GENERATION OF ULTRA-SHORT RELATIVISTIC-ELECTRON-BUNCH BY A LASER WAKEFIELD[*]

A. G. Khachatryan, K.-J. Boller, and F. A. van Goor,

Laser Physics & Nonlinear Optics Group, Dept. of Applied Physics, University of Twente, P.O. Box 217, 7500 AE Enschede, The Netherlands

Abstract

The possibility of the generation of an ultra-short (about one micron long) relativistic (up to a few GeVs) electron-bunch in a moderately nonlinear laser wakefield excited in an underdense plasma by an intense laser pulse is investigated. The ultra-short bunch is formed by trapping, effective compression (both in longitudinal and in transverse directions) and acceleration of an initially nonrelativistic (with kinetic energy of a few hundreds keVs) e-bunch that is injected in front of the laser pulse. The initial bunch may be of poor quality, may have a duration in the order of the laser pulse length or longer and can be generated by a laser-driven photo-cathode RF gun. Our 1D and 3D calculations predict that the accelerated ultra-short bunch will show a low energy spread of less than one percent and a low transverse emittance in the order of a nanometer. An energy gain in the GeV-range is feasible at an accelerating distance of a few centimetres. The total number of accelerated electrons is restricted by the beam loading effect only and can reach a value of 10^8–10^9.

INTRODUCTION

Electron bunches with the length in the order of a hundred microns have been produced using a photo-cathode radio-frequency (RF) gun [1], a thermionic RF gun [2], and a magnetic bunch compressor [3]. However, many applications, such as laser wake-field acceleration, inverse Cherenkov acceleration, inverse free electron laser (FEL), x-ray FEL, high energy physics and other applications, require much shorter electron bunches with a length in the order of a micrometer. Generation of such a short electron bunch is a difficult technical problem, new approaches to which are required. Recently, a new scheme of a laser wake-field accelerator (LWFA) has been proposed [4,5], which allows the generation of an ultra-short, ultra-relativistic, high quality electron bunch. The scheme utilizes a non(weakly)-relativistic bunch of electrons which is injected in front of a high-intensity laser pulse generating a nonlinear wake wave in an underdense plasma (see Fig. 1). Our results [4,5] showed that the electron bunch runs through the pulse, is trapped in the first accelerating maximum in the wake, essentially compressed both in longitudinal and transverse directions, and accelerated to an ultra-relativistic energy. Here we

show that the trapped electron bunch length is much less than the plasma wavelength. We also show that the generation of ultra-short relativistic bunches by our scheme is possible even when the initial bunch length is much longer than the plasma wavelength.

ONE-DIMENSIONAL (1D) THEORY

Because the trapped electrons are concentrated close to the axis, their longitudinal dynamics are well described by a 1D theory [5]. The 1D theory also allows a more detailed description of the problem. Suppose that an electron is initially ahead of the laser pulse (see Fig. 1) and moves with a velocity v_0 less than the group velocity
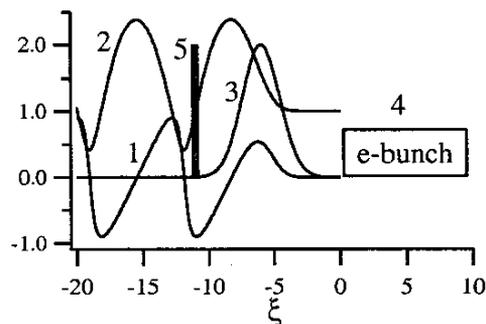

Figure 1: The laser wakefield scheme: the wake electric field E_z (1), the potential Φ (2), the laser pulse amplitude a (3), the initial (4) and trapped (5) bunches as a function of the position in a moving frame. $a_0=2$, $\sigma_z=2$, $\gamma_g=50$.

of the pulse v_g, which, in turn, is equal to the wake phase velocity. From the well known integral of motion $\gamma-\beta_g p-\Phi=$const (see, e.g., [4]; where γ and p are the gamma factor and the momentum of the electron normalized to $m_e c$, $\beta_g=v_g/c$, $\Phi(\xi)=1+e\varphi/m_e c^2\equiv1+\phi$ is the normalized wake potential, $\xi=k_p(Z-v_g t)\equiv z-\tau$, $k_p=\omega_p/v_g$, $\omega_p=(4\pi n_p e^2/m_e)^{1/2}$ is the plasma frequency, and n_p is the unperturbed plasma electron concentration) one can obtain an expression for the initial momentum of the trapped electron [4,5]:

$$p_0 = \gamma_g^2[\beta_g S - (S^2 - \gamma_g^{-2})^{1/2}], \qquad (1)$$

where $S=1/\gamma_g+1-\Phi_{tr}$, Φ_{tr} is the wake potential at the trapping point ξ_{tr}, at which $p=\beta_g\gamma_g\equiv p_g$, $\gamma_g=(1-\beta_g^2)^{-1/2}$. One

can see that the initial momentum of the trapped electron has a minimum, p_{min}, corresponding to the minimum of the wake potential, Φ_{min}; at this trapping point the wake electric field $E_z=-(1/\beta_g)^2 d\Phi/d\xi$ (normalized to $m_e c\omega_p/e$) is zero. The maximum value $p_{max}=p_g$ corresponds to the point in the wake where $\Phi=1$ ($E_z=-E_{z,max}<0$). In the nonlinear wakefield the longitudinal space interval, in which electrons can be trapped, decreases with the wake amplitude $E_{z,max}$ (though the nonlinear wake wavelength increases) and is less than a quarter of the linear wake wavelength $\lambda_p=2\pi/k_p$. Figure 2 shows numerical results for the minimum trapping momentum and the wakefield amplitude in dependence on the peak amplitude a_0 of a Gaussian laser pulse: $a=a_0\exp[-(\xi-\xi_c)^2/\sigma_z^2]$. As usual, the electric field amplitude a of the pulse is normalized to $m_e c\omega_L/e$, where ω_L is the laser frequency. In this paper values of $a_0=2$, $\gamma_g=50$, $\sigma_z=2$, and $\xi_c=3\sigma_z$ are used in the numerical calculations.
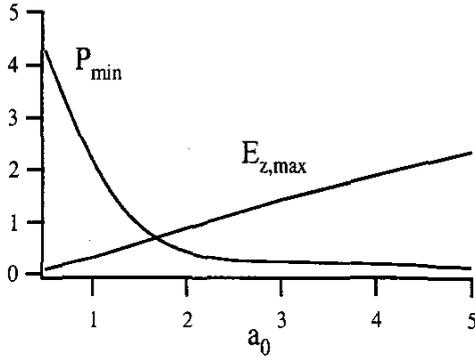

Figure 2. Calculated dependence of minimum initial momentum of trapped electrons, p_{min}, and wakefield amplitude, $E_{z,max}$, on the peak laser pulse amplitude, a_0.

For a moderately nonlinear wakefield with an amplitude much less than the wave-breaking field $[2(\gamma_g-1)]^{1/2}$, $|1-\Phi_{tr}|=|\phi_{tr}|>>1/\gamma_g<<1$. In this case, supposing that p_0 is not close to p_g, one can obtain from (1):

$$p_0 \approx (\phi_{tr} - 1/\phi_{tr})/2, \qquad (2)$$

where $-1<\phi_{tr}<0$. Then $p_{min}\approx(\phi_{min}-1/\phi_{min})/2$. In the linear wakefield $|\phi|<<1$, so $p_{min}\approx0.5/|\phi_{min}|>>1$. Thus, in the linear wakefield, only relativistic electrons can be trapped. However, for an initially relativistic electron, the trapping time increases proportional to γ_0^2.

The trapping time, τ_{tr}, which is the time interval, that is necessary for an electron to reach the trapping point $\xi_{tr}(p_0)$, depends on the initial momentum p_0 and on the initial position ξ_0:

$$\tau_{tr}(\xi_0,p_0)=\tau_{tr0}(p_0)+\xi_0/(1-\beta_0/\beta_g), \qquad (3)$$

where τ_{tr0} is the trapping time of an electron which is initially at $\xi=0$. The dependence $\tau_{tr}(p_0)$ is shown in Fig. 3 for $a_0=2$ and $\gamma_g=50$; in this case $E_{z,max}\approx0.9$ (see Fig. 1), the nonlinear plasma wavelength is $\Lambda_p\approx1.144\lambda_p$, and $p_{min}\approx0.53$. The trapping time has a minimum, which corresponds to $p_0\approx1$, or to a kinetic energy of about 200 keV. For the trapping time of an electron bunch, in which the electron momentum changes in the range $p_1\leq p_0\leq p_2$, and, supposing that the tail of the bunch is at $\xi=0$, we can write:

$$\Delta\tau_{tr} \approx (\tau_{tr,max} - \tau_{tr,min}) + L_0/(1-\beta_2/\beta_g), \qquad (4)$$

where $\tau_{tr,max(min)}$ is the maximum (minimum) value of τ_{tr0} for electrons in the range $p_1\leq p_0\leq p_2$, and L_0 is the initial bunch length.
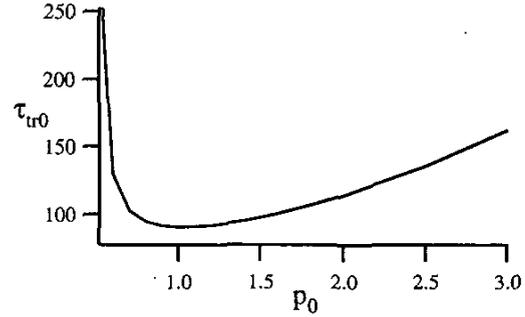

Figure 3. The trapping time, τ_{tr0}, as a function of the initial electron momentum, p_0.

As the trapped electron has reached a relativistic energy ($\gamma>>1$) in the wakefield, its dynamics becomes slow, and its spatial position in the wake changes on a normalized time scale of $\tau_{acc}=2\pi\gamma_g^2$. So, another electron, with the same p_0 but different ξ_0 will be trapped very close to the first one if $\Delta\tau_{tr}<<\tau_{acc}$. Test particle simulations showed that the trapped bunch length is much less than the plasma wavelength and undergoes practically no change during acceleration. One can see that the absolute and relative energy spread in the trapped bunch are $\delta\gamma\approx\Delta\tau_{tr}|E_z(\xi_{tr})|$ and $\varepsilon=\delta\gamma/\gamma\approx\Delta\tau_{tr}/(\tau-\Delta\tau_{tr})$ respectively [4,5]. In the case of finite momentum spread δp_0 in the initial bunch, electrons with different p_0 will be trapped at different trapping points. From Eq. (2) $\phi_{tr}\approx p_0-\gamma_0$. Then, for an electron bunch $\delta\phi_{tr}=\phi_{tr}(p_2)-\phi_{tr}(p_1)\approx|E_z|L\approx\delta\gamma_0-\delta p_0$, where E_z is some value of the wake electric field in the region occupied by the trapped bunch, L is the trapped bunch length. So, the trapped bunch length can be estimated as $L\approx(\delta\gamma_0-\delta p_0)/|E_z|$ when the effect of the finite initial bunch

length is less than the effect of the initial momentum spread.

In Fig. 4 the energy of trapped electrons is shown for the case of an initially mono-energetic bunch (in this case the trapped bunch length and the absolute energy spread are approximately proportional to the initial bunch length), $p_0=1$, and $L_0=10\lambda_p$. For example, at an acceleration length $l=400\lambda_p$ (corresponding to 2 cm for $\lambda_p=50$ μm), the trapped bunch length is about $7\times10^{-3}\lambda_p$ (≈0.35 μm in the case of $\lambda_p=50$ μm) when $L_0=10\lambda_p$ and 10 times less when $L_0=\lambda_p$; the trapped bunch length change is insignificant during acceleration. The energy of the electrons is about 1 GeV at $l=400\lambda_p$, the relative energy spread is $\approx0.93\%$ for $L_0=\lambda_p$ and $\approx9.5\%$ when $L_0=10\lambda_p$. The generation of good quality ultra-short relativistic e-bunches in the case of large initial momentum spread and $L_0\sim\lambda_p$ is also possible [4,5]. The initial bunch can be generated for example by a laser-driven photo-cathode RF gun.
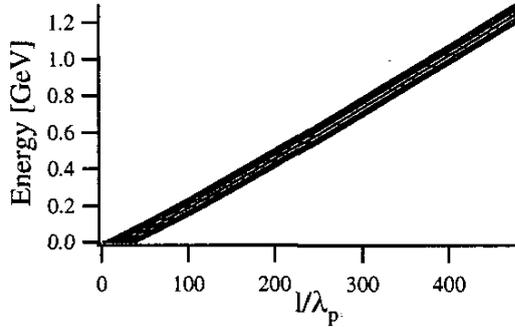

Figure 4. Energy of accelerating electrons, $p_0=1$, $L_0=10\lambda_p$.

THREE-DIMENSIONAL (3D) CASE

In a 3D linear wakefield, generated in an uniform plasma, the transverse force is defocusing in the trapping region. Fortunately, in the nonlinear wakefield and in the wake generated in a plasma channel the near-axis wakefield can be focusing over the entire accelerating region [5,6], so that our scheme can "work" in 3D as well. The trapped electrons are concentrated near the axis due to the focusing force. An example of the radial motion of the electrons with different initial radial positions is shown in Fig. 5 for the case of a laser wakefield excited in a plasma channel by a pulse with a Gaussian ($\sim\exp(-r^2/\sigma_r^2)$) radial profile. For these initial bunch parameters and $L_0=\lambda_p$, the trapped bunch length is $L\approx3.5\times10^{-3}\lambda_p$, the relative energy spread is $\varepsilon\approx3\%$ at the acceleration distance $l=400\lambda_p$, and $L\approx0.01\lambda_p$ and $\varepsilon\approx10\%$ when $L_0=10\lambda_p$. In the case of finite momentum spread in the initial bunch, namely, $0.9\leq p_{z0}\leq1.2$, $-0.02\leq p_{r0}\leq0.02$ ($1.35\leq\gamma_b\leq1.56$), and $L_0=0.8\lambda_p$, the trapped bunch length is about $0.02\lambda_p$ during acceleration. The average energy and the relative energy spread in the accelerated bunch are

≈1.07 GeV and $\approx4.5\%$ respectively, at $l=400\lambda_p$. The longitudinal dynamics and energy of the trapped electrons in 3D are approximately the same as in the 1D case.
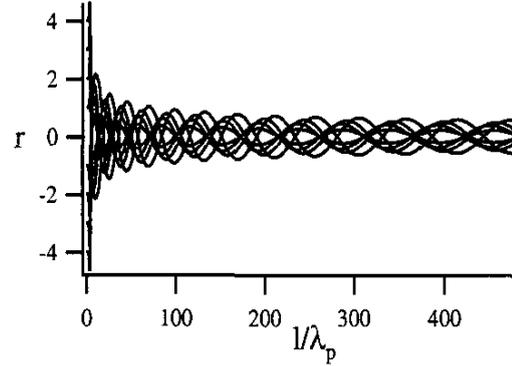

Figure 5. Transverse dynamics of trapped electrons, $p_{z0}=1$, $p_{r0}=0$, $a_0=2$, $\sigma_z=2$, $\sigma_r=5$.

In 3D, the normalized emittance was calculated to be $\varepsilon_n\sim(R^2\Omega/4\pi^2)\lambda_p$ [5], where $\Omega=(|\partial f_r/\partial r|/\gamma)^{1/2}$ is the betatron frequency, $R<<\sigma_r$ is the trapped bunch radius (Ω and R are in the normalized units), and is in the order of a nanometer in the ultra-relativistic regime. For example, $\varepsilon_n\sim3$ nm for the case presented in Fig. 5 (initially mono-energetic bunch) when $L_0=10\lambda_p$, $l=400\lambda_p$ and $\lambda_p=50$ μm; in the case of the finite initial momentum spread considered above, the emittance is approximately the same.

The total number of accelerated electrons is restricted by the beam loading effect, $N_{tot}<<3\times10^7\lambda_p[\mu$m] [5]. One can see that this number can reach a value of $\sim10^8$ when the plasma wavelength is in the order of tens of microns (that is typical for laser pulses with $\lambda_L\sim1$ μm) and $\sim10^9$ for λ_p in order of hundreds microns (that is typical for CO_2 laser pulses, $\lambda_L\approx10$ μm). The scheme can also be applied to the self-modulated LWFA case.

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