this way, the dependence of the lubrication transitions on the sum velocity and the relative velocity could be tested.

**Recommendations for the friction model**

1. In the equations of the friction model presented in this thesis, there is a correlation between the summit heights, the summit curvatures and the summit density of the solid surfaces by means of \( \eta \sigma_{s,c} / \rho \). The friction model could be improved by considering other contact mechanics models (for example, that of Greenwood [44] or Nayak [101], [102]) which use a statistical correlation between these parameters.

2. Near the lubrication transition from AL to ML, the gas film thickness equation could not be computed from the modified Reynolds equation, and was obtained by a curve fit at higher velocities and taking in consideration that at zero velocity the film thickness should be zero for all loads. (At the transition \( L_A \) the numerical computation did not converge properly.) The gas film thickness equation could be analysed further by using an improved model for very thin gas films as a function of load and velocity as well as slider dimensions.

3. It follows from the friction experiments on the glass substrate rigid disks that the CLA roughness parameter \( R_{a,z,c} \) in the lubrication number \( L \) is incomplete. The CLA roughness of these rigid disks varied only slightly, while the height of the protuberances at the disks varied significantly. A correlation has been found between the velocity \( v_A \) at the transition ML to AL and the height of the protuberances. A study could be carried out to find out which surface roughness parameters are of importance for the friction model.

**7.3 Suggestions for an Alternative Drive Design**

It can be concluded from this thesis that an HDI should operate at the transition from ML to AL. At this transition the COF is minimal and the HMS is low. However, current RDDs still operate at a considerable relative velocity and therefore at a high HMS. According to the lubrication transition diagram, moving the operating point towards the transition ML to AL can be done by adjusting the nominal pressure, the surface roughness or the relative velocity of the HDI. However, a modification of the surface roughness is not desirable as this affects the HMS. The pressure can be modified by adjusting the nominal area of contact (the dimensions of the slider). Modifying the relative velocity at which the interface operates will result in an alternative approach.
With two lubricated sliding solids the transitions from BL to ML and from ML to (E)HL are determined by the sum velocity $v_+ = v_1 + v_2$ and not by the relative velocity $v_- = v_1 - v_2$ (see Schipper [113]). The pressure $p(x,y)$ in the gas film of an HDI is obtained from the modified Reynolds equation (see, for example, Van der Stegen [127])

$$\nabla \cdot \left( \frac{pd^3_{g, \text{min}}}{12\eta_g} \nabla p \right) \left( 1 + 6C_1K_{n,a} \frac{p_a d_{g, \text{min}}}{pd_g} + 6C_2K_{n,a}^2 \left( \frac{p_a d_{g, \text{min}}}{pd_g} \right)^2 \right) - \frac{1}{2}v_+ pd_g - \frac{\partial (pd_g)}{\partial t} = 0, \quad (7.1)$$

in which $\eta_g$ is the dynamic viscosity of the gaseous lubricant, $p_a$ is the ambient pressure, $C_1$ and $C_2$ are surface correction coefficients, $d_{g, \text{min}}$ is the minimum thickness of the gaseous lubricant (at the trailing edge of the slider) and $K_{n,a}$ is the Knudsen number, given by

$$K_{n,a} = \frac{\lambda_a}{d_{g, \text{min}}}, \quad (7.2)$$

in which $\lambda_a$ is the molecular mean free path of the gas at ambient conditions. From (7.1) it follows that the pressure in the gas film depends only on the sum velocity $v_+$. The requirements for the velocities is illustrated in Fig. 7.1, which shows a diagram with the surface velocities $v_1$ and $v_2$, the sum velocity $v_+$ and the relative velocity $v_-$. Also shown are lines of constant sum velocity and constant relative velocity. The slip is defined as

$$S = \frac{|v_-|}{\frac{1}{2}v_+} \times 100\% = \frac{|v_1 - v_2|}{v_1 + v_2} \times 200\%. \quad (7.3)$$

With RDDs operating in track-following motion, the velocity $v_1$ of the slider is zero and thus, $S = 200\%$ (simple sliding).

From this thesis, the following conclusions regarding the velocities, can be drawn; The relative velocity $v_-$ should be high during normal operation of a RDD, because this will yield high access times. In order to achieve high linear bit densities and a sufficient output voltage pulse the HMS should be as low as possible. From the transition diagram it follows that the separation depends on the lubrication regime. The transitions between the lubrication regimes depend on the sum velocity $v_+$. With the pressures and roughness of current HDIs, it follows that the sum velocity at the transition from ML to AL should be reduced. Consequently, to make RDD operate at the transition from ML to AL, the slip $S$ of the HDIs should be as high as possible.
Fig. 7.1. The velocity field.
References


References


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A

Deformation and Wetting of Asperities

In this appendix the deformation and the wetting of the asperities at the HDI of the contact model described in Chapter 4 will be analysed. Firstly, the elastic deformation of a single asperity will be described on the basis of the elasticity theory of Hertz. Then the elastic and plastic deformation of the asperities of the contact model will be given. Finally, the forces at the wetted asperities will be derived.

A.1 The Elastic Deformation of a Single Asperity

If the initial loading and unloading process of an individual asperity is an elastic event, and if the strains are small and the materials are isotropic, the linear elasticity theory may be used. The asperities at the rough surface may then be considered as small elastic indenters. There exist numerous relations for the penetration of indenters into an elastic half space (see, for example, King [73], Loubet et al. [86], Pharr et al. [108] and Sneddon [124]). However, these are all based on rigid indenters and they are mainly used for determining the elastic behaviour in the initial unloading stage of nano indentation hardness measurements. All the relations predict that the stiffness $S$ of an indentation is given by

$$ S \equiv \frac{dP}{d\delta} = cE_c \sqrt{A} \approx 2E_c \sqrt{\frac{A}{\pi}}, \quad (A.1) $$

in which $c$ is a constant depending on the shape of the indenter, $P$ is the applied load at the indenter, $\delta$ is the indentation depth, $A$ is the projected area of contact and $E_c$ is the combined or reduced elastic modulus (the elasticity of the indenter and the specimen) given by

$$ \frac{1}{E_c} = \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2}, \quad (A.2) $$

where $E_1$ and $E_2$ are the elastic moduli of the indenter and specimen, respectively and $\nu_1$ and $\nu_2$ are the Poisson ratios of the indenter and specimen, respectively. The ratio $E_2/(1 - \nu_2^2)$ is also called the indentation modulus (see, for example, Vlassak and Nix [140], [141]). However, it can be shown that (A.1) follows as well from the elasticity theory of Hertz [55] for any nonconforming pair of contacting solid surfaces (see, for example, Moes [99]).
Fig. A.1. The contact geometry according to Hertz \( (a_x < a_y, \ k \equiv a_x/a_y \text{ and } m \equiv 1 - k^2) \).

If two nonconforming solid surfaces, each with a general curvature, are brought into contact with each other at a point, then the separation \( z \) in the neighbourhood of the point of contact can be written (by a suitable choice of Cartesian axes \( x \) and \( y \), see Fig. A.1) as

\[
z = \frac{1}{2} \rho_x x^2 + \frac{1}{2} \rho_y y^2, \quad \text{with} \quad \rho_x \geq \rho_y, \quad (A.3)
\]
in which the minor and major principal relative curvatures, \( \rho_x \) and \( \rho_y \), are given by

\[
\begin{align*}
\rho_x &= \rho_1 \cos^2 \Theta + \rho_2 \sin^2 \Theta + \rho_3 \sin \Theta \cos \Theta, \\
\rho_y &= \rho_1 \sin^2 \Theta + \rho_2 \cos^2 \Theta - \rho_3 \sin \Theta \cos \Theta,
\end{align*}
\]

where

\[
\begin{align*}
\rho_1 &= \rho_{11} + \rho_{21}; \quad \rho_2 = \rho_{12} + \rho_{22}; \quad \rho_3 = \rho_{13} + \rho_{23}, \\
\rho_1 &\geq 0, \quad \rho_2 \geq 0, \quad \rho_3 \leq 4 \rho_1 \rho_2. \quad (A.5)
\end{align*}
\]

Here, \( \rho_{11}, \rho_{12} \) and \( \rho_{13} \) are the curvatures of solid 1 in the \( x_1z, y_1z \) and \( x_1y_1 \) planes, respectively. Similarly, \( \rho_{21}, \rho_{22} \) and \( \rho_{23} \) are the curvatures of solid 2 in the \( x_2z, y_2z \) and \( x_2y_2 \) planes, respectively. The \( \rho_i \)'s are reciprocals of the corresponding radii of curvature, and each is taken to be positive if the centre of curvature for each instance lies within the solid. Furthermore, \( \Theta \) is the angle of the principal direction of the separation, given by
\[ \Theta = \frac{1}{2} \arccos \left( \frac{\rho_1 - \rho_2}{\sqrt{(\rho_1 - \rho_2)^2 + \rho_3^2}} \right) + \frac{\pi}{2} \mathcal{H}(-\rho_3), \]  

where \( \mathcal{H}(x) \) represents the Heaviside function, given by

\[ \mathcal{H}(x) = \begin{cases} 
0 & \text{if } x \leq 0 \\
1 & \text{if } x > 0.
\end{cases} \]  

If the solid surfaces are brought further into contact by applying a load and if the solids deform according to the linear elasticity theory, then the point of contact expands into an area of contact, generally of elliptical shape, bounded by the ellipse

\[ \left( \frac{x}{a_x} \right)^2 + \left( \frac{y}{a_y} \right)^2 = 1 \quad \text{with} \quad a_x \leq a_y, \]  

in which \( a_x \) and \( a_y \) are the minor and major semi axes of the contact ellipse, respectively. Hertz [55] showed that the dimensions of the contact ellipse, the mutual approach of the solid surfaces and the contact stresses in the neighbourhood of the contact are fully determined by the axis ratio

\[ \kappa \equiv \frac{a_x}{a_y}, \]  

of the contact ellipse as a function of the ratio of principal curvatures,

\[ \lambda \equiv \frac{\rho_y}{\rho_x}. \]  

The dimensions of the contact ellipse and the mutual approach of the solids are given by the following set of dimensionless parameters,

\[ \alpha \equiv a_x \sqrt[3]{\frac{2\rho E_c}{3P_{\text{a},1}}}; \quad \beta \equiv a_y \sqrt[3]{\frac{2\rho E_c}{3P_{\text{a},1}}}; \quad \gamma \equiv 2\delta \sqrt[3]{\frac{4E_c^2}{9\rho P_{\text{a},1}^2}}, \]  

where \( P_{\text{a},1} \) is the applied normal load, and \( \delta \) is the mutual approach of distant points in the two surfaces. Furthermore, \( \alpha, \beta, \gamma \) and \( \rho \) are defined as

\[ \alpha = \sqrt[3]{\frac{2\kappa E(m)}{\pi \pi}}; \quad \beta = \frac{\alpha}{\kappa}; \quad \gamma = \frac{2K(m)}{\pi \beta}; \quad \rho = \rho_x + \rho_y, \]  

in which \( m \equiv 1 - \kappa^2 \) and where \( K(m) \) and \( E(m) \) are the complete elliptic integrals of the first and second kind, respectively. The fundamental relation between \( \kappa \) and \( \lambda \) is given by Hertz [55] and can be written as

\[ \lambda = (1 - m) \frac{K(m) - E(m)}{E(m) - (1 - m)K(m)}. \]  

Thus, if the geometry of the solid surfaces is known, the main curvature \( \rho \), the minor and major relative curvatures \( \rho_x \) and \( \rho_y \) and their ratio \( \lambda \), can be calculated from (A.4) and (A.11). Next, the axis ratio \( \kappa \) of the contact ellipse can be calculated from (A.14) (see, for example, Dyson et al. [33]) or from the approximations given by Moes [99]. Then from (A.12) and (A.13) the dimensions \( a_x \) and \( a_y \) of the contact ellipse and the mutual approach \( \delta \) can be computed for any load \( P_{\text{a},1} \) and reduced elastic modulus \( E_c \).
A.2 The Effective Elastic Modulus

The elasticity theory of Hertz, which will be used for the deformation of the asperities, makes use of the reduced elastic modulus $E_e$ given by (A.2). However, an HDI consists of more than two materials. Due to the presence of the multilayers on the rigid disk, the reduced elastic modulus and the hardness may change, that is, the last term in (A.2) should be modified. As the reduced elastic modulus $E_e$ appears explicitly in the elasticity theory of Hertz, the term $(1 - \nu_e^2)/E_2$ may be replaced by $(1 - \nu_e^2)/E_e$, in which $E_e$ and $\nu_e$ are an effective elastic modulus and an effective Poisson ratio, respectively, which describe the mechanical properties of the multilayers and substrate.

Several relations have been proposed for an effective elastic modulus. Most of these relations are half theoretical and half empirical, and require some unknown parameter, for example, the equation proposed by Bhattacharya and Nix [9]. Nearly all of these equations are derived from indentation studies of indenters against thin films, that is, a rigid indenter against elastic surfaces. In this appendix, both indenting asperities are elastic. Furthermore, most of the equations found in the literature can only be used for single relatively thick films, and are not applicable for thin film rigid disks.

Gao et al. [39], however, describes an effective elastic modulus, which is suitable for a large range of thickness, and moreover, gives a relation for multilayers as well. They applied a moduli-perturbation method to construct a closed-form, first-order-accurate solution for the effective elastic modulus of a homogeneous medium with layered varying elastic moduli in the depth direction. One disadvantage is, that the relation is derived for a rigid cylindrical punch, indenting a multilayered linear elastic half space, and not for an arbitrary elastic indenter. Gao et al. [39] provided the following expression\(^1\) for the effective elastic modulus of a substrate coated with $n$ elastic dissimilar thin films,

\[
\frac{1 - \nu_e^2}{E_e} = \frac{1 - \nu_n - \sum_{i=1}^{n-1} I_1(\xi_i) \left( \nu_i - \nu_{i+1} \right)}{E_n \frac{E_n}{1 + \nu_n} + \sum_{i=1}^{n-1} I_0(\xi_i) \left( \frac{E_i}{1 + \nu_i} - \frac{E_{i+1}}{1 + \nu_{i+1}} \right) + I_0(\xi_n) \left( \frac{E_n}{1 + \nu_n} - \frac{E_n}{1 + \nu_n} \right)},
\]

(A.15)

in which $I_0(\xi)$ and $I_1(\xi)$ are given by

\[
I_0(\xi_i) = \frac{2}{\pi} \arctan \xi_i + \frac{1}{2\pi(1 - \nu)} \left[ (1 - 2\nu)\xi_i \ln \frac{1 + \xi_i^2}{\xi_i^2} - \frac{\xi_i}{1 + \xi_i^2} \right],
\]

(A.16)

\(^1\)The summations over the index $i$ in (18) of Gao et al. [39] are wrong, as they do not correspond with the thin films shown in Fig. 7 of the paper (no thin film exists with a thickness of $h_0$). The error has been corrected in this appendix by rewriting the equation in a somewhat different form.
and
\[
I_1(\xi) = \frac{2}{\pi} \arctan(\xi) + \frac{1}{\pi} \xi \ln \frac{1 + \xi^2}{\xi^2},
\]
respectively. The parameter \( \xi \) is defined as
\[
\xi_i = \sum_{k=1}^{i} \frac{t_k}{a_x},
\]
in which \( t_k \) is the thickness of thin film \( k \) and \( a_x \) is the contact radius of the contact circle. Furthermore, \( \nu_i \) and \( E_i \) are the Poisson ratio and the elastic modulus of thin film \( i \), respectively. A subscript \( s \) denotes the substrate. Both \( I_0(\xi_i) \) and \( I_1(\xi_i) \) vanish as \( \xi_i \to 0 \) and approach unity as \( \xi_i \to \infty \).

The function \( I_0 \) depends on the Poisson ratio \( \nu \) which can be taken as either one of the thin films or the substrate value. However, Gao et al. [39] found that this has a negligible effect because \( I_0 \) is rather insensitive to the choice of \( \nu \). Here, the average of the Poisson ratio’s of the thin films have been chosen for \( \nu \),
\[
\nu = \frac{1}{n} \sum_{i=1}^{n} \nu_i.
\]

As the reduced elastic modulus \( E_c \) occurs explicitly in the Hertzian equations, \( E_1 \) and \( \nu_1 \) in the reduced elastic modulus (A.2) can now be replaced by \( E_r \) and \( \nu_r \) of the slider and the term \( (1 - \nu_2^2)/E_2 \) can be replaced by the effective elastic modulus (A.15),
\[
\frac{1}{E_c} = \frac{1 - \nu_r^2}{E_r} + \frac{1 - \nu_e^2}{E_e},
\]
in which the latter term is given by (A.15). The reduced elastic modulus and therefore the dimensions of the contact ellipse, depends on the elastic properties of the surface of the slider, the thin films and the substrate.

A.3 The Elastic Deformation of Asperities

The behaviour of an individual asperity is known from (A.12), given in the previous section. The minor and major semi axes \( a_x \) and \( a_y \) of the contact ellipse of an asperity in contact with the smooth surface are,
\[
a_x = \alpha^3 \sqrt{\frac{3P_{n,1}}{2\rho E_c(a_x)}},
\]
and
\[
a_y = \beta^3 \sqrt{\frac{3P_{n,1}}{2\rho E_c(a_x)}},
\]
The approach \( \delta_0 \) (the distance which points outside the deforming zone move together during the deformation) of an asperity due to a load \( P_{n,1} \), is given by (A.12)
\[ \delta_s = \frac{1}{2} \gamma \left( \frac{9 \rho P_{s,1}^2}{4 E_c^2(a_s)} \right)^{\frac{3}{2}}. \]  

(A.23)

Note, that \( a_x, a_y \) and \( \delta_s \) depend on the reduced elastic modulus, which in turn depends on the radius \( a_s \). In order to proceed along the lines, set by Greenwood and Williamson [47], we need to express (A.21), (A.22) and (A.23) in terms of the approach \( \delta_s \). If (A.23) is substituted into (A.21) and (A.22), then \( a_s \) in terms of \( \delta_s \) becomes

\[ a_s = \alpha \sqrt{\frac{2 \delta_s}{\gamma \rho}}, \]  

(A.24)

and \( a_y \) becomes

\[ a_y = \beta \sqrt{\frac{2 \delta_s}{\gamma \rho}}, \]  

(A.25)

Thus, the reduced elastic modulus can be regarded as a function of the indentation, that is \( E_c(\delta_s) \). The contact area \( A_{s,1} \) of a microcontact follows from (A.21) and (A.22),

\[ A_{s,1} = \pi a_x a_y = \pi \alpha \beta \sqrt{\frac{3 P_{s,1}}{2 \rho E_c(\delta_s)}}. \]  

(A.26)

After substitution of (A.23) into (A.26) the contact area of a microcontact becomes

\[ A_{s,1} = 2 \pi \alpha \beta \frac{\delta_s}{\gamma \rho}. \]  

(A.27)

According to (A.23) the load \( P_{s,1} \) at a microcontact is given by

\[ P_{s,1} = \frac{4}{3} \sqrt{2E_c(\delta_s)} \sqrt{\frac{\delta_s^3}{\gamma^3 \rho}}, \]  

(A.28)

If the two surfaces come together until their reference planes are separated by a distance \( d_s \), then there will be contact at any asperity whose height was originally greater than \( d_s \). The probability of making contact at heights between \( z \) and \( z + dz \) is \( \phi(z)dz \), in which \( \phi(z) \) is the probability distribution of the summit heights of the rough surface (see Fig. 4.10, page 54). Thus, the probability of making contact at any given asperity of height \( z \) is,

\[ \Pr(z > d_s) = \int_{d_s}^{\infty} \phi(z)dz. \]  

(A.29)

If the nominal area of contact is \( A_a \) and the density of the asperities at the rough surface is \( \eta \), then there are \( \eta A_a \) asperities in all, and the expected number of microcontacts will be

\[ E\{n_s\} = \eta A_a \int_{d_s}^{\infty} \phi(z)dz, \]  

(A.30)
in which \( E \{ \} \) represents the expected value of the term inside the brackets. With \( \delta_n = z - d_n \), and (A.27), the expected contact area of an asperity becomes

\[
E \{ A_{r,1} \} = \int_{d_s}^{\infty} A_{n,1} \phi(z) \, dz = 2\pi \alpha \beta \frac{\alpha}{\gamma \rho} \int_{d_s}^{\infty} (z - d_n) \phi(z) \, dz, \quad (A.31)
\]

and the expected total real area of contact will be given by,

\[
E \{ A_{r,s} \} = \eta A_n E \{ A_{r,1} \} = 2\pi \eta A_n \frac{\alpha \beta}{\gamma \rho} \int_{d_s}^{\infty} (z - d_n) \phi(z) \, dz. \quad (A.32)
\]

Similarly, the expected load at an asperity becomes

\[
E \{ P_1 \} = \int_{d_s}^{\infty} P_{s,1} \phi(z) \, dz = \frac{4}{3} \sqrt{2} \gamma^{3/2} \rho^{1/2} \int_{d_s}^{\infty} E_c(\delta_n) \delta_n^{3/2} \phi(z) \, dz. \quad (A.33)
\]

Since, \( \delta_n = z - d_n \), this can be written as

\[
E \{ P_1 \} = \frac{4}{3} \sqrt{2} \gamma^{3/2} \rho^{1/2} \int_{d_s}^{\infty} E_c(z - d_n) (z - d_n)^{3/2} \phi(z) \, dz. \quad (A.34)
\]

The expected total load is,

\[
E \{ P_n \} = \eta A_n E \{ P_1 \} = \frac{4}{3} \sqrt{2} \gamma^{3/2} \rho^{1/2} \int_{d_s}^{\infty} E_c(z - d_n) (z - d_n)^{3/2} \phi(z) \, dz. \quad (A.35)
\]

The expected elastic work done by the deformation of an asperity is,

\[
E \{ W_{e,1} \} = \int_{d_s}^{\infty} P_{s,1} \delta_n \phi(z) \, dz = \frac{4}{3} \sqrt{2} \gamma^{3/2} \rho^{1/2} \int_{d_s}^{\infty} E_c(z - d_n) (z - d_n)^{5/2} \phi(z) \, dz. \quad (A.36)
\]

The expected total elastic work is,

\[
E \{ W_e \} = \eta A_n E \{ W_{e,1} \} = \frac{4}{3} \sqrt{2} \gamma^{3/2} \rho^{1/2} \int_{d_s}^{\infty} E_c(z - d_n) (z - d_n)^{5/2} \phi(z) \, dz. \quad (A.37)
\]

In contrast to Greenwood and Williamson, no standardised variables are introduced at this stage, but instead the following function is defined

\[
F_{m,n}(d_s) \overset{\text{def}}{=} \int_{d_s}^{\infty} E_c^m(z - d_n) (z - d_n)^n \phi(z) \, dz. \quad (A.38)
\]
Equations (A.30), (A.32), (A.35) and (A.37) can then be written as

\[ E\{n_s\} = \eta A_a F_{0,0}(d_s), \quad (A.39) \]

\[ E\{A_{r,s}\} = 2\pi \eta A_a \frac{\alpha \beta}{\gamma \rho} F_{0,1}(d_s), \quad (A.40) \]

\[ E\{P_s\} = \frac{4}{3} \sqrt{2} \eta A_a \frac{1}{\gamma^{3/2} \rho^{1/2}} F_{1.3/2}(d_s), \quad (A.41) \]

and

\[ E\{W_c\} = \frac{4}{3} \sqrt{2} \eta A_a \frac{1}{\gamma^{3/2} \rho^{1/2}} F_{1.5/2}(d_s). \quad (A.42) \]

The mean size \( \bar{A}_1 \) of a microcontact becomes,

\[ \bar{A}_1 = \frac{E\{A_{r,s}\}}{E\{n_s\}} = 2\pi \frac{\alpha \beta}{\gamma \rho} \frac{F_{0,1}(d_s)}{F_{0,0}(d_s)}. \quad (A.43) \]

The mean pressure \( \bar{p}_s \) follows from the quotient of (A.40) and (A.41), and is

\[ \bar{p}_s = \frac{E\{P_s\}}{E\{A_{r,s}\}} = \frac{2\sqrt{2}}{3\pi} \frac{1}{\alpha \beta} \sqrt{\frac{\rho}{\gamma}} \frac{F_{1.3/2}(d_s)}{F_{0,1}(d_s)}. \quad (A.44) \]

The nominal (or apparent) pressure \( p_a \) can be retrieved from (A.41), and is

\[ E\{P_s\} = p_a A_a = \bar{p}_s E\{A_{r,s}\} = \frac{4}{3} \sqrt{2} \eta A_a \frac{1}{\gamma^{3/2} \rho^{1/2}} F_{1.3/2}(d_s), \quad (A.45) \]

or

\[ p_a = \frac{4}{3} \sqrt{2} \eta \frac{1}{\gamma^{3/2} \rho^{1/2}} F_{1.3/2}(d_s). \quad (A.46) \]

A.4 The Plastic Deformation of Asperities

A microcontact will become plastic if the height of an asperity \( z \) is greater than \( d_s + w_p \), in which \( w_p \) is given by \( w_p = (1/\rho)(H/E_c)^2 \) (see Greenwood and Williamson [47]), with \( H \) the hardness of the asperity. The area of contact \( A_{p,1} \) of a plastically deformed asperity can be derived from (A.27). Since, \( \delta = z - d_s \), the expected contact area of a plastically deformed asperity is,

\[ E\{A_{p,1}\} = \int_{d_s + w_p}^{\infty} A_{s,1} \phi(z) \, dz = 2\pi \frac{\alpha \beta}{\gamma \rho} \int_{d_s + w_p}^{\infty} (z - d_s) \phi(z) \, dz, \quad (A.47) \]

and the expected total plastic area of contact will be given by;

\[ E\{A_{p,s}\} = \eta A_a E\{A_{p,1}\} = 2\pi \eta A_a \frac{\alpha \beta}{\gamma \rho} \int_{d_s + w_p}^{\infty} (z - d_s) \phi(z) \, dz, \quad (A.48) \]
Note, that in general
\[
\int_{d_s + w_p}^{\infty} (z - d_s) \phi(z) dz \neq F_{0,1}(d_s + w_p), \quad (A.49)
\]

but (A.48) can be written as
\[
E \{ A_{p,s} \} = 2\pi \eta A_a \frac{\alpha \beta}{\gamma \rho} \left( \int_{d_s + w_p}^{\infty} (z - (d_s + w_p)) \phi(z) dz + w_p \int_{d_s + w_p}^{\infty} \phi(z) dz \right), \quad (A.50)
\]
or
\[
E \{ A_{p,s} \} = 2\pi \eta A_a \frac{\alpha \beta}{\gamma \rho} \left( F_{0,1}(d_s + w_p) + w_p F_{0,0}(d_s + w_p) \right). \quad (A.51)
\]

The microcontacts of an HDI are thus specified by the following parameters: The height distribution of the summits \( \phi(z) \), the dimensionless minor and major semi axes \( \alpha \) and \( \beta \) of the contact ellipse of the microcontacts, the dimensionless main curvature \( \rho \) of the asperities and \( \gamma \) the dimensionless mutual approach of two asperities, the density \( \eta \) of the asperities, the reduced combined elastic modulus \( E_c \) and therefore, the elastic moduli, Poisson ratio’s and thickness of the thin films and substrate.

A.5 The Wetting of a Single Asperity

Due to the presence of a liquid lubricant layer at the HDI, there are two forces which may not be neglected in the friction model; these are the stiction and the viscous forces. In this section the equations for these forces will be derived.

A.5.1 The Stiction Force

If an individual asperity is touched by or immersed into a thin liquid lubricant layer, an attractive (adhesive) force arises between the asperity and the liquid. This force is ascribed to the pressure difference in the concave meniscus, that is generated around the asperity, by the liquid surface tension \( \gamma_t \). The pressure inside this meniscus is lower than outside, generating an attractive force which is usually referred to as the meniscus force or stiction force\(^2\).

Several equations have been proposed by various authors, from which the stiction force between an asperity and a surface can be calculated (see, for example, Chizhik [27], Gao et al. [37], [38], Komvopoulos and Yan [75], Li and Talke [81], [82], McFarlane and Tabor [90], Rabinowicz [111], and Streator [129]). Usually, these equations are derived by considering the Young-Laplace pressure in the liquid (see Adamson [2]) or by considering the total surface free energy of the interface. For example, Gao et al. [38], developed a model for the stiction force.

\(^2\)See also the footnote on stiction in Chapter 1 on page 11.
force at a single spherical asperity in the presence of a liquid film on a flat surface, which for a semi elliptical asperity becomes

\[ P_{t,1} = -2\pi \sqrt{\frac{1}{\rho_x\rho_y}} \gamma_t (1 + \cos \theta_t), \]  

(A.52)

in which \( \rho_x \) and \( \rho_y \) are the minor and major principle relative curvatures of the gap between the asperity and the surface of the liquid given in (A.4), \( \gamma_t \) is the surface tension of the liquid lubricant layer, and \( \theta_t \) is the meniscus angle, that is the contact angle between the lubricant and the asperity. The negative sign means that the stiction force is attractive. The stiction force acts on an elliptic area of

\[ A_{t,1} = \pi a_x a_y = 2\pi \sqrt{\frac{1}{\rho_x\rho_y}} \delta_t, \]  

(A.53)

and since, \( \delta_t = z - d_s + t_t \) (see Fig. 4.10), the area becomes

\[ A_{t,1} = 2\pi \sqrt{\frac{1}{\rho_x\rho_y}} (z - d_s + t_t). \]  

(A.54)

Therefore, the mean pressure below a wetted asperity is

\[ \overline{p}_{t,1} = \frac{P_{t,1}}{A_{t,1}} = -\frac{\gamma_t (1 + \cos \theta_t)}{z - d_s + t_t}. \]  

(A.55)

Similar equations for the pressure have been derived by the other authors mentioned above. Streator [129] rightly pointed out that these equations cannot be used for very thin liquid lubricant layers. When the separation \( d_s - z \) below an asperity approaches the thickness \( t_t \) of the liquid lubricant layer, the pressure can be very large. For example, for a typical HDI with \( \gamma_t = 2 \cdot 10^{-2} \, \text{N/m} \), \( \theta_t = 0 \), and \( z - d_s + t_t = 2 \, \text{nm} \), the pressure would become 20 MPa! In reality, the lubricant will cavitate or rupture long before this pressure is reached.

### A.5.2 The Viscous Force

With the same arguments, Streator et al. [130], [131] showed that the thin liquid lubricant layer of an HDI must exhibit non-Newtonian behaviour. For a typical HDI, operating at a velocity \( v_- \) of 1 m/s and a liquid lubricant layer thickness \( t_t \) of 1 nm, the rate of shearing \( \dot{\gamma} = v_- / t_t = 10^7 \, 1/\text{s} \). If the lubricant would exhibit Newtonian behaviour, the shear stress \( \tau_t \) would become \( \tau_t = \eta_t \dot{\gamma} = 1 \, \text{GPa} \), which, again, is an unrealistic value.

On the other hand, by applying the Reynolds equation to a fully liquid lubricated slider of an HDI, Streator et al. [131] derived the following equations for the friction force in the liquid lubricant layer of an HDI,

\[ F_t = \eta_t \frac{v_-}{t_t} A_h, \]  

(A.56)
where $A_n$ is the nominal (or apparent) area of contact. Furthermore, they showed that the effect of viscous heating on the viscosity is negligible. Thus for a fully liquid lubricated HDI, the viscous friction force is proportional to the nominal area of contact and directly proportional to the rate of shearing $\dot{\gamma} = v_-/t_{\ell}$.

### A.5.3 The Concept of Shear Thinning

The two dimensional stress state of the liquid lubricant at a particular point in the layer is described by the stress tensor $\sigma$

$$\sigma = \begin{pmatrix} -p & \tau_{\ell} \\ \tau_{\ell} & -p \end{pmatrix},$$  \hspace{1cm} (A.57)

in which $p$ is the pressure (relative to the ambient pressure $p_a$ and negative compared to a positive tensile stress) and $\tau_{\ell}$ is the shear stress. The principal stresses $\sigma_1$ and $\sigma_2$ follow from the characteristic equation

$$|\sigma - \sigma I| = 0, \quad \text{with} \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$  \hspace{1cm} (A.58)

in which $I$ is the unit matrix. Hence, the maximum and minimum principal (normal or tensile) stresses become

$$\sigma_1 = +\tau_{\ell} - p, \quad \text{and} \quad \sigma_2 = -\tau_{\ell} - p,$$  \hspace{1cm} (A.59)

respectively. The lubricants used for HDIs are usually perfluoropolyether (PFPE) fluids. Cantow et al. [22] studied the viscosity dependence of PFPEs on the rate of shearing. They found that these lubricants show shear thinning\(^3\) for rates of shearing higher than approximately 10 000 1/s. Winer and Bair [153] found from experiments carried out at low pressures that above certain high rates of shearing a limiting shear stress will be reached. This limiting shear stress is lower at lower pressures. The limiting shear stress is attributed to a critical normal stress $\sigma_m$, or a tensile strength, above which cavitation (void formation), yield or rupture, and the reduction of viscosity appears\(^4\). Thus, according to (A.59), $\sigma_1$ is limited by the tensile strength, $\sigma_m$, thus

$$\sigma_m \geq \tau_{\ell} - p,$$  \hspace{1cm} (A.60)

or

$$\tau_{\ell} = \eta \dot{\gamma} \leq p + \sigma_m.$$  \hspace{1cm} (A.61)

From this equation it follows that the maximum shear stress depends linearly on the pressure in the lubricant layer.

---

\(^3\) Shear thinning means that the viscosity of the lubricant decreases with increasing rate of shearing.

\(^4\) The shear strength of a liquid is the maximum shear stress sustainable by the liquid. Usually, the shear stress is expected to depend not only on the nature of the lubricant but also on the nature of the solid to which it is attached. The tensile strength, on the other hand, is a physical property of the liquid itself.
By using this concept of shear thinning and lubricant yield, Streator [129] derived upper bounds for the friction force of a thin lubricant layer such as in an HDI, which included the stiction at the interface. In another paper, Streator et al. [131] studied the lubrication of an HDI in the ML regime by using this concept of shear thinning (by means of rather thick lubricant layers of 20 to 80 nm, and at disk velocities of 0.25 mm/s to 0.25 m/s and at loads in the range of 60 to 280 mN). They concluded that the lubricant retains its bulk viscosity in lubricant layers as thin as 11 to 12 molecular diameters. They found also that the rheological state of the lubricant is determined by a parameter they introduced as the ‘Newtonian’ shear stress $\tau_0$, which is the product of the rate of shearing $\dot{\gamma}$ and the ‘low-shear’, or ‘zero-shear’ viscosity $\eta_0$, introduced by Cantow et al. [22],

$$\tau_0 \overset{\text{def}}{=} \eta_0 \dot{\gamma}.$$  \hfill (A.62)

In a recent paper, Streator [130] proposed the following equation for the shear stress of a thin liquid lubricant layer of PFPE (see Fig. A.2),

$$\log \tau_\ell (\tau_0) = \begin{cases} 
\log \tau_0 & \text{if } \tau_0 < \tau_s, \\
\frac{a_0 + a_1 \log \tau_0 + a_2 (\log \tau_0)^2}{n \log (a_3 \tau_0) + 1} & \text{if } \tau_s < \tau_0 < \tau_c, \\
\frac{n \log (a_3 \tau_0)}{n \log (a_3 \tau_0) + 1} & \text{if } \tau_0 \geq \tau_c,
\end{cases} \hfill (A.63)$$

where $\tau_\ell$ is the actual shear stress, $\tau_s$ is the value of Newtonian shear stress at which shear thinning begins, $\tau_c$ is the value of Newtonian shear stress that corresponds to the onset of the ‘rupture’ regime, and where $a_0$, $a_1$, $a_2$, $a_3$ and $n$ are empirically determined parameters, given by

$$a_0 = 0.221; \quad a_1 = 1.003; \quad a_2 = -1.74 \cdot 10^{-2}; \quad a_3 = 1.13 \cdot 10^{-15}; \quad n = -0.58. \hfill (A.64)$$

For PFPEs, $\tau_c = 5.0 \cdot 10^5$ Pa, $\tau_s = 4.4 \cdot 10^3$ Pa and $\eta_0 = 0.195$ Pa·s. In this thesis a curve fit has been derived for (A.63) which is

$$\frac{\tau_\ell}{\tau_0} = \frac{c_0 + c_2 \tau_0^{1/2} + c_4 \tau_0}{1 + c_1 \tau_0^{1/2} + c_3 \tau_0 + c_5 \tau_0^{3/2}}, \hfill (A.65)$$

in which the curve fit parameters $c_0$ to $c_5$ are given by

$$c_0 = 1.018; \quad c_1 = 2.616 \cdot 10^{-3}; \quad c_2 = -6.763 \cdot 10^{-6}; \quad c_3 = -7.001 \cdot 10^{-6}; \quad c_4 = 4.292 \cdot 10^{-11}; \quad c_5 = 9.693 \cdot 10^{-9}. \hfill (A.66)$$

The shear thinned viscosity $\eta_\ell$ is related to the shear stress $\tau_\ell$ by

$$\tau_\ell = \eta_\ell \dot{\gamma}, \hfill (A.67)$$

in which $\dot{\gamma}$ is the rate of shearing. From (A.62) and (A.67) it follows that $\eta_\ell / \eta_0 = \tau_\ell / \tau_0$. It will now be assumed that (A.60) and (A.63) can be used for each wetted lubricated asperity.
Fig. A.2. The dependency of the viscosity $\eta_l$ on the Newtonian shear stress $\tau_0$ for PFPEs, according to Streator [130].

### A.6 The Wetting of Asperities

The expected number of wetted asperities is now

$$E\{n_{\ell}\} = \eta A_a \int_{d_s - t_\ell}^{d_s} \phi(z) dz = \eta A_a G_{0,0}(d_s), \quad (A.68)$$

with $G_{0,0}(d_s)$ defined in (A.79). The wetted area of an asperity is given by (A.54). The expected wetted area of an asperity becomes

$$E\{A_{\ell,1}\} = \int_{d_s - t_\ell}^{d_s} A_{\ell,1} \phi(z) dz = 2\pi \sqrt{\frac{1}{\rho_x \rho_y}} \int_{d_s - t_\ell}^{d_s} (z - d_a + t_\ell) \phi(z) dz. \quad (A.69)$$

The expected total wetted area is now

$$E\{A_{\ell}\} = \eta A_a E\{A_{\ell,1}\} = 2\pi \eta A_a \sqrt{\frac{1}{\rho_x \rho_y}} \int_{d_s - t_\ell}^{d_s} (z - d_a + t_\ell) \phi(z) dz, \quad (A.70)$$

which can be written as
\[ E \{ A_{r,l} \} = 2\pi \eta A_n \sqrt{\frac{1}{\rho_x \rho_y}} G_{0,1}(d_s), \]  

(A.71)

in which \( G_{0,1}(d_s) \) is a function defined below in (A.79). The shear stress below a wetted asperity is given by (A.63) and with the Newtonian shear stress given by

\[ \tau_0 = \frac{\eta_0 v_-}{d_a - z}, \]

(A.72)

the shear stress becomes

\[ \tau_{\ell,1} = \tau_\ell(\eta_0, v_-, d_a - z). \]

(A.73)

The friction force at a wetted asperity, due to the shearing of the lubricant is

\[ F_{\ell,1} = \tau_{\ell,1} A_{\ell,1}. \]

(A.74)

After substituting (A.54) and (A.73) into (A.74) gives for the friction force at a wetted asperity

\[ F_{\ell,1} = 2\pi \sqrt{\frac{1}{\rho_x \rho_y}} (z - d_a + t_\ell) \tau_\ell(\eta_0, v_-, d_a - z). \]

(A.75)

The expected friction force due to a wetted asperity is then

\[
E \{ F_{\ell,1} \} = \int_{d_s-t_\ell}^{d_s} F_{\ell,1}(z) \phi(z) dz \\
= 2\pi \sqrt{\frac{1}{\rho_x \rho_y}} \int_{d_s-t_\ell}^{d_s} (z - d_a + t_\ell) \tau_\ell(\eta_0, v_-, d_a - z) \phi(z) dz.
\]

(A.76)

The expected total friction force due to the liquid lubricant layer will be given by

\[
E \{ F_{\ell} \} = \eta A_n E \{ F_{\ell,1} \} \\
= 2\pi \eta A_n \sqrt{\frac{1}{\rho_x \rho_y}} \int_{d_s-t_\ell}^{d_s} (z - d_a + t_\ell) \tau_\ell(\eta_0, v_-, d_a - z) \phi(z) dz,
\]

(A.77)

which can be written as

\[ E \{ F_{\ell} \} = 2\pi \eta A_n \sqrt{\frac{1}{\rho_x \rho_y}} G_{1,1}(d_s), \]

(A.78)

where \( G_{1,1}(d_s) \) is a function given by

\[ G_{m,n}(d_s) \overset{\text{def}}{=} \int_{d_s-t_\ell}^{d_s} \tau_\ell^m(\eta_0, v_-, d_a - z) (z - d_a + t_\ell)^n \phi(z) dz. \]

(A.79)
According to the limiting shear stress given by (A.60) it follows that the minimum pressure below a wetted asperity is

\[ p_{\ell,1} = \tau_{\ell,1} - \sigma_m, \quad (A.80) \]

in which \( \sigma_m \) is the maximum limiting tensile strength of the liquid lubricant layer. The minimum load carried by a wetted asperity is

\[ P_{\ell,1} = p_{\ell,1} A_{\ell,1} = (\tau_{\ell,1} - \sigma_m) A_{\ell,1}. \quad (A.81) \]

The expected load carried by a wetted asperity is now

\[ E\{P_{\ell,1}\} = \int_{d_s-t_{\ell}}^{d_a} P_{\ell,1}\phi(z)\,dz, \quad (A.82) \]

or

\[ E\{P_{\ell,1}\} = \int_{d_s-t_{\ell}}^{d_a} (\tau_{\ell,1} - \sigma_m) A_{\ell,1}\phi(z)\,dz, \quad (A.83) \]

The expected load carried by the liquid lubricant layer becomes

\[ E\{P_{\ell}\} = \eta A_n E\{P_{\ell,1}\} = \eta A_n \int_{d_s-t_{\ell}}^{d_a} (\tau_{\ell,1} - \sigma_m) A_{\ell,1}\phi(z)\,dz, \quad (A.84) \]

and after substitution of (A.54) and (A.73) the expected load carried by the liquid lubricant layer can be written as

\[ E\{P_{\ell}\} = E\{F_{\ell}\} - \sigma_m E\{A_{\ell}\}. \quad (A.85) \]

From this equation it follows that the expected load carried by the liquid lubricant layer becomes negative if

\[ E\{F_{\ell}\} \leq \sigma_m E\{A_{\ell}\}. \quad (A.86) \]

In that case the liquid introduces an adhesive force (stiction force) which contributes to the applied normal load.

### A.6.1 Gaussian Distribution of the Heights

In Chapter 3 it was mentioned that the summit height distribution of the surface of a slider and a rigid disk are Gaussian to a very good approximation. For a Gaussian summit height distribution, the probability density function \( \phi(z) \) is

\[ \phi(z) = \frac{1}{\sigma_{s,c}\sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{z}{\sigma_{s,c}} \right)^2}, \quad (A.87) \]
in which \( \sigma_{n,c} \) is the standard deviation of the summit heights and in which the mean value of the summit heights is assumed zero. Hence, (A.38) and (A.79) become

\[
F_{m,n}(d_s) = \frac{1}{\sigma_{n,c} \sqrt{2\pi}} \int_{d_s}^{\infty} E_c^m(z - d_s)(z - d_s)_n e^{-\frac{1}{2} \left( \frac{z}{\sigma_{n,c}} \right)^2} \, dz, \tag{A.88}
\]

and

\[
G_{m,n}(d_s) = \frac{1}{\sigma_{n,c} \sqrt{2\pi}} \int_{d_s - t_{\xi}}^{d_s} \tau_{\xi}^m(\eta_0, \nu, d_s - z)(z - d_s + t_{\xi})_n e^{-\frac{1}{2} \left( \frac{z}{\sigma_{n,c}} \right)^2} \, dz. \tag{A.89}
\]
B

The Shear Stress in a Thin Lubricating Gas Film

B.1 Problem Definition

In this appendix the equations for the velocity and the shear stress across a thin lubricating gas film will be derived. In Fig. B.1 two solid surfaces 1 and 2 are given which are entirely separated by a thin lubricating gas film. Both solid surfaces have an arbitrary shape, which are given by \( z_1 = f_1(x, y) \) and \( z_2 = f_2(x, y) \), respectively. Then, the separation at \((x, y)\) is \( d_g(x, y) = z_1 - z_2 \). In deriving the equation an Eulerian coordinate system will be used here, that is, the velocity of a passing volume element will be considered with respect to a fixed point at the solid-gas-solid interface. The velocities of solid surfaces 1 and 2 are \( \mathbf{v}_1 = (u_1, v_1, w_1) \) and \( \mathbf{v}_2 = (u_2, v_2, w_2) \), respectively, in which \( u_1, u_2, v_1, v_2, w_1, \) and \( w_2 \) are the components of the velocity vectors \( \mathbf{v}_1 \) and \( \mathbf{v}_2 \) and which are in general functions of \( x, y \) and \( z \). The velocity of the gas at \((x, y, z)\) is \( \mathbf{v} = (u, v, w) \), in which \( u, v \) and \( w \) are the components of the velocity vector \( \mathbf{v} \) in \( x, y, \) and \( z \) direction, respectively. The problem is to find the velocity \( \mathbf{v} \) as a function of the pressure gradient \( \nabla p(x, y) \) and the velocities \( \mathbf{v}_1 \) and \( \mathbf{v}_2 \) of the solid surfaces. It will be assumed that the gas flows continuous laminar, that the gas behaves Newtonian and that the viscosity of the gas is constant across the
gas film. Furthermore, it will be assumed that the gas flow can be considered isothermal.

**B.2 Boundary Conditions**

The velocity profile of a lubricating film is usually derived by assuming a continuous laminar flow. In this case, the boundary conditions are obtained by assuming that no slip exists at the gas-solid surface interfaces, that is

\[
\mathbf{v}\big|_{z=z_1} = \mathbf{v}_1, \\
\mathbf{v}\big|_{z=z_2} = \mathbf{v}_2. \tag{B.1}
\]

This assumption is reasonable for flows where the molecular mean free path\(^1\) \(\lambda\) of the gas is negligible compared to the separation \(d_g\) between the solid surfaces. However, in the case of an HDI, the separation \(d_g\) is on the order of 100 nm and the molecular mean free path \(\lambda\) of the gas is 63.5 nm. As a result, slip (see Kennard [67]) between the gas and the solid surfaces takes place, producing an effect similar to that which would be caused by a reduction in dynamic viscosity (usually called *molecular rarefaction*). Another effect is that the heat conduction is characterised by a discontinuity in temperature between the solid boundary and the gas. However, it can be shown that the difference between the temperature of the solid surfaces and the temperature of the gas film is small. Thus, the gas flow can be considered isothermal.

A measure for the rarefaction effects is the *Knudsen number* \(K_n\) defined as

\[
K_n \overset{\text{def}}{=} \frac{\lambda}{d_g}. \tag{B.2}
\]

As long as the Knudsen number \(0 \leq K_n \leq 1\), then, as a first approximation, the flow may still be treated by conventional continuum theories, but with modified boundary conditions. Instead of velocities \(\mathbf{v}\) vanishing at the boundaries, the concept of slip velocities is then introduced (see for example Burgdorfer [19], Hsia and Domoto [58] and Mitsuya [95]). Currently, there are three types of slip velocity models, which can be represented by

\[
\mathbf{v}_{\text{slip}} = C_1\lambda \frac{\partial \mathbf{v}}{\partial z} - \frac{1}{2} C_2 \lambda^2 \frac{\partial^2 \mathbf{v}}{\partial z^2}, \tag{B.3}
\]

in which \(C_1\) and \(C_2\) are surface correction coefficients. The boundary conditions in case of slip velocities are then

\[
\mathbf{v}\big|_{z=z_1} = \mathbf{v}_1 - C_1\lambda \frac{\partial \mathbf{v}}{\partial z}\big|_{z=z_1} - \frac{1}{2} C_2 \lambda^2 \frac{\partial \mathbf{v}}{\partial z}\big|_{z=z_1}, \\
\mathbf{v}\big|_{z=z_2} = \mathbf{v}_2 + C_1\lambda \frac{\partial \mathbf{v}}{\partial z}\big|_{z=z_2} - \frac{1}{2} C_2 \lambda^2 \frac{\partial \mathbf{v}}{\partial z}\big|_{z=z_2}, \tag{B.4}
\]

\[
p\big(x(t), y(t), z) = p_0\big(x(t), y(t)\big),
\]

\(^1\)The molecular mean free path is the distance a molecule of a gas travels in between two collisions.
in which $p_a$ is the ambient pressure. The first order slip velocity model is introduced by Burgdorfer [19] and is based on the kinetic gas theory. In this model, $C_1 = 1$ and $C_2 = 0$. The second order slip velocity model was introduced by Hsia and Domoto [58], and has coefficients $C_1 = 1$ and $C_2 = 1$. Mitsuya [95] pointed out that the second order slip velocity model was lacking a physical background and derived a one and a half order slip velocity model, which is represented by $C_1 = 1$ and $C_2 = 4/9$. Note that $\partial v/\partial z$ can be negative or positive, depending on the absolute values of $v_1$ and $v_2$. However, equations (B.4) are valid in both cases shown in Fig. B.2. And in both cases the velocity of the gas is lower than the velocity of the solid surfaces.

B.3 The Velocity Across the Gas Film

The motion of a Newtonian fluid can be derived from the Navier-Stokes equations, which describe the conservation of momentum

$$
\rho \frac{Dv}{Dt} = F - \nabla p + \nabla \left( \frac{1}{3} \eta_g \nabla \cdot v \right) + \eta_g \nabla^2 v , \tag{B.5}
$$

in which $\rho$ is the density of the gas, $p$ is the pressure, $\eta_g$ is the dynamic viscosity of the gas and $F$ is an external force field, for example the gravitational field. The operator $D/Dt$ represents the material derivative, defined as

$$
\frac{D}{Dt} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z} . \tag{B.6}
$$

When it is assumed that the mass inertia terms in (B.5) are much smaller than the viscous terms and that the thickness of the gas film is very thin compared to the dimensions in $x$ and $y$ direction, such that all derivatives of $u$, $v$, and $w$ in $x$ and $y$ direction are much smaller than those in $z$ direction then (B.5) reduce to

$$
\frac{d}{dz} \left( \eta_g \frac{dv}{dz} \right) = \nabla p - F , \tag{B.7}
$$

![Fig. B.2. Cases of first order slip at the surfaces for $\frac{\partial u}{\partial z} < 0$ (left) and $\frac{\partial u}{\partial z} > 0$ (right).]
in which \( p = p(x, y) \), thus a function of \( x \) and \( y \) only. Equations (B.7) contain four unknown quantities, namely, \( u, v, w, \) and \( p \). The velocity field can therefore be expressed in terms of the pressure gradient \( \nabla p \) and the velocities \( \mathbf{v}_1 \) and \( \mathbf{v}_2 \) of the solid surfaces.

The Navier-Stokes equations (B.7) may be integrated across the gas film using the specified boundary conditions (B.4),

\[
\frac{d\mathbf{v}}{dz} = \frac{1}{\eta_g} (\nabla p - \mathbf{F}) z + \mathbf{f}_1(x, y). \tag{B.8}
\]

Again, integrating (B.8) with respect to \( z \) yields

\[
\mathbf{v} = \frac{1}{2\eta_g} (\nabla p - \mathbf{F}) z^2 + \mathbf{f}_1(x, y) z + \mathbf{f}_2(x, y), \tag{B.9}
\]

in which \( \mathbf{f}_1 \) and \( \mathbf{f}_2 \) are functions of \( x \) and \( y \). From (B.7) it follows that

\[
\frac{d^2\mathbf{v}}{dz^2} \bigg|_{z=z_1} = \frac{d^2\mathbf{v}}{dz^2} \bigg|_{z=z_2} = \frac{1}{\eta_g} (\nabla p - \mathbf{F}), \tag{B.10}
\]

and according to (B.8)

\[
\frac{d\mathbf{v}}{dz} \bigg|_{z=z_1} = \frac{1}{\eta_g} (\nabla p - \mathbf{F}) z_1 + \mathbf{f}_1(x, y), \tag{B.11}
\]

\[
\frac{d\mathbf{v}}{dz} \bigg|_{z=z_2} = \frac{1}{\eta_g} (\nabla p - \mathbf{F}) z_2 + \mathbf{f}_1(x, y). \tag{B.11}
\]

Substitution of (B.10) and (B.11) into the boundary conditions (B.4) yields

\[
\mathbf{v} \bigg|_{z=z_1} = \mathbf{v}_1 - \frac{1}{2\eta_g} \left( 2C_1 \lambda z_1 + C_2 \lambda^2 \right) (\nabla p - \mathbf{F}) - C_1 \lambda \mathbf{f}_1(x, y),
\]

\[
\mathbf{v} \bigg|_{z=z_2} = \mathbf{v}_2 + \frac{1}{2\eta_g} \left( 2C_1 \lambda z_2 - C_2 \lambda^2 \right) (\nabla p - \mathbf{F}) + C_1 \lambda \mathbf{f}_1(x, y). \tag{B.12}
\]

But according to (B.9)

\[
\mathbf{v} \bigg|_{z=z_1} = \frac{1}{2\eta_g} (\nabla p - \mathbf{F}) z_1^2 + \mathbf{f}_1(x, y) z_1 + \mathbf{f}_2(x, y),
\]

\[
\mathbf{v} \bigg|_{z=z_2} = \frac{1}{2\eta_g} (\nabla p - \mathbf{F}) z_2^2 + \mathbf{f}_1(x, y) z_2 + \mathbf{f}_2(x, y). \tag{B.13}
\]

Comparing (B.12) with (B.13) it must follow that

\[
\mathbf{v}_1 - \frac{1}{2\eta_g} \left( 2C_1 \lambda z_1 + C_2 \lambda^2 \right) (\nabla p - \mathbf{F}) - C_1 \lambda \mathbf{f}_1(x, y)
\]

\[
= \frac{1}{2\eta_g} (\nabla p - \mathbf{F}) z_1^2 + \mathbf{f}_1(x, y) z_1 + \mathbf{f}_2(x, y), \tag{B.14}
\]

\[
\mathbf{v}_2 + \frac{1}{2\eta_g} \left( 2C_1 \lambda z_2 - C_2 \lambda^2 \right) (\nabla p - \mathbf{F}) + C_1 \lambda \mathbf{f}_1(x, y)
\]

\[
= \frac{1}{2\eta_g} (\nabla p - \mathbf{F}) z_2^2 + \mathbf{f}_1(x, y) z_2 + \mathbf{f}_2(x, y). \tag{B.14}
\]
Solving these equations for \( f_1(x, y) \) and \( f_2(x, y) \) gives

\[
\begin{align*}
  f_1(x, y) &= \frac{v_1 - v_2}{2C_1 \lambda + d_g} - \frac{1}{2 \eta_g} (z_1 + z_2) (\nabla p - F) , \\
  f_2(x, y) &= \frac{(C_1 \lambda - z_2) v_1 + (C_1 \lambda + z_1) v_2}{2C_1 \lambda + d_g} \\
 &\quad - \frac{1}{2 \eta_g} \left( C_1 \lambda d_g + C_2 \lambda^2 - z_1 z_2 \right) (\nabla p - F) ,
\end{align*}
\] (B.15)

where \( d_g = z_1 - z_2 \). Substitution of (B.15) into (B.9) gives for the velocity \( \mathbf{v} \)

\[
\begin{align*}
  \mathbf{v} &= \left( 1 - \frac{C_1 \lambda + z_1 - z}{2C_1 \lambda + d_g} \right) \mathbf{v}_1 + \left( 1 - \frac{C_1 \lambda + z - z_2}{2C_1 \lambda + d_g} \right) \mathbf{v}_2 \\
  &\quad + \frac{1}{2 \eta_g} \left( (z - z_1)(z - z_2) - C_1 \lambda d_g - C_2 \lambda^2 \right) (\nabla p - F) .
\end{align*}
\] (B.16)

### B.4 The Shear Stress Across the Gas Film

The shear stress across the gas film is defined as

\[
\tau_g = \eta_g \frac{\partial \mathbf{v}}{\partial z}, \quad \text{for} \quad z_1 \leq z \leq z_2 . \tag{B.17}
\]

After substituting (B.16) into (B.17) and assuming that no external force field is present \( (F = 0) \), the shear stress becomes

\[
\tau_g = \frac{\eta_g \mathbf{v}_-}{2C_1 \lambda + d_g} + \frac{1}{2} \nabla p \left( 2z - (z_1 + z_2) \right) , \tag{B.18}
\]

in which \( \mathbf{v}_- \) is the relative velocity between the solid surfaces,

\[
\mathbf{v}_- \overset{\text{def}}{=} \mathbf{v}_1 - \mathbf{v}_2 . \tag{B.19}
\]

The shear stress at solid surface 1 \( (z = z_1) \) becomes

\[
\tau_{g,1} = \frac{\eta_g \mathbf{v}_-}{2C_1 \lambda + d_g} + \frac{d_g}{2} \nabla p , \tag{B.20}
\]

and at solid surface 2 \( (z = z_2) \) the shear stress becomes

\[
\tau_{g,2} = \frac{\eta_g \mathbf{v}_-}{2C_1 \lambda + d_g} - \frac{d_g}{2} \nabla p . \tag{B.21}
\]

The friction force per unit of area at solid surface 1 then consists of a viscous component and a geometric component,

\[
f_{g,1} = -\tau_{g,1} - p \nabla z_1 , \tag{B.22}
\]

or
\[ f_{g,1} = -\frac{\eta_g v_-}{2C_1 \lambda + d_g} - \frac{d_g}{2} - p \nabla z_1, \]  

(B.23)

and, similarly, the friction force per unit of area, at solid surface 2 becomes

\[ f_{g,2} = -\tau_{g,2} - p \nabla z_2, \]  

(B.24)

or

\[ f_{g,2} = +\frac{\eta_g v_-}{2C_1 \lambda + d_g} - \frac{d_g}{2} \frac{-p \nabla z_2}{\text{squeezing geometry}}. \]  

(B.25)

At the TOV or the TDV, the solid surfaces are close together and the pressure flow \((d_g \nabla p)/2\) will everywhere be much smaller than the shear flow \((\eta_g v_-)/(2C_1 \lambda + d_g)\). Therefore, the pressure flow may be neglected and the shear stress at the slider becomes

\[ \tau_g = \frac{\eta_g v_-}{2C_1 \lambda + d_g}. \]  

(B.26)

Thus, in one direction of sliding, where \(v_- \overset{\text{def}}{=} v_1 - v_2\) is the relative velocity between the solid surfaces, the shear stress becomes

\[ \tau_g = \frac{\eta_g v_-}{2C_1 \lambda + d_g}. \]  

(B.27)

The molecular mean free path \(\lambda\) of the gas under pressure \(p_g\) is given by

\[ \frac{p_g}{p_a} = \frac{\rho}{\rho_a} = \frac{\lambda_a}{\lambda}, \]  

(B.28)

in which \(\lambda_a\) and \(\rho_a\) are the molecular mean free path and the density of the gas under ambient conditions, respectively. Then,

\[ \lambda = \lambda_a \frac{p_a}{p_g}. \]  

(B.29)

Substituting (B.29) into (B.27) gives for the shear stress

\[ \tau_g = \frac{\eta_g v_-}{2C_1 \lambda_a p_a / p_g + d_g}. \]  

(B.30)

From (B.30) it can be seen that for \(K_n = \lambda_a / d_g \ll p_g / (2C_1 p_a)\) the rarefaction effects can be neglected, in which case the familiar shear stress equation

\[ \tau_g = \frac{\eta_g v_-}{d_g}, \]  

(B.31)

appears.
C
The Force Measured by the Friction Force Sensor

A friction force sensor has been build for measuring the friction force between the slider and the disk. In this appendix it will be shown that if the sensor is being acted upon by an arbitrary load it only measures the force in the Z direction. Also an equation will be derived which shows that the measured force is directly proportional to the output voltage of the sensor.

The friction force sensor consists of a silicon table and a frame, interconnected by four equal beams, B₁, B₂, B₃, and B₄ (see Fig. 5.7). Silicon table, frame and beams are one part and are micro machined out of a silicon wafer and therefore, have equal thickness. The frame is fixed to a PCB. It will be assumed that the table and the frame are rigid and that the four beams are elastic. Both beams B₃ and B₄ are provided with two rectangular piezo-resistive polycrystalline silicon strain gauges (S₁, S₂, S₃ and S₄ in Fig. C.3). Each strain gauge has a length ℓₚ. If a strain gauge is stretched or compressed, its resistance will change because of dimensional changes (length and cross-sectional area) and because of a fundamental property of materials called *piezo-resistance*, which indicates a dependence of resistivity on the mechanical strain. With polycrystalline silicon strain gauges, the change in resistance due to the piezo-resistance is much larger than the change due to the change in dimensions. If the direction of the electric field (that is the voltage difference across the strain gauge) is applied in length direction of the strain gauge, then the relative change in resistance \( \frac{\Delta R_{i,s}}{R_{i,s}} \) of a polycrystalline silicon strain gauge Sᵢ is (see, for example, Burger [21] and Doebelin [32])

\[
\frac{\Delta R_{i,s}}{R_{i,s}} = G\epsilon_{i,t},
\]

in which \( G \) is the gauge factor and \( \epsilon_{i,t} \) is the strain in longitudinal direction of the strain gauge. The strain gauges form the four elements of a Wheatstone bridge (see Fig. C.1). In order to measure the friction force in the Z direction only, the four strain gauges are arranged in the bridge in such a way that the output voltage \( U_0 \) of the bridge is given by

\[
U_0 = \frac{U_i}{4} \left( \frac{\Delta R_{1,s}}{R_{1,s}} - \frac{\Delta R_{2,s}}{R_{2,s}} + \frac{\Delta R_{3,s}}{R_{3,s}} - \frac{\Delta R_{4,s}}{R_{4,s}} \right),
\]

where \( U_i \) is the supplied voltage across the bridge, and \( \Delta R_{1,s}, \Delta R_{2,s}, \Delta R_{3,s} \) and \( \Delta R_{4,s} \) are the small changes in resistance of the strain gauges S₁, S₂, S₃ and S₄,
respectively. It will be assumed that all four strain gauges have equal gauge factors $G$. Substituting (C.1) into (C.2) gives the output voltage $U_o$ of the Wheatstone bridge as a function of the longitudinal strain in the strain gauges,

$$U_o = U_i \frac{G}{4} (\epsilon_1 - \epsilon_2 + \epsilon_3 - \epsilon_4).$$  \hfill (C.3)

Hence, the output voltage of the sensor depends on the strain induced in the strain gauges, by the load acting at the sensor. In the next sections the relationship between the strain in the strain gauges and the load at the sensor will be derived.

### C.1 The Strain in the Strain Gauges

Let the sensor be subjected to an arbitrary load with components $F_x$, $F_y$, $F_z$, $M_x$, $M_y$, and $M_z$ acting at the centre of the table (see Fig. C.2). The friction force sensor is a statically indeterminate structure, that is, the equations of static equilibrium are not sufficient to determine all reaction forces and moments. However, since the friction force sensor has two planes of symmetry (the $XY$ plane and the $XZ$ plane), the symmetry and asymmetry of the structure and the load, and the deformation of the beams can be used to find the reaction forces and moments at the end of each beam. The reaction forces and moments acting at the end of the beams B3 and B4 are shown in Fig. C.3, and are

$$F_{3,x} = \frac{1}{4} \left( F_x - \frac{M_y}{d_z} + \frac{M_z}{d_y} \right); \quad F_{3,y} = \frac{1}{4} \left( F_y + \frac{M_z}{d_y} \right); \quad F_{3,z} = \frac{1}{4} F_z;$$

$$M_{3,x} = -\frac{1}{8} F_x \ell; \quad M_{3,y} = 0; \quad M_{3,z} = \frac{1}{8} \left( F_x \ell + M_z \frac{\ell}{d_y} \right),$$

and

$$F_{4,x} = \frac{1}{4} \left( F_x - \frac{M_y}{d_z} - \frac{M_z}{d_y} \right); \quad F_{4,y} = \frac{1}{4} \left( F_y + \frac{M_z}{d_y} \right); \quad F_{4,z} = \frac{1}{4} F_z;$$

$$M_{4,x} = \frac{1}{8} F_x \ell; \quad M_{4,y} = 0; \quad M_{4,z} = -\frac{1}{8} \left( F_x \ell - M_z \frac{\ell}{d_y} \right),$$

\hfill (C.5)
Fig. C.2. The load acting at the centre of the table of the friction force sensor.

Fig. C.3. The reaction forces and moments acting upon the beams B₃ and B₄.
in which \( \ell \) is the length of each beam and \( d_y \) and \( d_z \) are the distances between the loading point, that is the centre of the table, and the centre of the cross-sectional area of the beams in \( Y \) and \( Z \) direction, respectively (see Fig. C.2).

### C.1.1 The Strain Due to \( F_y \) and \( M_x \)

The tensile strain \( \epsilon_{1,\ell} \) in the longitudinal direction in the strain gauge \( S_1 \), due to a force \( F_{3,y} \) is given by (see Figs. C.4 and C.5)

\[
\epsilon_{1,\ell} = \frac{F_{3,y}}{EA},
\]  

(C.6)

in which \( A = wh \) is the cross-sectional area of the beam. With \( F_{3,y} \) given in (C.4), the strain in strain gauge \( S_1 \) becomes

\[
\epsilon_{1,\ell} = \frac{1}{4Ewh} \left( F_y + \frac{M_x}{d_z} \right).
\]  

(C.7)

Strain gauge \( S_2 \) will be stretched in the same way as strain gauge \( S_1 \), and therefore,

\[
\epsilon_{2,\ell} = \epsilon_{1,\ell}.
\]  

(C.8)

Furthermore, due to the compressive force \( F_{4,y} \), the compressive strain \( \epsilon_{3,\ell} \) in the strain gauge \( S_3 \) is

\[
\epsilon_{3,\ell} = \frac{1}{4Ewh} \left( F_y + \frac{M_x}{d_z} \right)
\]  

(C.9)

and the compressive strain in strain gauge \( S_4 \) becomes

\[
\epsilon_{4,\ell} = \epsilon_{3,\ell}.
\]  

(C.10)

---

**Fig. C.4.** Bending of the beam \( B_3 \) due to a moment in \( Z \) direction (Front view).

**Fig. C.5.** Bending of the beam \( B_3 \) due to a moment in \( X \) direction (Top view).
C.1.2 The Strain Due to \( F_x, M_y \) and \( M_z \)

The compressive strain \( \epsilon_{3,z}(y_B) \) due to a bending moment \( M_z(y_B) \) in beam \( B_3 \) at a distance \( h/2 \) from the neutral line of the beam is given as (see Fig. C.4)

\[
\epsilon_{3,z}(y_B) = \frac{M_z(y_B)h}{2EI_z},
\]

in which \( E \) is the elastic modulus of the beam, \( h \) is the height of the beam and \( I_z \) is the moment of inertia of the cross-sectional area of the beam at \( y_B \) with respect to the \( z_B \) axis, which is given by

\[
I_z = \frac{wh^3}{12},
\]

where \( w \) is the width of the beam. The bending moment \( M_z(y_B) \) at \( y_B \) is given by

\[
M_z(y_B) = F_{3,z}(\ell - y_B) - M_{3,z}.
\]

The average strain in the strain gauge can be found by integrating (C.11) over the length \( \ell_g \) of the strain gauge, thus

\[
\epsilon_{1,\ell} = \frac{1}{\ell_{g}} \int_0^{\ell_g} \epsilon_{3,z}(y_B)dy_B = \frac{1}{\ell_{g}} \int_0^{\ell_g} \frac{M_z(y_B)h}{2EI_z}dy_B.
\]

After substituting (C.13) into (C.14) the strain in strain gauge \( S_1 \) becomes

\[
\epsilon_{1,\ell} = \frac{h}{2EI_z} \left( F_{3,z} \left( \ell - \frac{1}{2}\ell_g \right) - M_{3,z} \right),
\]

and with \( F_{3,z}, M_{3,z} \) and \( I_z \) given by (C.4) and (C.12) the average strain in strain gauge \( S_1 \) becomes

\[
\epsilon_{1,\ell} = \frac{3}{4wh^2} \left( \left( F_x + \frac{M_z}{d_y} \right) (\ell - \ell_g) - \frac{M_y}{d_x} (2\ell - \ell_g) \right).
\]

When subjected to a bending moment \( M_z(y_B) \) the strain gauge \( S_2 \) bends in the same direction as strain gauge \( S_1 \) and the average strain in strain gauge \( S_2 \) is therefore also compressive,

\[
\epsilon_{2,\ell} = \epsilon_{1,\ell}.
\]

The strain induced in the strain gauges \( S_3 \) and \( S_4 \) of beam \( B_4 \) by the load \( F_x, M_y \) and \( M_z \) can be found by the same method as shown above. The strains induced in the strain gauges \( S_3 \) and \( S_4 \) are in that case

\[
\epsilon_{3,\ell} = \frac{3}{4wh^2} \left( \left( F_x - \frac{M_z}{d_y} \right) (\ell - \ell_g) - \frac{M_y}{d_x} (2\ell - \ell_g) \right),
\]

and obviously,

\[
\epsilon_{4,\ell} = \epsilon_{3,\ell}.
\]
C.1.3 The Strain Due to $F_z$

The compressive strain $\epsilon_{3,x}(y_B)$ due to a bending moment $M_x(y_B)$ in beam $B_3$ at a distance $a$ from the neutral line is (see Fig. C.5)

$$\epsilon_{3,x}(y_B) = \frac{M_x(y_B)a}{EI_x}, \quad (C.20)$$

where $I_x$ is the moment of inertia of the cross-sectional area at $y_B$ with respect to the $x_B$ axis, which is given by

$$I_x = \frac{y^3h}{12}. \quad (C.21)$$

The bending moment $M_x(y_B)$ at $y_B$ is given by

$$M_x(y_B) = F_{3,x}(\ell - y_B) + M_{3,x}. \quad (C.22)$$

The average strain in the strain gauge can be found by integrating (C.20) over the length $\ell_g$ of the strain gauge

$$\epsilon_{1,t} = \frac{1}{\ell_g} \int_0^{\ell_g} \epsilon_{3,x}(y_B)dy_B = \frac{1}{\ell_g} \int_0^{\ell_g} \frac{M_x(y_B)a}{EI_x} dy_B. \quad (C.23)$$

After substituting (C.22) into (C.23) the strain in strain gauge $S_1$ becomes

$$\epsilon_{1,t} = \frac{a}{EI_x} \left( F_{3,x} \left( \ell - \frac{1}{2}\ell_g \right) + M_{3,x} \right), \quad (C.24)$$

and with $F_{3,x}$, $M_{3,x}$ and $I_x$ given by (C.4) and (C.21) the average strain in strain gauge $S_1$ becomes

$$\epsilon_{1,t} = \frac{3a}{2Ew^3h} F_x(\ell - \ell_g). \quad (C.25)$$

Because strain gauge $S_2$ bends in the same direction as that of strain gauge $S_1$ when subjected to a bending moment, $M_x(y_B)$, the strain in strain gauge $S_2$ will be tensile, thus

$$\epsilon_{2,t} = -\epsilon_{1,t}. \quad (C.26)$$

The strains induced in the strain gauges $S_3$ and $S_4$ of beam $B_4$ by the load $F_z$ can be found by the same method described above. The strains induced in the strain gauges $S_3$ and $S_4$ are in this case

$$\epsilon_{3,t} = \frac{3a}{2Ew^3h} F_x(\ell - \ell_g), \quad (C.27)$$

and also,

$$\epsilon_{4,t} = -\epsilon_{3,t}. \quad (C.28)$$
C.2 Conclusions

From the previous sections, it follows that the strains in the strain gauges due to a load $F_y$ and a moment $M_x$ are

$$
\epsilon_{1,t} = \epsilon_{2,t} = \epsilon_{3,t} = \epsilon_{4,t} = \frac{1}{4Ewh} \left( F_y + \frac{M_x}{d_z} \right). \tag{C.29}
$$

If this result is substituted into (C.3) it follows that the changes in resistance of the strain gauges due to a force $F_y$ cancel each other out. The strains in the strain gauges due to a load $F_x$, $M_y$ and $M_z$ are

$$
\epsilon_{1,t} = \epsilon_{2,t} = \frac{3}{4Ewh^2} \left( \left( F_x + \frac{M_z}{d_y} \right) (l - \ell_g) - \frac{M_y}{d_z} (2l - \ell_g) \right),
\epsilon_{3,t} = \epsilon_{4,t} = \frac{3}{4Ewh^2} \left( \left( F_x - \frac{M_z}{d_y} \right) (l - \ell_g) - \frac{M_y}{d_z} (2l - \ell_g) \right). \tag{C.30}
$$

Again, if this result is substituted into (C.3) it follows that the changes in resistance of the strain gauges induced by a force $F_x$, $M_y$ and $M_z$ have no effect on the output voltage of the friction force sensor. Finally, the strains in the strain gauges due to a load $F_x$ are

$$
\epsilon_{1,t} = -\epsilon_{2,t} = \epsilon_{3,t} = -\epsilon_{4,t} = \frac{3a}{2Ewh^3} F_x (l - \ell_g). \tag{C.31}
$$

If this result is substituted into (C.3) the output voltage $U_o$ of the Wheatstone bridge becomes

$$
U_o = U_i \frac{3G_a F_x (l - \ell_g)}{2Ewh^3}. \tag{C.32}
$$

Consequently, the friction force sensor measures only a force in the $Z$ direction and forces in the $X$ and $Y$ direction as well as moments in $X$, $Y$ and $Z$ direction have no effect on the measurements. As any arbitrary load working at the table of the sensor can be resolved into components $F_x$, $F_y$, $F_z$, $M_x$, $M_y$ and $M_z$, acting on the centre of the table, it has been proven in this appendix that the friction force sensor measures only the friction force $F_z$ in the $Z$ direction. Furthermore, it follows from (C.32) that the output voltage $U_o$ of the sensor is directly proportional to the friction force $F_z$.

If the friction force sensor bridge circuit has an amplification of $A_F$, the output voltage $U_F$ of the friction force sensor bridge circuit becomes,

$$
U_F = A_F U_o = A_F G U_i \frac{3F_a (l - \ell_g)}{2Ewh^3}, \tag{C.33}
$$

that is the friction force $F_z$ is given by

$$
F_z = \frac{1}{A_F G U_i} \frac{U_F}{3a (l - \ell_g)} \frac{2Ewh^3}{3}. \tag{C.34}
$$
C. The Force Measured by the Friction Force Sensor
The Attractive Force Between the Plates of the Parallel Plate Capacitor

A parallel plate capacitor is used for measuring the displacements of the probe in the X direction (see Fig. D.1). In order to reduce edge effects the diameter of plate $C_1$ is larger than that of plate $C_2$. Because of the voltage $U(t)$ applied to the plates, the plates are oppositely charged. As a consequence, they attract each other. In this appendix it will be shown that the attractive force between the plates can be neglected in the analysis of the dynamics of the pendulum. It will be assumed that the electric field across the plates remains constant as the separation of the capacitor plates changes. Gauss’ divergence theorem states that the electric field $E$ across the plates is given by

$$\nabla \cdot E = 0. \quad (D.1)$$

Because of the radial symmetry of the plates, the electric field $E$ depends on the $x$ coordinate only. Therefore (D.1) becomes

![Diagram of parallel plate capacitor](Fig.D.1. The parallel plate capacitor.)
\[ E_x = E = \text{constant}. \quad (D.2) \]

Furthermore,
\[ \mathbf{E} = -\nabla U, \quad (D.3) \]

in which \( U \) is the potential voltage between the two plates. After integration of (D.3) and substitution of (D.2) the voltage \( U \) becomes
\[ U - U_1 = E(x - x_1). \quad (D.4) \]

With \( U_1 = 0, x_1 = 0 \) and the gap between the plates given by \( g_c = x - x_1 \), the voltage across the plates becomes
\[ U = Eg_c. \quad (D.5) \]

According to the electromagnetic theory of Maxwell the dielectric displacement vector \( \mathbf{D} \) perpendicular to the surface of plate \([C_2]\), which is surrounded by a dielectric (air), is equal to the surface charge density \( \sigma \) at the plate, that is
\[ \sigma = \mathbf{D} \cdot \mathbf{e}_n, \quad (D.6) \]

in which \( \mathbf{e}_n \) is the unit vector normal to the surface of the plate. In linear and isotropic dielectrics
\[ \mathbf{D} = \varepsilon \mathbf{E}, \quad (D.7) \]

with \( \varepsilon = \varepsilon_\varepsilon \), in which \( \varepsilon_\varepsilon \) is the permittivity of free space and \( \varepsilon_\varepsilon \) is the relative permittivity. Substitution of (D.7) into (D.6) results in
\[ \sigma = \varepsilon E. \quad (D.8) \]

An element of charge \( \sigma dA \) on the surface of a plate experiences the electric field of all the other charges, and is therefore subjected to an electric force perpendicularly to the surface of the plate. The electric force on the element of area \( dA \) of the plate \([C_2]\) is (see, for example, Lorrain et al. [85])
\[ dF_p = -\frac{1}{2}\sigma EdA = -\frac{\sigma^2}{2\varepsilon} dA. \quad (D.9) \]

The attractive force on the plate of area \( A \) is then
\[ F_p = -\int_A \frac{\sigma^2}{2\varepsilon} dA = -\frac{\sigma^2}{2\varepsilon} A. \quad (D.10) \]

and with \( A = \pi r_p^2 \), the surface area of the smallest plate, and with (D.5) and (D.8) the attractive force becomes
\[ F_p = -\frac{1}{2}\pi \varepsilon \varepsilon_\varepsilon \left( \frac{U r_p}{g_c} \right)^2. \quad (D.11) \]

The above equation is only valid when the electric field remains constant as the separation of the capacitor plates changes. According to (D.11) an external force
of $F = -F_p$ should be applied to maintain the plates at a distance $g_c$ from each other. With $U = 1$ Volt, $r_p = 13.5$ mm, $g_{c_{\text{min}}} = 0.5$ mm, $\varepsilon_r = 1.00054$ and $\varepsilon_o = 8.8542 \cdot 10^{-12}$ F/m, the attractive force between the capacitor plates becomes 10 nN. This force is many orders smaller than the other forces acting upon the pendulum beam, and, consequently, it may be neglected in the analysis of the equilibrium position of the pendulum.
D. The Attractive Force Between the Plates of the Parallel Plate Capacitor
Estimate of the Extremes of the Angular Deflection, $\theta$

The maximum outward movement of the probe, and thereby the angular deflection $\theta$, is defined by the limit stop (see Fig. 5.3). This limit stop defines also the orientation (the sign of $\theta$) of the pendulum beam when a load is applied. Its position is manually adjusted with a micrometer (see Fig. 5.1 on page 102). In general, the position is such that when the pendulum rests against the limit stop, the capacitance bridge is in the linear portion of the output voltage (see Fig. 5.12). An estimate of the angular deflection $\theta$ can be made by considering the extremes of the outward (negative $\theta$) and inward (positive $\theta$) movement of the probe by means of the measured gap $g_c$ at the centre of the capacitor plates (points 8 and 9 in Fig. 5.3).

The position $\mathbf{r}'_i$ of any point $i$ of the pendulum, after an angular deflection $\theta$ in the $xyz$ coordinate system, is (see Fig. E.1)

$$\mathbf{r}'_i = M_\theta \mathbf{r}_i,$$  \hspace{1cm} (E.1)

where $M_\theta$ is the transformation matrix given by

![Diagram](image)

**Fig. E.1.** The position vector $\mathbf{r}'_i$ of any point $i$ of the pendulum after an angular deflection $\theta$. 
\[ M_\theta = \begin{pmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{pmatrix}. \] 

(E.2)

The gap \( g_c \) is

\[ g_c = r'_{8,x} - r_{9,x}, \] 

(E.3)

and with \( r'_{8,x} \) and \( r_{9,x} \) from (5.2) and with (E.1) the gap becomes

\[ g_c = g_{c,0} + \ell_{11}(1 - \cos \theta) - \ell_3 \sin \theta, \] 

(E.4)

where \( g_{c,0} = \ell_{12} - \ell_{11} \) is the gap at \( \theta = 0 \). The maximum gap \( g_{c,max} \) occurs when the pendulum rests against the limit stop \([O]\) (when \( \theta = \theta_{\text{min}} \)). In that case the gap \( g_{c,max} \) is

\[ g_{c,max} = g_{c,0} + \ell_{11}(1 - \cos \theta_{\text{min}}) - \ell_3 \sin \theta_{\text{min}}. \] 

(E.5)

In practice \( g_{c,max} < 1.5 \text{ mm} \) and with this value it follows from (E.5) that \( \theta_{\text{min}} > -25 \text{ mrad} \) \(( -1.5 \text{ degrees}) \). The minimum gap \( g_{c,min} \) occurs when the bottom of the movable capacitor plate \([C_2]\) touches the fixed capacitor plate \([C_1]\). In that case

\[ g_{c,min} = r_p \sin \theta_{\text{max}}, \] 

(E.6)

in which \( r_p \) is the radius of plate \([C_2]\). Substitution of (E.6) into (E.4) yields

\[ r_p \sin \theta_{\text{max}} = g_{c,0} + \ell_{11}(1 - \cos \theta_{\text{max}}) - \ell_3 \sin \theta_{\text{max}}. \] 

(E.7)

And with \( r_p = 13.5 \text{ mm} \) the maximum angular deflection becomes \( \theta_{\text{max}} < +10 \text{ mrad} \) \(( +0.6 \text{ degrees}) \). Therefore, the possible range of the angular deflection \( \theta \) is \(-25 < \theta < 10 \text{ mrad} \) \((-1.5 < \theta < 0.6 \text{ degrees}) \). During measurements the angular deflection will be close to zero, because, after the probe is brought into contact with the specimen, the pendulum will be withdrawn from the limit stop \([O]\), by moving the XYZ table some increments further in negative X direction, such that \( \theta \) decreases (see Section 5.3.3).

Because of the small angular deflection \( \theta \) it is allowed to linearise (E.2) and (E.4) by using the approximations \( \cos \theta \approx 1 \) and \( \sin \theta \approx \theta \). The position \( r'_i \) of any point \( i \) of the pendulum after an angular deflection of \( \theta \) is then given by (E.1) with

\[ M_\theta = \begin{pmatrix} 1 & 0 & \theta \\ 0 & 1 & 0 \\ -\theta & 0 & 1 \end{pmatrix}. \] 

(E.8)

Finally, the gap \( g_c \) given in (E.4) then becomes

\[ g_c = g_{c,0} - \ell_3 \theta, \] 

(E.9)

with \( g_{c,0} = \ell_{12} - \ell_{11} \).
The Forces at the Pendulum

In this appendix the forces acting on the pendulum beam will be derived with respect to the $XYZ$ coordinate system. For this purpose, a third Cartesian coordinate system $X'Y'Z'$ will be introduced, which has its $Y'$ axis along the velocity vector $\mathbf{v}$ of the specimen (see Fig. 5.11). As a result of friction between the probe and the specimen surface, the friction force $\mathbf{F}$ acting on the probe, will be in the same direction as the velocity vector $\mathbf{v}$. The normal load $\mathbf{P}$ at the probe, that is, the resultant of the load at the specimen, is perpendicular to the surface of the specimen and also perpendicular to the friction force $\mathbf{F}$. In the $X'Y'Z'$ coordinate system the velocity vector $\mathbf{v}'$ and the friction force $\mathbf{F}'$ are given by

$$
\mathbf{v}' = \begin{pmatrix} 0 \\ v \\ 0 \end{pmatrix}; \quad \mathbf{F}' = \begin{pmatrix} 0 \\ F \\ 0 \end{pmatrix},
$$

and the normal $\mathbf{n}'$ to the specimen surface and the normal load $\mathbf{P}'$ are given by

**Fig. F.1.** The coordinates used in the rotation of the $XYZ$ coordinate system to the $X''Y''Z''$ coordinate system (Top view).
Fig. F.2. The coordinates used in the rotation of the $X'' Y''' Z'''$ coordinate system to the $X'' Y'' Z''$ coordinate system (Front view).

Fig. F.3. The coordinates used in the rotation of the $X'' Y''' Z'''$ coordinate system to the $X' Y' Z'$ coordinate system (Side view).
\[ \mathbf{n'} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}; \quad \mathbf{P'} = \begin{pmatrix} -P \\ 0 \\ 0 \end{pmatrix}. \quad (F.2) \]

The rotation from the \(XYZ\) coordinate system to the \(X'Y'Z'\) coordinate system can be thought of as three successive rotations performed in a specific sequence. The sequence employed here is stated by rotating the \(XYZ\) system of axes by an angle \(\beta\) anticlockwise about the \(Z\) axis (see Fig. F.1). The intermediate axes \(X''Y''Z''\) are then rotated anticlockwise about the \(Y''\) axis by an angle \(\gamma\) to produce a second intermediate set of axes \(X''Y''Z''\) (see Fig. F.2). This second system \(X''Y''Z''\) is then rotated anticlockwise about the \(X''\) axis by an angle \(\alpha\) to produce the coordinate system \(X'Y'Z'\) given in Figs. 5,11 and F.3. The complete rotation from the \(XYZ\) coordinate system to the \(X'Y'Z'\) coordinate system can be described by the transformation matrix \(\mathbf{M}\), which can be obtained by writing it as a triple product of the separate rotations, each of which has a relatively simple matrix form. The first rotation around the \(Z\) axis can be described by a matrix \(\mathbf{M}_Z\) (see Fig. F.1)

\[
\mathbf{M}_Z = \begin{pmatrix}
\cos \beta & \sin \beta & 0 \\
-\sin \beta & \cos \beta & 0 \\
0 & 0 & 1
\end{pmatrix}. \quad (F.3)
\]

Similarly, the transformation of the second type is given by the matrix \(\mathbf{M}_Y\) (see Fig. F.2)

\[
\mathbf{M}_Y = \begin{pmatrix}
\cos \gamma & 0 & -\sin \gamma \\
0 & 1 & 0 \\
\sin \gamma & 0 & \cos \gamma
\end{pmatrix}. \quad (F.4)
\]

The third transformation is a rotation about the \(X''\) axis and has the form (see Fig. F.3)

\[
\mathbf{M}_X = \begin{pmatrix}
1 & 0 & 0 \\
0 & \cos \alpha & \sin \alpha \\
0 & -\sin \alpha & \cos \alpha
\end{pmatrix}. \quad (F.5)
\]

The complete transformation matrix \(\mathbf{M}\) is then the product of \(\mathbf{M}_X\), \(\mathbf{M}_Y\) and \(\mathbf{M}_Z\), thus

\[
\mathbf{M} = \mathbf{M}_X \mathbf{M}_Y \mathbf{M}_Z = \\
\begin{pmatrix}
\cos \beta \cos \gamma & \sin \beta \cos \gamma & -\sin \gamma \\
\sin \alpha \cos \beta \sin \gamma & \sin \alpha \sin \beta \sin \gamma & \sin \alpha \cos \gamma \\
-\cos \alpha \sin \beta & + \cos \alpha \cos \beta & \\
\cos \alpha \cos \beta \sin \gamma & \cos \alpha \sin \beta \sin \gamma & \cos \alpha \cos \gamma \\
+ \sin \alpha \sin \beta & - \sin \alpha \cos \beta
\end{pmatrix}. \quad (F.6)
\]

The inverse transformation from coordinates \(X'Y'Z'\) to coordinates \(XYZ\) is then given by
\[ \mathbf{r} = \mathbf{M}^{-1} \mathbf{r}' . \]  \hspace{1cm} (F.7)

The determinant \( \det(\mathbf{M}) \) of an orthogonal matrix \( \mathbf{M} \) is 1, and its inverse is identical to its transpose, \( \mathbf{M}^{-1} = \mathbf{M}' \). Therefore, the inverse matrix \( \mathbf{M}^{-1} \) becomes

\[
\mathbf{M}^{-1} = 
\begin{pmatrix}
\cos \beta \cos \gamma & \sin \alpha \cos \beta \sin \gamma & \cos \alpha \cos \beta \sin \gamma \\
-\cos \gamma \sin \beta & + \sin \alpha \sin \beta \\
\sin \beta \cos \gamma & \sin \alpha \sin \beta \sin \gamma & \cos \alpha \sin \beta \sin \gamma \\
+ \cos \alpha \cos \beta & - \sin \alpha \cos \beta \\
-\sin \gamma & \sin \alpha \cos \gamma & \cos \alpha \cos \gamma
\end{pmatrix} . \hspace{1cm} (F.8)
\]

The normal \( \mathbf{n} \) to the specimen surface, the velocity vector \( \mathbf{v} \), the friction force \( \mathbf{F} \), and the applied normal load \( \mathbf{P} \), with respect to the XYZ coordinate system are now given by

\[
\mathbf{v} = \mathbf{M}^{-1} \mathbf{v}' = \begin{pmatrix} \frac{v (\sin \alpha \cos \beta \sin \gamma - \cos \alpha \sin \beta)}{v \sin \alpha \cos \gamma} \\ \frac{v (\cos \alpha \cos \beta + \sin \alpha \sin \beta \sin \gamma)}{v \sin \alpha \cos \gamma} \\ \frac{v \sin \alpha}{v \sin \alpha \cos \gamma} \end{pmatrix} , \hspace{1cm} (F.9)
\]

\[
\mathbf{F} = \mathbf{M}^{-1} \mathbf{F}' = \begin{pmatrix} \frac{F (\sin \alpha \cos \beta \sin \gamma - \cos \alpha \sin \beta)}{F \sin \alpha \cos \gamma} \\ \frac{F (\cos \alpha \cos \beta + \sin \alpha \sin \beta \sin \gamma)}{F \sin \alpha \cos \gamma} \\ \frac{F \sin \alpha \cos \gamma}{F \sin \alpha \cos \gamma} \end{pmatrix} , \hspace{1cm} (F.10)
\]

\[
\mathbf{n} = \mathbf{M}^{-1} \mathbf{n}' = \begin{pmatrix} -\cos \beta \cos \gamma \\ -\sin \beta \cos \gamma \\ \sin \gamma \end{pmatrix} , \hspace{1cm} (F.11)
\]

\[
\mathbf{P} = \mathbf{M}^{-1} \mathbf{P}' = \begin{pmatrix} -P \cos \beta \cos \gamma \\ -P \sin \beta \cos \gamma \\ P \sin \gamma \end{pmatrix} . \hspace{1cm} (F.12)
\]

Here, only small angles \( \beta \) and \( \gamma \) are considered (the normal to the specimen surface is nearly perpendicular to the \( YZ \) plane), therefore, the approximations \( \cos \beta \approx 1 \), \( \sin \beta \approx \beta \), \( \cos \gamma \approx 1 \) and \( \sin \gamma \approx \gamma \) may be used and (F.9) to (F.12) reduce to

\[
\mathbf{v} = \begin{pmatrix} \frac{v (\gamma \sin \alpha - \beta \cos \alpha)}{v \sin \alpha} \\ \frac{v (\cos \alpha + \beta \gamma \sin \alpha)}{v \sin \alpha} \end{pmatrix} ; \quad \mathbf{F} = \begin{pmatrix} \frac{F (\gamma \sin \alpha - \beta \cos \alpha)}{F \sin \alpha} \\ \frac{F (\cos \alpha + \beta \gamma \sin \alpha)}{F \sin \alpha} \end{pmatrix} , \hspace{1cm} (F.13)
\]

and

\[
\mathbf{n} = \begin{pmatrix} -1 \\ -\beta \\ \gamma \end{pmatrix} ; \quad \mathbf{P} = \begin{pmatrix} -P \beta \\ -P \gamma \end{pmatrix} . \hspace{1cm} (F.14)
\]
The Load Generated at the Coil of the Pendulum

The coil and permanent magnet shown in Fig. G.1 form in principle an electro-mechanical actuator. The purpose of this actuator is to convert an electrical variable, the input voltage $U_c(t)$, into a mechanical variable the load $F_c(t)$. In this appendix a model of the actuator will be given and the constitutional equations which describe this model will be derived.

The actuator consists of a circular coil $L$ on top of the pendulum, which moves back and forth through a magnetic field in response to an electrical input. The magnetic field is supplied by the circular permanent magnet $E$ having concentric north and south poles, which result in radial lines of magnetic flux directed outwards from the axis of the magnet. The magnetic flux also extends outside the gap. It is this flux that will be used for the generation of the load at the coil. The angular deflection of the pendulum is small (see Appendix E) and therefore, the flux density $B$ of the magnetic field of the permanent magnet can

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**Fig. G.1.** Schematic of the coil and permanent magnet ($E$ = Permanent magnet, $L$ = Circular coil).
be considered constant. The coil has \( n \) windings, each of which has a radius of \( R \). The coil resistance is \( R_c \) and is in series with the inductance of the coil, which is represented by the inductor \( L \) located externally to the coil (see Fig. G.1). Let \( x_1, y_1, z_1 \) be a Cartesian coordinate system of the coil, with its origin at point 1 of the coil. Then, the unit vector \( e_1 \), which is tangential to the axis of the coil wire, the magnetic flux density \( B \) and the velocity of point 1 of the coil are given by

\[
e_1 = \begin{pmatrix} 0 \\ -\sin \phi \\ \cos \phi \end{pmatrix} ; \quad B = \begin{pmatrix} B_x \\ B_r \cos \phi \\ B_r \sin \phi \end{pmatrix} \quad \text{and} \quad v_1 = \begin{pmatrix} r'_1 \\ 0 \\ 0 \end{pmatrix}, \quad (G.1)
\]

respectively. Here, \( B_r \) and \( B_x \) are the magnetic flux densities in the radial and \( x \) direction, respectively, and \( r'_1 \) is the horizontal velocity of the coil. With a current \( i \) present in the coil, in the direction of \( e_1 \) shown in Fig. G.1, a magnetic force, that is a load \( F_c \), acts upon the coil, which, according to the electromagnetic theory, is given by

\[
F_c = \int \ell e_1 \times B d\ell, \quad (G.2)
\]

in which \( \ell \) is the total length of the coil wire, which is \( \ell = 2\pi nR \). The magnetic force \( F_c \) is perpendicular to both \( e_1 \) and \( B \). To obtain the total magnetic force \( F_c \), (G.2) must be integrated along the length \( \ell \) of the coil wire. From (G.1) it follows that

\[
e_1 \times B = \begin{pmatrix} -B_r \\ B_x \cos \phi \\ B_x \sin \phi \end{pmatrix}. \quad (G.3)
\]

Substitution of this vector into (G.2) yields

\[
F_c = \begin{pmatrix} -2\pi nRB_r \ell \\ 0 \\ 0 \end{pmatrix}. \quad (G.4)
\]

Hence, as a result of the radial symmetry of the coil and the magnetic flux lines in and surrounding the gap, only the flux density \( B_r \) in radial direction contributes to the magnetic force. The voltage induced in the coil wire of length \( \ell \) moving with velocity \( v_1 \) in the magnetic field of flux density \( B \) is given by

\[
E_m = \int \ell (v_1 \times B) \cdot e_1 d\ell. \quad (G.5)
\]

The total induced voltage \( E_m \) is obtained by integrating (G.5) along the length \( \ell \) of the coil wire. From (G.1) it follows that

\[
v_1 \times B = \begin{pmatrix} 0 \\ r'_1 \times B_r \\ 0 \end{pmatrix} \quad \text{and} \quad (v_1 \times B) \cdot e_1 = r'_1 \times B_r. \quad (G.6)
\]

From this equation, (G.5) is to be found as
\[ E_m = 2\pi nRB_{1,i_x}. \]  
(E.7)

Applying Kirchoff's voltage law and using (E.7) for the circuit that makes up the electrical part of the actuator gives

\[ L \frac{di}{dt} + R_c i = U_c(t) - E_m, \]  
(E.8)

or after substitution of (E.7)

\[ L \frac{di}{dt} + R_c i + 2\pi nRB_{1,i_x} = U_c(t). \]  
(E.9)

From (5.2) and (5.3) it follows that the velocity \( r_{1,x}' \) is

\[ r_{1,x}' = (\ell_5 \sin \theta + \ell_1 \cos \theta) \dot{\theta}, \]  
(E.10)

so that

\[ L \frac{di}{dt} + R_c i + 2\pi nRB_{1,i_x} (\ell_5 \sin \theta + \ell_1 \cos \theta) \dot{\theta} = U_c(t). \]  
(E.11)

For small \( \theta \)'s the approximations \( \cos \theta \approx 1 \) and \( \sin \theta \approx \theta \) may be used and therefore

\[ L \frac{di}{dt} + R_c i + 2\pi nRB_{1,i_x} (\ell_5 \theta + \ell_1) \dot{\theta} = U_c(t). \]  
(E.12)

Equations (E.4) and (E.12) constitute the complete model of the actuator. Together with the equations in Appendix H, these equations can be written in dimensionless form as follows,

\[ \begin{pmatrix} \Phi_c \\ 0 \end{pmatrix} = \begin{pmatrix} -F_c \\ 0 \end{pmatrix} = \begin{pmatrix} -2\pi NI \\ 0 \end{pmatrix}, \]  
(E.13)

and

\[ \mathcal{L} \dot{I} + I + 2\pi N(L_0 \theta + 1) \dot{\theta}(\tau) = \mathcal{U}_c(\tau), \]  
(E.14)

in which \( \dot{I} \overset{\text{def}}{=} \frac{dI}{d\tau} \) and in which \( \Phi_c \) is the dimensionless load at the coil in the \( x_1 \) direction. Furthermore, the dimensionless groups are given by (H.22) and by

\[ \begin{align*}
\mathcal{U}_c &\equiv U_c \sqrt{\frac{J}{R_c^2 (M_cg)^3}}; \quad I \equiv i \sqrt{\frac{JR_c^2}{(M_cg)^3}}; \\
N &\equiv nRB_{1,i_x} \sqrt{\frac{1}{R_c^2 JM_cgR_x}}; \quad \mathcal{L} \equiv \frac{L_{r,c}}{R_c} \sqrt{\frac{M_cgR_x}{J}}.
\end{align*} \]  
(E.15)

In practical situations, the inductance \( L \) of the coil is often sufficiently small so that the term \( \mathcal{L} \dot{I} \) in (E.14) may be neglected compared to all other terms. Then

\[ I + 2\pi N (L_0 \theta + 1) \dot{\theta}(\tau) = \mathcal{U}_c(\tau), \]  
(E.16)

or
\[ I = \bar{U}_c(\tau) - 2\pi N (L_5 \theta + 1) \dot{\theta}(\tau). \tag{G.17} \]

Substitution of (G.17) into (G.13) gives

\[ \bar{F}_c = 2\pi N \left( \bar{U}_c(\tau) - 2\pi N (L_5 \theta + 1) \dot{\theta}(\tau) \right). \tag{G.18} \]

For small angular deflections \( \theta \), \( L_5 \theta \ll 1 \) and therefore

\[ \bar{F}_c = 2\pi N \left( \bar{U}_c(\tau) - 2\pi N \dot{\theta}(\tau) \right). \tag{G.19} \]

Thus, the force at the coil depends on the number of windings, the applied voltage at the coil, and the angular velocity of the pendulum.
The Equations of Motion of the Pendulum

In this appendix the equations of motion of the pendulum will be derived from Lagrangian mechanics, in case the unloaded pendulum is in mechanical balance, in case of a load calibration and in case of measurements. The Lagrange equations which will be employed here to derive the equations of motion of the pendulum are given by (see, for example, Goldstein [43])

$$\frac{d}{dt} \left( \frac{\partial \mathcal{L}}{\partial \dot{q}_i} \right) - \frac{\partial \mathcal{L}}{\partial q_i} = Q_i, \quad (i = 1, 2, ..., n),$$  \hspace{1cm} (H.1)

in which the $q_i$'s are the generalised coordinates, $n$ is the number of generalised coordinates, $\dot{q}_i \overset{\text{def}}{=} \frac{dq_i}{dt}$, $Q_i$'s are the generalised forces, and $\mathcal{L}$ is the Lagrangian defined as

$$\mathcal{L} \overset{\text{def}}{=} T - V,$$ \hspace{1cm} (H.2)

in which $T$ is the kinetic energy and $V$ is the potential energy of the system. The pendulum has one degree of freedom, and, therefore, let the angular deflection $\theta$ be the generalised coordinate. The Lagrange equations then reduce to a single equation for the motion of the pendulum

$$\frac{d}{dt} \left( \frac{\partial \mathcal{L}}{\partial \dot{\theta}} \right) - \frac{\partial \mathcal{L}}{\partial \theta} = Q.$$ \hspace{1cm} (H.3)

In order to obtain the equations of motion of the pendulum, it is sufficient to derive the kinetic and potential energies and the generalised force. In this appendix, first a general equation of motion of the pendulum will be derived, in case all forces and masses are applied to the pendulum and when the probe is in contact with the specimen. The forces then acting on the pendulum are the load at the coil, the gravitational force due to the calibration mass and the friction force and load at the probe tip. The equations of motion to be derived for the three cases mentioned above can then be regarded as special cases of this general equation of motion.

H.1 The General Equation of Motion of the Pendulum

The kinetic energy of the pendulum is
\[ T(\dot{\theta}) = \frac{1}{2} (J + I_m) \dot{\theta}^2, \]  

(H.4)
in which \( J \) is the moment of inertia of the pendulum system \textit{about the axis of rotation} (\( y \) axis), given by

\[ J = I_c + I_p + I_w, \]  

(H.5)
and where \( I_c, I_p, I_w \) and \( I_m \) are the moment of inertia about the axis of rotation of the pendulum, the probe, the balance weight and the calibration weight, respectively. The potential energy of the pendulum is the work done by the conservative forces acting on the pendulum \textit{from a general configuration} \( A \) \textit{to a reference configuration} \( B \), and consists of the gravitational potential energy determined by the pendulum masses and the calibration weight, and the elastic potential energy determined by the spring of the elastic hinge. If the \( xy \) plane is taken as the reference plane where the potential energies are zero, then the potential energy with respect to this plane is

\[
V(\theta) = - \int_{R_z^p}^{R_z^r} M_c g dz - \int_{r_{A,z}^p}^{r_{A,z}^r} mg dz - \int_\theta^0 K_p \theta d\theta,
\]  

(H.6)
and after integration

\[
V(\theta) = M_c g R_z^r + m g r_{A,z}^r + \frac{1}{2} K_p \theta^2,
\]  

(H.7)
in which \( M_c \) is the total mass of the pendulum system, given by (5.9), \( m \) is the mass of the calibration weight, \( K_p \) is the spring constant of the elastic hinge and \( g \) is the gravitational acceleration (9.807 m/s\(^2\)). And with \( r_{A,z}^r \) and \( R_z^r \) derived from (5.2), (5.3) and (5.10) the potential energy becomes

\[
V(\theta) = (M_c R_z - m \ell_2) g \cos \theta - (M_c R_x + m \ell_9) g \sin \theta + \frac{1}{2} K_p \theta^2.
\]  

(H.8)
The Lagrangian \( \mathcal{L} \) defined in (H.2) is then

\[
\mathcal{L}(\theta, \dot{\theta}) = \frac{1}{2} (J + I_m) \dot{\theta}^2 - (M_c R_z - m \ell_2) g \cos \theta
\]

\[ + (M_c R_x + m \ell_9) g \sin \theta - \frac{1}{2} K_p \theta^2. \]  

(H.9)
The virtual work \( \delta W \) done by the pendulum due to a virtual displacement \( \delta \theta \) of the pendulum is

\[ \delta W = Q \cdot \delta \theta, \]  

(H.10)
in which \( Q \) and \( \theta \) are given by

\[
Q = \begin{pmatrix} Q_x \\ Q_y \\ Q_z \end{pmatrix} \quad \text{and} \quad \theta = \begin{pmatrix} \theta \\ 0 \\ 0 \end{pmatrix}.
\]  

(H.11)
Thus,
\[ \delta W = Q_y \delta \theta = \left[ r'_1 \times F_c + r'_7 \times F + r'_7 \times P - C_p \dot{\theta} \right] \delta \theta. \tag{H.12} \]

Writing out this equation gives
\[ \delta W = Q_y \delta \theta = \left( -F_c(t) r'_{1,z} - (F_x + P_z) r'_{7,x} + (F_x + P_x) (r'_{7,z} - C_p \dot{\theta}) \right) \delta \theta, \tag{H.13} \]
in which \( Q_y \) is the generalised force acting on the pendulum and \( C_p \) is the viscous damping constant of the pendulum. And with \( F, F_x, F_z, P_x \) and \( P_z \) given by (5.7), (5.11) and (5.12), the work becomes
\[ \delta W = \left( -F_c(t) r'_{1,z} - (\mu P \sin \alpha + P \gamma) r'_{7,x} \right. \]
\[ + \left( \mu P (\gamma \sin \alpha - \beta \cos \alpha) - P \right) (r'_{7,z} - C_p \dot{\theta}) \delta \theta. \tag{H.14} \]
Furthermore, the positions \( r'_{1,z}, r'_{7,x} \) and \( r'_{7} \) are given by (5.2) and (5.3) so that
\[ \delta W = \left( -F_c(t) (\ell_2 \sin \theta + \ell_1 \cos \theta) + (\mu P \sin \alpha + P \gamma) (\ell_{10} \cos \theta + \ell_3 \sin \theta) \right. \]
\[ + \left. (\mu P (\gamma \sin \alpha - \beta \cos \alpha) - P) (\ell_{10} \sin \theta - \ell_3 \cos \theta) - C_p \dot{\theta} \right) \delta \theta. \tag{H.15} \]

Thus, the generalised force \( Q_y \) becomes
\[ Q_y = -F_c(t) (\ell_2 \sin \theta + \ell_1 \cos \theta) + P \left( (\mu \sin \alpha + \gamma) (\ell_{10} \cos \theta + \ell_3 \sin \theta) \right. \]
\[ + \left. (\mu (\gamma \sin \alpha - \beta \cos \alpha) - 1) (\ell_{10} \sin \theta - \ell_3 \cos \theta) \right) - C_p \dot{\theta}. \tag{H.16} \]
Substitution of (H.9) and (H.16) into (H.3) gives the general equation of motion of the pendulum
\[ (J + I_m) \ddot{\theta} + C_p \dot{\theta} - (M_c R_x + m \ell_9) g \cos \theta - (M_c R_x - m \ell_2) g \sin \theta + K_p \theta \]
\[ = -F_c(t) (\ell_2 \sin \theta + \ell_1 \cos \theta) + P (A \sin \theta + B \cos \theta), \tag{H.17} \]
in which \( A \) and \( B \) are defined as
\[ A = (\mu \sin \alpha + \gamma) \ell_3 + (\mu (\gamma \sin \alpha - \beta \cos \alpha) - 1) \ell_{10}, \]
\[ B = (\mu \sin \alpha + \gamma) \ell_{10} - (\mu (\gamma \sin \alpha - \beta \cos \alpha) - 1) \ell_3. \tag{H.18} \]
By applying a similarity analysis (according to Moe [98]) to the set of equations (G.4), (G.12), (H.4), (H.8) and (H.17), these equations can be written in dimensionless form as
\[ T(\dot{\theta}) = \frac{1}{2} (1 + T_m) \dot{\theta}^2, \tag{H.19} \]
\[ \nabla(\theta) = (1 - M_m L_2) \cos \theta - (\xi + M_m L_9) \sin \theta + \frac{1}{2} (\omega^2 + 1) \theta^2, \tag{H.20} \]
and
\[(1 + T_m) \ddot{\theta} + 2 \zeta \dot{\theta} - (\xi + M_m L_9) \cos \theta - (1 - M_m L_2) \sin \theta + (\omega^2 + 1) \theta = -T_c(\tau)(L_5 \sin \theta + \cos \theta) + \mathcal{F}(A \sin \theta + \mathcal{B} \cos \theta), \quad (H.21)\]

respectively, in which the dimensionless groups are given by

\[
\tau = t \sqrt{\frac{M_c g R_z}{J}}; \quad \omega = \sqrt{\frac{K_p}{M_c g R_z} - 1}; \quad \zeta = \frac{C_p}{2} \sqrt{\frac{1}{J M_c g R_z}}; \quad \xi = \frac{R_x}{R_z}; \quad \mathcal{F}_c = \frac{F_c \ell_1}{M_c g R_z}; \quad \mathcal{F} = \frac{P}{M_c g}; \quad \bar{I}_m = \frac{I_m}{J} = \frac{I_c}{I_c + I_p + I_w}; \quad \bar{A} = \frac{A}{R_z}; \quad \bar{B} = \frac{B}{R_z}; \quad (H.22)
\]

\[
L_2 = \frac{\ell_2}{R_z}; \quad L_3 = \frac{\ell_3}{R_z}; \quad L_5 = \frac{\ell_5}{\ell_1}; \quad L_9 = \frac{\ell_9}{R_z}; \quad L_{10} = \frac{\ell_{10}}{R_z}; \quad M_m = \frac{m}{M_c}; \quad \mathcal{T} = \frac{T}{M_c g R_z}; \quad \mathbf{V} = \frac{V}{M_c g R_z},
\]

and with

\[
\bar{A} = (\mu \sin \alpha + \gamma) L_3 + (\mu (\gamma \sin \alpha - \beta \cos \alpha) - 1) L_{10}, \quad \bar{B} = (\mu \sin \alpha + \gamma) L_{10} - (\mu (\gamma \sin \alpha - \beta \cos \alpha) - 1) L_3. \quad (H.23)
\]

Furthermore,

\[
M \mathbf{X}_c + M_p L_{10} - M_w L_8 = -\xi, \quad M \mathbf{Z}_e - M_p L_3 - M_w L_2 = 1, \quad M + M_p + M_w = 1, \quad \bar{I}_c + \bar{I}_p + \bar{I}_w = 1, \quad (H.24)
\]

in which

\[
M = \frac{m_c}{M_c}; \quad M_p = \frac{m_p}{M_c}; \quad M_w = \frac{m_w}{M_c}; \quad \mathbf{X}_c \equiv \frac{x_c}{R_z}; \quad \mathbf{Z}_c \equiv \frac{z_c}{R_z}; \quad L_8 \equiv \frac{\ell_8}{R_z}; \quad \mathbf{T}_c \equiv \frac{I_c}{J}; \quad \mathbf{T}_p \equiv \frac{I_p}{J}; \quad \mathbf{T}_w \equiv \frac{I_w}{J}. \quad (H.25)
\]

In Appendix E it has been shown that the angular deflection \( \theta \) of the pendulum beam is very small \((-1.5 \leq \theta \leq 0.6 \text{ degrees})\). For small angular deflections, the approximations \( \cos \theta \approx 1 \) and \( \sin \theta \approx \theta \), may be used. Therefore, the linearised equation of motion of the pendulum follows from \((H.21)\) as

\[
(1 + T_m) \ddot{\theta}(\tau) + 2 \zeta \dot{\theta}(\tau) + (\omega^2 + M_m L_2) \theta(\tau) - \xi - M_m L_9
\]

\[
= -\mathcal{F}_c(\tau)(L_5 \theta(\tau) + 1) + \mathcal{F}(\bar{A} \theta(\tau) + \bar{B}). \quad (H.26)
\]
H.2 Motion of the Mechanically Balanced Unloaded Pendulum

In case of the mechanically balanced unloaded pendulum, there is no calibration weight, no load at the coil and the probe is not in contact with the specimen. Thus, $M_m = 0$, $I_m = 0$, $F_c = 0$, $F = 0$, and from (H.26) the equation of motion of the mechanically balanced unloaded pendulum becomes

$$\ddot{\theta}(\tau) + 2\zeta \dot{\theta}(\tau) + \omega^2 \theta(\tau) = \xi,$$  \hspace{1cm} (H.27)

with the initial conditions

$$\theta(0) = 0 \quad \text{and} \quad \dot{\theta}(0) = 0.$$  \hspace{1cm} (H.28)

This is the equation of motion of the pendulum when the length $\xi$, thus, length $\ell_8$ of the balance weight $[W]$ has been set, and the pendulum swings correctly. Thereafter, a pendulum test will be carried out and no change in the position $\ell_8$ of the weight $[W]$ will be done. If $\ell_8$ were to change, then $R_x$ would change and this would affect the motion of the pendulum. Equation (H.27) is a typical equation for a damped forced vibration of an inverted pendulum. Furthermore, the dimensionless kinetic and potential energies follow from (H.19) and (H.20) are given by

$$\overline{T}(\dot{\theta}) = \frac{1}{2} \dot{\theta}^2,$$  \hspace{1cm} (H.29)

and

$$\overline{V}(\theta) = \cos \theta - \xi \sin \theta + \frac{1}{2} (\omega^2 + 1) \theta^2,$$  \hspace{1cm} (H.30)

respectively. The equation of motion of the pendulum (H.27) is a linear ordinary differential equation, which will be solved here by means of a Laplace transformation. On taking the Laplace transform of both sides of (H.27) an algebraic equation is obtained,

$$s^2 \Theta(s) - s \theta(0) - \dot{\theta}(0) + 2\zeta \left( s \Theta(s) - \theta(0) \right) + \omega^2 \Theta(s) = \frac{\xi}{s},$$  \hspace{1cm} (H.31)

in which $\Theta(s)$ is the Laplace transform of $\theta(\tau)$. Given the initial conditions (H.28) the algebraic equation becomes

$$s^2 \Theta(s) + 2\zeta s \Theta(s) + \omega^2 \Theta(s) = \frac{\xi}{s}.$$  \hspace{1cm} (H.32)

Solving for the Laplace transform $\Theta(s)$ gives

$$\Theta(s) = \frac{\xi}{s(s^2 + 2\zeta s + \omega^2)},$$  \hspace{1cm} (H.33)

which can be rewritten as

$$\Theta(s) = \frac{\xi \omega^2}{s^2} \left( \frac{1}{s} - \frac{1}{s + \zeta^2 + \omega^2} \right).$$  \hspace{1cm} (H.34)
The required solution is now obtained by finding the inverse Laplace transform of (H.34). Three cases can be distinguished, which will be discussed below.

**Case 1,** \( \omega^2 - \zeta^2 > 0 \). In this case, the dimensionless angular deflection \( \theta(\tau) \) is given by

\[
\frac{\theta(\tau)}{\theta_0} = 1 - e^{-\zeta \tau} \left( \cos \left( \tau \sqrt{\omega^2 - \zeta^2} \right) + \frac{\zeta}{\sqrt{\omega^2 - \zeta^2}} \sin \left( \tau \sqrt{\omega^2 - \zeta^2} \right) \right),
\]

(H.35)

in which

\[
\theta_0 = \frac{\zeta}{\omega^2}.
\]

(H.36)

The motion of the pendulum is called **damped oscillatory**. The pendulum oscillates about the deflection angle \( \theta_0 \), the magnitude of each oscillation becoming smaller with each swing (see Fig. H.1). The frequency \( f \) of the oscillations is given by

\[
f = \frac{1}{2\pi} \sqrt{\omega^2 - \zeta^2}.
\]

(H.37)

The frequency \( f_n \) corresponding to \( \zeta = 0 \), that is when no damping is present, is called the **natural frequency**,

\[
f_n = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{K_p}{M_c g R_x}} - 1.
\]

(H.38)

The motion without damping present, then reduces to

\[
\frac{\theta(\tau)}{\theta_0} = 1 - \cos(\omega \tau).
\]

(H.39)

**Case 2,** \( \omega^2 - \zeta^2 = 0 \). In this case, the dimensionless angular deflection \( \theta(\tau) \) is given by

\[
\frac{\theta(\tau)}{\theta_0} = 1 - e^{-\zeta \tau} (1 + \zeta \tau).
\]

(H.40)

Here, the pendulum does not oscillate indefinitely about the angular deflection \( \theta_0 \). Instead, it approaches \( \theta_0 \) gradually but never reaches it (see Fig. H.1). The pendulum motion is called **critically damped motion** since any decrease in the damping constant \( \zeta \) would produce oscillations.

**Case 3,** \( \omega^2 - \zeta^2 < 0 \). In this case, the dimensionless angular deflection \( \theta(\tau) \) can be written as

\[
\frac{\theta(\tau)}{\theta_0} = 1 - e^{-\zeta \tau} \left( \cos \left( \tau \sqrt{\zeta^2 - \omega^2} \right) + \frac{\zeta}{\sqrt{\zeta^2 - \omega^2}} \sin \left( \tau \sqrt{\zeta^2 - \omega^2} \right) \right),
\]

(H.41)

in which \( i = \sqrt{-1} \), the imaginary unit. And with \( \cos(iz) = \cosh(z) \) and \( \sin(iz) = -i^{-1} \sinh(z) \) this can be written as
\[
\frac{\theta(\tau)}{\theta_0} = 1 - e^{-\zeta \tau} \left( \cosh \left( \tau \sqrt{\zeta^2 - \omega^2} \right) + \frac{\zeta}{\sqrt{\zeta^2 - \omega^2}} \sinh \left( \tau \sqrt{\zeta^2 - \omega^2} \right) \right). \quad \text{(H.42)}
\]

The motion of the pendulum is called *overdamped motion* and is non-oscillatory. The graph is similar to that of critically damped motion (see Fig. H.1).

### H.3 Motion of the Pendulum During Load Calibration

In case of a load calibration, there is no contact between probe and specimen. Thus, \( \mathbf{T} = 0 \) and from (H.26) the equation of motion of the pendulum during load calibration becomes

\[
(1 + T_m) \ddot{\theta}(\tau) + 2 \zeta \dot{\theta}(\tau) + (\omega^2 + M_m L_2) \theta(\tau) - \xi - M_m L_9 \\
= -T_c(\tau) \left( L_5 \theta(\tau) + 1 \right), \quad \text{(H.43)}
\]

in which the dimensionless groups are given by (H.22). Finally, the dimensionless kinetic and potential energies follow from (H.19) and (H.20), and are given by

\[
\mathcal{T}(\dot{\theta}) = \frac{1}{2} (1 + I_m) \dot{\theta}^2, \quad \text{(H.44)}
\]

and

\[
\mathcal{V}(\theta) = (1 - M_m L_2) \cos \theta - (\xi + M_m L_9) \sin \theta + \frac{1}{2} (\omega^2 + 1) \theta^2, \quad \text{(H.45)}
\]

respectively.

![Graph](https://via.placeholder.com/150)

**Fig. H.1.** The dimensionless angular deflection \( \theta/\theta_0 \) as a function of dimensionless time \( \tau \) in case of the mechanically balanced unloaded pendulum (\( \omega = 4.0 \) and \( \xi = 0.1 \)).
H.4 Motion of the Pendulum During Contact Between Probe and Specimen

In case the probe tip is in contact with the specimen surface during measurements, there is no calibration weight, thus $M_m = 0$, and $\overline{I}_m = 0$ and the equation of motion of the pendulum follows from (H.26) as

$$\ddot{\theta}(\tau) + 2\zeta \dot{\theta}(\tau) + \left( F_e(\tau) - \omega^2 \right) \theta(\tau) + \overline{F}_m(\tau) - \xi = \overline{F}(A\dot{\theta}(\tau) + B),$$  \hspace{1cm} (H.46)

and with $\overline{A}$ and $\overline{B}$ given in (H.23). Finally, the dimensionless kinetic and potential energies follow from (H.19) and (H.20), and are given by

$$\mathcal{T}(\dot{\theta}) = \frac{1}{2} \dot{\theta}^2,$$  \hspace{1cm} (H.47)

and

$$\mathcal{V}(\theta) = \cos \theta - \xi \sin \theta + \frac{1}{2}(\omega^2 + 1)\theta^2,$$  \hspace{1cm} (H.48)

respectively.
I

Surface Topography Measurements

An optical phase shift interferometry profiler has been used for the measurements of the surface topography of the rigid disks and the sliders used in the friction experiments\(^1\). This instrument measures the light intensity distribution of two interfering light beams, one of which is reflected by the measurement area and the other by a reference mirror. The splitted beams experience a phase shift upon reflection, resulting in interference fringe patterns. Moving the reference mirror relative to the surface area during measurements allows the use of a dynamic phase shift technique. With this technique the light intensity is measured at several phase shifts (positions of the reference mirror). From these light intensity measurements the phase relationship between the two interfering light beams can be quantified. In turn, the surface height distribution \(z_r(x, y)\) can be calculated from this phase relationship. By analysing this distribution several statistical roughness parameters of the surface and summit heights can be obtained. It has been found that the instrument can measure the surface height distribution to within 0.1 nm. The results of some actual measurements obtained with the instrument are given in Figs. 3.8 and 3.11 on pages 38 and 42, respectively.

Prior to the calculation of the parameters the data were digitally preprocessed, in order to remove the overall surface shape, or any ‘trend’ or slope due, for example, to tilt of the specimen. Removing these features, implies the fitting of a suitable reference shape which reveals the microstructure of the surface from which the surface heights can be measured. The reference shape that has been applied in the preprocessing of the data is a so called quartic, defined as

\[
\begin{align*}
z_0(x, y) &= c_0 + c_1x + c_2y + c_3xy + \\
c_4x^2 + c_5y^2 + c_6x^3 + c_7y^3 + c_8xy^2 + c_9x^2y + \\
c_{10}x^4 + c_{11}y^4 + c_{12}xy^3 + c_{13}x^2y^2 + c_{14}x^3y,
\end{align*}
\]

(I.1)

whereby the coefficients \(c_0\) to \(c_{14}\) were obtained from the fitting procedure. The surface heights then become

\[
z(x, y) = z_r(x, y) - z_0(x, y).
\]

Finally, the instrument represents the surface heights \(z_{ij} = z(x_i, y_j)\) at equidistant coordinates \(x_i\) and \(y_j\) (see Fig. I.1).

\(^1\)A stylus profiler cannot be used where the surface topography of smooth thin film rigid disks must be determined, because any contact between stylus and surface may deform and perhaps damage the surface during a measurement.
Fig. I.1. General geometry for the surface topography measurements on the rigid disks and sliders. The surface heights \( z_{i,j} = z(x_i, y_j) \) are taken at equidistant coordinates \( x_i = (i-1) \Delta x \) and \( y_j = (j-1) \Delta y \), where \( \Delta x = x_{i+1} - x_i \), \( \Delta y = y_{j+1} - y_j \), \( i = 1, 2, \ldots, n \) and \( j = 1, 2, \ldots, m \).

I.1 Statistics of the Surface Heights

After removing the reference shape from each measurement \( k \), the following statistical parameters of the surface heights were calculated.

1. The mean of the surface heights \( z_{k,x} \), which is given by

   \[
   z_{k,x} = \frac{1}{nm} \sum_{i=1}^{n} \sum_{j=1}^{m} z_{i,j},
   \]  

   in which \( n \) and \( m \) are the number of readings in the \( x \) and \( y \) directions, respectively.

2. The centre-line-average\(^2\) (CLA) roughness parameter \( R_{a,k,x} \), which is defined as

   \[
   R_{a,k,x} = \frac{1}{nm} \sum_{i=1}^{n} \sum_{j=1}^{m} | z_{i,j} - z_{k,x} |.
   \]

\(^2\)Perhaps a better name for this parameter in this case would be centre-plane-average.
3. The root-mean-square (RMS) roughness parameter $\sigma_{k,x}$, defined as

$$\sigma_{k,x} = \sqrt{\frac{1}{nm-1} \left( \sum_{i=1}^{n} \sum_{j=1}^{m} \frac{z_{i,j}^2}{\sigma_{i,j}^2} - n m \frac{\sigma_{k,x}^2}{\sigma_{k,x}^2} \right)}.$$ (I.5)

4. The mean curvature $\rho_{k,x,x}$ in the $x$ direction, which is given by

$$\rho_{k,x,x} = \frac{1}{nm} \sum_{i=1}^{n} \sum_{j=1}^{m} \rho_{i,j,x,x},$$ (I.6)

and the mean curvature $\rho_{k,y,x}$ in the $y$ direction, given by

$$\rho_{k,y,x} = \frac{1}{nm} \sum_{i=1}^{n} \sum_{j=1}^{m} \rho_{i,j,y,x},$$ (I.7)

in which $\rho_{i,j,x,x}$ and $\rho_{i,j,y,x}$ are the local curvatures in the $x$ direction

$$\rho_{i,j,x,x} = \frac{1}{\Delta x^2} (z_{i,j+1} - 2z_{i,j} + z_{i,j-1}),$$ (I.8)

and in the $y$ direction

$$\rho_{i,j,y,x} = \frac{1}{\Delta y^2} (z_{i+1,j} - 2z_{i,j} + z_{i-1,j}),$$ (I.9)

respectively. Finally, the mean curvature $\rho_{k,x}$ is given by

$$\rho_{k,x} = \frac{1}{2} (\rho_{k,x,x} + \rho_{k,y,x}).$$ (I.10)

A series of measurements were carried out, each on a different location at the rigid disk or slider. For each measurement $k$ the statistical parameters given in (I.3) to (I.10) were calculated. From these parameters, the mean values were calculated, giving the mean of the surface heights $\overline{z}$, the mean CLA roughness $R_{a,z}$, the mean RMS roughness $\sigma_{z,x}$, the mean curvature $\overline{\rho}_{z,x}$ in the $x$ direction, the mean curvature $\overline{\rho}_{z,y}$ in the $y$ direction and the mean curvature $\overline{\rho}_{z}$, respectively. Also, the standard deviation of each of these parameters was calculated.

### I.2 Statistics of the Summit Heights

A point at the surface is defined as a summit when its height lies above the height of all its eight neighbouring points (i.e. according to Greenwood [44] a ‘nine points summit’, see Fig. I.1), that is when

$$z_{i,j} > z_{i-1,j-1}, z_{i-1,j}, z_{i-1,j+1}, z_{i,j-1}, z_{i,j+1}, z_{i+1,j-1}, z_{i+1,j}, z_{i+1,j+1}.$$ (I.11)
With this definition of a summit, the same statistical parameters given in (I.3) to (I.10) were used for the height distribution of the summits, whereby the index \( z \) (height) was replaced by the index \( s \) (summit), and the summations were taken over the number of summits. Then the mean values of these parameters for a series of measurements were determined, giving the mean of the summit heights \( \bar{z}_s \), the mean CLA roughness \( R_{a,s} \), the mean RMS roughness \( \sigma_s \), the mean curvature \( \bar{p}_{x,s} \) in the \( x \) direction, the mean curvature \( \bar{p}_{y,s} \) in the \( y \) direction and the mean curvature \( \bar{p}_s \), respectively. Furthermore, the density \( \eta_s \) of the summits, given by

\[
\eta_s = \frac{N_s}{A_m},
\]

was also calculated, in which \( A_m = nm \Delta x \Delta y \) is the measurement area, and \( N_s \) is the number of summits at the measurement area. Again, the standard deviation of each of these parameters was also calculated.

I.3 Results of the Measurements

Tables I.1 to I.4 give the statistical parameters determined in the previous section, for all the disks and sliders used in the friction experiments. The index \( r \) (rails) and \( d \) (disk) in these tables indicate the slider and rigid disk, respectively (conform Fig. 4.10 on page 54). The curvatures of the surface heights \( \bar{p}_{x,s} \), \( \bar{p}_{y,s} \) and \( \bar{p}_s \) can be neglected compared to the curvatures of the summit heights \( \bar{p}_{x,s} \), \( \bar{p}_{y,s} \) and \( \bar{p}_s \) and therefore, they are not given in the tables. The mean curvature \( \bar{p}_{x,s} \) of the summits in the \( x \) direction of the disk types 1 and 2 is larger than the mean curvature \( \bar{p}_{y,s} \) in the \( y \) direction. This illustrates the circumferential texture at disks 1 and 2. The mean curvatures in the \( x \) and \( y \) direction of the disks 3 to 7 are almost the same. The last column in Tables I.1 and I.2 shows the parameter \( \eta_s \sigma_s / \bar{p}_s \) given by Whitehouse and Archard [151, page 118]. For the sliders this parameter is larger than for the rigid disks. The curvatures and the summit densities are much lower than those reported by Poon and Bhushan [109]. This is probably due to a difference in the sampling resolution or due to a different definition of a summit (i.e. a ‘five points summit’ instead of a ‘nine points summit’ in this thesis).

For the sliders, the following relationships have been found between the statistical parameters (see Table I.3),

\[
\begin{align*}
\bar{z}_{s,r} - \bar{z}_{z,r} &\approx 0.49 R_{a,s,r}; & R_{a,z,r} &\approx 0.84 \sigma_{s,r};
\sigma_{z,r} &\approx 1.26 R_{a,z,r}; & \sigma_{s,r} &\approx 1.28 R_{a,s,r};
\end{align*}
\]

whereas for the rigid disks (see Table I.4),

\[
\begin{align*}
\bar{z}_{s,d} - \bar{z}_{z,d} &\approx 0.85 R_{a,s,d}; & R_{a,z,d} &\approx 0.83 \sigma_{s,d};
\sigma_{z,d} &\approx 1.26 R_{a,z,d}; & \sigma_{s,d} &\approx 1.24 R_{a,s,d}.
\end{align*}
\]

In case the surface and summit heights are Gaussian distributed it can be shown that \( \sigma_z = \sqrt{\pi/2} R_{a,x} \) and \( \sigma_s = \sqrt{\pi/2} R_{a,s} \). Thus, as already is indicated in Chapter 3, Section 3.2.2, the rigid disks and sliders have a Gaussian height and summit distribution. From (I.13) and (I.14) it follows that \( R_{a,z,r} \approx 1.08 R_{a,s,r} \) and
\(R_{a,x,d} \approx 1.03R_{a,s,d}\). Thus the CLA roughnesses of the surface heights and the summit heights are practically equal. Whitehouse and Archard [151, page 118] found similar relationships, but with different constants. For a ‘fine structure’ they found

\[
\overline{z}_s - \overline{z}_z = 0.47 \sigma_z; \quad \sigma_s = 0.90 \sigma_z, \tag{I.15}
\]

and for a Gaussian height and summit distribution, this gives

\[
\overline{z}_s - \overline{z}_z = 0.66R_{a,s}, \tag{I.16}
\]

which is in between the values for the rigid disks and the sliders.

Finally, the combined surface roughness parameters for each pair of slider and rigid disk used in the friction experiments were derived from the individual calculated surface roughness parameters of the corresponding rigid disk and slider, and are given in Table I.5, whereby

\[
\overline{\sigma} = \left(\overline{z}_{s,r} - \overline{z}_{z,r}\right) + \left(\overline{z}_{s,d} - \overline{z}_{z,d}\right); \quad \eta = \eta_{h,r} + \eta_{h,d}, \tag{I.17}
\]

and for the surface heights

\[
\begin{align*}
R_{a,x,c} &= \sqrt{R_{a,x,r}^2 + R_{a,x,d}^2}; \quad \sigma_{x,c} = \sqrt{\sigma_{x,r}^2 + \sigma_{x,d}^2}; \\
\overline{p}_{x,c} &= \overline{p}_{x,r} + \overline{p}_{x,d}; \quad \overline{p}_{y,c} = \overline{p}_{y,r} + \overline{p}_{y,d},
\end{align*}
\tag{I.18}
\]

and for the summit heights

\[
\begin{align*}
R_{a,s,c} &= \sqrt{R_{a,s,r}^2 + R_{a,s,d}^2}; \quad \sigma_{s,c} = \sqrt{\sigma_{s,r}^2 + \sigma_{s,d}^2}; \\
\overline{p}_{x,c} &= \overline{p}_{s,x,r} + \overline{p}_{s,x,d}; \quad \overline{p}_{y,c} = \overline{p}_{s,y,r} + \overline{p}_{s,y,d}.
\end{align*}
\tag{I.19}
\]
Table I.1. Summary of the topography of the sliders.

<table>
<thead>
<tr>
<th>Slider no.</th>
<th>( \bar{z}_{r,r} ) (nm)</th>
<th>( \sigma_{z,r} ) (nm)</th>
<th>( R_{a,z,r} ) (nm)</th>
<th>( \bar{z}_{s,r} ) (nm)</th>
<th>( \sigma_{s,r} ) (nm)</th>
<th>( R_{a,s,r} ) (m(^{-1}))</th>
<th>( \bar{p}_{k,r} ) (m(^{-1}))</th>
<th>( \bar{p}_{s,s,r} ) (m(^{-1}))</th>
<th>( \bar{p}_{y,s,r} ) (m(^{-1}))</th>
<th>( N_{kr} )</th>
<th>( \eta_{kr} )</th>
<th>( \eta_{kr} \sigma_{kr} / \bar{p}_{k,r} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Mean</td>
<td>-0.00</td>
<td>1.16</td>
<td>0.88</td>
<td>0.68</td>
<td>1.25</td>
<td>0.80</td>
<td>5.3</td>
<td>6.6</td>
<td>4.0</td>
<td>116</td>
<td>390</td>
<td>0.092</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.21</td>
<td>0.26</td>
<td>0.16</td>
<td>0.27</td>
<td>0.37</td>
<td>0.13</td>
<td>1.0</td>
<td>1.9</td>
<td>0.6</td>
<td>14</td>
<td>57</td>
<td>0.020</td>
</tr>
<tr>
<td>2 Mean</td>
<td>-0.07</td>
<td>2.06</td>
<td>1.59</td>
<td>1.13</td>
<td>1.75</td>
<td>1.34</td>
<td>6.2</td>
<td>8.9</td>
<td>3.5</td>
<td>90</td>
<td>288</td>
<td>0.077</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>1.06</td>
<td>0.61</td>
<td>0.47</td>
<td>1.24</td>
<td>0.44</td>
<td>0.33</td>
<td>1.6</td>
<td>3.0</td>
<td>0.4</td>
<td>33</td>
<td>106</td>
<td>0.012</td>
</tr>
<tr>
<td>3 Mean</td>
<td>0.03</td>
<td>1.50</td>
<td>1.15</td>
<td>1.00</td>
<td>1.19</td>
<td>0.95</td>
<td>5.4</td>
<td>7.5</td>
<td>3.3</td>
<td>105</td>
<td>338</td>
<td>0.075</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.09</td>
<td>0.18</td>
<td>0.14</td>
<td>0.27</td>
<td>0.25</td>
<td>0.20</td>
<td>1.4</td>
<td>1.7</td>
<td>1.4</td>
<td>29</td>
<td>91</td>
<td>0.020</td>
</tr>
<tr>
<td>4 (A) Mean</td>
<td>-0.02</td>
<td>0.69</td>
<td>0.55</td>
<td>0.37</td>
<td>0.65</td>
<td>0.51</td>
<td>2.6</td>
<td>2.4</td>
<td>2.7</td>
<td>91</td>
<td>291</td>
<td>0.073</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.38</td>
<td>0.28</td>
<td>0.23</td>
<td>0.39</td>
<td>0.33</td>
<td>0.27</td>
<td>0.1</td>
<td>0.2</td>
<td>0.2</td>
<td>7</td>
<td>25</td>
<td>0.036</td>
</tr>
<tr>
<td>4 (B) Mean</td>
<td>0.00</td>
<td>1.08</td>
<td>0.87</td>
<td>0.339</td>
<td>1.03</td>
<td>0.83</td>
<td>2.7</td>
<td>2.7</td>
<td>2.7</td>
<td>85</td>
<td>279</td>
<td>0.103</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.79</td>
<td>0.37</td>
<td>0.89</td>
<td>0.37</td>
<td>0.37</td>
<td>0.29</td>
<td>0.2</td>
<td>0.3</td>
<td>0.5</td>
<td>17</td>
<td>54</td>
<td>0.019</td>
</tr>
<tr>
<td>5 (A) Mean</td>
<td>0.02</td>
<td>0.59</td>
<td>0.47</td>
<td>0.30</td>
<td>0.58</td>
<td>0.46</td>
<td>2.8</td>
<td>2.6</td>
<td>3.0</td>
<td>137</td>
<td>436</td>
<td>0.090</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.18</td>
<td>0.06</td>
<td>0.06</td>
<td>0.16</td>
<td>0.07</td>
<td>0.07</td>
<td>0.3</td>
<td>0.2</td>
<td>0.4</td>
<td>9</td>
<td>28</td>
<td>0.017</td>
</tr>
<tr>
<td>5 (B) Mean</td>
<td>-0.01</td>
<td>1.27</td>
<td>0.99</td>
<td>0.44</td>
<td>1.16</td>
<td>0.91</td>
<td>3.2</td>
<td>3.8</td>
<td>2.5</td>
<td>102</td>
<td>326</td>
<td>0.120</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.92</td>
<td>0.12</td>
<td>0.06</td>
<td>0.93</td>
<td>0.06</td>
<td>0.09</td>
<td>0.5</td>
<td>1.1</td>
<td>0.3</td>
<td>14</td>
<td>46</td>
<td>0.011</td>
</tr>
</tbody>
</table>
### Table I.2: Summary of the topography of the disks.

<table>
<thead>
<tr>
<th>Disk no.</th>
<th>$\bar{Z}_{x,y}$ (nm)</th>
<th>$\sigma_{x,y}$ (nm)</th>
<th>$\bar{P}_{x,y}$ (m$^{-1}$)</th>
<th>$\sigma_{P_{x,y}}$ (m$^{-1}$)</th>
<th>$P_{\text{av},x,y}$ (m$^{-1}$)</th>
<th>$\sigma_{P_{\text{av},x,y}}$ (m$^{-1}$)</th>
<th>$N_{x,y}$</th>
<th>$\bar{P}<em>{x,y}/N</em>{x,y}$</th>
<th>$\bar{P}<em>{\text{av},x,y}/N</em>{x,y}$</th>
<th>$P_{x,y}/N_{x,y}$</th>
<th>$P_{\text{av},x,y}/N_{x,y}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (side 1)</td>
<td>0.24</td>
<td>0.08</td>
<td>0.42</td>
<td>0.33</td>
<td>4.45</td>
<td>1.07</td>
<td>118.0</td>
<td>0.45</td>
<td>30.4</td>
<td>118.0</td>
<td>0.045</td>
</tr>
<tr>
<td>1 (side 2)</td>
<td>0.50</td>
<td>0.42</td>
<td>1.71</td>
<td>0.86</td>
<td>3.23</td>
<td>0.34</td>
<td>3.3</td>
<td>0.45</td>
<td>118.0</td>
<td>0.045</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.05</td>
<td>0.08</td>
<td>0.71</td>
<td>0.35</td>
<td>4.45</td>
<td>1.07</td>
<td>118.0</td>
<td>0.45</td>
<td>30.4</td>
<td>118.0</td>
<td>0.045</td>
</tr>
<tr>
<td>3</td>
<td>0.00</td>
<td>0.00</td>
<td>0.71</td>
<td>0.71</td>
<td>4.45</td>
<td>1.07</td>
<td>118.0</td>
<td>0.45</td>
<td>30.4</td>
<td>118.0</td>
<td>0.045</td>
</tr>
<tr>
<td>4</td>
<td>0.11</td>
<td>0.10</td>
<td>0.71</td>
<td>0.71</td>
<td>4.45</td>
<td>1.07</td>
<td>118.0</td>
<td>0.45</td>
<td>30.4</td>
<td>118.0</td>
<td>0.045</td>
</tr>
<tr>
<td>5</td>
<td>0.11</td>
<td>0.11</td>
<td>0.71</td>
<td>0.71</td>
<td>4.45</td>
<td>1.07</td>
<td>118.0</td>
<td>0.45</td>
<td>30.4</td>
<td>118.0</td>
<td>0.045</td>
</tr>
<tr>
<td>6</td>
<td>0.00</td>
<td>0.00</td>
<td>0.71</td>
<td>0.71</td>
<td>4.45</td>
<td>1.07</td>
<td>118.0</td>
<td>0.45</td>
<td>30.4</td>
<td>118.0</td>
<td>0.045</td>
</tr>
<tr>
<td>7</td>
<td>0.00</td>
<td>0.00</td>
<td>0.71</td>
<td>0.71</td>
<td>4.45</td>
<td>1.07</td>
<td>118.0</td>
<td>0.45</td>
<td>30.4</td>
<td>118.0</td>
<td>0.045</td>
</tr>
</tbody>
</table>

I.3 Results of the Measurements
### Table I.3. Summary of the topography of the sliders.

<table>
<thead>
<tr>
<th>Slider no.</th>
<th>$\overline{z}<em>{3, r} - \overline{z}</em>{3, f}$</th>
<th>$R_{a, 3, r}$</th>
<th>$\sigma_{z, r}$</th>
<th>$R_{a, 3, f}$</th>
<th>$\sigma_{a, r}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.55</td>
<td>0.70</td>
<td>1.32</td>
<td>1.57</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.69</td>
<td>0.91</td>
<td>1.30</td>
<td>1.30</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.83</td>
<td>0.97</td>
<td>1.31</td>
<td>1.25</td>
<td></td>
</tr>
<tr>
<td>4 a</td>
<td>0.60</td>
<td>0.86</td>
<td>1.24</td>
<td>1.26</td>
<td></td>
</tr>
<tr>
<td>4 b</td>
<td>0.33</td>
<td>0.84</td>
<td>1.24</td>
<td>1.25</td>
<td></td>
</tr>
<tr>
<td>5 a</td>
<td>0.49</td>
<td>0.82</td>
<td>1.24</td>
<td>1.26</td>
<td></td>
</tr>
<tr>
<td>5 b</td>
<td>0.38</td>
<td>0.85</td>
<td>1.29</td>
<td>1.28</td>
<td></td>
</tr>
<tr>
<td>Mean:</td>
<td>0.49</td>
<td>0.84</td>
<td>1.26</td>
<td>1.28</td>
<td></td>
</tr>
<tr>
<td>Std. Dev.:</td>
<td>0.21</td>
<td>0.06</td>
<td>0.04</td>
<td>0.09</td>
<td></td>
</tr>
</tbody>
</table>

### Table I.4. Summary of the topography of the disks.

<table>
<thead>
<tr>
<th>Disk no.</th>
<th>$\overline{z}<em>{3, d} - \overline{z}</em>{3, d}$</th>
<th>$R_{a, 3, d}$</th>
<th>$\sigma_{z, d}$</th>
<th>$R_{a, 3, d}$</th>
<th>$\sigma_{a, d}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (side 1)</td>
<td>1.00</td>
<td>0.94</td>
<td>1.25</td>
<td>1.26</td>
<td></td>
</tr>
<tr>
<td>1 (side 2)</td>
<td>0.83</td>
<td>0.89</td>
<td>1.25</td>
<td>1.25</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.62</td>
<td>0.73</td>
<td>1.26</td>
<td>1.24</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.67</td>
<td>0.66</td>
<td>1.26</td>
<td>1.21</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.73</td>
<td>0.66</td>
<td>1.27</td>
<td>1.23</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.64</td>
<td>0.65</td>
<td>1.26</td>
<td>1.20</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.69</td>
<td>0.57</td>
<td>1.27</td>
<td>1.20</td>
<td></td>
</tr>
<tr>
<td>2 a</td>
<td>0.86</td>
<td>0.88</td>
<td>1.24</td>
<td>1.26</td>
<td></td>
</tr>
<tr>
<td>2 b</td>
<td>0.91</td>
<td>0.91</td>
<td>1.25</td>
<td>1.25</td>
<td></td>
</tr>
<tr>
<td>Mean:</td>
<td>0.85</td>
<td>0.83</td>
<td>1.26</td>
<td>1.24</td>
<td></td>
</tr>
<tr>
<td>Std. Dev.:</td>
<td>0.17</td>
<td>0.14</td>
<td>0.01</td>
<td>0.02</td>
<td></td>
</tr>
</tbody>
</table>
Table I.5. Summary of the topography of the combined surfaces of the disks and the sliders used in the experiments.

<table>
<thead>
<tr>
<th>Disk no.</th>
<th>Slider no.</th>
<th>$\sigma$ (nm)</th>
<th>$\sigma_{z,c}$ (nm)</th>
<th>$R_{a,z,c}$ (nm)</th>
<th>$\sigma_{a,c}$ (nm)</th>
<th>$R_{a,s,c}$ (nm)</th>
<th>$\overline{\gamma}_{x,s,c}$ (m$^{-1}$)</th>
<th>$\overline{\gamma}_{y,s,c}$ (m$^{-1}$)</th>
<th>$\eta$ (mm$^{-2}$)</th>
<th>$z_B$ (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 a</td>
<td>1</td>
<td>5.12</td>
<td>5.35</td>
<td>4.26</td>
<td>4.62</td>
<td>3.64</td>
<td>496.7</td>
<td>121.9</td>
<td>3455</td>
<td>—</td>
</tr>
<tr>
<td>1 a</td>
<td>2</td>
<td>5.64</td>
<td>5.61</td>
<td>4.46</td>
<td>4.78</td>
<td>3.79</td>
<td>498.9</td>
<td>121.4</td>
<td>3352</td>
<td>—</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>1.64</td>
<td>1.79</td>
<td>1.39</td>
<td>1.60</td>
<td>1.28</td>
<td>65.8</td>
<td>64.4</td>
<td>3983</td>
<td>10–15</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>2.02</td>
<td>1.99</td>
<td>1.54</td>
<td>1.97</td>
<td>1.61</td>
<td>92.7</td>
<td>81.5</td>
<td>3419</td>
<td>15–20</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>1.97</td>
<td>1.89</td>
<td>1.46</td>
<td>1.81</td>
<td>1.46</td>
<td>87.6</td>
<td>72.2</td>
<td>3208</td>
<td>20–25</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>1.91</td>
<td>1.92</td>
<td>1.49</td>
<td>1.89</td>
<td>1.55</td>
<td>72.0</td>
<td>72.7</td>
<td>3150</td>
<td>25–30</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>2.26</td>
<td>2.02</td>
<td>1.56</td>
<td>2.21</td>
<td>1.82</td>
<td>106.2</td>
<td>87.1</td>
<td>2765</td>
<td>30–35</td>
</tr>
<tr>
<td>2 a</td>
<td>4 a</td>
<td>2.63</td>
<td>2.95</td>
<td>2.38</td>
<td>2.69</td>
<td>2.15</td>
<td>33.6</td>
<td>11.5</td>
<td>697</td>
<td>—</td>
</tr>
<tr>
<td>2 a</td>
<td>5 a</td>
<td>2.52</td>
<td>2.93</td>
<td>2.36</td>
<td>2.68</td>
<td>2.13</td>
<td>33.9</td>
<td>11.8</td>
<td>842</td>
<td>—</td>
</tr>
</tbody>
</table>
I. Surface Topography Measurements
Acknowledgements

There are many people I like to thank for their contribution to this thesis.

The work has been initiated within the CAMST\textsuperscript{1} project at the University of Salford, Department of Physics. I have been particularly fortunate that I had the opportunity to spend four years with the Applied Magnetic Materials Group (AMM). I am especially indebted to prof. dr. P.J. Grundy who took me on in his group. He introduced me into the mysteries of magnetics and magnetic recording, and he gave me insight in the techniques for depositing thin films, and the excellent analysing techniques in which his group is specialised. Without his support and encouragements I could never have acquired the foundation that was necessary for the study of friction of thin film rigid disks. I am particularly grateful that he and prof. dr. P.G. Arnell of the Department of Aeronautical Engineering, at the University of Salford, made it possible to ship the NanoTest instrument from Salford to Twente, so that I could continue working with it, while a new instrument was built for the University of Twente. My thanks go also to his co-workers, especially dr. C. Faunce, who was very helpful with the analysis of the disk and slider materials. I can still hear his urgent warning ‘go out of harm’s way!’; to every one in the laboratory when I hurried in. I also thank dr. M. Toy, with whom I shared the office at Salford, and who was always prepared to explain some tacit parts of the English language that I did not understand.

My thanks go to dr. J.F. Smith, managing director at Micro Materials Ltd., and his staff for the useful information about the NanoTest instrument they have given me freely, and also for their collaboration with the construction of a modified pendulum for this instrument. We spent many hours discussing how to extend the instrument and how to make it useful for friction measurements. Especially, dr. S. Goodness and I had many useful conversations on how to write the software for the friction measurements.

I am very grateful to my promotor prof. ir. A.J.W. de Gee, who was present as a supervisor at the background. His fundamental comments and critics were like ‘chewing gum’; They, unexpectedly stuck to the surface. After a while, I usually realised that they were not merely about words, but about genuine problems, to be dealt with by discussion and rewriting. At that time, they usually disappeared, even unexpectedly as they came.

This thesis owes much to my mentor dr. ir. D.J. Schipper. His basic ideas of the transitions of lubrication regimes were also applied in this thesis for the lubrication of thin film rigid disks. From a pure mechanical engineering approach, he always tried to persuade me to take the shortest distance from A to B, in all kinds of decisions that had to be made. Now I have reached the end of the main path, I recognise that this was not an easy task; to convince someone who has a persistent sense of detail, along whatever (shortest) path he travels, must be arduously. Without going into detail, he also arranged that I could finish this thesis, which pleases me greatly.

\textsuperscript{1}CAMST: Concerted Action on Magnetic Storage Technology.
I thank the various people of the Tribology Group at the University of Twente. Particularly, I like to thank dr. ir. R.H.M. van der Stegen who kindly supplied the software and the numerical results for the gas film thickness computations. I also record my thanks to ing. E.G. de Vries for the many roughness measurements he carefully carried out on the specimens, and for our discussions about the modifications of the measurement techniques. I thank mr. L. Tiemersma of the machine shop for the perfect ‘tiny bits’ which only he could make for the NanoTest instrument.

I am very grateful to dr. ir. G.J. Burger, dr. ir. J.F. Burger and ing. E. Berenschot, at the time working in the MESA Research Institute, who employed their expertise in the field of micro machining by realising my ideas of a sensitive micro machined friction force sensor for the NanoTest instrument. Especially, dr. ir. J.F. Burger spent many hours in helping me with testing and preparing the sensor for the friction experiments.

My thanks are due to dr. C.F. Baumberger, at the time consultant for Applied Magnetics in Paris, France, and dr. D. Lea of Applied Magnetics in Ireland who provided all the sliders used in this study. I also wish to acknowledge a person whom I have never met: dr. Nader of Nashua Corporation in Santa Clara, California who supplied rigid disks. I gratefully acknowledge the supply of the special glass disks by dr. D. Hemingway at Pilkington, Special Glass Division, in Wales.

To my partner, dr. Cisca Joldersma I owe more words than can be described. Let me attempt to do so by saying that she not only encouraged me continuously, but also applied all her strength, talents and means to support me. With her strong analytical powers, she was able to discuss and criticise my work in a positive way, even when we were separated by hundredths of miles. Everywhere, she showed me that difficulties can be overcome by regarding them as opportunities instead of mountains that cannot be taken. She also learned me to concentrate on efforts that result in large instead of small effects.

Above all I want to express my thanks to my father and mother. I am sure that their loving care, believes and interest went far beyond the normal conversations. Finally, I thank my grandmother, who showed me her fascinating perseverance and common sense. I wish that both of these can also be found in this thesis.

Remains me to say the following reassuring words to those readers of this thesis who might immediately run off to their computer and become worried about the ‘wear and tear’ of their rigid disk drives: commercially available computer rigid disk drives are working reasonably well, even after many years.

Harm Visscher
Enschede, March 2001
About the Author

Harm Visscher was born in Dedemsvaart (Avereest), The Netherlands, on 9 March 1960. From his twelfth year onwards he was educated in the field of mechanical engineering. First he went to the Lagere Technische School at Hardenberg, where he received a diploma in 1976. After completing a three-year program at the Middelbare Technische School at Hoogeveen, he went to the Hogere Technische School at Zwolle. After two years at this college of technology he was admitted, by means of a law ‘Wet Wederzijdse Doorstroming’, to the University of Twente. During his Final Year project in the Tribology Group, he studied the influence of laser hardening on the wear resistance of steel surfaces. He received an Msc in mechanical engineering from the University of Twente in 1988. On graduation he served the armed forces in 1988 and 1989, where he developed software.

Early 1990 he began to work as a Research Assistant at the University of Salford, Department of Physics (Applied Magnetic Materials Group), Greater Manchester, England. Here, he worked within the European CAMST project on the friction and wear of magnetic recording media. His research emphasis was on the newly developed nano-indentation hardness instrument in close collaboration with Micro Materials Ltd. (Wales). He later continued the work as a Visiting Fellow on the friction of the head-disk interface, from where his Ph.D. study began within the Tribology Group at the University of Twente. In close collaboration with Micro Materials Ltd., he analysed and developed the instrument further, in order to carry out friction measurements on rigid disks. In addition, with the Micro Electronics and Materials Engineering (MESA) Research Institute at the University of Twente, he designed and tested a novel micro sensor for this instrument.

\[\text{CAMST: Concerted Action on Magnetic Storage Technology.}\]