Influence of poloidal flow on TAE modes

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Abstract

Toroidal flow has been shown [1] to significantly influence the continuous MHD spectrum of tokamaks by the creation of new Toroidal Flow-induced Alfvén Eigenmodes (TFAE's). Unlike Toroidal Alfvén Eigenmodes (TAE's) of static plasmas, these modes occur at low frequencies so that they also influence stability, through the occurrence of centrifugal and Coriolis effects. The investigation of the effects of poloidal flows on the TAE's is the subject of this paper.

We have completed the construction of two state-of-the-art numerical programs for the computation of the stationary equilibria (called FINESSE) and of the spectrum and eigenfunctions of the waves and instabilities (called PHOENIX). FINESSE computes the equilibrium of an axisymmetric plasma with both toroidal and poloidal flows in three different flow regimes (sub-slow, sub-Alfvénic, and super-Alfvénic) by solving the coupled system of flux equation and Bernoulli equation by means of bicubic Hermite finite elements. PHOENIX computes the full ideal and resistive spectrum for these equilibria by means of the new parallel Jacobi-Davidson algorithm. This permits us to zoom in onto a target eigenvalue and then produce all eigenvalues in a wide neighborhood of it, with unprecedented accuracy.

We here present the first results (ever) on the effects of poloidal flows on TAE modes obtained by means of these new tools.

1 Introduction

The ideal continuous spectrum of static tokamak plasmas has Alfvén gaps where global toroidal Alfvén eigenmodes (TAE's) may appear. The concept of static plasma is unrealistic for modern tokamaks where neutral beam heating and divertor pumping are used. Flow can seriously influence the global dynamics of tokamaks. As shown in [1], toroidal flow enlarges the Alfvén gaps of the ideal static MHD spectra and induces new global waves inside the gaps. These toroidal-flow-induced Alfvén eigenmodes (TFAE's) are—in contrast to the TAE's—in the low-frequency range and thus important for stability. In this paper, we consider the effects of poloidal flow on the TAE's.

2 The model and tools

Our initial equilibrium model is determined by the stationary ideal MHD equations

$$
\begin{aligned}
&\nabla \cdot (\rho \mathbf{v}) = 0, \\
&\rho \nabla \left(\frac{1}{2} v^2 + \frac{\gamma}{\gamma - 1} \rho^{\gamma - 1} S \right) - \frac{1}{\gamma - 1} \rho^{\gamma} \nabla S - \rho \mathbf{v} \times \boldsymbol{\omega} - \mathbf{J} \times \mathbf{B} = 0, \\
&\mathbf{v} \cdot \nabla S = 0, \\
&\nabla \times (\mathbf{v} \times \mathbf{B}) = 0, \\
&\nabla \cdot \mathbf{B} = 0,
\end{aligned}
\tag{1}
$$

where the density ρ, velocity $\mathbf{v}$, entropy S, and magnetic field $\mathbf{B}$ are the unknowns. Here $\boldsymbol{\omega} \equiv \nabla \times \mathbf{v}$ is the vorticity, and $\mathbf{J} \equiv \nabla \times \mathbf{B}$ is the electric current density ($\mu_0 = 1$ in the normalization used). The pressure p is related to S via $p \equiv S\rho^{\gamma}$, where γ is the ratio of specific heats of the plasma.

Using a cylindrical coordinate system (R, Z, φ) and axisymmetry $(\partial/\partial\varphi \equiv 0)$, and introducing the poloidal magnetic flux function ψ and the poloidal stream function χ, one can show [2, 3] that the system (1) can be reduced to a system of equations in two unknowns: ψ and the squared poloidal Mach number $M^2 \equiv \rho v_p^2/B_p^2$, whereas the other unknowns can be expressed via five functions of ψ that can be chosen arbitrarily. One of the equations of the new reduced system appears to be a nonlinear PDE in ψ for known M^2. The other equation is a nonlinear algebraic equation in M^2 for known ψ. The reduced system is efficiently and accurately solved with a special iterative process in a code called FINESSE (FINite Element Solver for Stationary Equilibria) which provides plasma equilibria with flow.

The accuracy of the computed equilibria is necessary to permit a stability analysis. It is done with the help of another new code called PHOENIX. In it, the linearized ideal or resistive MHD equations are discretized by quadratic or cubic Hermite finite elements in the radial direction and by a Fourier spectral method in the poloidal direction. This leads to a large-scale generalized eigenvalue problem

$$
Ax = \lambda Bx, \qquad \lambda \equiv -i\omega,
$$

that is solved by the recently developed iterative Jacobi-Davidson method [4]. This method is of the Krylov subspace type. It allows to find eigenvalues in a region of

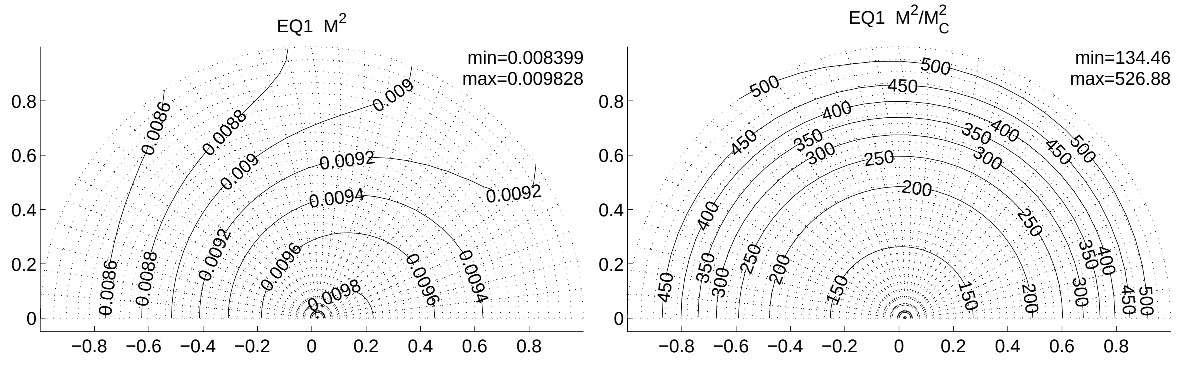

Figure 1: Top half of a tokamak cross-section with the distribution of the squared poloidal Alfvén Mach number, M^2, and the ratio M^2/M_c^2.

interest and the corresponding eigenvectors fast and with unprecedented accuracy. The implementation of the Jacobi-Davidson method for PHOENIX is parallel.

3 Results

We present here spectral results for a tokamak equilibrium delivered by FINESSE in the slow flow region (see [2, 3]). The tokamak has a circular cross-section and inverse aspect ratio $\epsilon = 0.05$.

In Fig. 1, the squared poloidal Alfvén Mach number M^2 together with the ratio M^2/M_c^2 are shown, where $M_c^2 \equiv \gamma p/(\gamma p + B^2)$. The plots are superposed on a straight field line coordinate grid (ψ, ϑ) where ϑ is a poloidal angle constructed in such a way that the equilibrium magnetic field is straight on each magnetic surface: $B^\vartheta/B^\varphi = q(\psi)$, where q denotes the safety factor. In Fig. 1, we clearly see that M^2 is a strongly varying function of ϑ in the slow flow domain. Note that $M_c^2 < M^2 < 1$. This is characteristic for the slow flow region where the poloidal Alfvén Mach number has exceeded the critical value M_c.

A part of the ideal continuous spectrum for this equilibrium as computed by the PHOENIX code is plotted in Fig. 2. A gap in the $m = 1, 2$ Alfvén branches occurs at $s \approx 0.82$. The resistive spectrum at the position of this gap is shown in Fig. 3a. As can be seen in Fig. 4a, the eigenvalue A of Fig. 3a corresponds to a typical resistive wave with a rapidly oscillating eigenfunction achieving maximum amplitude around $s \approx 0.15$. On the other hand, for vanishing resistivity, a new eigenvalue (indicated by B in Fig. 3b) appears in the gap of the continuous spectrum. It is a new global Alfvén eigenmode driven by the poloidal flow with global $m = 1, 2$ character (Fig. 4b).

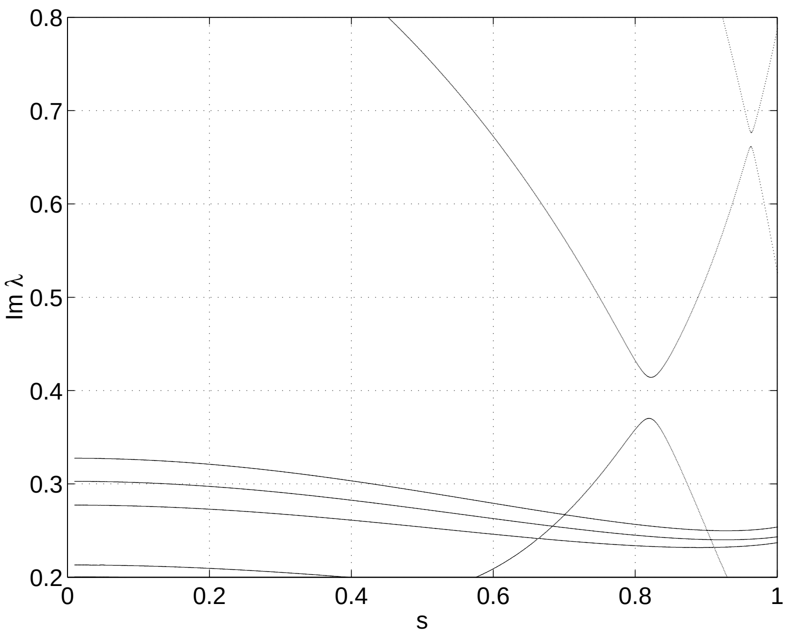

Figure 2: Gap in the continuous spectrum of the tokamak equilibrium plotted against $s \equiv \sqrt{\psi}$.

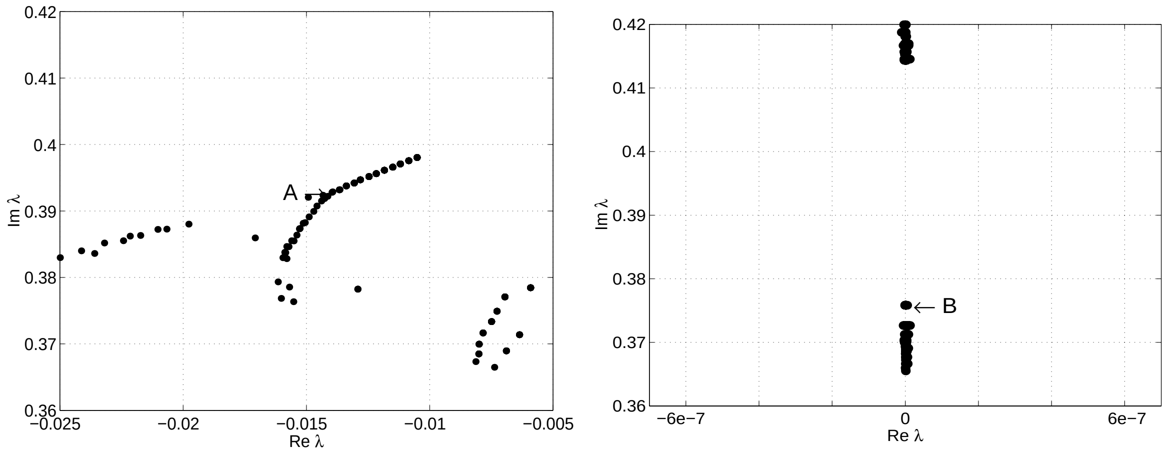

Figure 3: Resistive spectrum in the gap for resistivity $\eta = 10^{-5}$ (left plot) and ideal continuous spectrum with discrete eigenmodes (right plot). For the indicated eigenvalues A and B, the eigenfunctions are presented in Fig. 4. The new ideal global mode B in the gap of the continuous spectrum is driven by transonic poloidal flows.

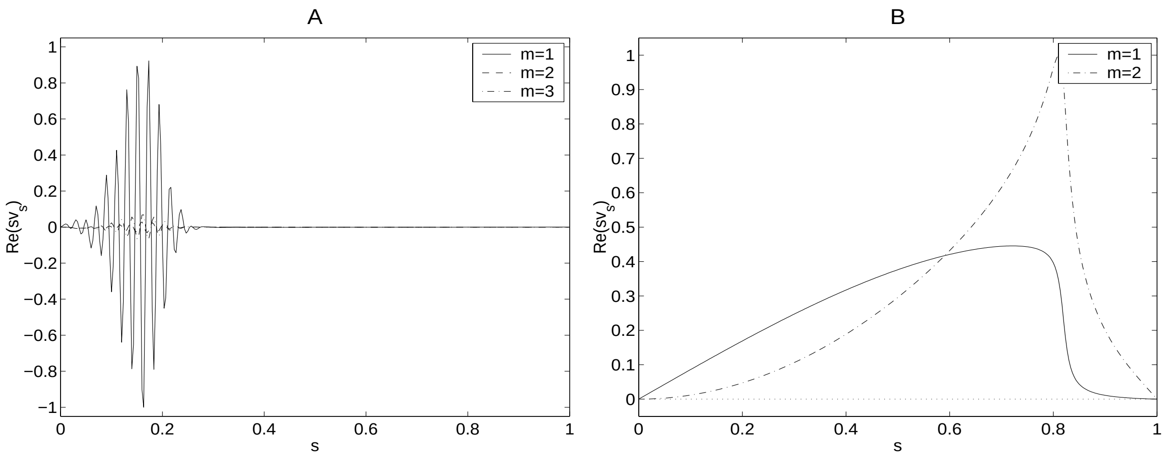

Figure 4: Eigenfunctions of the resistive eigenvalue A (left plot) and the new ideal global gap mode B (right plot).

4 Conclusions

We have studied the influence of poloidal flow in tokamak plasmas on the toroidal Alfvén eigenmodes (TAE's). This has been done with two new MHD codes, FINESSE and PHOENIX, that compute plasma equilibria with flow and resistive or ideal spectra of these equilibria, respectively. New global, poloidal-flow-induced, Alfvén eigenmodes have been found in the gaps of the modified continua for transonic slow flows.

References

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