WeP24

SIMPLIFIED CALCULUS FOR THE DESIGN OF A CRYOGENIC CURRENT COMPARATOR

J. Sesé[1], E. Bartolomé[1], A. Camón[3], J. Flokstra[1], G. Rietveld[2] and C. Rillo[3]

[1]Low Temperature Division, Applied Physics Department, University of Twente, 7500 AE Enschede, The Netherlands

[2]NMi Van Swinden Laboratorium, Dept. Electricity and Magnetism, 2600 AR Delft, The Netherlands

[3]ICMA, CSIC-University of Zaragoza, 50009 Zaragoza, Spain

Abstract

The calculation of inductances of superconducting structures like the Cryogenic Current Comparator (CCC) is not straightforward due to image effects. We have found a "rule of thumb" that maximizes the value of an inductance inside a superconducting shield. With this rule, the design of an optimum CCC is simplified.

Introduction

The cryogenic current comparator [1] allows controlling the ratio of two currents I_1, I_2 with high precision: $I_1/I_2=N_2/N_1$ where N_1 and N_2 are integer numbers.

Resistance bridges based on the CCC allows making resistance comparisons at metrological level. They are currently widely used to transfer the value of the quantum Hall resistance standard to high stable resistors [2]. More recently, its use for the amplification of the very small currents produced by Single Electron Tunneling devices, towards the realization of a quantum current standard is being investigated [3-5].

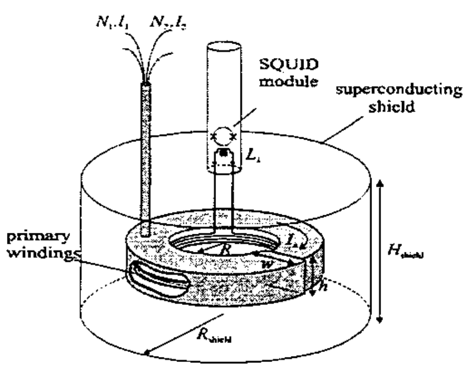

Figure 1. Schematics of a CCC. The overlapped tube has cross section hxw and internal radius R. The flux created by the Meissner current $I_e=N_1I_1-N_2I_2$ is picked-up by a sensing coil connected to a SQUID. The whole is surrounded by a superconducting shield with dimensions R_{shield} and H_{shield}.

0-7803-7242-5/02/$10.00 ©2002 IEEE

Design of an optimum CCC

The ultimate current resolution of a cryogenic current comparator (CCC) per turn in the primary windings that is ideally coupled to a SQUID is given by [6]:

$$\langle i_p^2 \rangle = \frac{8\varepsilon_{SQ}}{k_{SQ}^2 L_{CCC.eff}}$$

where $\varepsilon_{SQ}/k_{SQ}^2$ is the extrinsic energy resolution of the SQUID and $L_{CCC.eff}$ is the effective inductance of the CCC overlapped tube inside the superconducting shield, see figure 1. For a fixed number of primary turns and a limited volume, the optimum CCC will be the one with maximum $L_{CCC.eff}$ for the available dimensions. This problem has been the subject of papers based on different numerical methods [7-10]. Figure 2 shows a typical calculation of $L_{CCC.eff}$ vs. R following the method described in reference [7]. An initial increase of $L_{CCC.eff}$ vs. R is observed until a maximum is reached, then it rapidly decreases towards zero when the overlapped tube approaches the shield. The limit value of R is R_{shield}-w.

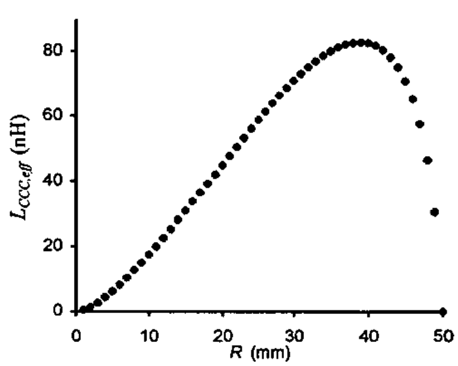

Figure 2. Calculated values of $L_{CCC.eff}$ vs. the internal radius R for h=w=10 mm, H_{shield}=80 mm and R_{shield}=60 mm. The maximum value for R is R_{shield} -w=50 mm.

Numerical methods can be applied to any geometry and have proven to give accurate results. However it would be desirable to have a simple "rule of thumb" that is valid for the typical dimensions of a CCC. Then the design of a CCC will be considerably simplified. In this work, a geometrical "rule of thumb" is deduced using physical arguments and its range of validity for the typical dimensions of CCCs verified numerically.

"Rule of thumb"

Any current in the CCC tube generates a flux, $\Phi = L_{CCC,eff}$ x I_e, in the internal area of the CCC, $A_{\text{int}} = \pi R^2$. This flux should be able to return through the area A_{ext} comprised between the CCC outer side and the shield: $A_{ext} = \pi R_{shield}^2 - \pi (R+w)^2$. So the flux follows a path that has a magnetic conductance determined by these areas. When $A_{int} << A_{ext}$, the magnetic conductance is mainly determined by A_{int} and $L_{CCC,eff}$ will increase when A_{int} increases (so when R increases). When $A_{ext} << A_{int}$ the magnetic conductance will now be determined by A_{ext} and $L_{CCC,eff}$ will decrease when A_{ext} decreases (so when R increases). Since the available area is restricting the flow of flux lines, both generation and return of flux should be 'as easy', and therefore the maximum value for $L_{CCC,eff}$ is expected when the inner area is equal to the outer area: $A_{int}=A_{ext}$. This will occur for a radius R given by:

$$R_{opt} = \frac{\sqrt{2R_{shield}^2 - w^2} - w}{2}$$

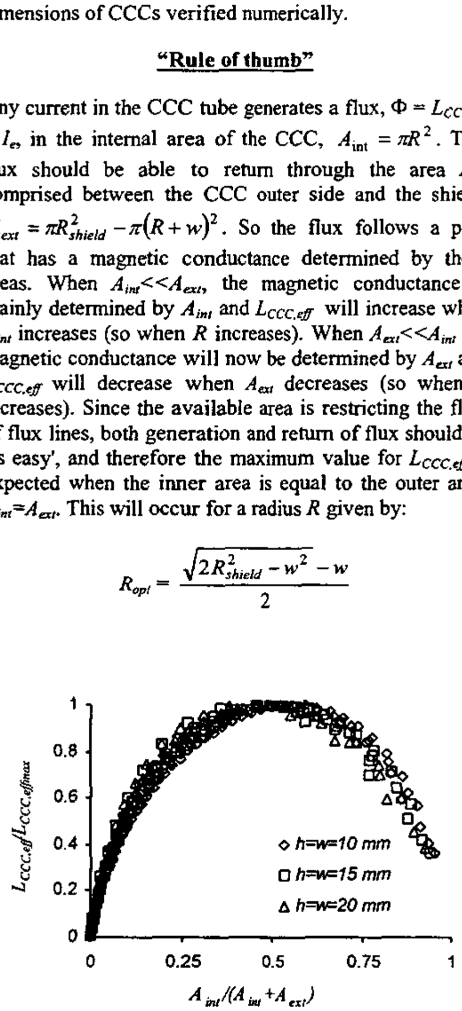

Figure 3. $L_{CCC,eff}$ for typical CCC dimensions. $L_{CCC,eff}$ is normalized to its maximum $L_{CCC,effmax}$. In all studied cases the maximum is reached when $A_{int}=A_{ext}$.

Our approach is exactly valid when a homogeneous flux density exists inside the areas A_{int} and A_{ext}. This will not be the case for arbitrary dimensions. For example when h and w are much smaller than R and R_{shield}, the magnetic flux density will be higher near the overlapped tube, decreasing inversely with the distance. Nevertheless, the dimensions of typical CCCs are such that h, w, R, R_{shield} are of the same order. So, to validate this intuitive approach we have calculated numerically the dependence of $L_{CCC,eff}$ vs. $A_{int}/(A_{int}+A_{ext})$ for several cases with the dimensions that are typical in the construction of a CCC following ref. [7]. The results are shown in figure 3. Indeed it is seen that the maximum value for $L_{CCC,eff}$ occurs when $A_{int}=A_{ext}$ in all studied cases.

Conclusions

The maximum value of the overlapping tube inductance of a CCC inside a superconducting shield occurs when the internal and external areas of the CCC are equal. We have validated this condition using numerical methods for typical dimensions of CCC. This new rule of thumb simplifies the calculations for the design of an optimum CCC.

References

[1] I. K. Harvey, "A precise low temperature dc ratio transformer," Rev. Sci. Instrum., vol. 43, pp. 1626-1629, 1972.

[2] B. Jeckelmann and B. Jeanneret, "The Quantum Hall effect as an electrical resistance standard," submitted Rep. Prog. Phys., 2001.

[3] A. Hartland, "Development of a cryogenic current comparator for the measurement of small currents," Conf. Electrom. Meas. BEMC, vol. 18, pp. 1-4, 1993.

[4] Y. de Wilde, F. Gay, F. Piquemal and G. Geneves, "Measurements of single electron transistor devices combined with a CCC: progress report," IEEE Trans. Instrum. Meas, vol. 50, no. 2, pp. 231-234, 2001.

[5] E. Bartolomé, J. Sesé, G. Rietveld, J. Flokstra, C. Rillo, A. Camón, H. Rogalla, "1:30000 Cryogenic current comparator with optimum SQUID readout," presented at this conference CPEM2002.

[6] J. Sesé, A. Camón, G. Rietveld and C. Rillo, "Ultimate current resolution of a cryogenic current comparator," IEEE Trans. Instrum. Meas., vol. 48, no. 6, pp. 1306-1313, 1999.

[7] J. Sesé, F. Lera, A. Camón and C. Rillo, "Calculation of effective inductances of superconducting devices. Application to the cryogenic current comparator," IEEE Trans. Appl. Supercon., vol. 9, no. 1, pp. 58-62, 1999.

[8] G.T. Symm, "Design of a Cryogenic Current Comparator," Proc. Boundary Elements XIV: Field problems and applications, C.A. Brabbia, J. Dominguez and F. Paris eds., vol. 1, pp. 519-526, 1992.

[9] M. D. Early and K. Jones, "Optimum sensitivity of an externally shielded cryogenic current comparator," IEEE Trans. Instrum. Meas., vol. 46, pp. 459-462, 1997.

[10] E.T. Fratnsuz, "On forward estimate of the sensitivity of a cryogenic current comparator," Metrologia, vol. A85, no. 1-3, pp. 847-851, 1998.