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Internal stabilization and external L_p stabilization of linear systems subject to constraints

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1 Introduction

During the last decade stabilization and other control design problems for linear systems subject to constraints, especially those with actuator saturation, have received much attention. Whenever the magnitude of the control input is bounded, internal stabilization in either global or semiglobal sense is possible if and only if the open-loop system is asymptotically null controllable with bounded control (i.e. if and only if the open-loop system is stabilizable and all its poles are in the closed left-half plane). During the 1990's, control design problems for linear systems with actuator saturation were mostly studied in the framework of semiglobal and global stabilization and hence attentions were focused only on asymptotically null controllable systems.

Besides internal stabilization, another control design problem of interest is external stabilization or the requirement of L_p stability, i.e. the requirement of the controlled output being in L_p whenever the external signals are in L_p and the initial conditions are zero. However, in many cases internal stabilization by itself does not automatically guarantee external stabilization. One has to design a new controller for simultaneous internal and external stabilization. A number of simultaneous internal and external stabilization problems in either global or semiglobal or regional sense have been formulated and studied (see [2, 5] and the references cited there). A standard topology in all these works pertains to the case where the external signal w is input additive. For such a configuration, the study of different types of simultaneous internal and external stabilization problems with or without finite gain (finite gain implies that the induced norm of the mapping from the external input to the controlled output is finite) is complete [2, 5] when state feedback controllers are used, and leads to the following result:

> For asymptotically null controllable systems and for input additive configuration, as depicted in Figure 1, simultaneous internal and external L_p stabilization in either global or semiglobal sense for $1 \leqslant p \leqslant \infty$ can be achieved, and moreover the induced norm of the mapping from external signal w to the controlled output or state x can be rendered arbitrarily small (i.e. almost disturbance rejection where the external signal is viewed as a disturbance). Furthermore, in order to design appropriate state feedback controllers that achieve such results, one can utilize methodologies involving various low-gain, low-high gain, scheduled low-high gain designs.

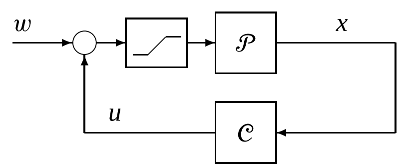

Figure 1

On the other hand, whenever the external signal is not input additive, as depicted in Figure 2 (non-input additive configuration), simultaneous internal and external stabilization is profoundly different from the input-additive situation and more complicated. As one can see, in the case of input-additive configuration, it is quite easy to reduce the influence of the external signal on the controlled output, compared to the case of disturbances which are not input-additive. As the available results show, the controller can make use of its full capacity to counteract the external signal if it is input-additive. Even more, the state trajectory starting from the origin can be controlled in some compact invariant set for any arbitrary disturbance in some functional space, say L_p space. However, for the non-input-additive case, the control capability is clearly limited by the magnitude constraint on the input. This naturally leads to some performance deficiency that can never be overcome by whatever control laws one can use. For example, in general it becomes impossible to keep the state trajectroy starting from the origin to be inside some

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compact invariant set for any arbitrary L_p disturbance unless some restrictive conditions are made. A recent paper [8] makes some pioneering contribution to the external behavior with non-input-additive disturbance:

> Even for asymptotically null controllable systems, whenever the external signal w is non-input additive, (see Figure 2), simultaneous internal and external L_p stabilization with a finite gain in a global sense for $1 \leqslant p \leqslant \infty$ is in general not possible, in particular when the signal w can excite the unstable dynamics of the plant. However, one can achieve simultaneous internal and external L_p stabilization without finite gain in a global sense. Moreover, simultaneous internal and external L_p stabilization with finite gain in a semiglobal sense can be achieved for any $p \in [1, 2]$.

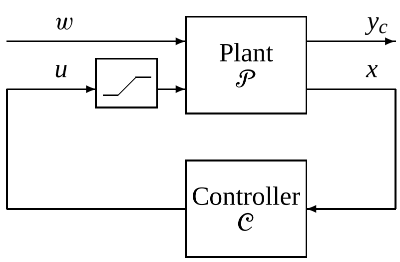

Figure 2

All the existing literature including [2, 5, 8] addresses linear systems with magnitude constraints *only* on input signals. Having studied during the last decade several aspects of several control design problems for linear systems subject to magnitude and rate constraints on control variables, during the last two years the research thrust of the authors and their students has broadened to include magnitude constraints on control variables as well as state variables. Recent work [1, 3, 4] considered linear systems in a general framework for constraints including both input magnitude constraints as well as state magnitude constraints. In particular, [3, 4] consider internal stabilization while [1] considers output regulation in different frameworks, namely a global, semiglobal, and regional framework.

2 Preliminaries

Consider the linear system Σ,

$$\begin{aligned} \dot{x} &= Ax + Bu + Ew \\ z &= C_z x + D_z u \\ y &= Cx + Du \end{aligned} \tag{2.1}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the control input, $w \in \mathbb{R}^s$ is the external signal, $y \in \mathbb{R}^p$ is the controlled output, and $z \in \mathbb{R}^\ell$ is the constraint output subjected to the constraint $z(t) \in \mathcal{S}$ for all $t \geqslant 0$.

Based on the system model (2.1), the goal of this paper is to establish solvability conditions and develop locally Lipschitz control laws so that any external signal in L_p space or in a subset of the L_p space produces an controlled output y in L_p, meanwhile, the constraints are not violated and internal semiglobal or global stabilization is achieved if the external signal is zero. Furthermore, we may impose that the resulting input/output mapping from w to y has a finite induced L_p norm, so called *finite gain*. In general, this latter requirement yields more restrictive solvability conditions compared to the case without a finite L_p gain.

We make a general assumption on the constrained output equation:

Assumption 2.1 *The set $\mathcal{S}$ is bounded, convex and contains 0 as an interior point. Moreover, we assume $C_z^T D_z = 0$ and*

$$\mathcal{S} = (\mathcal{S} \cap \operatorname{im} C_z) + (\mathcal{S} \cap \operatorname{im} D_z) \tag{2.2}$$

This assumption is not restrictive. In fact, it is a general reflection of the separability of input constraints and state constraints.

Definition 2.2 (Admissible set of initial conditions) *Let the system (2.1) and the constraint set $\mathcal{S}$ be given. We define*

$$\mathcal{X}(\mathcal{S}) := \left\{ x_0 \in \mathbb{R}^p \mid C_z x_0 \in \mathcal{S} \right\} \tag{2.3}$$

as the admissible set of initial conditions.

Remark. Note that the set $\mathcal{X}(\mathcal{S})$ could also have been defined as

$$\mathcal{X}(\mathcal{S}) := \left\{ x_0 \in \mathbb{R}^p \mid \exists\, u_0 \text{ such that } C_z x_0 + D_z u_0 \in \mathcal{S} \right\}.$$

Because of assumption 2.1, the two definitions turn out to be equivalent.

3 Taxonomy of Constraints

We recall in this section in detail the *taxonomy* of constraints developed earlier in [3]. Such a taxonomy is a consequence of the structural properties of the subsystem Σ_{zu} characterized by (A, B, C_z, D_z).

The first categorization is based on whether Σ_{zu} is right invertible or not. We have the following definition regarding the first categorization of constraints which is based on whether Σ_{zu} is right invertible or not.

Definition 3.1 *The constraints are said to be*

- right invertible constraints *if Σ_{zu} is right invertible.*
- non-right invertible constraints *if Σ_{zu} is non-right invertible.*

The second categorization is based on the location of the invariant zeros of Σ_{zu}). Because of their importance, we specifically label the invariant zeros of Σ_{zu} as the *constraint invariant zeros* of the plant.

Definition 3.2 (Constraint invariant zeros) *The invariant zeros of Σ_{zu} are called the constraint invariant zeros of the plant associated with the constrained output z.*

We have the following definition regarding the second categorization of constraints. Let $\mathbb{C}$, $\mathbb{C}^+$, $\mathbb{C}^-$ and $\mathbb{C}^0$ denote respectively the entire complex plane, the open right-half complex plane, the open left-half complex plane, and the imaginary axis. Then, the constraints are said to be

- *minimum phase constraints* if all the constraint invariant zeros are in $\mathbb{C}^-$.
- *weakly minimum phase constraints* if all the constraint invariant zeros are in $\mathbb{C}^- \cup \mathbb{C}^0$ with the restriction that at least one such constraint invariant zero is in $\mathbb{C}^0$ and any such constraint invariant zero in $\mathbb{C}^0$ is simple.
- *weakly non-minimum phase constraints* if all the constraint invariant zeros are in $\mathbb{C}^- \cup \mathbb{C}^0$ and at least one constraint invariant zero in $\mathbb{C}^0$ is not simple.
- *at most weakly non-minimum phase constraints* if all the constraint invariant zeros are in $\mathbb{C}^- \cup \mathbb{C}^0$.
- *strongly non-minimum phase constraints* if one or more of the constraint invariant zeros are in $\mathbb{C}^+$.

The third categorization is based on the order of the infinite zeros of Σ_{zu}. See [7] for a definition of infinite zeros of a system. Because of their importance, we specifically label the infinite zeros of Σ_{zu} as the *constraint infinite zeros* of the plant.

Definition 3.3 (Constraint infinite zeros) *The infinite zeros of Σ_{zu} are called the constraint infinite zeros of the plant associated with the constrained output z.*

We have the following definition regarding the third categorization of constraints.

Definition 3.4 *The constraints are said to be*

- type 1 constraints *if the order of all constraint infinite zeros is less than or equal to one*

As we said in introduction, the above taxonomy of constraints plays a critical role in connection with internal stabilization. As we shall see, the new notions of *external constraint invariant zeros* and *external constraint infinite zeros* to be introduced later on will further broaden the taxonomy of constraints discussed above.

4 Statement of problems

In this section, we formulate clearly the problems considered in this paper. We denote $L_p(D) := \{w \in L_p : \|w\|_{L_p} \leqslant D\}$ for any $D > 0$.

Problem 4.1 Consider the system (2.1) along with the constraint set $\mathcal{S}$. The problem of **global internal stabilization and global external L_p stabilization**, i.e. the (G_i/G_e) problem is to find a possibly nonlinear and dynamic feedback law $u = \mathcal{F}(x)$ such that the following properties hold:

(i) In the absence of external input w, the equilibrium point $x = 0$ of the closed-loop system is globally asymptotically stable without violating the constraint, that is, the region of attraction is the admissible set $\mathcal{X}(\mathcal{S})$ and, for any initial condition in $\mathcal{X}(\mathcal{S})$, we have $z(t) \in \mathcal{S}$ for all $t \geqslant 0$.

(ii) For any $w \in L_p$ and $x(0) = 0$, we have $y \in L_p$ and $z(t) \in \mathcal{S}$ for all $t \geqslant 0$.

If in addition to items (i) and (ii),

(iii) There exists a $\gamma > 0$ such that for any $w \in L_p$ and $x(0) = 0$, $\|y\|_{L_p} \leqslant \gamma\|w\|_{L_p}$ and $z(t) \in \mathcal{S}$ for all $t \geqslant 0$,

then the problem is said to be (G_i/G_e) with *finite gain* and is labeled as $(G_i/G_e)_{\text{fg}}$.

Problem 4.2 Consider the system (2.1) along with the constraint set $\mathcal{S}$. The problem of **global internal stabilization and semiglobal external L_p stabilization**, i.e. the (G_i/SG_e) problem is for any $D > 0$ to find a possibly nonlinear and dynamic feedback law $u = \mathcal{F}(x)$ such that the following properties hold:

(i) In the absence of external input w, the equilibrium point $x = 0$ of the closed-loop system is globally asymptotically stable without violating the constraint, that is, the region of attraction is the admissible set $\mathcal{X}(\mathcal{S})$ and, for any initial condition in $\mathcal{X}(\mathcal{S})$, we have $z(t) \in \mathcal{S}$ for all $t \geqslant 0$.

(ii) For any $w \in L_p(D)$ and $x(0) = 0$, we have $y \in L_p$ and $z(t) \in \mathcal{S}$ for all $t \geqslant 0$.

If in addition to items (i) and (ii),

(iii) There exists a $\gamma > 0$ such that for any $w \in L_p(D)$ and $x(0) = 0$, $\|y\|_{L_p} \leqslant \gamma\|w\|_{L_p}$ and $z(t) \in \mathcal{S}$ for all $t \geqslant 0$,

then the problem is said to be (G_i/SG_e) with *finite gain* and is labeled as $(G_i/SG_e)_{\text{fg}}$.

Problem 4.3 Consider the system (2.1) along with the constraint set $\mathcal{S}$. The problem of **semiglobal internal stabilization and global external L_p stabilization**, i.e. the (SG_i/G_e) problem, is for any given compact set $\mathcal{K}_1$ contained in the interior of $\mathcal{X}(\mathcal{S})$ to find a possibly nonlinear and dynamic feedback law $u = \mathcal{F}(x)$ such that the following properties hold:

(*i*) In the absence of external input w, the equilibrium point $x = 0$ of the closed-loop system is asymptotically stable, with the region of attraction containing $\mathcal{K}_1$ and, for any initial condition in $\mathcal{K}_1$, we have $z(t) \in \mathcal{S}$ for all $t \geqslant 0$.

(*ii*) For any $w \in L_p$ and $x(0) = 0$, we have $y \in L_p$ and $z(t) \in \mathcal{S}$ for all $t \geqslant 0$.

(*iii*) For $x(0) = 0$ and any $w \in L_p$ for which there exists $T > 0$ such that $w(t) = 0$ for $t > T$ we have that $x(t) \to 0$ as $t \to \infty$.

If in addition to items (*i*) and (*ii*),

(*iii*) There exists a $\gamma > 0$ such that for any $w \in L_p$ and $x(0) = 0$, $\|y\|_{L_p} \leqslant \gamma \|w\|_{L_p}$ and $z(t) \in \mathcal{S}$ for all $t \geqslant 0$,

then the problem is said to be (SG_i/G_e) with *finite gain* and is labeled as $(SG_i/G_e)_{\text{fg}}$.

Problem 4.4 Consider the system (2.1) along with the constraint set $\mathcal{S}$. The problem of **semiglobal internal stabilization and semiglobal external L_p stabilization**, i.e. the (SG_i/SG_e) problem, is for any given compact set $\mathcal{K}_1$ contained in the interior of $\mathcal{X}(\mathcal{S})$ and for any $D > 0$ to find a possibly nonlinear and dynamic feedback law $u = \mathcal{F}(x)$ such that the following properties hold:

(*i*) In the absence of external input w, the equilibrium point $x = 0$ of the closed-loop system is asymptotically stable, with the region of attraction containing $\mathcal{K}_1$ and, for any initial condition in $\mathcal{K}_1$, $z(t) \in \mathcal{S}$ for all $t \geqslant 0$.

(*ii*) For any $w \in L_p(D)$ and $x(0) = 0$, we have $y \in L_p$ and $z(t) \in \mathcal{S}$ for all $t \geqslant 0$.

(*iii*) For $x(0) = 0$ and any $w \in L_p$ for which there exists $T > 0$ such that $w(t) = 0$ for $t > T$ we have that $x(t) \to 0$ as $t \to \infty$.

If in addition to items (*i*) and (*ii*),

(*i*) There exists a $\gamma > 0$ such that for any $w \in L_p(D)$ and $x(0) = 0$, $\|y\|_{L_p} \leqslant \gamma \|w\|_{L_p}$, and $z(t) \in \mathcal{S}$ for all $t \geqslant 0$,

then the problem is said to be (SG_i/SG_e) with *finite gain* and is labeled as $(SG_i/SG_e)_{\text{fg}}$.

5 Main results

Our development here focuses on right invertible constraints, and moreover relies largely on the structure of the underlying system. For this reason, the original system Σ, given by (2.1) needs to be rewritten in a special coordinate basis so that the system properties involving invariant zeros and infinite zeros are revealed naturally. This will greatly facilitate the design of appropriate controllers. A detailed special coordinate basis (scb) is presented in [6, 7]. Utilizing the scb format for Σ_{zu} that is characterized by (A, B, C_z, D_z), and noting the fact that Σ_{zu} is right invertible, one can show that there exist a state transformation Γ_s, an input transformation Γ_i, and a prefeedback law: $\tilde{x}^{\mathrm{T}} = (x_a^{\mathrm{T}}, x_c^{\mathrm{T}}, x_f^{\mathrm{T}})$, $\tilde{u} = \Gamma_i u$, $\tilde{u} = -F\tilde{x} + v$, and $v^{\mathrm{T}} = (v_0^{\mathrm{T}}, v_c^{\mathrm{T}}, v_f^{\mathrm{T}})$ such that the original system Σ given by (2.1) can be rewritten as

$$\begin{aligned}
\begin{pmatrix}\dot{x}_a\\ \dot{x}_c\\ \dot{x}_f\end{pmatrix} &= \begin{pmatrix}A_{aa} & 0 & 0\\ 0 & A_{cc} & 0\\ 0 & 0 & A_{ff}\end{pmatrix}\begin{pmatrix}x_a\\ x_c\\ x_f\end{pmatrix} + \begin{pmatrix}0 & 0 & 0\\ 0 & B_c & 0\\ 0 & 0 & B_f\end{pmatrix}\begin{pmatrix}v_0\\ v_c\\ v_f\end{pmatrix}\\
&\qquad + \begin{pmatrix}K_1\\ K_2\\ K_3\end{pmatrix} z + \begin{pmatrix}E_1\\ E_2\\ E_3\end{pmatrix} w\\
z &= \begin{pmatrix}z_0\\ z_f\end{pmatrix} = \begin{pmatrix}0 & 0 & 0\\ 0 & 0 & C_f\end{pmatrix}\begin{pmatrix}x_a\\ x_c\\ x_f\end{pmatrix} + \begin{pmatrix}I & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}v_0\\ v_c\\ v_f\end{pmatrix}\\
y &= \begin{pmatrix}C_1 & C_2 & C_3\end{pmatrix}\begin{pmatrix}x_a\\ x_c\\ x_f\end{pmatrix} + \begin{pmatrix}D_1 & D_2 & D_3\end{pmatrix}\begin{pmatrix}v_0\\ v_c\\ v_f\end{pmatrix}
\end{aligned} \tag{5.1}$$

Furthermore, from the properties of scb given in [6,7], we can deduce the following properties:

- The subsystem characterized by the quadruple $(A_{ff}, B_f, C_f, 0)$ has no finite invariant zeros and is invertible.
- The constraint invariant zeros are equal to the eigenvalues of the pair A_{aa}.
- The dynamics exhibited by x_a and x_c is the zero dynamics of the system with respect to the constrained output z in the absence of external signal w.

Based on system (5.1) we introduce several new concepts that are related to the structural properties for external stabilization. We will first need to introduce the zero dynamics if we impose the constraint $z = 0$. From [6,7] we know there exists a state feedback F_f and matrices $A_0, \ldots, A_r$ such that if we choose:

$$v_f = F_f x_f + \sum_{i=0}^{r} A_i w^{(i)}, \qquad v_0 = 0$$

where $w^{(i)}$ denotes the i'th derivative of w, then we have $z = 0$ when $x_f(0) = 0$ for all disturbances w and all inputs v_c. We obtain the following zero dynamics:

$$\begin{aligned}\begin{pmatrix}\dot{x}_a\\ \dot{x}_c\end{pmatrix} &= \begin{pmatrix}A_{aa} & 0\\ 0 & A_{cc}\end{pmatrix}\begin{pmatrix}x_a\\ x_c\end{pmatrix} + \begin{pmatrix}0\\ B_c\end{pmatrix} v_c + \begin{pmatrix}E_1\\ E_2\end{pmatrix} w\\ y &= \begin{pmatrix}C_1 & C_2\end{pmatrix}\begin{pmatrix}x_a\\ x_c\end{pmatrix} + D_2 v_c + \sum_{i=0}^{r} D_3 A_i w^{(i)}\end{aligned} \tag{5.2}$$

5.1 More on the Taxonomy of Constraints

We identify below a subset of the constraint invariant zeros which are labeled as *external constraint invariant zeros* as defined below:

Definition 5.1 *Given the system* (2.1) *we can construct the zero dynamics as in* (5.2)*. The poles of the transfer matrix from w to y which cannot be influenced by v_c are called the* ***external constraint invariant zeros.***

We have the following definition regarding the fourth categorization of constraints.

Definition 5.2 *The constraints are said to be*

- externally minimum phase constraints *if the external constraint invariant zeros are in $\mathbb{C}^-$.*

We present the solvability conditions for the four problems defined in Section 4 in the following four subsections.

5.2 The (G_i/G_e) and $(G_i/G_e)_{\text{fg}}$ problems

Theorem 5.3 *Consider the system* (2.1) *and the constraint set $\mathcal{S}$ that satisfies Assumption 2.1. Let the constraints be right-invertible, and the system* (2.1) *be stabilizable (i.e. the pair (A, B) is stabilizable). We decompose the system according to* (5.1)*. Then the problem of global internal stabilization and global external L_p stabilization* without finite-gain *(i.e. the (G_i/G_e) problem) is solvable for any $p \in [1, \infty)$ via a feedback $u = \mathcal{F}(x)$ if and only if the following conditions hold:*

(*i*) $E_3 = 0$.

(*ii*) *The constraints are at most weakly non-minimum phase.*

(*iii*) *The constraints are of type 1.*

Remark. The above conditions are still necessary but obviously not sufficient when $p = \infty$.

Theorem 5.4 *Consider the system* (2.1) *and the constraint set $\mathcal{S}$ that satisfies Assumption 2.1. Let the constraints be right-invertible, and the system* (2.1) *be stabilizable (i.e. the pair (A, B) is stabilizable). We decompose the system according to* (5.1)*. Then the problem of global internal stabilization and global external L_p stabilization* with finite gain *(i.e. the $(G_i/G_e)_{fg}$ problem) is solvable via a feedback $u = \mathcal{F}(x)$ for any $p \in [1, \infty]$ only if the following conditions hold:*

(*i*) $E_3 = 0$.

(*ii*) *The constraints are at most weakly non-minimum phase.*

(*iii*) *The constraints are of externally minimum phase.*

(*iv*) *The constraints are of type 1.*

Remark. It is easily shown that the above conditions are not sufficient for $p = \infty$. However, it is an open problem whether these conditions are sufficient for $p \in [1, \infty)$.

5.3 The (G_i/SG_e) and $(G_i/SG_e)_{\text{fg}}$ problems

Theorem 5.5 *Consider the system* (2.1) *and the constraint set $\mathcal{S}$ that satisfies Assumption 2.1. Let the constraints be right-invertible, and the system* (2.1) *be stabilizable (i.e. the pair (A, B) is stabilizable). Then the problem of global internal stabilization and semiglobal external L_p stabilization* without finite-gain *(i.e. the (G_i/SG_e) problem) is solvable for any $p \in [1, \infty)$ via a feedback $u = \mathcal{F}(x)$ if and only if the following conditions hold:*

(*i*) *The constraint are at most weakly non-minimum phase.*

(*ii*) *The constraints are of type 1.*

(*iii*) $E_3 = 0$ *when* $p = 1$.

Remark. For $p = \infty$ the above conditions are still necessary but no longer sufficient for solvability.

Theorem 5.6 *Consider the system* (2.1) *and the constraint set $\mathcal{S}$ that satisfies Assumption 2.1. Let the constraints be right-invertible, and the system* (2.1) *be stabilizable (i.e. the pair (A, B) is stabilizable). Then the problem of global internal stabilization and semiglobal external L_p stabilization* with finite gain *(i.e. the $(G_i/SG_e)_{fg}$ problem) is solvable for any $p \in [1, 2]$ via a feedback $u = \mathcal{F}(x)$ if and only if the following conditions hold:*

(*i*) *The constraints are at most weakly non-minimum phase.*

(*ii*) *The constraints are of type 1.*

(*iii*) $E_3 = 0$ *when* $p = 1$

Remark. For $p > 2$ the above conditions are still necessary for solvability of the problem of global internal stabilization and semiglobal external L_p stabilization with finite gain. For $p = \infty$ the conditions are not sufficient but it is not clear whether for $p \in (2, \infty)$ the conditions are sufficient.

5.4 The (SG_i/G_e) and $(SG_i/G_e)_{\text{fg}}$ problems

Theorem 5.7 *Consider the system* (2.1) *and the constraint set $\mathcal{S}$ that satisfies Assumption 2.1. Let the constraints be right-invertible, and the system* (2.1) *be stabilizable (i.e. the pair* (A, B) *is stabilizable). Then the problem of semi-global internal stabilization and global external L_p stabilization* without finite-gain *(i.e. the* (SG_i/G_e) *problem) is solvable for any* $p \in [1, \infty)$ *via a feedback* $u = \mathcal{F}(x)$ *via a controller which is globally defined if and only if*

(*i*) $E_3 = 0$.

(*ii*) *The constraints are at most weakly non-minimum phase.*

Remark. Again, for $p = \infty$ the above condition is still necessary but no longer sufficient for solvability.

Theorem 5.8 *Consider the system* (2.1) *and the constraint set $\mathcal{S}$ that satisfies Assumption 2.1. Let the constraints be right-invertible, and the system* (2.1) *be stabilizable (i.e. the pair* (A, B) *is stabilizable). Then the problem of semiglobal internal stabilization and global external L_p stabilization* with finite gain *(i.e. the* $(SG_i/G_e)_{fg}$ *problem) is solvable for any* $p \in [1, \infty]$ *via a feedback* $u = \mathcal{F}(x)$ *only if the following conditions hold:*

(*i*) $E_3 = 0$.

(*ii*) *The constraints are at most weakly non-minimum phase.*

(*iii*) *The constraints are of externally minimum phase.*

5.5 The (SG_i/SG_e) and $(SG_i/SG_e)_{\text{fg}}$ problems

Theorem 5.9 *Consider the system* (2.1) *and the constraint set $\mathcal{S}$ that satisfies Assumption 2.1. Let the constraints be right-invertible, and the system* (2.1) *be stabilizable (i.e. the pair* (A, B) *is stabilizable). Then the problem of semi-global internal stabilization and semi-global external L_p stabilization* without finite-gain *(i.e. the* (SG_i/SG_e) *problem) is solvable for any* $p \in [1, \infty)$ *via a feedback* $u = \mathcal{F}(x)$ *if and only if*

- *The constraints are at most weakly non-minimum-phase,*
- $E_3 = 0$ *when* $p = 1$

Remark. Again, for $p = \infty$ the above condition is still necessary but no longer sufficient for solvability.

Theorem 5.10 *Consider the system* (2.1) *and the constraint set $\mathcal{S}$ that satisfies Assumption 2.1. Let the constraints be right-invertible, and the system* (2.1) *be stabilizable (i.e. the pair* (A, B) *is stabilizable). Then the problem of semi-global internal stabilization and semi-global external L_p stabilization* with finite-gain *(i.e. the* $(SG_i/SG_e)_{fg}$ *is solvable for any* $p \in [1, 2]$ *via a feedback* $u = \mathcal{F}(x)$ *if and only if*

- *The constraints are at most weakly non-minimum-phase,*
- $E_3 = 0$ *when* $p = 1$

Remark. Note that the problem with finite gain has the same solvability condition as the problem without finite gain. However, with the requirement of finite gain, the problem for $p \in (2, \infty)$ remains a major open problem. For $p = \infty$ the conditions are obviously not sufficient.

6 Conclusion

It is clear that these problems as defined in this paper require very strong solvability conditions. Therefore a main focus for future research should focus on finding a controller with a large domain of attraction and some good rejection properties for disturbances restricted to some bounded set. Even semiglobal external stabilization is in many cases simply to much to ask for.

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