DIRECT NUMERICAL SIMULATION OF THE LIFT FORCE IN BUBBLY FLOWS

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ABSTRACT

It is well-known that the lift force is responsible for the segregation of small and large bubbles encountered in bubbly flows through pipes and bubble columns: in the case of up-flow small spherical bubbles move to the wall, while larger deformed bubbles move to the core region. Depending on the fluid properties there is a transition at a certain bubble diameter, which is extremely critical if one wants to predict the correct circulation pattern and gas-holdup. However, until now quantitative knowledge about this force is limited to spherical bubbles (Legendre & Magnaudet, 1998) and deformed bubbles in moderately viscous liquids (Tomiyama, 1998). Therefore, this work focuses on extending the knowledge on the lift force, bridging the gap towards a wide range of bubble diameters as well as less viscous liquids, such as the industrially important air-water system, using direct numerical simulations (DNS).

To enable numerical simulation of small bubbles at high density ratios, the surface tension treatment of a 3D Front Tracking model has been significantly improved. Also its numerical implementation has been carefully optimized to reduce computation time, to be able to efficiently run the large number of cases required in this study. The numerical simulations have been carried out using a cubic computational domain consisting of one million grid cells, which yields good resolution at reasonable calculation time (typically two weeks on a single CPU). The initially spherical bubble is placed in the centre of the computational domain and a window shifting technique assures that it keeps this position. The top, left and right boundaries are used to enforce the linear shear field, using inflow and no-slip boundary conditions respectively. They are supplemented by a prescribed pressure outflow boundary at the bottom, where the liquid is free to exit the domain, and free-slip boundaries at the front and rear.

First of all, the results confirm that small spherical bubbles move to the high negative velocity side (wall region), while large deformed bubbles move in the opposite direction. The transition in the lift force is accompanied by a slanted wake structure behind the larger bubbles. Secondly, the numerical values of the lift coefficient show a good agreement with Tomiyama et al. (2002) for moderate to low viscosity liquids.

Surprisingly, at higher viscosities there is a very significant discrepancy. Finally, it was found that both the drag and lift force coefficients are not a function of the shear rate.

Keywords: CFD, DNS, hydrodynamics.

NOMENCLATURE

A Cross-sectional area of the bubble [m²]
C Coefficient
d Equivalent bubble diameter [m]
D Distribution function
\(E_o\) Eötvös number
\(E_{o_h}\) Horizontal Eötvös number
\(F\) Phase fraction
\(F\) Force [N], Force density (Navier-Stokes) [N·m⁻³]
\(g\) Gravitational constant/vector [m·s⁻²]
\(G\) Phase fraction gradient [m⁻¹]
h Height [m]
\(J\) Function of the Re and Sr
n Normal vector
\(Re\) Reynolds number
S Surface area
\(Sr\) Shear ratio
\(t\) Time [s]
\(u\) Continuous phase velocity [m·s⁻¹]
\(v\) Bubble velocity [m·s⁻¹]
\(V\) Bubble volume [m³]
w Width [m]
\(\alpha\) Angle of attack [rad]
\(\Delta x\) Grid size [m]
\(\mu\) Viscosity [Pa·s]
\(\rho\) Density [kg·m⁻³]
\(\sigma\) Surface tension coefficient [N·m⁻¹]
\(\tau\) Viscous stress tensor [N·m⁻³]
\(\omega\) Shear rate [s⁻¹]

Subscripts
B Buoyancy
D Drag force
G Gas
H Horizontal
L Liquid, lift force
m Marker
\(x, y\) Horizontal directions
z Vertical direction
\(\sigma\) Surface tension
INTRODUCTION

Multiphase gas/liquid and gas/liquid/liquid flows are widely encountered, in natural phenomena as well as in industry. For instance, the oil industry has to deal with complex flows consisting of oil droplets and gas bubbles dispersed in water. More examples include Fischer-Tropsch and other important chemical processes. Because of the wide range of length and time scales, it is virtually impossible to capture all the details of the flow field with currently available computational resources. Therefore a successful description of multiphase flows therefore has to be based on a sound multi-level modeling approach (van Sint Annaland et al., 2003):

![Multi-level modeling approach](image)

**Figure 1:** Multi-level modeling approach for multiphase dispersed gas-liquid flow. The exchange of information is indicated by arrows.

At the smallest time and length scale a Direct Numerical Simulation (DNS) is used to study the behavior of a single or a few gas bubbles or liquid droplets. These simulations backed up by dedicated detailed experiments can be used to derive closures for the bubble-liquid interaction, which can then be used in higher level models. One step up, the Euler-Lagrange model can be used to study the interactions between a large number of bubbles and the influence of these interactions on the macroscopic flow structure. In this model each bubble is represented in a discrete fashion and the forces on each bubble are computed from closure equations. In this approach a large number of bubbles (~100,000) can be simulated with acceptable computation time. However, in industrial applications multi-phase flows with even a much higher number of dispersed elements are encountered, which requires a continuum approach. At this highest level of modeling the Euler-Euler or multi-fluid continuum models, bubbles lose their discrete identity, which enables the simulation of very large systems and study large-scale heterogeneous structures in the flow.

It has proven to be a daunting task to accurately describe the behavior of gas-liquid or liquid-liquid systems with the higher level models, because detailed knowledge on the behavior of single bubbles or droplets in complex flow fields is lacking. For example, even the behavior of a single air bubble rising in quiescent water is not yet completely understood: not only physical properties like the density, viscosity and surface tension affect the behavior of the bubbles, but also small amounts of surface active impurities (Clift et al., 1976). More recently, Wu and Gharib (2002) and Tomiyama et al. (2002) independently pointed out that the initial shape of the bubble can affect its terminal rise velocity. This illustrates the intrinsic complexities in performing dedicated experiments.

The problem in the description of the motion of a single bubble or droplet arises from the complex interaction between the bubble shape dynamics and the flow field in its vicinity. This is particularly difficult at high Reynolds numbers, which are encountered in the industrially important case of dispersed elements in water. With the advances that have been made in CFD during the last decades, now the shape and interface dynamics can be studied in great detail. In this study DNS is used to study the behavior of air bubbles rising in water.

When it comes to DNS several models have been proposed and used in the literature, where it is important to realize that every model has its own strong and weak points (van Sint Annaland et al., 2006). By far the most popular model is the Volume of Fluid (VOF) model, which typically involves reconstruction of the interface using the spatial distribution of the volume fraction of the phases. The major advantage of this model is that it is relatively easy to implement and the volume of the dispersed elements is very well conserved. However these advantages come at a high cost: the interface is not explicitly tracked, but has to be reconstructed from the phase fractions. First of all this causes problems when calculating the surface tension force, which is a singular force acting on the interface. Secondly a poor interface reconstruction combined with a large density ratio may cause the numerical method to become unstable. Also parasitic currents in the vicinity of the interface may develop. These drawbacks of the VOF method are especially limiting for small air bubbles (~1 mm) in water, where a high density ratio and a high surface tension force are combined.

In this work a full 3D Front Tracking (FT) model is used, based on the work of Unverdi and Tryggvason (1992). The advantage of this model is that the interface is explicitly tracked by interconnected points which form triangular markers. In sharp contrast with VOF this makes it possible to describe the shape and location of the interface with a very high accuracy. The first benefit is that the accuracy of the surface tension force calculation can be improved (Popinet and Zaleski, 1998). Secondly, because there is no interface reconstruction, parasitic currents are greatly reduced. However, this comes at a price: the volume of the dispersed phases is not intrinsically conserved and because of deformation, marker points have to be periodically added and removed (surface remeshing). For a detailed comparison of different DNS methods the
interested reader is referred to Scardovelli and Zaleski (1999).

3D Front Tracking was used in this work to calculate the lift force directly, without the need for any kind of closures. The lift force is responsible for the segregation of small and large bubbles in bubbly flows: in the case of up flow small bubbles move to the wall, while large bubbles move towards the core region (Fig. 2). It has been demonstrated numerically (Legendre & Magnaudet, 1998) that for spherical bubbles at low Reynolds numbers, the lift force results from both viscous and pressure effects. For spherical bubbles at high Reynolds numbers, the pressure effect dominates and the sign of the lift force coefficient is positive. On the other hand, for large deformed bubbles it is well-known that the sign of the lift force coefficient becomes negative. Very little is known about the mechanism of this lift inversion, which may be caused by a combination of effects related to the bubble shape, its orientation and modification of the wake structure.

The results of the numerical simulations are compared to lift force closures from literature, which were obtained from analytical theory, numerical simulations and experiments. In all of our simulations realistic physical properties were used, for instance a density ratio of 800 for air bubbles in water. Before this was possible, some modifications had to be made to the original model, in order to improve mass conservation for small air bubbles in water. These modifications were extensively verified using standard test cases as reported by van Sint Annaland et al. (2006).

### Existing Lift Correlations

Equations for the lift force for the two limiting cases of low and high Reynolds numbers have been analytically derived for spherical bubbles. Saffman (1965) was the first to derive an expression for the lift force acting on a slowly rotating sphere in the limit of zero Reynolds number and infinite shear ratio (Sr). This result was extended to arbitrary shear rates by McLaughlin (1991), which allows it to be used for practical purposes (Eq. 1). It can be seen that the lift coefficient at low Reynolds numbers is strongly dependant on both the Reynolds number ($Re$) as well as the shear ratio (Sr).

$$C_L = 1.37 \frac{J(Re, Sr)}{Re Sr} \quad Re \ll 1$$  \hspace{1cm} (1)

Mei & Klausner (1994) showed – using the same methodology as McLaughlin – that the lift force for a bubble is simply $2/3$rd of that of a rigid sphere. Legendre & Magnaudet (1997) found an error in their derivation and derived that the lift force coefficient for a bubble should be $4/9$th of the rigid sphere solution:

$$C_L = 0.255 \frac{6 J(Re, Sr)}{Re Sr} + 0.2 \frac{Re}{Sr} \quad Re \ll 1$$  \hspace{1cm} (2)

At the high Reynolds limit, Auton (1987) showed that the lift coefficient of a spherical bubble or particle is equal to a constant value of 1/2. Legendre & Magnaudet (1998) bridged the gap between these two limiting analytical solutions (Eq. 3) by simulating spherical bubbles in a weak linear shear field at Reynolds numbers between 0.1 and 500. Especially the low Reynolds limit proved to be very challenging, because of the strong influence of the size of the computational domain. Their empirical correlation reduces to the analytical solutions at both extremes and is valid for all Reynolds numbers.

$$C_L = \left( \frac{6J(Re, Sr)}{\pi \sqrt{Re Sr}} \right)^{2/3} \left( \frac{1}{2} \frac{Re + 16}{Re + 29} \right) \quad Sr \ll 1$$  \hspace{1cm} (3)

Results for the lift force on deformed bubbles on the other hand are extremely scarce, since the level of complexity increases substantially. Analytically, the analysis by Auton was extended to ellipsoidal bubbles moving along their minor axis by Naciri (1992), who found a positive lift coefficient, demonstrating that deformation by itself does not change the direction of the lift force.

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**Figure 2:** Two limiting cases of the lift force: small spherical bubbles move to the high negative velocity side, while large deformed bubbles move in the opposite direction.

In the following paragraphs, first the most important lift force correlations from literature are described and their applicability is discussed. Secondly, based on a force balance for a discrete bubble in a shear flow, equations are derived to obtain the drag and lift force coefficients. In the remainder of this chapter, the numerical aspects are described, followed by the results.
Experimentally, the most significant work has been carried out by Tomiyama et al. (2002), who used a linear shear field in viscous liquids ($10^{0.5} \leq \text{Mo} \leq 10^{2.8}$). The corresponding closure for the lift force (Eq. 4) is based on a modified Eötvös number (Eq. 5) and reduces to zero for very small bubbles. Hibiki & Ishii (2007) combined the experimental data by Tomiyama et al. with Eq. 3, aiming at extending the range of applicability of this correlation. Unfortunately, in their paper they have to conclude that there is no literature data available to test the validity at higher Reynolds numbers (i.e. air-water system).

$$C_i = \text{MIN} \left( 0.288 \tanh \left( 0.121 \text{Re} \right), f \left( E_{o_n} \right) \right)$$

$$f \left( E_{o_n} \right) = 0.00105 E_{o_n} - 0.0159 E_{o_n}^2 - 0.0204 E_{o_n} + 0.474$$

$$E_{o_n} = \frac{(\rho_i - \rho_o) g d_n^2}{\sigma}$$

Numerically, Bothe et al. (2006) obtained lift coefficients slightly lower than predicted by the correlation by Tomiyama, using similar viscous liquids. Secondly, they conclude by cubic extrapolation that the lift force coefficient at low Eötvös numbers approaches 0.5, which is the analytical solution for a spherical bubble at high Reynolds numbers. It would be interesting to see whether the lift coefficient also approaches ½ for the case of a much lower viscosity liquid (i.e. water). For more viscous liquids, there should be no limiting value for the lift coefficient, as it depends strongly on the Reynolds number (Eq. 3).

**FRONT TRACKING MODEL**

**Governing equations**

In the FT model the Navier-Stokes equations are solved together with the continuity equation for incompressible media:

$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial}{\partial t} (\rho \mathbf{u}) + \nabla \cdot \rho \mathbf{u} \mathbf{u} = -\nabla p + \rho \mathbf{g} +$$

$$\nabla \cdot \left[ \left( \nabla \mathbf{u} + (\nabla \mathbf{u})^T \right) \right] + \mathbf{F}_\sigma$$

where the density $\rho$ and the viscosity $\mu$ are locally averaged over all the phases present, based on the phase fraction $F_i$. The surface tension force is included as a volumetric force density $\mathbf{F}_\sigma$ acting only in the vicinity of the interface.

The Navier Stokes equations are solved on a staggered Cartesian mesh with a finite volume technique using an implicit treatment of the pressure gradient and an explicit treatment of the convection and diffusion terms. For the convection term a second order flux delimited Barton scheme is used (Centrella and Wilson, 1984) and for the diffusion term a standard second order finite difference scheme is used. To be able to simulate large density ratios, the Navier-Stokes equations are rewritten in their non-conservative form using the continuity equation (Van Sint Annaland et al., 2003):

$$\rho \left[ \frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot \mathbf{u} \mathbf{u} \right] = -\nabla p + \rho \mathbf{g} +$$

$$\nabla \cdot \left[ \left( \nabla \mathbf{u} + (\nabla \mathbf{u})^T \right) \right] + \mathbf{F}_\sigma$$

A two step projection-correction method is used to solve the two equations: first the velocity is calculated using all the explicit terms in the Navier-Stokes equations and secondly a robust ICCG method is used to calculate the pressure correction to satisfy the incompressibility constraint.

**Average fluid properties**

For the local density linear weighing of all the phase fractions is used:

$$\rho = \sum_{i=1}^{n} F_i \rho_i$$

where $F_i$ represents the fraction of phase i. Usually the viscosity is also linearly averaged, but here a more fundamental approach is used based on harmonic averaging of the kinematic viscosities (Prosperetti, 2001):

$$\frac{\rho}{\mu} = \sum_{i=1}^{n} F_i \frac{\rho_i}{\mu_i}$$

**Surface tension**

Making direct use of the triangulation of the interface, the surface tension force acting on marker $m$ is calculated via a contour integral over the tensile forces (see Fig. 3):

$$\mathbf{F}_{\sigma,m} = \frac{\sigma}{\Delta x \Delta y \Delta z} \oint (\mathbf{t}_m \times \mathbf{n}_m) dl$$

where $\mathbf{t}_m$ is the counter clockwise unit tangent vector along the edges of the marker $m$ and $l$ is the length of these tangent vectors (the perimeter of the marker).

**Figure 3**: Schematic illustration of the direct surface tension force calculation.
This method avoids the computation of the numerically inaccurate curvature and can be used for surfaces with a very high curvature with less numerical instability and better accuracy. The surface tension force is mapped on to the Eulerian grid using a summation over all the markers and their edges l:

\[ F_m = \sum_{m} \sum_{l} D(x - x_m) \sigma \left( t_{m,l} \times n_{m,l} \right) \Delta x \Delta y \Delta z \]  

(12)

where \( t_{m,l} \) is the tangential vector and \( D \) is the distribution kernel, for which in this work density weighing (Deen et al., 2004) is used. Density weighing avoids mapping the surface tension force to a cell with a low mass, which can cause large distortions of the velocity field near the interface. Tryggvason et al. (1992) is followed. Traditionally, in the FT model the phase fractions are calculated by a method proposed by Unverdi and Tryggvason (1992):

\[ \nabla^2 F = \nabla \cdot G \\
G = \sum_{m} D(x - x_m) n_m \Delta x_m \]  

(15)

where \( n_m \) is the outwards pointing normal and \( \Delta x_m \) is the surface area of the marker. First the gradient \( G \) is calculated from the interface markers, after which an ICCG method is used to solve this Poisson equation. It was found that this method smears out the phase fraction near the interface and it creates over- and undershoots, which have to be filtered out because of stability issues. To improve on this, our improved FT model uses a simple geometrical procedure to calculate the exact volume under the interface triangulation in each cell. This ensures that the phase fraction field remains sharp near the interface and also reduces volume losses. Finally, the computationally expensive iterative procedure to solve the Poisson equation (Eq. 15) can be replaced by a simple explicit algorithm, which saves valuable computational time.

Updating the interface

Once the flow field has been found on the Eulerian grid, each marker point of the interface triangulation is moved with the local flow field. For the velocity interpolation to the marker points a 3rd order spline is used and the points are moved with a 4th order Runge-Kutta scheme. This combination of higher order methods ensures that the interface stays smooth and the volume error due to moving the mesh is negligible. After some time the surface grid will become deformed. Some markers will become too large or too stretched, while others become too small. To maintain an adequate resolution, points will have to be added at some places and removed at other places. In this work a similar approach as described by Unverdi and Tryggvason (1992) is followed.

DERIVATION OF INTERFACE FORCES

The drag and lift coefficients for a bubble in a linear shear flow (Fig. 4) can be obtained from a steady-state force balance, including buoyancy \( (F_b) \), drag \( (F_d) \) and lift \( (F_l) \) forces. A spherical-equivalent bubble diameter is used, so that the resulting closures can be applied without any additional equations, such as e.g. for the actual bubble shape. For bubbles experiencing path-instability, time-averaging is necessary to compute the drag and lift coefficients. The time-averaged forces are chosen as a basis, so that the time-averaged momentum exchange between the bubbles and the liquid phase is correct:
0 = \langle F_B \rangle + \langle F_L \rangle + \langle F_D \rangle
0 = \frac{\pi}{6} d^{4} (\rho_1 - \rho_w) g \frac{\pi}{8} d^{4} \rho_1 C_D \langle [v - u] \cdot (v - u) \rangle \quad (16)
- \frac{\pi}{6} d^{4} \rho_1 C_L \langle (v - u) \times (\nabla \times u) \rangle

Figure 4: Force balance on a bubble rising in a linear shear field, indicating the direction of the buoyancy, lift and drag forces (in clockwise order).

Because the drag and lift forces are perpendicular to each other, they can easily be separated. Since the lift force is much weaker than the drag force, it can be assumed that the drag force works in the z-direction and the lift force in the x-direction. First of all, the drag coefficient follows from the component in the z-direction:

\[ \frac{4 d \left( \rho_1 - \rho_w \right) g \left\langle v_z - u_z \right\rangle}{3 \rho_1 \left\langle [v - u] \cdot (v - u) \right\rangle} \]

Secondly, the lift coefficient can be derived from the transverse component of the force balance:

\[ \frac{\left\langle \rho_1 - \rho_w \right\rangle g \langle \sin \alpha \rangle - \frac{\pi}{6} d^{4} \rho_1 C_L \frac{du}{dx} \langle [v - u] \rangle}{\rho_1 \frac{du}{dx} \langle [v - u] \rangle} \quad (17) \]

SIMULATION SETTINGS

The numerical simulations have been carried out with the improved FT model using a cubic computational domain consisting of one million cells, which yields good resolution at reasonable calculation time (typically two weeks). The initially spherical bubble is placed horizontally in the center of the computational domain and a window shifting technique assures that it keeps this position. Depending on the viscosity of the liquid, the bubble diameter is equal to 15 or 20 Eulerian cells, which is the best trade-off between acceptable domain size and required resolution. The top, left and right boundaries enforce the linear shear field, using inflow and no-slip boundary conditions respectively. They are supplemented by a prescribed pressure boundary at the bottom, where the liquid is free to exit the domain, and free-slip boundaries at the front and rear. The physical properties of the liquids used, are given in Table 1.

<table>
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<th>( \mu ) [mPa·s]</th>
<th>( \rho ) [kg·m(^{-3})]</th>
<th>( \sigma ) [mN·m(^{-1})]</th>
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<tr>
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</table>

Table 1: Physical properties for the water/glycerine mixtures used in the numerical simulations.

As an example, the determination of the drag and lift force coefficients from a numerical simulation is shown for an 8 mm air bubble in liquid VI. The instantaneous drag and lift coefficient (Fig. 5) are used to determine the appropriate period for the time-averaging. In this particular case, the bubble takes about 0.7 seconds to move along a stable helical path, which is responsible for the periodic fluctuations in the drag and lift force. The time-averaging was carried out excluding this initial period. Because the magnitude of the oscillation \((-6 < C_L < 5)\) is much larger than the average \((-0.66)\), this interval is adjusted so that the time-averaging starts and stops with the bubble in the same state. Also a sufficient number of oscillation periods was used (typically 5-10), in order to give an accurate value.

RESULTS

Lift force coefficient

First of all, the lift force is studied for nearly spherical bubbles (E>0.95) in Fig. 6. It is found that there is a good agreement with the equation by Legendre & Magnaudet (1998), provided that the liquid is not too viscous. Fortunately, the lift force is relatively unimportant at low Reynolds numbers (Legendre & Magnaudet, 1998) and Eq. 3 works well in this region,
so that these differences are not very important. Also, it can be seen that there is a significant difference with the Reynolds-dependent part of the correlation by Tomiyama et al. (2002), which is not surprising as this does not reduce to the analytical solution at neither low nor high Reynolds numbers.

![Figure 6: Lift force coefficient for spherical bubbles (E>0.95) in pure liquids in a linear shear field, showing that the simulations agree well with the correlation by Legendre & Magnaudet (1998) at higher Reynolds numbers.](image1)

Figure 6: Lift force coefficient for spherical bubbles (E>0.95) in pure liquids in a linear shear field, showing that the simulations agree well with the correlation by Legendre & Magnaudet (1998) at higher Reynolds numbers.

![Figure 7: Lift force coefficient for pure deformed bubbles in a linear shear field at different liquid viscosities.](image2)

Figure 7: Lift force coefficient for pure deformed bubbles in a linear shear field at different liquid viscosities.

For deformed bubbles, the numerical results are compared to the Eötvös-dependent part of the correlation by Tomiyama (Eq. 4). Fig. 7 shows that for the most viscous liquid (I) the simulation results are quite different from the correlation and the other simulation results, which may be related to the low Reynolds numbers, similar to the minimum of the lift force for spherical bubbles around Re=1. However, more simulations (using a very large domain size) at low Reynolds number have to be conducted in order to investigate this phenomenon further.

At similar conditions as Tomiyama used (II-IV) and beyond, a good agreement with his correlation is found, although the lift force is always slightly lower in the numerical simulations. The small scatter in the numerical data points is mainly caused by path-instability or wobbling motion of the bubbles, which cause the lift force coefficient to oscillate with very large amplitude (see Fig. 5). This makes a very accurate determination of its average value extremely challenging, moreover because it can take a very long time to reach a steady state.

Finally, the effect of increasing the shear rate from 2.0 to 4.0 s⁻¹ was determined. It was found that there is no influence of the shear rate on the drag and lift force coefficient, except – as expected – at very low Reynolds numbers.

**Wake structure**

From literature it was found that large deformed bubbles have a slanted wake structure, which may be a key factor in explaining the negative lift force coefficient. Fig. 8 shows that as soon as the lift coefficient turns negative (6 mm air bubble in liquid IV), a distinct asymmetric wake structure appears.

![Figure 8: Bubble and streamlines around 2 and 6 mm bubbles (liquid IV, \(\omega=4\) s⁻¹). Note that the 6 mm bubble has a distinct asymmetric (slanted) wake, which is typical for deformed bubbles.](image3)
CONCLUSION
In this paper the lift force has been studied using numerical simulations, which have been carried out at Reynolds numbers above 1, because of the very strong effect of the walls at lower Re numbers. The results show a good agreement with the correlation by Legendre & Magnaudet (1998) for spherical bubbles at sufficiently high Reynolds numbers, contradicting the Reynolds-dependent part of Tomiyama’s closure (2002). For larger bubbles there is a good agreement with the correlation by Tomiyama, although the lift coefficient was consistently somewhat lower. Finally, it was shown that the shear rate has no influence on the drag and lift coefficients.

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