INTRODUCTION
Braiding is a very old technique, mostly used in the textile and cable industry. Automated braiding is also a suitable process for manufacturing reproducible preforms for resin transfer moulding. Due to the highly interlaced structure of a braid, components with sharp curvatures and non-circular cross-sections can be covered. Furthermore, the interlaced nature of braids provides high levels of impact strength. So far, it was by no means trivial to predict the mechanical properties of a braid reinforced product, firstly because the fibre directions could not be predicted in advance. Different models were developed for simple circular braiding (1), (2). These models cannot be used for complex braided shapes such as the preforms indicated. This paper presents a model for braiding complex shapes.

PROCESS DESCRIPTION
An illustration of a braiding machine is given in fig. 1. The mandrel, supported by a holder (not shown in the illustration), is located between the spools. The mandrel moves with an axial velocity, \( V \). The yarns are driven by spools in the spool plane. In the braiding operation, all yarns move with their spools and warp around the surface of the mandrel. One group of yarns, the warp yarns, move clockwise and the weft yarns move counter-clockwise, both with an angular velocity of \( \pm \omega \). These two yarn groups interlock to form a biaxial fabric on the mandrel. A guide ring with radius \( R_g \) leads the yarns towards the mandrel. The yarns converge to the mandrel and touch the mandrel at a distance \( H \) from the guide ring. The point where they touch the mandrel is called the fell point.

![Diagram of a braiding machine](image)

*Fig.1: Braiding machine*
MODEL ASSUMPTIONS
A mathematical model was developed to describe the braiding process and to predict the resulting fibre directions on a complex shape. This model is based on the following assumptions:
1. The yarn trajectory is continuous and differentiable. The yarn trajectory has a certain smoothness without any kinks.
2. The fibre paths have to lie on the mandrel surface.
3. After the yarns touch the mandrel, relative motion between the yarns stops. The interlacing pattern permits the yarns to follow non-geodesic curves on the mandrel (1).
4. The yarns are straight in the convergence zone. The interaction between the yarns in the convergence zone is neglected. The model presented in (2) showed that this is close to reality for the braider, used in the experiments.
5. At the fell point, the yarns have to be tangent to the surface of the mandrel (2).

DEFINITIONS
Fig. 2 shows a model of a braiding machine with a complex mandrel. Because it is assumed that the yarns have no interaction, it is sufficient to describe one single yarn at a time. In this case, it can also be assumed that the spools rotate over the guide ring with radius $R_{g}$, instead of rotating on the spool plane. In fig. 2, the position of the spool, $\tilde{q}$, the surface of the mandrel, $Q$, and the fell point of the yarn, $\tilde{p}$, can be seen. The angle between the path of the yarn and the tangent line of the surface in z-direction is the braid angle, $\alpha$.

![Fig. 2: Model of a braiding machine with a complex mandrel](image)

The surface of the mandrel can be defined as a function $Q$ with for every point $\tilde{x}$ on the surface:

$$O(\tilde{x}) = 0 \quad (1)$$

The path of the yarn on the mandrel, as shown in fig. 2, can be defined as a trajectory $\tilde{f}$. The fell point, $\tilde{p}(t)$, moves along this trajectory in time. The fell point is therefore given by:

$$\tilde{f}(\tilde{p}(t)) = \tilde{0} \quad (2)$$

The coordinate system applied in fig. 2 is fixed to the mandrel. So the fell point, $\tilde{p}$, does not change when the mandrel is displaced or rotated.
MATHEMATICAL RELATIONS

The five assumptions can be translated into three mathematical relations. First of all, the yarn path has to be continuous and differentiable. This implies that the direction of the fibre path at the fell point, \( \tilde{p} \), has to be equal to the direction of the yarn path in the convergence zone:

\[
\frac{\partial}{\partial t} \tilde{p} = \lambda \cdot (\tilde{p} - \tilde{q})
\]

(3)

Secondly, the fibre path has to lie on the mandrel. This implies:

\[ Q(\tilde{p}) = 0 \]

(4)

At the fell point, the relative motion stops. At this point the yarn is tangent to the surface. This can be written as:

\[ (\tilde{p} - \tilde{q}) \cdot \nabla Q(\tilde{p}) = 0 \]

(5)

MODEL IMPLEMENTATION

The three mathematical relations can be solved numerically, resulting in the positions, \( \tilde{p} \), of the yarns along their trajectory. The model is implemented using the MATLAB programming environment. A geometric simplification is used to find the next fell point. By estimating the \( z \)-coordinate of the fell point in advance (using a forward Euler scheme), it is sufficient to find a tangent line on a curve in a plane, instead of finding a tangent line on the mandrel surface in three-dimensional space. The tangent line on the cross-section, and the direction of the yarn path, both in the previous fell point, give a first prediction of the next fell point. This prediction is corrected with the position of the tangent line at the predicted fell point position. If these values differ much, the calculation can be performed again, until the difference is smaller than the allowable tolerance. The model shows quadratic convergence with decreasing step size.

![Figure 3: Test mandrels](image)

EXPERIMENTS

Experiments with two mandrels have been carried out to test the numerical model. A 96-spool braider of EuroCarbon in the Netherlands was used for these experiments. A 3-D view of the two mandrels is given in fig. 3. The first mandrel has both a circular and square cross-section and is clamped eccentrically in the braiding machine. The second mandrel is more complicated. The cross-section of this mandrel changes its size and shape, and the mandrel has both centric and eccentric parts. The corners of both mandrels were rounded off with a 15 mm radius.
Figure 4 shows the experimental and numerical results for the two mandrels. The braid angles on the mandrels are plotted along the side, where \( y=0 \) and \( x>0 \). The mandrels are symmetric around the \( x \)-axis, so the results of the warp and weft yarns should be the same. Figure 4 shows that the experimental results are close to the numerical predicted ones.

**Fig.4a: Results of mandrel 1**

**Fig.4b: Results of mandrel 2**

The results of mandrel 2 show that at \( z=200 \ldots 350 \) the experimental values are higher than the predicted ones. During the experiments it was observed that the yarns slid downwards due to a low degree of coverage (the percentage of yarn surface on the mandrel), resulting in an increase of the braid angle. This movement of the yarns, after they are deposited, is not taken into account in the model and is therefore not predicted.

**CONCLUSIONS**

For designing braided composite structures, it would be desirable to determine the braiding machine parameters out of the product properties. A first step for such a model is to predict the braid structure of preforms out of the machine parameters. Such a model was developed and presented in this paper. The experimental results show good agreement with the calculated values. Problems can occur when the braid structure has a low fibre density. The yarns can then still move after they are deposited. The current model is unable to predict these displacements. In practice, the degree of coverage will be much higher than in these experiments. For these practical cases, the developed model gives a good prediction of the fibre directions in the braided composite, even if the product has non-circular cross-sections and sharp curvatures.

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**REFERENCES**