Raptor Codes for Use in Opportunistic Error Correction

T. Zijnge X. Shao
R. Schiphorst C.H. Slump
University of Twente
Fac. EEMCS, Dept. Signals and Systems
P.O. Box 217, 7500 AE Enschede, The Netherlands
tzijnge@alumnus.utwente.nl x.shao@utwente.nl
r.schiphorst@utwente.nl c.h.slump@utwente.nl

Abstract

In this paper a Raptor code is developed and applied in an opportunistic error correction (OEC) layer for Coded OFDM systems. Opportunistic error correction [3] tries to recover information when it is available with the least effort. This is achieved by using Fountain codes in a COFDM system, which means that each subcarrier contains a Fountain coded packet. The property of a Fountain code that any missing packet can be replaced by any other packet allows the receiver to discard packets with low SNR. As a result ADC resolution and therefore energy consumption can be reduced.

In this paper the possibilities of using Raptor codes in an OEC system are investigated. Raptor codes are a type of Fountain codes with an efficient encoding and decoding algorithm. By simulating an OEC system on a wireless indoor channel model, it is shown that Raptor codes can save up to 70% in energy consumption with respect to a traditional system. Furthermore it is found that a feedback channel can reduce energy consumption even more. In this setting 85% of energy consumption can be saved.

1 Introduction

In [3], Shao et al. propose a novel cross layer scheme based on Fountain coding to reduce the energy consumption of *Analog to Digital Converters* (ADCs) in OFDM systems. Current OFDM systems often use fixed, high resolution ADCs in order to recover information from OFDM subcarriers for a wide range of *Signal-to-Noise Ratios* (SNR), and traditional error correction methods. This has two disadvantages. First, ADC resolution is designed for a worst case situation, resulting in an over dimensioned system for most of the time. Since high resolution ADCs can consume up to 50% of the baseband energy in an OFDM system, energy can be saved by replacing them with resolution-adaptive ADCs. Second, all information must be processed by the entire receiver chain. Burst errors introduced in the wireless channel are spread out over a message by means of interleaving in frequency and time. This results in a relatively high average error rate, for which a channel code has to be designed. By discarding bad parts of the received information, remaining parts have high quality, i.e. low *Bit Error Rate* (BER). This allows the dynamic range of the ADCs in the receiver to be reduced, resulting in a reduction of ADC energy consumption.

A Fountain encoder encodes source data into a stream of encoded packets. A receiver needs to recover a fixed number of these packets, which ones is not important. In an OEC system, a Fountain encoded packet is transmitted on one subcarrier. If this subcarrier experiences a bad channel, the entire packet is lost. This is not harmful to the Fountain code as the lost packet can be replaced by any other packet. In this

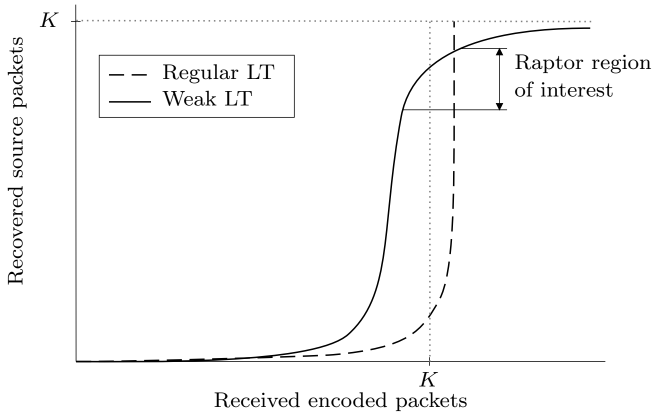


Figure 1: Typical decoding curves of the regular and weak LT code.

situation it is not necessary to recover all packets correctly, which allows for lower ADC resolution. The energy consumption of an ADC is proportional to the number of quantization levels. Lower resolution therefore means lower energy consumption.

In previous work by Shao [3] it was shown that it is possible to reduce ADC energy consumption using OEC with LT codes. A reduction of 72% was achieved but the used LT decoding algorithm was rather complex. In this paper it is investigated if similar results can be attained using Raptor codes, for which an efficient encoding and decoding algorithm exists. Furthermore the possibilities of using feedback of *Channel State Information* (CSI) is investigated.

2 Fountain codes

Fountain codes are a class of rateless erasure correcting codes, first described by Byers et al. in [1]. They are rateless because the encoder generates a potentially limitless stream of encoded packets from a source file. The receiver is able to recover the data when a number of packets is received that amounts to a size only slightly larger than the source file. The order of received packets is not important; as long as enough packets are received, the decoder can recover the message. Fountain codes can combine a low overhead with robustness to many kinds of errors. Furthermore they are near optimal on an erasure channel and have many advantages in multicast systems.

2.1 LT codes

LT codes are a subclass of Fountain codes and are the first practical implementation of a Fountain code. They are described by Luby in [2]. In an LT code, an encoded packet is created by splitting a message into K source packets and then adding (modulo 2) a random number of randomly picked packets to create an encoded packet. The number of source packets that is used for an encoded packet is called the degree and is chosen from a degree distribution. Luby developed the Robust Soliton distribution, which gives good results in a practical system. Due to the average degree of $\log K$ of this distribution, the decoding (using a message passing algorithm) scales logarithmically with K.

An LT code based on a truncated Robust Soliton distribution is called a weak LT code. Figure 1 shows the typical decoding behaviour of a regular LT code and a weak LT code. It can be seen that for both codes the decoding only starts after a large

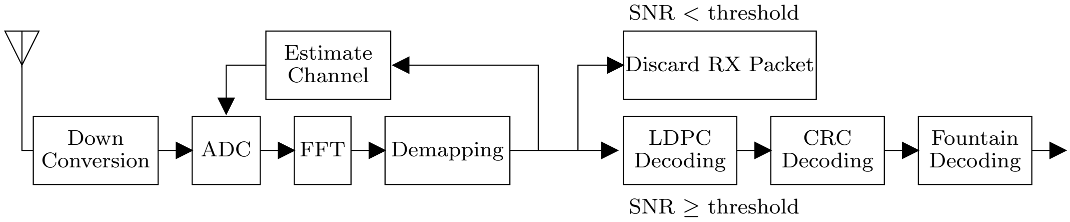


Figure 2: Simplified schematic of an OEC receiver. The ADC is resolution adaptive.

number of encoded packets has been received. After this point the decoding curve becomes very steep, marking successful recovery of all source packets for a regular LT code. For a weak LT code however, the decoding curve flattens again after the initial steep ascent. This means that many encoded packets are still needed to recover the source packets. This is not a desirable property, as the code overhead (i.e. the ratio of encoded packets and source packets) is very large. On the other hand the weak LT code can be decoded in linear time using a message passing algorithm.

2.2 Raptor codes

In this section a Raptor code is described, which uses the low decoding complexity of the weak LT code but circumvents the high overhead by combining it with a pre-code.

Shokrollahi discovered Raptor codes in 2001 and described them in [5]. He used a weak LT code in combination with an LDPC pre-code. LDPC codes, like the weak LT code, have a decoding complexity (using message passing) that scales linearly with the block length. Therefore the combination of these codes also has linear decoding complexity. Such a code is called a Raptor code and works as follows. Figure 1 shows the region of interest of a weak LT code that is used in a Raptor code. It can be seen that in this region, the weak LT code can recover for example 90% of the source packets. The remaining 10% can then be recovered by pre-coding the source data with an erasure correcting code that can correct 10% erasures.

3 Research framework

The reference OFDM system in this research is IEEE 802.11a WLAN. This system is taken as a starting point for the new Opportunistic Error Correction layer. In IEEE 802.11a data is transmitted in MAC frames. A MAC frame consists of at most 500 OFDM symbols and lasts 2 ms. The system uses OFDM with 48 data carriers. The throughput of such a system depends on the type of modulation and forward error correction. As an example, a WLAN system using 16-QAM and a code rate of 0.5 can achieve a throughput of $500 \cdot 48 \cdot 4 \cdot 0.5 \cdot 2^{-3} = 24$ Mbit/s.

As described in Section 1, the new Opportunistic Error Correction system can transfer information at the same data rate as a regular system but has a lower energy consumption in the receiver. This is achieved by using resolution adaptive AD converters and transmitting a packet on one subcarrier instead of spreading it over all subcarriers. Packets transmitted on bad subcarriers are discarded by the receiver. A block diagram of such a system is given in Figure 2.

Fountain Type	$\mathcal{C}_{\text{analytical}}$	K	Overhead %		$\mathcal{C}_{\text{evaluated}}$
			mean	max	
Raptor + LDPC	$\mathcal{O}(K_{\text{pre}})+\mathcal{O}(K)$	1296	19.8	26.6	2376
		1944	18.1	22.9	3564
LT + MP	$\mathcal{O}(K \log_e K)$	1296	11.8	29.5	9288
		1944	9.9	24.4	14721
LT + MP + GE	$\mathcal{O}(K_1 \log_e K_1)+$	500	N/A	~ 3	63880
	$\mathcal{O}\left((K-K_1)^2\right)$	1296	N/A	~ 3	424099
		1944	N/A	~ 3	951471

Table 1: Comparison of three different Fountain codes in overhead and complexity ($\mathcal{C}$). K_{pre} is the block length of the pre-code, in this case 5/6 of K. All data about the LT code with Gaussian elimination and message passing is taken from [3] with permission. K_1 is the number of packets not recovered by Gaussian elimination and has a value of 250.

4 Raptor code analysis

In this section a Raptor code is compared to two similar LT codes, one using the message passing algorithm, the other using a combination of Gaussian elimination (GE) and message passing (MP). The latter is taken from [3] and uses $K = 500$ source packets because it was shown that for this decoding method the overhead is constant for $K \geq 500$. The other two decoding methods use a larger value of K because of the decreased Fountain overhead at larger K. The three codes are compared in terms of overhead and complexity.

To investigate the performance of the codes and decoding methods, a fixed amount of data is encoded into a stream of encoded packets. At the decoder side, encoded packets are collected and the source data is recovered. The overhead is determined as the percentage of additional encoded packets that is needed to recover all source data. Because encoding of Fountain codes is a random process, a large number (10^3) of transmissions is simulated.

Results are shown in Table 1. It can be seen that the Raptor code has low complexity but a relatively large overhead. The LT code from [3] on the other hand, has a very small overhead but a high complexity. Even though the decoding method partially uses the efficient message passing algorithm and K is much smaller than for the Raptor code, the complexity is many times larger.

Table 1 shows a significant difference between average and maximum overhead for the Raptor code (almost 5%). The maximum overhead indicates the number of encoded packets that is needed for the recovery of source data with high probability. This measure is useful when there is no possibility to acknowledge successfully received data. In most cases however, the source data will be recovered from much less encoded packets. This is indicated by the average overhead which is useful in a system with feedback from receiver to transmitter. This difference is used later on in the calculation of energy consumption.

5 Channel state information feedback

In this chapter the use of CSI feedback in an OFDM system is investigated. In Section 4 it was found that acknowledging a correctly received file decreases the overhead of the investigated Fountain code. This requires a feedback channel which may then also be used to inform the transmitter with CSI from the receiver. Here it is researched whether it is possible to dynamically adapt the per-packet channel code rate to optimally suit channel conditions as observed from the receiver. This can lead to increased throughput or reduced energy consumption. Here, the emphasis is on energy consumption of the wireless receiver ADC. Therefore it is first explained how ADC energy consumption is calculated and the influence it has on the throughput of the system. After that, a description and the results of the new system are given. In total four different scenarios are compared. The first is IEEE 802.11a WLAN, which uses neither OEC nor CSI feedback. The second is the OEC system developed in [3], which uses the rather inefficient LT codes. The third system uses OEC in combination with Raptor codes and the fourth system adds CSI feedback to this.

5.1 ADC energy consumption

ADC power is equal to the number of quantization levels (N_q) times the sampling rate R_s [6]. Instead of using ADC power to compare different transmission schemes, it is better to use the energy per source bit. This way it is easier to compare systems with slightly different throughputs. Since power is defined as the rate of energy consumption, energy per source bit E_b is obtained by dividing the ADC power by effective throughput T.

$$E_b = \frac{P_{\text{ADC}}}{T} = \frac{N_q R_s}{T} \tag{1}$$

The number of quantization levels N_q depends on the quantization step size Δ. This relation is given in Equation 2.

$$N_q = 2\left\lceil \frac{C}{\Delta} \right\rceil \tag{2}$$

Here, the constant C is a limit to the number of samples that are smaller than the quantization step Δ and can therefore not be quantized correctly [3].

It is shown in [4] that the frequency domain noise is determined by the resolution of the ADC and has a Gaussian distribution with zero mean and a variance of $\Delta^2/6$. The signal-to-noise ratio for carrier k is determined by the magnitude of the channel frequency response at carrier k ($|H_k|^2$) and the variance of the noise in the frequency domain. Conversely, a minimum required Δ can be calculated when a channel code (requiring a certain SNR) is chosen (Equation 3). Both equations are from [3].

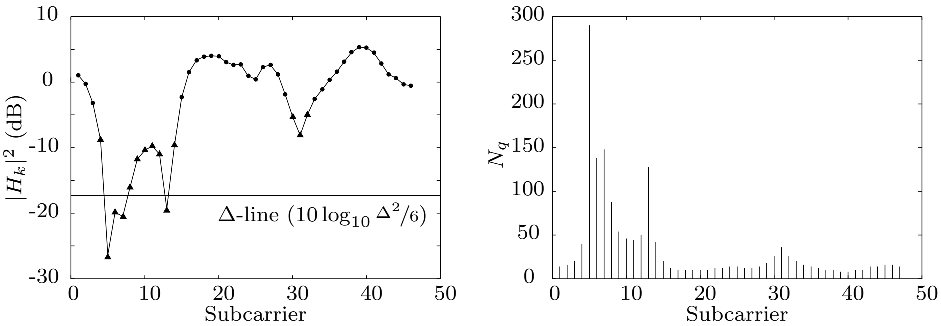
$$\text{SNR}_k = \frac{|H_k|^2}{\Delta^2/6} \Leftrightarrow \Delta = \sqrt{6\frac{|H_k|^2}{\text{SNR}_k}} \tag{3}$$

Equation 1 shows that the energy per source bit depends on the effective throughput T of the system. For IEEE 802.11a, T is calculated as follows.

$$T = \frac{\text{Nr. source bits per MAC frame}}{\text{MAC frame duration}} = \frac{N_s N_c N_b R \alpha_F}{t_m} \tag{4}$$

with N_s the number of OFDM symbols per MAC frame, N_c the effective number of data carriers per OFDM symbol, N_b the number of bits per carrier (determined by the modulation type), R the rate of the channel code, α_F a throughput factor and t_m the MAC frame duration. The throughput factor α_F can be used to describe allowed packet loss or the throughput factor of a Fountain code.

The above equations describe an inverse relation between the energy consumption of an OFDM system and the frequency domain SNR. A signal can be AD converted with a low probability of error at the price of high energy consumption, and vice versa.



(a) **(b)**

Figure 3: Typical frequency response (3a) and the corresponding number of quantization levels (3b) required for an SNR of 15 dB on every carrier. In 3a the 14 worst subcarriers are marked with a triangle. The Δ line is 15 dB below the 15th lowest subcarrier. Setting the ADC with this value of Δ results in an SNR of at least 15 dB for the remaining 34 subcarriers.

5.2 Analysis

Figure 3a shows a typical wireless channel frequency response. Corresponding values of N_q are shown in Figure 3b. Because AD conversion is performed in the time domain, N_q is constant for the duration of at least one OFDM symbol. The value of N_q that is needed for a certain SNR on every carrier is determined by the highest peak in Figure 3b. From this figure it is clear that most subcarriers however require a much smaller N_q. Using OEC it is possible to choose N_q such that not all subcarriers have the required SNR. These carriers are discarded. Because of the use of Fountain codes in OEC, data transmitted on these carriers can be easily replaced. There is a trade-off between the amount of data that can be transmitted on the used carriers and the ADC energy consumption.

In a system without CSI feedback the number of discarded subcarriers is fixed, based on a model of the channel. Otherwise the number of discarded subcarriers depends on the state of the channel. In this case per-carrier adaptive coding and modulation can be applied to use the channel capacity optimally. This can be shown graphically with a typical channel frequency response, such as the one in Figure 3a. In this graph a horizontal line called the Δ-line is drawn at $10 \log_{10} \Delta^2/6$. According to Equation 3 (converted to the log domain), the distance between the frequency response and the Δ-line is the SNR of the transmitted signal. The Δ-line is inversely proportional to the energy consumption. A higher Δ-line means lower energy consumption.

When the channel frequency response is known, the most energy efficient ADC setting that still maintains the desired throughput can be found in an iterative process. In this process the Δ-line is placed somewhere high where the desired throughput is not achieved and then gradually shifted down until the desired throughput is met.

When CSI feedback is used the transmitter already knows which carriers will be discarded by the receiver. Power control can therefore be used at the transmitter to distribute power of unused carriers over used carriers. This way the SNR for these carriers is increased at the receiver, which therefore requires less ADC quantization levels and decreases energy consumption.

Scenario	**I**	**II**	**III**	**IV**
Description	IEEE 802.11a	OEC+LT +GE+MP	OEC+Raptor	OEC+Raptor +CSI feedback
Error protection	Conv. codes	LT+LDPC +CRC	Raptor +LDPC	Raptor +LDPC
R	0.5	0.66	0.8	-
Modulation	16-QAM	16-QAM	16-QAM	-
$\mathbf{N_c}$	48	34	34	-
$\alpha_\mathbf{F}$	0.9	0.97	0.81	0.81
T (Mbit/s)	21.6	21.77	22.03	21.81
$\mathbf{E_b}$	65.07	16.34	19.23	10.04

Table 2: Properties of the four scenarios (top) and the resulting throughput and bit energy (bottom).

5.3 Results

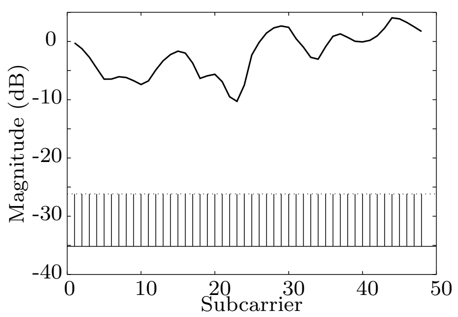
As mentioned at the beginning of this section, four scenarios are compared at a throughput of approximately 21.6 Mbit/s. This is achieved with the parameters listed in Table 2. Figure 4 shows how each of the scenarios handles a typical channel response. It can be seen that for Scenario I SNR requirements are easily met at the cost of high energy consumption (Δ-line at -35 dB). Scenarios II and III already perform much better. Both systems discard the 14 worst subcarriers and by doing so allow for lower ADC energy consumption (Δ-line at -17 dB). In Scenario IV a distinct transmission mode (i.e. a combination of code rate and modulation type that assures a certain SNR) is used for every subcarrier such that the space between the Δ-line and the frequency response is completely used. This scenario has the lowest energy consumption. The lower part of Table 2 shows the resulting throughput and energy consumption for each scenario.

6 Conclusions

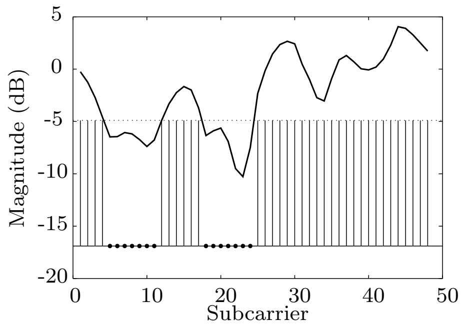
Simulations on a wireless indoor channel model have shown that is possible to use Raptor codes in an opportunistic error correction system. At an equal data rate such a system allows for a lower ADC energy consumption. Compared to a standard IEEE 802.11a WLAN system, a reduction of 70% was achieved. This is somewhat less than Shao's method from [3] (Scenario II) which achieved a reduction of 75% on the same channel. The system with the Raptor code however has a much smaller decoding complexity, which is an important issue for high speed applications. The use of a feedback channel was also investigated. Using a feedback channel it is possible to acknowledge a successfully received file and inform the transmitter about the state of the channel. This way a total energy reduction of 85% was achieved.

Fountain codes are very robust against all kinds of errors that occur during the transmission of digital data. Raptor codes add the property of encoding and decoding in linear time. This makes Raptor codes a very attractive error correcting solution. Raptor codes used in this paper work on a relatively small number of packets. It is well known that Fountain codes perform best on a very large number of packets. A larger number of source packets would decrease Raptor overhead and reduce energy consumption even more.

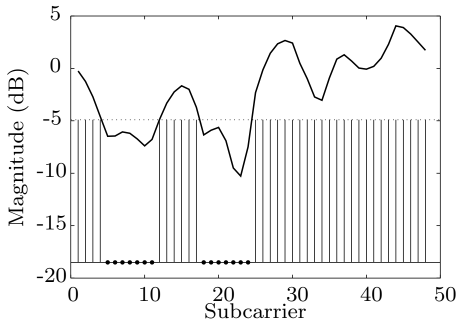
Further research focuses on a more advanced feedback channel and steps toward a



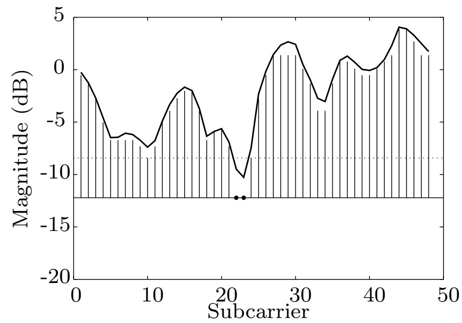
(a) Scenario I



(b) Scenario II



(c) Scenario III



(d) Scenario IV

Figure 4: All four Scenarios from Table 2 on the same channel realization. Note that Figure 4a has a different scale.

real implementation.

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