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Dynamic Balancing of Mechanisms by using an Actively Driven Counter-Rotary Counter-Mass for Low Mass and Low Inertia

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Abstract: *Dynamic balancing of mechanisms still goes together with a considerable increase of mass and inertia. The goal of this article is to actively balance various useful 1-, 2-, and 3-degree-of-freedom planar and spatial, serial and parallel mechanisms and to show that active balancing is a good alternative for low mass and low inertia dynamic balancing. It is proposed to force balance mechanisms with the minimum number of counter-masses and to use the inertia of these counter-masses to balance the moment by actively controlling them with an additional actuator. The counter-masses then are driven such that they counter-rotate with respect to the mechanism and the dynamic balance is obtained. Herewith the advantages for low mass and low inertia of the counter-rotary counter-mass (CRCM-) principle and the principle of duplicate mechanisms (DM) where a mechanism is balanced altogether (instead of link by link), are combined. A double pendulum is actively balanced, compared with other balancing principles, and used for the synthesis of various actively balanced manipulators.*

It was found that dynamic balancing by active control of the CRCM (ACRCM) results into a better total mass-inertia relation then balancing with nonactive CRCMs or using separate counter-rotations for the moment balance. The DM-principle still is better, however the size of the ACRCM-balanced mechanism is considerably smaller. For a low mass and low inertia addition, the ACRCM should have a large inertia and a low mass. Active control of the ACRCM has the advantage of being able to compensate for disturbances that affect the moment balance, such as drift, belt elasticity or external forces. Disadvantages are the addition of a controlled actuator and difficulty to handle high accelerations as for example due to impact. It is shown that a planar 3-RRR parallel manipulator and a spatial 3-RRR parallel manipulator can be dynamically balanced with respectively one and two ACRCMs. It is also shown that a 3-DOF planar 1-RRR serial manipulator can be completely dynamically balanced by a single ACRCM.

1 Introduction

Dynamic balancing of mechanisms with a low addition of mass and a low addition of inertia is an important target. More inertia means that more power is needed to drive the mechanism while more mass means more power to lift and control the object in free space and an increase of material costs (Van der Wijk, Herder and Demeulenaere, 2008). To dynamically balance a mechanism, often counter-masses are added to force balance mechanism links and separate counter-rotations (SCR) (Kamenskii, 1968; Berkof, 1973), or counter-rotary counter-masses (CRCMs) (Berestov, 1975; Herder and Gosselin, 2004) are used to balance the moment of the links. With the SCR-principle, a mechanism link is force balanced first, then by adding a separate counter-rotating inertia element, the moment of the link is balanced.

With the CRCM-principle the moment of a mechanism link is balanced by counter-rotating the counter-mass that is used for the force balance of the link. The CRCM-principle has proven to be more advantageous for low mass and low inertia dynamic balancing than the SCR-principle (Herder and Gosselin, 2004; Van der Wijk, Herder and Demeulenaere, 2008; Van der Wijk *et al.*, 2008). It is also possible to balance the complete mechanism altogether instead of link by link, by duplicating it two times (Lowen and Berkof, 1968). This duplicate mechanisms (DM-)principle proved to be the best for low mass and low inertia dynamic balancing (Van der Wijk *et al.*, 2008), but is generally a complex and space consuming balancing principle.

With the target to find ways to balance mechanisms altogether when few space is available, Hilpert's solution to force balance 4R-four bar mechanisms by adding a pantograph with a single counter-mass (Hilpert, 1968), is useful. It was shown that the COM of a 4R-four bar mechanism can be materialized with additional parallel links. By connecting one end of the pantograph to the center of mass (COM) and connecting the pivot of the pantograph to the base, the overall COM could be made stationary.

This implies that any mechanism of which the COM can be materialized can be force balanced with a pantograph and a single counter-mass.

Since in planar mechanisms the shaking moment exists solely in one plane, it is possible to balance the moment of any planar mechanism by only one counter-rotating element. To keep the addition of mass and inertia low, it seems obvious to use the counter-mass also for the moment balance by having it counter-rotate with respect to the mechanism.

Contrary to the configurations in Herder and Gosselin (2004); Van der Wijk, Herder and Demeulenaere (2008); Van der Wijk *et al.* (2008), balancing a more than 1-degree of freedom (DOF) mechanism with a single counter-mass leads to a mass distribution (reduced inertia of the mechanism (VDI2149, 1999)) that depends on the position of the mechanism. Then it is not possible to balance the mechanism by driving the CRCM with, for instance, a pair of gears since the counter-rotation of the CRCM then solely depends on the velocity of the mechanism. Therefore it is proposed to drive the counter-rotation actively. Then, next to the actuators that are used to drive the mechanism, an extra actuator is included that solely actuates the counter-rotary counter-mass (ACRCM).

The goal of this article is to actively balance various useful 1-, 2-, and 3-degree-of-freedom planar and spatial, serial and parallel mechanisms and to compare the results with known passive (i.e. non-active) balancers with the focus on the mechanical system.

The approach is to balance a double pendulum with an ACRCM first and then use this balanced double pendulum for the synthesis of new ACRCM-balanced mechanisms. With the momentum equations the conditions for the dynamic balance and the equations of the total mass and the reduced inertia of the mechanism are obtained. A numerical example is carried out and the results are compared to the passive (non-active) balancing principles. At the end it is shown that also a planar 3-DOF serial mechanism can be balanced actively with just a single ACRCM.

2 Balanced Double Pendulum with one ACRCM

In Van der Wijk, Herder and Demeulenaere (2008) a double pendulum (also called dyad (Tsai and Roth, 1972)) was found to be an important building element in the synthesis of mechanisms. In addition it was found to be a suitable mechanism for a comparative study of balancing principles regarding the addition of mass and the addition of inertia.

Figure 1 shows a dynamically balanced double pendulum with a single ACRCM. The initial double pendulum before balancing consists of link 1 with length l_1 and link 2 with length l_2. At the endpoint of link 2 there is a lumped mass m with inertia I. This lumped mass can represent a payload, the mass and inertia of link 2, or both.

The mechanism has two degrees of freedom which are described by θ_1 and θ_2. These are the relative angles between two connecting links. The absolute angle of link 2 with the reference frame is α_2. The x-axis of the reference frame is chosen to be along the base link for which the absolute angle of link 1 is equal to θ_1.

For the force balance of the double pendulum, two parallel

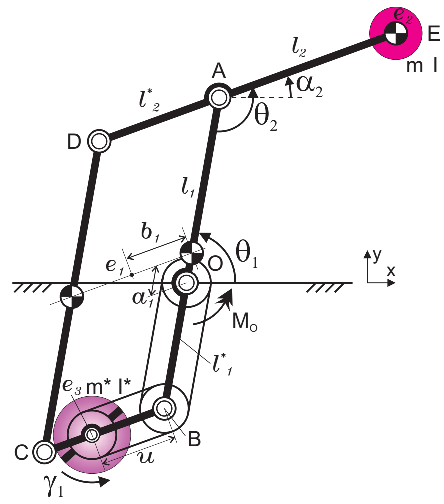

Figure 1: Completely Balanced Double Pendulum by using a single Actively Driven Counter-Rotary Counter-Mass (ACRCM); An additional Actuator drives the Gear at O by which the ACRCM Counter-Rotates with respect to the Mechanism

links are added such that the double pendulum is changed into a pantograph mechanism. The ACRCM with mass m^* and inertia I^* is placed at link BC at a distance u from B such that the COM of the complete mechanism becomes stationary at the origin O.

For the moment balance, the ACRCM is driven by a belt transmission along the gears at B and O. The gear at O is not fixed to the base, but is driven by an actuator that applies a torque M_O to this gear.

For the calculations, the combined mass of link 1 and its parallel link CD m_{e_1} is at e_1, the location of the lumped mass is e_2 and the position of the ACRCM is e_3. For the ease of calculation, the mass of link BC is neglected. However including it is possible. The positions of e_1, e_2 and e_3 can be written in vector notation $[x,y,z]^T$ as:

$$\mathbf{r}_{e_1} = \begin{bmatrix} a_1\cos\theta_1 - b_1\cos\alpha_2 \\ a_1\sin\theta_1 - b_1\sin\alpha_2 \\ 0 \end{bmatrix}$$

$$\mathbf{r}_{e_2} = \begin{bmatrix} l_1\cos\theta_1 + l_2\cos\alpha_2 \\ l_1\sin\theta_1 + l_2\sin\alpha_2 \\ 0 \end{bmatrix} \tag{1}$$

$$\mathbf{r}_{e_3} = \begin{bmatrix} -l_1^*\cos\theta_1 - u\cos\alpha_2 \\ -l_1^*\sin\theta_1 - u\sin\alpha_2 \\ 0 \end{bmatrix}$$

in which α_2 is related to θ_1 and θ_2 by $\alpha_2 = \theta_1 + \theta_2 - \pi$. From the conservation of momentum method (Herder and Gosselin, 2004; Van der Wijk, Herder and Demeulenaere, 2008) it is known that a mechanism is force balanced if the linear momentum of the mechanism is constant and that a mechanism is moment balanced if the angular momentum of the mechanism is constant. With the derivatives of the position vectors, the linear momentum of the

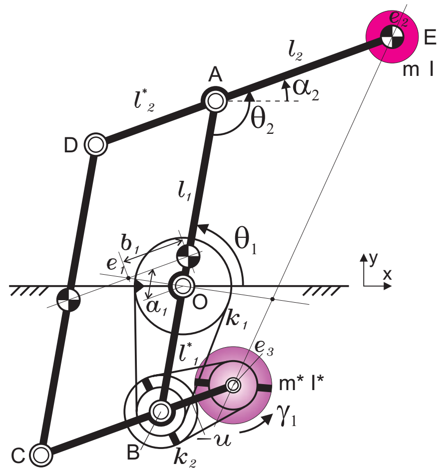

Figure 2: CRCM Balanced Double Pendulum with the Gear at O being fixed to the Base and with Transmission Ratios k_1 and k_2; m and m^* are balanced by the Parallel Link CD

mechanism can be written as:

$$\mathbf{p}_O = m_{e_1}\dot{\mathbf{r}}_{e_1} + m\dot{\mathbf{r}}_{e_2} + m^*\dot{\mathbf{r}}_{e_3} = \begin{bmatrix} (-m_{e_1}a_1 - ml_1 + m^*l_1^*)\dot{\theta}_1\sin\theta_1 + \\ \quad (m_{e,1}b_1 - ml_2 + m^*u)\dot{\alpha}_2\sin\alpha_2 \\ (m_{e_1}a_1 + ml_1 - m^*l_1^*)\dot{\theta}_1\cos\theta_1 + \\ \quad (-m_{e,1}b_1 + ml_2 - m^*u)\dot{\alpha}_2\cos\alpha_2 \\ 0 \end{bmatrix} \quad (2)$$

A constant linear momentum is obtained for the following conditions:

$$m^*l_1^* = m_{e_1}a_1 + ml_1 \quad (3)$$
$$ml_2 = m_{e,1}b_1 + m^*u \quad (4)$$

The angular momentum can be written as:

$$\begin{aligned}\mathbf{h}_{O,z} &= I\dot{\alpha}_2 + I^*\dot{\gamma} + \mathbf{r}_{e_1}\times m_{e_1}\dot{\mathbf{r}}_{e_1} + \mathbf{r}_{e_2}\times m\dot{\mathbf{r}}_{e_2} + \mathbf{r}_{e_3}\times m^*\dot{\mathbf{r}}_{e_3} \\ &= I\dot{\alpha}_2 + I^*\dot{\gamma} + (ml_1^2 + m_{e_1}a_1^2 + m^*l_1^{*2})\dot{\theta}_1 + \\ &\quad (ml_1l_2 - m_{e_1}a_1b_1 + m^*l_1^*u)(\dot{\theta}_1 + \dot{\alpha}_2)\cos(\theta_1 - \alpha_2) + \\ &\quad (ml_2^2 + m_{e_1}b_1^2 + m^*u^2)\dot{\alpha}_2 \end{aligned} \quad (5)$$

It is possible to balance this double pendulum passively, e.g. with the gear at O being fixed to the base and without an extra actuator. The angular momentum then must solely depend on angular velocities and not on the position of the mechanism. Therefore the cosine term in the angular momentum equations must be constant or eliminated, which is the case if at least one of the following conditions is met:

$$\theta_1 - \alpha_2 = constant \quad (6)$$
$$\dot{\theta}_1 + \dot{\alpha}_2 = 0 \quad (7)$$
$$ml_1l_2 - m_{e_1}a_1b_1 + m^*l_1^*u = 0 \quad (8)$$

The first condition implies that link 2 does not move with respect to link 1 for which $\theta_2 = 0$, while the second condition means that the angular velocity of link 2 is equal to that of link 1 but in opposite direction. The third condition implies that the masses m and m^* are balanced by mass m_{e_1} of the parallel linkage, as indicated in Fig. 2. Moment balance is possible for some specific transmission ratios k_1 and k_2 as described in Van der Wijk and Herder (2008).

Contrarily however, it is the intention to balance the mass of the mechanism (linkage with the lumped mass at link 2) with the ACRCM. Moreover, in practice the parallel linkage may need to be as small as possible and therefore have a low mass and not be suitable to use for the force balance.

The moment of the mechanism can be balanced by actively driving the ACRCM. Therefore the angular momentum of the mechanism has to be constant. This means that the ACRCM has to have a specific angular momentum which can be written from Eqn. (5) and is:

$$\begin{aligned} I^*\dot{\gamma} &= -(ml_1^2 + m_{e_1}a_1^2 + m^*l_1^{*2})\dot{\theta}_1 - \\ &\quad (ml_1l_2 - m_{e_1}a_1b_1 + m^*l_1^*u)(\dot{\theta}_1 + \dot{\alpha}_2)\cos(\theta_1 - \alpha_2) - \\ &\quad (I + ml_2^2 + m_{e_1}b_1^2 + m^*u^2)\dot{\alpha}_2 + C \end{aligned} \quad (9)$$

in which C is the constant value of the angular momentum. The ACRCM must be driven with a rotational velocity of:

$$\begin{aligned} \dot{\gamma} &= -\frac{(ml_1^2 + m_{e_1}a_1^2 + m^*l_1^{*2})\dot{\theta}_1}{I^*} - \\ &\quad \frac{(ml_1l_2 - m_{e_1}a_1b_1 + m^*l_1^*u)(\dot{\theta}_1 + \dot{\alpha}_2)\cos(\theta_1 - \alpha_2)}{I^*} - \\ &\quad \frac{(I + ml_2^2 + m_{e_1}b_1^2 + m^*u^2)\dot{\alpha}_2 - C}{I^*} \end{aligned} \quad (10)$$

Driving the ACRCM can be accomplished by controlling the actuator, which is mounted to the base, to drive the gear at O and have it rotate with a prescribed angular velocity. If the motion of the manipulator is known in advance, the angular velocity function of the ACRCM, Eqn. (10), can be precalculated and the ACRCM can be driven with feedforward control. By continuous and accurate detection of the position and velocity of the mechanism, also realtime control is possible. However this is more sensible to distortions for quick alternating motion.

To accelerate the ACRCM, the actuator has to apply a torque M_O to the gear at O. Often it is easier to control the torque of an actuator than its output velocity, since e.g. the torque of a motor is related to the current. The torque that has to be applied to the gear at O can be calculated from the velocity function of the CRCM, Eqn. (10), as:

$$\begin{aligned} M_O &= I^*\ddot{\gamma} \\ &= -(ml_1^2 + m_{e_1}a_1^2 + m^*l_1^{*2})\ddot{\theta}_1 - \\ &\quad (ml_1l_2 - m_{e_1}a_1b_1 + m^*l_1^*u)(\ddot{\theta}_1 + \ddot{\alpha}_2)\cos(\theta_1 - \alpha_2) + \\ &\quad (ml_1l_2 - m_{e_1}a_1b_1 + m^*l_1^*u)(\dot{\theta}_1^2 - \dot{\alpha}_2^2)\sin(\theta_1 - \alpha_2) - \\ &\quad (I + ml_2^2 + m_{e_1}b_1^2 + m^*u^2)\ddot{\alpha}_2 \end{aligned} \quad (11)$$

In fact, this torque is equal but opposite to the shaking moment that the force balanced mechanism exerts to the base, which generally can be obtained from:

$$M_{sh} = \frac{d}{dt}\mathbf{h}_{O,z} \quad (12)$$

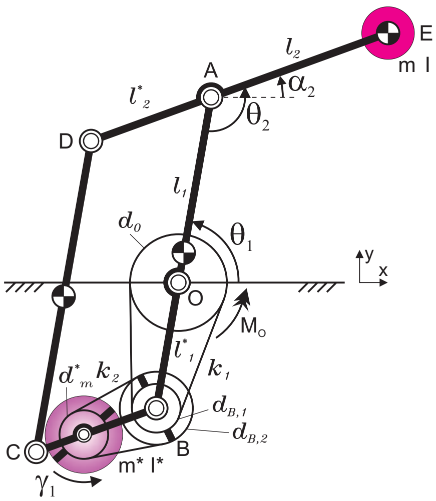

Figure 3: For Active Balancing with Transmission Ratios, the Output Velocity of the Actuator is Reduced

For this case, the shaking moment is zero with M_O of Eqn. (11) and the mechanisms is moment balanced.

2.1 Transmission

By using an ACRCM, the transmission of the motion from the gear at O to the ACRCM can be simple. For instance by parallel belt drives for which the dimensions of the gears are equal as was shown in Fig. 1. However it is also possible to use transmissions with different gears as shown in Fig. 3. The angular velocity of the ACRCM then is influenced by the motion of the linkage and depends on the gear ratios. The angular velocity of the ACRCM dependent on the motion of the mechanism can be calculated by imagining the gear at O being fixed for rotation and becomes:

$$
\begin{aligned}
\dot{\gamma} &= (1 - \frac{d_O}{d_{B,1}})\frac{d_{B,2}}{d^*_m}\dot{\theta}_1 + (1 - \frac{d_{B,2}}{d_{m^*}})\dot{\theta}_2 \\
&= k_1(1-k_2)\dot{\theta}_1 + k_2\dot{\theta}_2 \qquad (13)
\end{aligned}
$$

with d_O, $d_{B,1}$, $d_{B,2}$ and d_{m^*} being the diameter of the gear at O, the small gear at B, the large gear at B, and the gear at ACRCM m^* respectively. k_1 and k_2 are the transmission ratios of the belt transmission of each link. For a parallel transmission, $d_O = d_{B,1}$ and $d_{B,2} = d_{m^*}$ and Eqn. (13) becomes zero. The motion of the ACRCM then is not influenced by the motion of the linkage.

For gear diameters that differ and if the ACRCM is driven by controlling the angular velocity of the actuator, the angular velocity of the actuator is different from the angular velocity of the ACRCM. The angular velocity of the actuator can be calculated by adding Eqn. (13) to the right term of the velocity function of Eqn. (10). The resulting equation can be rewritten by which the angular velocity of the actuator becomes:

$$
\begin{aligned}
\dot{\gamma}_{act} &= \left\{-\frac{ml_1^2 + m_{e_1}a_1^2 + m^* l_1^{*2}}{I^*} - k_1(1-k_2)\right\}\dot{\theta}_1 - \\
&\quad \frac{(ml_1l_2 - m_{e_1}a_1b_1 + m^* l_1^* u)(\dot{\theta}_1 + \dot{\alpha}_2)\cos(\theta_1 - \alpha_2)}{I^*} + \\
&\quad \left\{-\frac{I + ml_2^2 + m_{e_1}b_1^2 + m^* u^2}{I^*}\right\}\dot{\alpha}_2 - k_2\dot{\theta}_2 + \frac{C}{I^*} \qquad (14)
\end{aligned}
$$

For counter-rotations, the transmission ratios k_1 and k_2 are negative and the output velocity of the actuator is reduced. However the moment that the actuator has to apply to the gear at O does not change. This is since the shaking moment depends solely on the mechanism by Eqn. (12) and is not influenced by the design of the transmissions.

The actuator itself however can influence the dynamic balance of the mechanism. If the actuator is a motor, then for an alternating velocity the angular momentum of the rotor is not constant. Hence the motor exerts a shaking moment to the base.

If a motor is driving the gear at O directly, e.g the gear at O is attached to the shaft of the motor, the rotor rotates in the same direction as the ACRCM. This means that the momentum of the ACRCM can be smaller, which can be done by decreasing its inertia or decreasing its angular velocity. If the motor is driving the ACRCM such that it counter-rotates with respect to the ACRCM, then the ACRCM must compensate and must have an increased angular momentum.

2.2 Reduced Inertia and Total Mass

The inertia of the mechanism is defined as the reduced inertia I^{red} (VDI2149, 1999). This is the inertia moment of all elements reduced to the input angles of the mechanism. Since the reduced inertia is an essential characteristic of a mechanism (VDI2149, 1999), it can be used to calculate the increase of inertia by balancing and by comparing different balancing principles.

The inertia of the double pendulum can be reduced to the two input angles θ_1 and θ_2. The reduced inertia per input angle then is defined as the inertia of the moving elements when all other input angles are constant. The reduced inertias of the double pendulum can be calculated by writing the kinetic energy equation of the manipulator for each input angle:

$$
\begin{aligned}
T_O &= \frac{1}{2}I^{red}_{\theta_1}\dot{\theta}_1^2 \qquad (15) \\
T_A &= \frac{1}{2}I^{red}_{\theta_2}\dot{\theta}_2^2 \qquad (16)
\end{aligned}
$$

in which $I^{red}_{\theta_1}$ and $I^{red}_{\theta_2}$ are the reduced inertia moments about O and A respectively. To calculate the reduced inertia about O, the kinetic energy of the complete balanced manipulator can be written as:

$$
\begin{aligned}
T &= \frac{1}{2}I\dot{\alpha}_2^2 + \frac{1}{2}I^*\dot{\gamma}^2 + \frac{1}{2}m_{e_1}[\dot{\mathbf{r}}_{e_1}]^T[\dot{\mathbf{r}}_{e_1}] + \\
&\quad \frac{1}{2}m[\dot{\mathbf{r}}_{e_2}]^T[\dot{\mathbf{r}}_{e_2}] + \frac{1}{2}m^*[\dot{\mathbf{r}}_{e_4}]^T[\dot{\mathbf{r}}_{e_4}] \qquad (17)
\end{aligned}
$$

The squared angular velocity of the ACRCM $\dot{\gamma}^2$ can be written from Eqn. (10) as:

$$
\begin{aligned}
\dot{\gamma}^2 &= \left\{\frac{U\dot{\theta}_1 + V\dot{\theta}_2 + W(2\dot{\theta}_1 + \dot{\theta}_2)\cos\theta_2 + C}{-I^*}\right\}^2 \qquad (18) \\
&= \left\{\frac{X}{-I^*}\right\}^2
\end{aligned}
$$

with:

$$
\begin{aligned}
U &= I + m(l_1^2 + l_2^2) + m_{e_1}(a_1^2 + b_1^2) + m^*(l_1^{*2} + u^2) \\
V &= I + ml_2^2 + m_{e_1}b_1^2 + m^*u^2 \\
W &= -ml_1l_2 + m_{e_1}a_1b_1 - m^*l_1^*u
\end{aligned}
$$

By substituting Eqn. (18) into (17), the kinetic energy can be rewritten as:

$$
T = \frac{1}{2}U\dot{\theta}_1^2 + \frac{X^2}{2I^*} + \frac{1}{2}V(\dot{\theta}_2^2 + 2\dot{\theta}_1\dot{\theta}_2) + W(\dot{\theta}_1^2 + \dot{\theta}_1\dot{\theta}_2)\cos\theta_2 \tag{19}
$$

Assuming the constant angular momentum to be zero ($C = 0$), the kinetic energy T_O is obtained by substituting $\dot{\theta}_2 = 0$ into Eqn. (19) and becomes:

$$
T_O = \frac{1}{2}\left\{U + 2W\cos\theta_2 + \frac{(U + 2W\cos\theta_2)^2}{I^*}\right\}\dot{\theta}_1^2 \tag{20}
$$

The reduced inertia of the mechanism about O then is:

$$
I_{\theta_1}^{red} = U + 2W\cos\theta_2 + \frac{(U + 2W\cos\theta_2)^2}{I^*} \tag{21}
$$

For the reduced inertia about A, the kinetic energy of link 2 must be calculated with $\dot{\theta}_1 = 0$. This equation is written as:

$$
T_A = \frac{1}{2}\left\{V + \frac{V^2}{I^*}\right\}\dot{\theta}_2^2 \tag{22}
$$

and the reduced inertia of the mechanism about A becomes:

$$
I_{\theta_2}^{red} = V + \frac{V^2}{I^*} \tag{23}
$$

Generally for the (A)CRCM-principle, the equations for the reduced inertia can be written in the form:

$$
I_{\theta}^{red} = I_{link} + k^2 I^*
$$

in which I_{link} is the inertia of the force balanced linkage about its joint, I^* the inertia of the (A)CRCM and k the transmission ratio. Comparing this equation to Eqn. (21) and (23), the transmission ratios of the ACRCM can be derived and are:

$$
k_1 = \frac{U + 2W\cos\theta_2}{I^*} \tag{24}
$$

$$
k_2 = \frac{V}{I^*} \tag{25}
$$

The total mass of the ACRCM-balanced double pendulum is calculated with:

$$
m_{tot} = m + m^* + m_{e_1} \tag{26}
$$

2.3 Numerical Example and Comparison

With a numerical example, the ACRCM-balanced double pendulum is compared to the SCR-, CRCM-, and DM-principle. The results for these balancing principles were obtained from Van der Wijk, Herder and Demeulenaere (2008). For a fair comparison, the same parameter values were chosen and are shown in Table 1. Also the mass m_{e_1} of the parallel linkage was chosen to be zero and the mass m and ACRCM m^* were modeled as discs with thickness t and density ρ. The mass and the inertia of the ACRCM then are related as:

Table 1: Parameter Values

$m = 0.3$ [kg]	$l_1 = 0.25$ [m]	$t = 0.01$ [m]
$m_{e_1} = 0$ [kg]	$l_2 = 0.25$ [m]	$\rho = 7800$[kgm^{-3}]
$I = 184$ [kgmm^2]		

Table 2: Results of an ACRCM-Balanced Double Pendulum

m* [kg]	m_{tot} [kg]	I^{red} [kgm^2]	k [-]	l_1*=u [m]
0.9	1.2	$2.04e^{-4} \le I^{red}_{\theta1} \le 6.17$	$0.11 \le k_1 \le 60.6$	0.083
		$I^{red}_{\theta2}=0.41$	$k_2=15.2$	
2.79	3.09	$1.86e^{-4} \le I^{red}_{\theta1} \le 0.52$	$0.01 \le k_1 \le 5.24$	0.027
		$I^{red}_{\theta2}=0.05$	$k_2=1.3$	
3.83	4.13	$1.85e^{-4} \le I^{red}_{\theta1} \le 0.30$	$0.01 \le k_1 \le 2.71$	0.020
		$I^{red}_{\theta2}=0.03$	$k_2=0.7$	

$$
I^* = \frac{m^{*2}}{2\rho\pi t} \tag{27}
$$

Table 2 shows the results for some specific choices of mass m^*. For comparison, Table 3 shows the results for the three balancing principles. The choice for $m^* = 0.9kg$ results into a total mass of $m_{tot} = 1.2kg$, which is equal to the total mass of the DM-principle, the lowest total mass of all principles. However, the maximum inertia $I_{\theta_1}^{red}$ in this case is more than 20 times larger than that of the DM principle. Inertia $I_{\theta_2}^{red}$ is lower than that of the DM-principle and is equal to $I_{\theta_2}^{red}$ of the (passive) CRCM-principle for $k_1 = k_2 = -1$, the lowest of all.

For $m^* = 2.79kg$, the total mass of the ACRCM-principle is equal to that of the CRCM-principle for $k_1 = k_2 = -16$. In this case the maximum inertia for $I_{\theta_1}^{red}$ is more than five times smaller than that of the CRCM-principle while inertia $I_{\theta_2}^{red}$ is about ten times smaller.

The DM principle has the smallest maximum $I_{\theta_1}^{red}$ of all the three passive principles. For an equal maximum value for $I_{\theta_1}^{red}$ of $0.30kgm^2$, the total mass for the active ACRCM-principle becomes $m_{tot} = 4.13kg$. This is about three and a half times more than the DM principle.

From the force balance conditions of Eqn. (3) and (4) and with m^* being known, the dimensions of l_1^* and u can be calculated. These values are also shown in Table 2 and are relatively small. This means that with ACRCMs the mechanism remains compact. For $m_{tot} = 4.13kg$, an ACRCM-balanced double pendulum is drawn to scale in Fig. 4. Since a belt transmission in this figure would be very small and therefore unclear, it was not drawn.

Table 3: Results of Passive Balancing Principles obtained from Van der Wijk, Herder and Demeulenaere (2008)

$k_1=k_2=-1$		CRCM	Separate CR	Duplicate Mechanisms
Total Mass [kg]		16.01	38.78	1.20
Total Inertia [kgm^2]	$I^{red}_{\theta 1}$	0.640	1.362	$7.35e^{-4} \leq I^{red}_{\theta 1} \leq 0.30$
	$I^{red}_{\theta 2}$	0.041	0.083	0.076
$k_1=k_2=-4$				
Total Mass [kg]		6.89	14.33	
Total Inertia [kgm^2]	$I^{red}_{\theta 1}$	0.992	1.275	
	$I^{red}_{\theta 2}$	0.112	0.140	
$k_1=k_2=-8$				
Total Mass [kg]		4.68	9.36	
Total Inertia [kgm^2]	$I^{red}_{\theta 1}$	1.484	1.699	
	$I^{red}_{\theta 2}$	0.213	0.239	
$k_1=k_2=-16$				
Total Mass [kg]		3.09	6.13	
Total Inertia [kgm^2]	$I^{red}_{\theta 1}$	2.650	2.531	
	$I^{red}_{\theta 2}$	0.488	0.461	

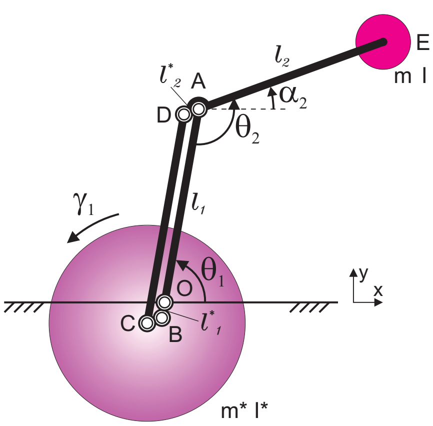

Figure 4: ACRCM-Balanced Double Pendulum with $m^* = 3.83kg$, drawn to scale

3 Evaluation

With the active control of a single counter-rotary counter-mass to balance the mechanism altogether, it was tried to combine the advantages of both the (passive) CRCM-principle and the DM-principle. The former has the advantage to be compact and efficient since the counter-masses are also used as counter-rotations. The latter has the advantage to have the lowest addition of mass and inertia of all balancing principles, however it is a complex and space consuming principle.

Figure 4, which was drawn to scale (with m being a disc with thickness t too) , showed that with the ACRCM-principle the size of the mechanism remains considerably smaller than the size that would be obtained by duplicating the double pendulum twice. Compared to passive balancing principles that need a counter-mass at link 2, the space required for the ACRCM-principle is the smallest of all.

The number of additional elements to balance a double pendulum with the ACRCM-principle is reduced to a minimum. The results of the numerical example showed this is advantageous for the reduction of the additional mass and additional inertia. The ACRCM-principle has a better total mass-inertia relation than balancing with passive CRCMs, or with separate counter-rotations. The mass-inertia relation did however not win from the DM-principle.

The main reason for this is that due to the chosen disc configuration, by reducing the mass of the ACRCM, the inertia of the ACRCM becomes smaller too. Hence the ACRCM must rotate with a higher angular velocity to obtain the necessary angular momentum. This means that the transmission ratio becomes higher which effects the inertia quadratically.

To improve the performance of the ACRCM-principle, the design of the ACRCM should be such that its inertia is high but its mass is low, for instance by using a ring shaped ACRCM. A disadvantage of this is that then the size of the balanced mechanism will become larger. However it is likely that for such a configuration for equal performance of the DM-principle, the size still is much smaller.

4 2-DOF Parallel Manipulator

Figure 5a shows a configuration of a passively balanced planar 2-DOF 2-RRR parallel manipulator which was derived in Van der Wijk and Herder (2008). It has two counter-masses and two CRCMs and the links form a parallelogram. The CRCMs are driven by a belt transmission with a gear at O which is fixed to the base and cannot rotate.

A new configuration is shown in Fig. 5b where the two CRCMs are combined to one ACRCM which is driven by a belt transmission by an actuator at O. This ACRCM balances the complete manipulator. The two counter-masses in the configuration of Fig. 5a can be taken away. They were needed to maintain the reduced inertia about O constant, which is not anymore necessary if active balancing is applied. In the remainder of this section the balancing conditions for this manipulator will be derived.

For the ease of calculation, it is assumed that the combined mass m_{e_1} of the links 1, 2, 3 and 4 is at e_1 and the combined

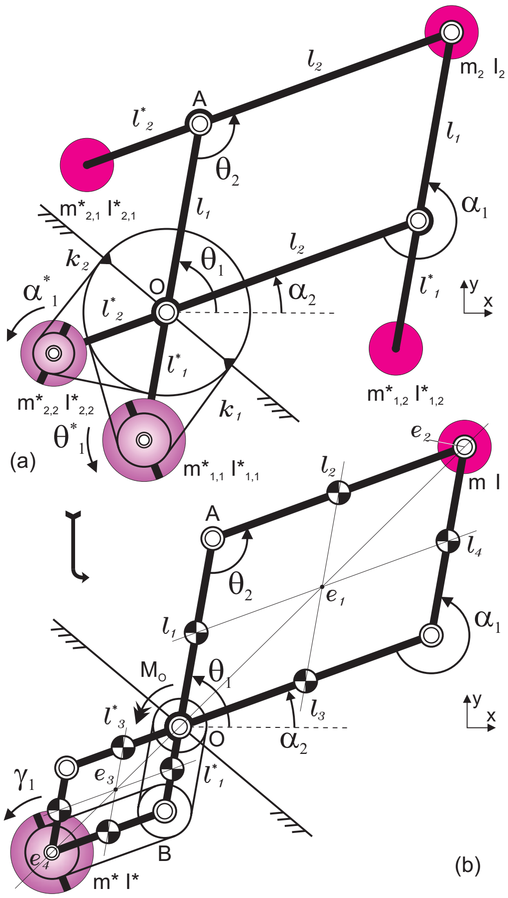

Figure 5: A) CRCM-Balanced Planar 2-DOF 2-RRR Parallel Manipulator (Herder *et al.*, 2006); B) ACRCM-Balanced Configuration by combining the two CRCMs to one ACRCM

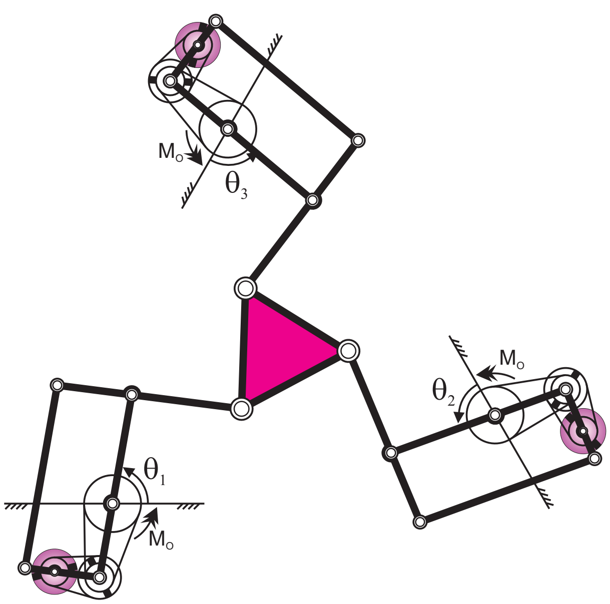

Figure 6: ACRCM-Balanced Planar 3-RRR Parallel Manipulator Synthesized from three ACRCM-Balanced Double Pendula

mass m_{e_3} of the small parallelogram is at e_3. Mass m is at e_2 and the ACRCM m^* is at e_4. To derive the conditions for which the manipulator is force balanced, the positions of e_1, e_2, e_3 and e_4 can be written in vector notation $[x, y, z]^T$ as:

$$\mathbf{r}_{e_1} = \begin{bmatrix} \frac{l_1}{2}\cos\theta_1 + \frac{l_2}{2}\cos\alpha_2 \\ \frac{l_1}{2}\sin\theta_1 + \frac{l_2}{2}\sin\alpha_2 \\ 0 \end{bmatrix}$$

$$\mathbf{r}_{e_2} = \begin{bmatrix} l_1\cos\theta_1 + l_2\cos\alpha_2 \\ l_1\sin\theta_1 + l_2\sin\alpha_2 \\ 0 \end{bmatrix}$$

$$\mathbf{r}_{e_3} = \begin{bmatrix} -\frac{l_1^*}{2}\cos\theta_1 - \frac{l_3^*}{2}\cos\alpha_2 \\ -\frac{l_1^*}{2}\sin\theta_1 - \frac{l_3^*}{2}\sin\alpha_2 \\ 0 \end{bmatrix}$$

$$\mathbf{r}_{e_4} = \begin{bmatrix} -l_1^*\cos\theta_1 - l_3^*\cos\alpha_2 \\ -l_1^*\sin\theta_1 - l_3^*\sin\alpha_2 \\ 0 \end{bmatrix}$$

With the derivatives of the position vectors, the linear momentum of the mechanism can be written as:

$$\begin{aligned} \mathbf{p}_O &= m_{e_1}\dot{\mathbf{r}}_{e_1} + m\dot{\mathbf{r}}_{e_2} + m_{e_3}\dot{\mathbf{r}}_{e_3} + m^*\dot{\mathbf{r}}_{e_4} \\ &= \begin{bmatrix} (-ml_1 - \frac{1}{2}m_{e_1}l_1 + \frac{1}{2}m_{e_3}l_1^* + m^*l_1^*)\dot{\theta}_1\sin\theta_1 + \\ \quad (-ml_2 - \frac{1}{2}m_{e_1}l_2 + \frac{1}{2}m_{e_3}l_3^* + m^*l_3^*)\dot{\alpha}_2\sin\alpha_2 \\ (ml_1 + \frac{1}{2}m_{e_1}l_1 - \frac{1}{2}m_{e_3}l_1^* - m^*l_1^*)\dot{\theta}_1\cos\theta_1 + \\ \quad (ml_2 + \frac{1}{2}m_{e_1}l_2 - \frac{1}{2}m_{e_3}l_3^* - m^*l_3^*)\dot{\alpha}_2\cos\alpha_2 \\ 0 \end{bmatrix} \end{aligned} \tag{28}$$

The conditions for which the mechanism has a constant linear momentum for any motion and is force balanced are:

$$(m + \frac{1}{2}m_{e_1})l_1 = (\frac{1}{2}m_{e_3} + m^*)l_1^* \tag{29}$$

$$(m + \frac{1}{2}m_{e_1})l_2 = (\frac{1}{2}m_{e_3} + m^*)l_3^* \tag{30}$$

If half of the inertia I is assumed to be at link 2 and link 4 (different distributions are possible), the angular momentum of the manipulator can be written as:

$$\begin{aligned} \mathbf{h}_{O,z} &= \frac{1}{2}I\dot{\theta}_1 + \frac{1}{2}I\dot{\alpha}_2 + I^*\dot{\gamma} + \mathbf{r}_{e_1} \times m_{e_1}\dot{\mathbf{r}}_{e_1} + \\ &\quad \mathbf{r}_{e_2} \times m\dot{\mathbf{r}}_{e_2} + \mathbf{r}_{e_3} \times m_{e_3}\dot{\mathbf{r}}_{e_3} + \mathbf{r}_{e_4} \times m^*\dot{\mathbf{r}}_{e_4} \\ &= \frac{1}{2}I\dot{\theta}_1 + \frac{1}{2}I\dot{\alpha}_2 + I^*\dot{\gamma} + \\ &\quad (ml_1^2 + \frac{1}{4}m_{e_1}l_1^2 + \frac{1}{4}m_{e_3}l_1^{*2} + m^*l_1^{*2})\dot{\theta}_1 + \\ &\quad (ml_1l_2 + \frac{1}{4}m_{e_1}l_1l_2 + \frac{1}{4}m_{e_3}l_1^*l_2^* + m^*l_1^*l_2^*) \\ &\quad (\dot{\theta}_1 + \dot{\alpha}_2)\cos(\theta_1 - \alpha_2) + \\ &\quad (ml_2^2 + \frac{1}{4}m_{e_1}l_2^2 + \frac{1}{4}m_{e_3}l_2^{*2} + m^*l_2^{*2})\dot{\alpha}_2 \end{aligned}$$

For the conditions of Eqn. (6) and (7) the manipulator can be balanced in a passive way. By active control of the ACRCM the manipulator is balanced for any motion if the angular momentum of the ACRCM is:

$$\begin{aligned} I^*\dot{\gamma} &= -(\frac{1}{2}I + ml_1^2 + \frac{1}{4}m_{e_1}l_1^2 + \frac{1}{4}m_{e_3}l_1^{*2} + m^*l_1^{*2})\dot{\theta}_1 - \\ &\quad (ml_1l_2 + \frac{1}{4}m_{e_1}l_1l_2 + \frac{1}{4}m_{e_3}l_1^*l_2^* + m^*l_1^*l_2^*) \\ &\quad (\dot{\theta}_1 + \dot{\alpha}_2)\cos(\theta_1 - \alpha_2) - \\ &\quad (\frac{1}{2}I + ml_2^2 + \frac{1}{4}m_{e_1}l_2^2 + \frac{1}{4}m_{e_3}l_2^{*2} + m^*l_2^{*2})\dot{\alpha}_2 + C \end{aligned} \tag{31}$$

The ACRCM then has to be driven with a rotational velocity of:

$$\begin{aligned} \dot{\gamma} &= -\frac{(\frac{1}{2}I + ml_1^2 + \frac{1}{4}m_{e_1}l_1^2 + \frac{1}{4}m_{e_3}l_1^{*2} + m^*l_1^{*2})\dot{\theta}_1}{I^*} - \\ &\quad \frac{(ml_1l_2 + \frac{1}{4}m_{e_1}l_1l_2 + \frac{1}{4}m_{e_3}l_1^*l_2^* + m^*l_1^*l_2^*)}{I^*} \\ &\quad (\dot{\theta}_1 + \dot{\alpha}_2)\cos(\theta_1 - \alpha_2) - \\ &\quad \frac{(\frac{1}{2}I + ml_2^2 + \frac{1}{4}m_{e_1}l_2^2 + \frac{1}{4}m_{e_3}l_2^{*2} + m^*l_2^{*2})\dot{\alpha}_2 - C}{I^*} \end{aligned} \tag{32}$$

The torque that needs to be applied to the gear at O can be calculated with:

$$\begin{aligned} M_O &= I^*\ddot{\gamma} \\ &= -(\frac{1}{2}I + ml_1^2 + \frac{1}{4}m_{e_1}l_1^2 + \frac{1}{4}m_{e_3}l_1^{*2} + m^*l_1^{*2})\ddot{\theta}_1 - \\ &\quad (ml_1l_2 + \frac{1}{4}m_{e_1}l_1l_2 + \frac{1}{4}m_{e_3}l_1^*l_2^* + m^*l_1^*l_2^*) \\ &\quad (\ddot{\theta}_1 + \ddot{\alpha}_2)\cos(\theta_1 - \alpha_2) + \\ &\quad (ml_1l_2 + \frac{1}{4}m_{e_1}l_1l_2 + \frac{1}{4}m_{e_3}l_1^*l_2^* + m^*l_1^*l_2^*) \\ &\quad (\dot{\theta}_1^2 - \dot{\alpha}_2^2)\sin(\theta_1 - \alpha_2) - \\ &\quad (\frac{1}{2}I + ml_2^2 + \frac{1}{4}m_{e_1}l_2^2 + \frac{1}{4}m_{e_3}l_2^{*2} + m^*l_2^{*2})\ddot{\alpha}_2 \end{aligned} \tag{33}$$

5 3-DOF Parallel Manipulators

The configurations of Fig. 3 and 5b can be used for the synthesis of various dynamically balanced 3-DOF planar and spatial manipulators. For instance the planar 3-RRR parallel manipulator with one rotation and two translations of Fig. 6 or the spatial 3-RRR parallel manipulator of Fig. 7 with two rotations and one translation.

As described in Wu and Gosselin (2007), the platforms of these manipulators can be modeled by lumped masses at their joints, maintaining its original mass, the location of the center of mass, and the inertia tensor. This allows each leg to be balanced individually for which their combination is balanced too.

Since the configuration of Fig. 6 rotates within a single plane, one ACRCM can be used to balance the moment of the complete manipulator. As shown in Fig. 8, two of the three ACRCMs can be fixed to their links. This means that only one additional actuator is necessary for balancing this manipulator.

For the configuration of Fig. 7, the rotations of the platform and the links are in two planes. Therefore one of the three ACRCMs can be fixed to its link, as shown in Fig. 9.

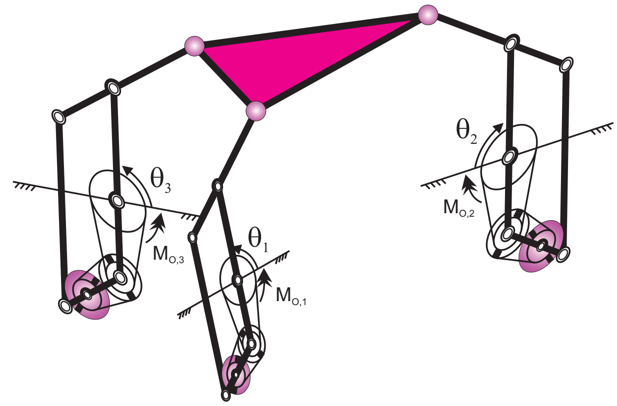

Figure 7: ACRCM-Balanced Spatial 3-RRR Parallel Manipulator Synthesized from three ACRCM-Balanced Double Pendula

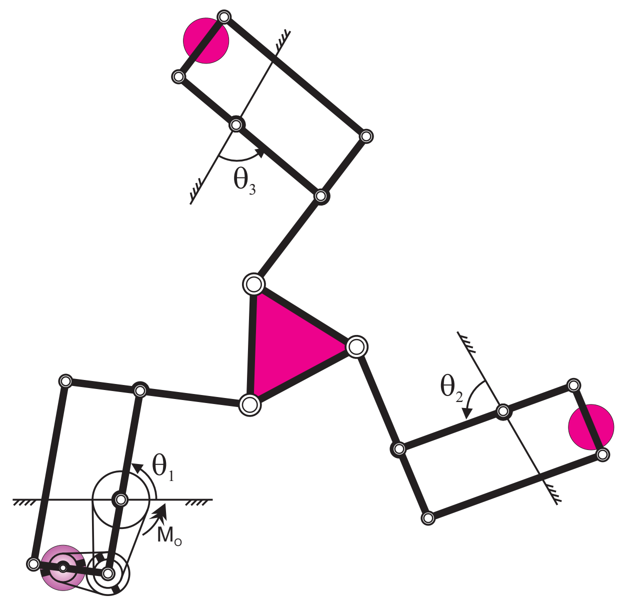

Figure 8: ACRCM-Balanced Planar 3-RRR Parallel Manipulator with one ACRCM for the complete Moment Balance

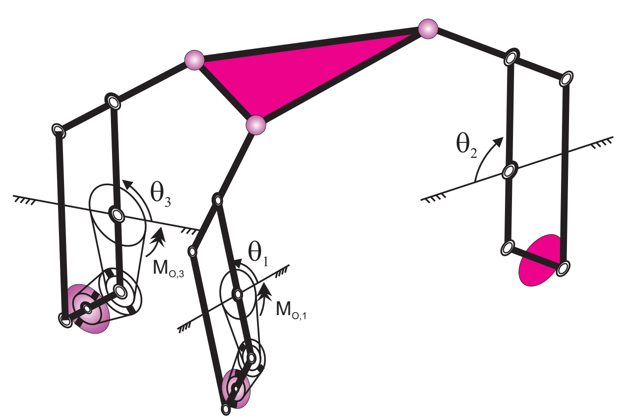

Figure 9: ACRCM-Balanced Spatial 3-RRR Parallel Manipulator with two ACRCMs for the complete Moment Balance

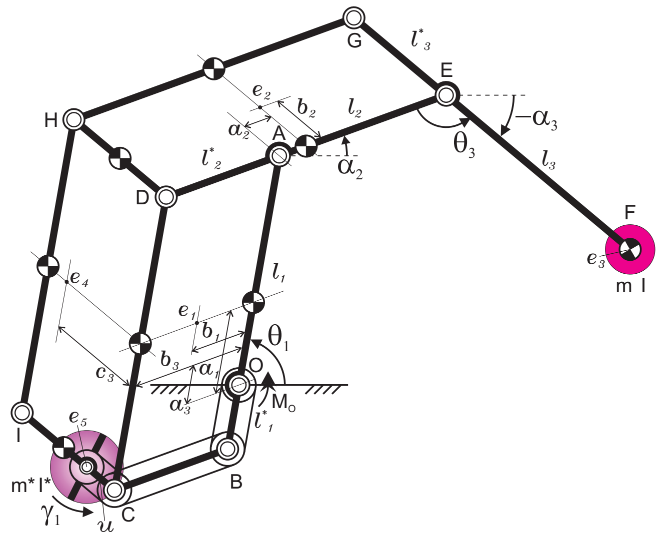

Figure 10: ACRCM-Balanced Planar 3-DOF 1-RRR Serial Manipulator

By fixing some of the ACRCMs, the force balance conditions do not change. However the angular momentum of the remaining ACRCM(s) must be suitable to have the angular momentum of the complete manipulator be constant. For the planar 3-RRR manipulator the angular momentum of the remaining ACRCM has to be the sum of all three former ACRCMs, minus the angular momentum that the former ACRCMs still produce by being fixed to their links. To calculate the angular momentum of each ACRCM of the spatial 3-RRR manipulator is more complicated since the planes in which the ACRCMs move are at an angle.

6 3-DOF Serial Manipulator

As a final example it is illustrated how a 3-DOF planar 1-RRR serial manipulator can be balanced by using a single ACRCM. The configuration of this manipulator is shown in Fig. 10. The initial 'triple pendulum' consists of the links with lengths l_1, l_2 and l_3. A lumped mass m with inertia I is positioned at the endpoint of link 3. The ACRCM with mass m^* and inertia I^* is placed on link CI at e_5 with at a distance u from C and is driven with a belt transmission along gears at C, at B and at O. The gear at O is actively controlled by an additional actuator.

For the calculations, the combined mass m_{e_1} of links AB and CD is at e_1, the combined mass m_{e_2} of links DE and HG is at e_2, the lumped mass is at e_3 and the combined mass m_{e_4} of links CI, IH and DH is at e_4. For the ease of calculation, the mass of link CB in neglected, however including this mass is possible.

To obtain the conditions for which this manipulator is force balanced, the positions of e_1, e_2, e_3, e_4 and e_5 are written in vector notation $[x, y, z]^T$ as:

$$\mathbf{r}_{e_1} = \begin{bmatrix} a_1\cos\theta_1 - b_1\cos\alpha_2 \\ a_1\sin\theta_1 - b_1\sin\alpha_2 \\ 0 \end{bmatrix}$$

$$\mathbf{r}_{e_2} = \begin{bmatrix} l_1\cos\theta_1 + a_2\cos\alpha_2 - b_2\cos\alpha_3 \\ l_1\sin\theta_1 + a_2\sin\alpha_2 - b_2\sin\alpha_3 \\ 0 \end{bmatrix}$$

$$\mathbf{r}_{e_3} = \begin{bmatrix} l_1\cos\theta_1 + l_2\cos\alpha_2 + l_3\cos\alpha_3 \\ l_1\sin\theta_1 + l_2\sin\alpha_2 + l_3\sin\alpha_3 \\ 0 \end{bmatrix}$$

$$\mathbf{r}_{e_4} = \begin{bmatrix} a_3\cos\theta_1 - b_3\cos\alpha_2 - c_3\cos\alpha_3 \\ a_3\sin\theta_1 - b_3\sin\alpha_2 - c_3\sin\alpha_3 \\ 0 \end{bmatrix}$$

$$\mathbf{r}_{e_5} = \begin{bmatrix} -l^*_1\cos\theta_1 - l^*_2\cos\alpha_2 - u\cos\alpha_3 \\ -l^*_1\sin\theta_1 - l^*_2\sin\alpha_2 - u\sin\alpha_3 \\ 0 \end{bmatrix}$$

With the derivatives of the position vectors, the linear momentum of the mechanism can be written as:

$$\begin{aligned} \mathbf{p}_O &= m_{e_1}\dot{\mathbf{r}}_{e_1} + m_{e_2}\dot{\mathbf{r}}_{e_2} + m\dot{\mathbf{r}}_{e_3} + m_{e_4}\dot{\mathbf{r}}_{e_4} + m^*\dot{\mathbf{r}}_{e_5} \\ &= \begin{bmatrix} (-ml_1 - m_{e_1}a_1 - m_{e_2}l_1 - m_{e_3}a_3 + m^*l^*_1) \\ \quad \dot{\theta}_1\sin\theta_1 + (-ml_2 + m_{e_1}b_1 - m_{e_2}a_2 + m_{e_3}b_3 + \\ \quad m^*l^*_2)\dot{\alpha}_2\sin\alpha_2 + (-ml_3 + m_{e_2}b_2 + m_{e_3}c_3 + \\ \quad m^*u)\dot{\alpha}_3\sin\alpha_3 \\ (ml_1 + m_{e_1}a_1 + m_{e_2}l_1 + m_{e_3}a_3 - m^*l^*_1) \\ \quad \dot{\theta}_1\sin\theta_1 + (ml_2 - m_{e_1}b_1 + m_{e_2}a_2 - m_{e_3}b_3 - \\ \quad m^*l^*_2)\dot{\alpha}_2\sin\alpha_2 + (ml_3 - m_{e_2}b_2 - m_{e_3}c_3 - \\ \quad m^*u)\dot{\alpha}_3\sin\alpha_3 \\ 0 \end{bmatrix} \end{aligned} \tag{34}$$

The conditions for which the mechanism has a constant linear momentum for any motion and is force balanced then are:

$$m^*l^*_1 = ml_1 + m_{e_1}a_1 + m_{e_2}l_1 + m_{e_3}a_3 \tag{35}$$

$$m^*l^*_2 = ml_2 - m_{e_1}b_1 + m_{e_2}a_2 - m_{e_3}b_3 \tag{36}$$

$$m^*u = ml_3 - m_{e_2}b_2 - m_{e_3}c_3 \tag{37}$$

With these conditions and since the manipulator is planar, the angular momentum does not have to be calculated to prove that this 3-DOF serial manipulator can be balanced by a single ACRCM. The procedure to calculate the velocity function and the torque function of this ACRCM is equivalent to the procedure of the ACRCM-balanced double pendulum.

7 Discussion

The former sections showed how an ACRCM-balanced double pendulum can be used kinematically and dynamically to synthesize and balance various mechanisms. The discussion of the balancing conditions and the results of a numerical example and a comparative study of the of the ACRCM-balance double pendulum with non-actively balanced double pendula, was done in the section 'evaluation'. The active control of the ACRCM however, was not yet discussed. This active control has some advantages and disadvantages

If the links collide with each other or with the base (internal collisions), accelerations can be very high. With the passive CRCM balanced configurations, this was not a problem since the CRCM was mechanically constrained to move with the right velocity for which the momentum of the mechanism was conserved. However controllers need time to detect changes and the more rapid situations change, the more difficult it is to interact.

On the other hand, if external forces act on the mechanism, the CRCM could be used to compensate the resulting shaking moment by changing its velocity. External forces can be for instance forces due to the transportation of cables that come from the environment and are necessary for the end effector. In fact, by actively driving the CRCM there are two separate mechanisms. The linkage is one mechanism and the ACRCM is a separate mechanism of which the shaking moment can be controlled such that it balances (or compensates) the shaking moment of the force balanced linkage.

Drift of the angular velocity of the ACRCM could influence the moment balance of the mechanism. For instance if the actuator is controlling the velocity of the ACRCM and without input of the actuator the ACRCM is already rotating with a constant velocity (offset). A constant velocity itself does not influence the moment balance, since the angular momentum then is constant. However, to reach a prescribed velocity of the ACRCM, the acceleration will be different and hence the applied torque will not compensate the shaking moment and will lead to unbalance.

This problem does not occur when the ACRCM is driven by controlling the torque applied to the ACRCM. If the torque applied to the ACRCM is as prescribed, the angular velocity of the ACRCM is not of importance. Since the torque balances the shaking moment, the ACRCM can rotate with any velocity offset.

Another advantage of an ACRCM by using a prescribed torque is the ability to compensate for the elasticity, e.g. the elasticity of the belts of the transmissions. Due to this elasticity the acceleration of the ACRCM lags. However since the prescribed torque already balances the shaking moment, this does not anymore matter.

Experiments on these subjects together with the design of the controller (which was not treated here) will be topics of future research, with the aim to build a prototype of a ACRCM-balanced double pendulum and test it.

8 Conclusion

This article proposed to force balance mechanisms with the minimum number of counter-masses and to use the inertia of these counter-masses for the moment balance by actively controlling their rotations. It was shown how a single actively driven counter-rotary counter-mass (ACRCM) can be used to dynamically balance a double pendulum. The force balance of the complete mechanism is obtained by adding a single counter-mass. By having this counter-mass counter-rotate with respect to the mechanism with the right angular velocity, the moment of the mechanism is balanced. The angular velocity of the ACRCM is controlled by an additional actuator which is mounted on the base.

The velocity-function and the torque-function of the actuator were calculated. The ACRCM-principle was compared to other balancing principles and by using the ACRCM-balanced double pendulum as building element, various useful ACRCM-balanced 1-, 2-, and 3-degree-of-freedom planar and spatial, serial and parallel mechanisms were synthesized.

The relation between the total mass and the reduced inertia of the ACRCM-principle is better than balancing with nonactive counter-rotary counter-masses or with counter-masses and separate counter-rotations. A trade off between the addition of mass and the addition of inertia remains also for the ACRCM-principle. The relation between the total mass and the reduced inertia by duplicating the mechanism still is better, however the ACRCM-principle can be improved by changing the design of the ACRCM. In addition, the size of the balanced mechanism with an ACRCM is considerably smaller than by duplicating the mechanism.

Another advantage of the ACRCM-principle is the ability to compensate for disturbances that effect the moment balance. By controlling the applied torque to drive the ACRCM, drift and influence of elasticity within the transmission do not cause unbalance. Disadvantages are the addition of a controlled actuator and difficulties for high accelerations as for example occur due to impact.

A planar 3-RRR parallel manipulator and a 3-DOF planar 1-RRR serial manipulator were dynamically balanced with a single ACRCM. A spatial 3-RRR parallel manipulator was dynamically balanced with two ACRCMs.

Nomenclature

I inertia
I^{red} reduced inertia
m mass
l link length
α absolute angle of link with respect to reference frame
θ relative angle between two links
γ ACRCM angle
$\mathbf{r}$ position vector
d gear diameter
e COM of a combination of masses
$(.)^*$ balance property
$\mathbf{p}_O$ linear momentum about the origin
$\mathbf{h}_O$ angular momentum about the origin
k transmission ratio
T mechanism's kinetic energy
M applied torque
M_{sh} shaking moment

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