A MIXED ELASTOPLASTIC / RIGID PLASTIC MATERIAL MODEL

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ABSTRACT: A new integration algorithm for plastic deformation is derived in combination with the anisotropic Hill'49 yield criterion. The algorithm degenerates to the Euler forward elastoplastic material model for small deformations and to the rigid plastic material model for large strain increments. The new model benefits from the advantages of both the elastoplastic and rigid plastic material models: accuracy and fast convergence over a large range of strain increments. The performance of the new algorithm is tested by a deep drawing simulation of a rectangular product. It can be concluded that the new algorithm performs well: the plastic thickness strain distribution of the mixed model inclines towards the elastoplastic material model.

1. INTRODUCTION

The plastic deformations of a material are described by constitutive relations and usually these are based on rate equations. For use in an incremental procedure, the plastic strain must be integrated to yield a plastic strain increment. Many of these algorithms are based on a elastic predictor / plastic corrector scheme (Wilkins,1964) , (Rice,1973). The direction of the plastic flow can be interpolated between the directions calculated at the start and the end of a strain increment. The magnitude of the plastic flow is determined by a consistency relation, i.e. the stress state remains on the yield surface. However these elastoplastic algorithms can give rise to numerical instabilities due to the transition from elastic to plastic behavior which is incorporated in these models.

When the strain increments are large, then the elastic part of the strain can be neglected without a serious lack of accuracy. In that case the plastic strain equals the total strain which is better known as rigid plastic material behavior.

In deep drawing simulations, the rigid plastic material model is widely used because of its fast and numerically robust behavior. The model yields accurate results for large strain increments compared to the elastic limit strain. in cases where the strain increments are small, for example in dead metal zones, the model becomes unstable or inaccurate. Another drawback of the rigid plastic approach is that elastic phenomena such as springback cannot be described.

Huétink (Huétink,1998) developed a new integration algorithm for large plastic deformations in combination with Mises material behavior. The algorithm degenerates to the Euler Forward elastoplastic material model for small strain increments and to the rigid plastic material model for large strain increments. The new model benefits from the advantages of both the elastoplastic and rigid plastic material model: accuracy and fast convergence over a large range of plastic strain increments.

In this paper the new integration algorithm is derived for large plastic deformations in combination with the Hill'49 yield criterion. The performance of the new model is verified by a set of deep drawing simulations of a rectangular product.

2. HILL'49 YIELD CRITERION

The Hill'49 yield criterion (Hill,1950) in its most general form is given in equation (1). The yield function ϕ defines the stress states at which a material starts to deform plastic. Plastic deformation occurs if $\phi = 0$ and $\dot{\phi} = 0$, elastic deformation occurs if $\phi < 0$ or $\dot{\phi} < 0$.

$$\phi = F(\sigma_y - \sigma_z)^2 + G(\sigma_z - \sigma_x)^2 + H(\sigma_x - \sigma_y)^2 + 2L\sigma_{yz}^2 + 2M\sigma_{zx}^2 + 2N\sigma_{xy}^2 - 1 = 0 \quad (1)$$

The material x-, y- and z-direction coincide with the axes of orthotropy. The parameters F, G, H, L, M and N describe the anisotropic behavior of the material. Equation (1) can alternatively written as:

$$\phi = \sqrt{\underline{\sigma}^T : \underline{\underline{P}} : \underline{\sigma}} - \sqrt{\tfrac{2}{3}(F+G+H)}\sigma_y = \sqrt{\underline{\sigma}^T : \underline{\underline{P}} : \underline{\sigma}} - \sqrt{\xi}\sigma_y = 0 \quad (2)$$

in which the fourth order tensor $\underline{\underline{P}}$ contains the above mentioned material parameters. The transposed Cauchy stress tensor equals the Cauchy stress tensor due to its symmetry. Therefore the index ^{T} will be dropped for convenience. The derivative of ϕ with respect to $\underline{\sigma}$ is:

$$\frac{\partial\phi}{\partial\underline{\sigma}} = \frac{\underline{\underline{P}} : \underline{\sigma}}{\sqrt{\underline{\sigma} : \underline{\underline{P}} : \underline{\sigma}}} \quad (3)$$

In this material description only associative plasticity according to Drucker is concerned:

$$\underline{\dot{\varepsilon}}^p = \dot{\lambda}\frac{\partial\phi}{\partial\underline{\sigma}} = \dot{\lambda}\frac{\underline{\underline{P}} : \underline{\sigma}}{\sqrt{\underline{\sigma} : \underline{\underline{P}} : \underline{\sigma}}} = \frac{\dot{\lambda}}{\sqrt{\xi}\sigma_y}\underline{\underline{P}} : \underline{\sigma} \quad (4)$$

with $\dot{\lambda}$ is the plastic multiplier. The relation between the equivalent plastic strain $\dot{\kappa}$ and the plastic multiplier $\dot{\lambda}$ is derived from the plastic deformation energy and equation (4):

$$\sigma_y \cdot \dot{\kappa} = \underline{\sigma} : \underline{\dot{\varepsilon}}^p = \frac{\dot{\lambda}}{\sqrt{\xi}\sigma_y}\underline{\sigma} : \underline{\underline{P}} : \underline{\sigma} = \dot{\lambda}\sqrt{\xi}\sigma_y \quad \Rightarrow \quad \dot{\kappa} = \sqrt{\xi}\dot{\lambda} \quad (5)$$

The definition of the equivalent plastic strain $\dot{\kappa}$ follows from the plastic deformation energy and equation (4) also:

$$\dot{\kappa} = \sqrt{\xi \underline{\dot{\varepsilon}}^p : \underline{\underline{P}}^{-1} : \underline{\dot{\varepsilon}}^p} \quad (6)$$

3. STRESS-STRAIN RELATION

The elastic strain tensor is related to the Cauchy stress tensor by the elasticity tensor $\underline{\underline{E}}$. When the total strain is decomposed into a plastic and an elastic part, the elastic stress-strain relation in incremental form yields:

$$\Delta\underline{\sigma} = \underline{\underline{E}} : (\Delta\underline{\varepsilon} - \Delta\underline{\varepsilon}^p) \quad (7)$$

The incremental plastic strain $\Delta\underline{\varepsilon}^p$ can be written as:

$$\Delta\underline{\varepsilon}^p = \int_{t_0}^{t_1} \underline{\dot{\varepsilon}}^p dt = (\underline{\underline{H}} - \underline{\underline{\alpha}}) : \underline{\dot{\varepsilon}}_0^p \Delta t + \underline{\underline{\alpha}} : \underline{\dot{\varepsilon}}_1^p \Delta t \quad (8)$$

where t_0 and t_1 bound a time increment. The direction of the incremental plastic strain is determined by weighting the plastic strain rate directions at the begin and end of the time increment with the tensor $\underline{\underline{\alpha}}$. The plastic strain rate at time $t=t_0$ and $t=t_1$ is, see equation (4):

$$t = t_0: \quad \underline{\dot{\varepsilon}}_0^p = \frac{\dot{\lambda}_0}{\sqrt{\xi}\sigma_{y,0}}\underline{\underline{P}} : \underline{\sigma}_0 \qquad t = t_1: \quad \underline{\dot{\varepsilon}}_1^p = \frac{\dot{\lambda}_1}{\sqrt{\xi}\sigma_{y,1}}\underline{\underline{P}} : \underline{\sigma}_1 \quad (9)$$

Suppose $\frac{\dot{\lambda}_0}{\sigma_{y,0}} = \frac{\dot{\lambda}_1}{\sigma_{y,1}}$. Then, with $\kappa = \sqrt{\xi}\dot{\lambda}$:

$$\Delta\underline{\varepsilon}^p = \frac{\Delta\kappa}{\xi\sigma_y}\left((\underline{\underline{H}} - \underline{\underline{\alpha}}) : \underline{\underline{P}} : \underline{\sigma}_0 + \underline{\underline{\alpha}} : \underline{\underline{P}} : \underline{\sigma}_1\right) \quad (10)$$

Substitution of (10) in (7) yields the expression for the actual stress state:

$$\left(\underline{\underline{H}} + \frac{\Delta\kappa}{\xi\sigma_y}\underline{\underline{E}} : \underline{\underline{\alpha}} : \underline{\underline{P}}\right) : \underline{\sigma}_1 = \left(\underline{\underline{H}} - \frac{\Delta\kappa}{\xi\sigma_y}\underline{\underline{E}} : (\underline{\underline{H}} - \underline{\underline{\alpha}}) : \underline{\underline{P}}\right) : \underline{\sigma}_0 + \underline{\underline{E}} : \Delta\underline{\varepsilon} \quad (11)$$

4. STRESS UPDATE ALGORITHM

Within a numerical time increment, only the total strain at the begin and the end of the time increment are determined. The stress at the end of the time increment must be calculated. The task of the stress integration is only to calculate the stress for a prescribed stress increment. If the calculated stress does not match the weighted equilibrium at a global level, a new strain increment has to be calculated.

The new stress state $\underline{\sigma}_1$ is determined with an elastic predictor - plastic corrector method. The elastic predictor defines a trail stress state $\underline{\sigma}_t$:

$$\underline{\sigma}_t = \underline{\sigma}_0 + \underline{\underline{E}}:\Delta\underline{\varepsilon} \tag{12}$$

If $\phi(\underline{\sigma}_t,\kappa) > 0$ a plastic corrector is used to determine the new stress state $\underline{\sigma}_1$ which lies on the yield surface. Equation (11) can be written as a function of the trial stress state:

$$\left(\underline{\underline{H}} + \frac{\Delta\kappa}{\xi\sigma_y}\underline{\underline{E}}:\underline{\underline{\alpha}}:\underline{\underline{P}}\right):\underline{\sigma}_1 = \underline{\sigma}_t - \frac{\Delta\kappa}{\xi\sigma_y}\underline{\underline{E}}:(\underline{\underline{H}} - \underline{\underline{\alpha}}):\underline{\underline{P}}:\underline{\sigma}_0 \tag{13}$$

Define a fourth order tensor $\underline{\underline{A}}$:

$$\underline{\underline{A}} = \left(\underline{\underline{H}} + \frac{\Delta\kappa}{\xi\sigma_y}\underline{\underline{E}}:\underline{\underline{\alpha}}:\underline{\underline{P}}\right), \tag{14}$$

then equation (13) can be written as:

$$\underline{\sigma}_1 = \underline{\underline{A}}^{-1}:\underline{\sigma}_t - \frac{\Delta\kappa}{\xi\sigma_y}\underline{\underline{A}}^{-1}:\underline{\underline{E}}:(\underline{\underline{H}} - \underline{\underline{\alpha}}):\underline{\underline{P}}:\underline{\sigma}_0 \tag{15}$$

The incremental equivalent plastic strain $\Delta\kappa$ must be known to calculate the new stress state $\underline{\sigma}_1$. The yield function depends on $\Delta\kappa$:

$$\begin{aligned}\phi(\Delta\kappa) &= \left(\underline{\sigma}_1 : P : \underline{\sigma}_1\right)^{\frac{1}{2}} - \sqrt{\xi}\sigma_y = \\ &\left(\left(\underline{\sigma}_t:\underline{\underline{A}}^{-1} - \frac{\Delta\kappa}{\xi\sigma_y}\underline{\sigma}_0:\underline{\underline{P}}:(\underline{\underline{H}} - \underline{\underline{\alpha}}):\underline{\underline{E}}:\underline{\underline{A}}^{-1}\right):\underline{\underline{P}}: \left(\underline{\underline{A}}^{-1}:\underline{\sigma}_t - \frac{\Delta\kappa}{\xi\sigma_y}\underline{\underline{A}}^{-1}:\underline{\underline{E}}:(\underline{\underline{H}} - \underline{\underline{\alpha}}):\underline{\underline{P}}:\underline{\sigma}_0\right)\right)^{\frac{1}{2}} \\ &- \sqrt{\xi}\sigma_y\end{aligned} \tag{16}$$

Note that the tensor $\underline{\underline{A}}$ is symmetric for Hill anisotropic yielding, which means that the transposed form of $\underline{\underline{A}}$ is identical to $\underline{\underline{A}}$. Here σ_y and the tensor $\underline{\underline{A}}$ are functions of the equivalent plastic strain increment $\Delta\kappa$, therefore (16) is a non-linear relation that has to be solved by an iterative procedure, e.g. by a Newton-Raphson method:

$$\begin{array}{l}\Delta\kappa_0 = 0 \quad ; \quad k = 0 \\ \text{while } (\|\phi(\Delta\kappa_k\|) > error) \text{ do} \\ \quad k{:}{=}k{+}1 \\ \quad \text{determine } \left(\dfrac{d\phi}{d\kappa}\right) \\ \quad \Delta\kappa_{k+1} = \Delta\kappa_k - \left(\dfrac{d\phi}{d\kappa}\right)^{-1}\phi(\Delta\kappa) \\ \text{end while}\end{array} \tag{17}$$

5. CONSISTENT STIFFNESS

For a fast convergence of the global equilibrium a consistent stiffness matrix must be derived. To obtain this, equation (13) must be written in differential form:

$$\begin{aligned}&\underline{\underline{A}}:d\underline{\sigma}_1 + d\underline{\underline{A}}:\underline{\sigma}_1 = \\ &\quad -\frac{1}{\xi\sigma_y}\left(1 - \frac{h\Delta\kappa}{\sigma_y}\right)\underline{\underline{E}}:(\underline{\underline{H}} - \underline{\underline{\alpha}}):\underline{\underline{P}}:\underline{\sigma}_0 d\kappa + \\ &\quad \frac{\Delta\kappa}{\xi\sigma_y}\underline{\underline{E}}:\underline{\underline{\psi}}:\underline{\underline{P}}:\underline{\sigma}_0 d\kappa + \underline{\underline{E}}:d\underline{\varepsilon}\end{aligned} \tag{18}$$

in which $h = \dfrac{d\sigma_y}{d\kappa}$ is the hardening modulus, $\underline{\underline{\psi}} = \dfrac{d\underline{\underline{\alpha}}}{d\kappa}$ a to be specified fourth order tensor and with $d\underline{\underline{A}}$:

$$d\underline{\underline{A}} = \left(\frac{1}{\xi\sigma_y}\left(1 - \frac{h\Delta\kappa}{\sigma_y}\right)\underline{\underline{E}}:\underline{\underline{\alpha}}:\underline{\underline{P}} + \frac{\Delta\kappa}{\xi\sigma_y}\underline{\underline{E}}:\underline{\underline{\psi}}:\underline{\underline{P}}\right)d\kappa \tag{19}$$

Combining equation (18) and (19) gives:

$$d\underline{\sigma}_1 = \underline{\underline{A}}^{-1}:\left(\underline{\underline{E}}:d\underline{\varepsilon} - \underline{\underline{U}}d\kappa\right) \tag{20}$$

with the second order tensor $\underline{U}$:

$$\underline{U} = \left(\frac{1}{\xi\sigma_y}\left(1-\frac{h\Delta\kappa}{\sigma_y}\right)\underline{\underline{E}}:\underline{\underline{\alpha}}:\underline{\underline{P}} + \frac{\Delta\kappa}{\xi\sigma_y}\underline{\underline{E}}:\underline{\underline{\psi}}:\underline{\underline{P}}\right):\underline{\sigma}_1 + \left(\frac{1}{\xi\sigma_y}\left(1-\frac{h\Delta\kappa}{\sigma_y}\right)\underline{\underline{E}}:(\underline{\underline{H}}-\underline{\underline{\alpha}}):\underline{\underline{P}} - \frac{\Delta\kappa}{\xi\sigma_y}\underline{\underline{E}}:\underline{\underline{\psi}}:\underline{\underline{P}}\right):\underline{\sigma}_0 \quad (21)$$

An expression for $d\kappa$ as a function of $d\underline{\varepsilon}$ can be derived from the derivative of the yield function:

$$d\phi = \frac{1}{\sqrt{\xi}\sigma_y}\underline{\sigma}_1:\underline{\underline{P}}:d\underline{\sigma}_1 - h\sqrt{\xi}d\kappa = 0 \quad \Rightarrow$$
$$\underline{\sigma}_1:\underline{\underline{P}}:d\underline{\sigma}_1 = h\xi\sigma_y d\kappa \quad (22)$$

which gives, together with equation (20), an expression for $d\kappa$:

$$d\kappa = \frac{\underline{\sigma}_1:\underline{\underline{P}}:\underline{\underline{A}}^{-1}:\underline{\underline{E}}}{h\xi\sigma_y + \underline{\sigma}_1:\underline{\underline{P}}:\underline{\underline{A}}^{-1}:\underline{U}}:d\underline{\varepsilon} \quad (23)$$

Finally, adding equation (23) to (20) yields the consistent stiffness matrix for elastic plastic material behavior:

$$d\underline{\sigma}_1 = \left[\underline{\underline{A}}^{-1}:\underline{\underline{E}} - \frac{\left(\underline{\underline{A}}^{-1}:\underline{U}\right)\left(\underline{\sigma}_1:\underline{\underline{P}}:\underline{\underline{A}}^{-1}:\underline{\underline{E}}\right)}{h\xi\sigma_y + \underline{\sigma}_1:\underline{\underline{P}}:\underline{\underline{A}}^{-1}:\underline{U}}\right]:d\underline{\varepsilon} \quad (24)$$

6. MIXED ELASTOPLASTIC / RIGID PLASTIC MATERIAL MODEL

The purpose of this new model is to combine the accuracy of the elastoplastic material model and the rigid plastic material model over a large range of strain increments. The starting point of the mixed model is equation (11). The value of the tensor $\underline{\underline{\alpha}}$ depends on the value of the equivalent plastic strain increment $\Delta\kappa$. It is demanded that the initial stress $\underline{\sigma}_0$ must vanish for large strain increments:

$$\lim_{\Delta\kappa\to\infty}\left(\underline{\underline{H}} - \frac{\Delta\kappa}{\xi\sigma_y}\underline{\underline{E}}:(\underline{\underline{H}}-\underline{\underline{\alpha}}):\underline{\underline{P}}\right) = \underline{\underline{0}} \quad (25)$$

which gives a definition for $\underline{\underline{\alpha}}$:

$$\underline{\underline{\alpha}} = \left(\underline{\underline{H}} - \frac{\xi\sigma_y}{\Delta\kappa}\underline{\underline{E}}^{-1}:\underline{\underline{P}}^{-1}\right) \quad (26)$$

A reference strain increment κ_{ref} is introduced which corresponds to an elastic stress increment from zero to the yield stress. For the new model the next is stated:

$$\Delta\kappa \le \kappa_{ref}: \qquad \underline{\underline{\alpha}} = \underline{\underline{0}}$$
$$\Delta\kappa > \kappa_{ref}: \qquad \underline{\underline{\alpha}} = \left(\underline{\underline{H}} - \frac{\xi\sigma_y}{\Delta\kappa}\underline{\underline{E}}^{-1}:\underline{\underline{P}}^{-1}\right) \quad (27)$$

which holds that for $\Delta\kappa \le \kappa_{ref}$ Euler forward integration is used for the stress update.
For $\Delta\kappa \le \kappa_{ref}$ the stress state yields, see equation (13):

$$\underline{\sigma}_1 = \left(\underline{\underline{H}} - \frac{\Delta\kappa}{\xi\sigma_y}\underline{\underline{E}}:\underline{\underline{P}}\right):\underline{\sigma}_0 + \underline{\underline{E}}:\Delta\underline{\varepsilon} \quad (28)$$

For $\Delta\kappa > \kappa_{ref}$ the stress state yields, see equation (13):

$$\left(\underline{\underline{H}} + \frac{\Delta\kappa}{\xi\sigma_y}\underline{\underline{E}}:\left(\underline{\underline{H}} - \frac{\xi\sigma_y}{\Delta\kappa}\underline{\underline{E}}^{-1}:\underline{\underline{P}}^{-1}\right):\underline{\underline{P}}\right):\underline{\sigma}_1 =$$
$$\left(\underline{\underline{H}} - \frac{\Delta\kappa}{\xi\sigma_y}\left(\underline{\underline{H}} - \underline{\underline{E}}:\left(\underline{\underline{H}} - \frac{\xi\sigma_y}{\Delta\kappa}\underline{\underline{E}}^{-1}:\underline{\underline{P}}^{-1}\right)\right):\underline{\underline{P}}\right):\underline{\sigma}_0 \quad (29)$$
$$+\underline{\underline{E}}:\Delta\underline{\varepsilon} \qquad \Rightarrow$$
$$\underline{\sigma}_1 = \frac{\xi\sigma_y}{\Delta\kappa}\underline{\underline{P}}^{-1}:\Delta\underline{\varepsilon}$$

Mark that for $\Delta\kappa > \kappa_{ref}$ the elastoplastic material description degenerates to the rigid plastic material description.

6.1 Consistent stiffness tensor

The consistent stiffness tensor has to be derived for both parts of the mixed model. First the consistent stiffness tensor for Euler forward integration is derived. Since $\underline{\underline{\alpha}} = 0$, one can write:

$$\underline{\underline{\psi}} = \underline{\underline{0}} \quad \text{and} \quad \underline{\underline{A}} = \underline{\underline{H}} \quad (30)$$

Filling in these conditions into equation (24) yields the consistent stiffness tensor for Euler forward integration:

$$d\underline{\sigma}_1 =$$

$$\left[\underline{\underline{E}} - \frac{\frac{1}{\xi\sigma_y}\left(1-\frac{h\Delta\kappa}{\sigma_y}\right)\left(\underline{\underline{E}}:\underline{\underline{P}}:\underline{\sigma}_0\right)\left(\underline{\sigma}_1:\underline{\underline{P}}:\underline{\underline{E}}\right)}{h\xi\sigma_y + \frac{1}{\xi\sigma_y}\left(1-\frac{h\Delta\kappa}{\sigma_y}\right)\underline{\sigma}_1:\underline{\underline{P}}:\underline{\underline{E}}:\underline{\underline{P}}:\underline{\sigma}_0}\right]:d\underline{\varepsilon}$$

$$= \left[\underline{\underline{E}} - \frac{\left(\underline{\underline{E}}:\underline{\underline{P}}:\underline{\sigma}_0\right)\left(\underline{\sigma}_1:\underline{\underline{P}}:\underline{\underline{E}}\right)}{\frac{h\xi^2\sigma_y^3}{(\sigma_y - h\Delta\kappa)} + \underline{\sigma}_1:\underline{\underline{P}}:\underline{\underline{E}}:\underline{\underline{P}}:\underline{\sigma}_0}\right]:d\underline{\varepsilon} \qquad (31)$$

Second the consistent stiffness tensor is derived for the rigid plastic part of the new model ($\Delta\kappa > \kappa_{ref}$). Using the second part of equation (27), one can write:

$$\underline{\underline{A}} = \left(\underline{\underline{H}} + \frac{\Delta\kappa}{\xi\sigma_y}\underline{\underline{E}}:\underline{\underline{\alpha}}:\underline{\underline{P}}\right) = \frac{\Delta\kappa}{\xi\sigma_y}\underline{\underline{E}}:\underline{\underline{P}} \qquad (32)$$

$$\underline{\underline{\psi}} = \frac{\xi}{\Delta\kappa}\left(\frac{\sigma_y}{\Delta\kappa} - h\right)\underline{\underline{E}}^{-1}:\underline{\underline{P}}^{-1} \qquad (33)$$

$$\underline{U} = \frac{1}{\xi\sigma_y}\left(1-\frac{h\Delta\kappa}{\sigma_y}\right)\underline{\underline{E}}:\underline{\underline{P}}:\underline{\sigma}_1 \qquad (34)$$

Combining equations (32)-(34) with equation (24) yields the consistent stiffness tensor for the rigid plastic part of the new model:

$$d\underline{\sigma}_1 = \left[\frac{\xi\sigma_y}{\Delta\kappa}\underline{\underline{P}}^{-1} - \left(\frac{\sigma_y}{\Delta\kappa} - h\right)\frac{\underline{\sigma}_1\underline{\sigma}_1}{\sigma_y^2}\right]:d\underline{\varepsilon} \qquad (35)$$

7. APPLICATION

The deep drawing of a rectangular product will be used to illustrate the behavior of the rigid plastic material model, the elastoplastic material model and the mixed elastoplastic / rigid plastic material model. Simulations are performed with the implicit finite element code DiekA.

The geometry of the rectangular tools is given in Figure 1. The product depth is 100 mm. The used blank is 600 mm * 470 mm and has a thickness of 0.7 mm.

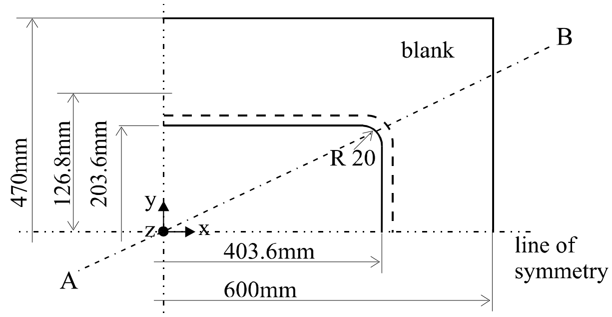

Figure 1. Tool geometry

A first set of simulations is performed with an incremental step size of 0.3 mm in which the three material models are applied separately. The plastic thickness strain distributions in the rectangular product after 75 mm deep drawing are depicted in Figures 3-5 for the different material models.

One can mark that the used material model influences the plastic thickness strain distribution drastically. The thickness reduction is the highest for the rigid plastic material model and the lowest for the elastoplastic material model. The plastic thickness strain distribution of the mixed material model inclines towards the plastic thickness strain distribution of the elastoplastic material model.

The convergence behavior of the simulations differs significantly as well. The mechanical unbalance ratio is set at 2 percent. The simulation with the rigid plastic material model needs 1 iteration per incremental step for convergence. The simulation with the elastoplastic material model needs 1 to 5 iterations per step for convergence. The simulation with the mixed material model needs 3 iterations per step for convergence.

The final product depth of 100 mm is successfully reached in the simulation with the rigid plastic material model and the mixed material model. However, the simulation with the elastic-plastic material model crashes after 93 mm deep drawing due to unstable behavior.

The plastic thickness strain distribution after 100 mm deep drawing along line A-B (see Figure 1) is depicted in Figure 2 for the rigid plastic material model and the mixed material model. It can be seen clearly that the plastic thickness strain is case of the rigid plastic material model is higher than the plastic thickness strain in case of the mixed material model, especially in the bottom of the product. This can be explained as follows. The denominator of the rigid plastic material model consists of the equivalent plastic strain. Problems arise when no plastic strain occurs in some parts of a product during the deep drawing simulation. To avoid this problem, a small amount of fictive plastic strain is assumed when no plastic strain occurs. This yields for the rectangular product in plastic strain generation in the bottom of the product and in some parts of the flange when the rigid plastic material model is used.

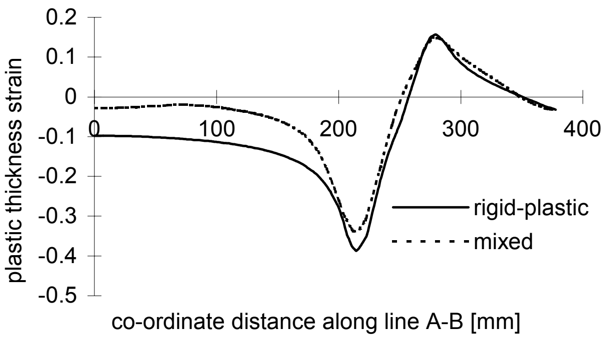

Figure 2. The plastic thickness strain distribution along line A-B

8. CONCLUDING REMARKS

- A new integration method for elastoplastic material models is developed which degenerates to the rigid plastic model for large strain increments.
- The new model appears to be more stable and as accurate as the conventional elastoplastic material model

9. REFERENCES

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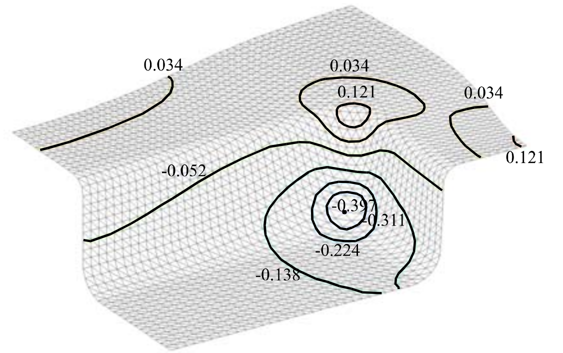

Figure 3. Plastic thickness strain distribution (rigid plastic material model)

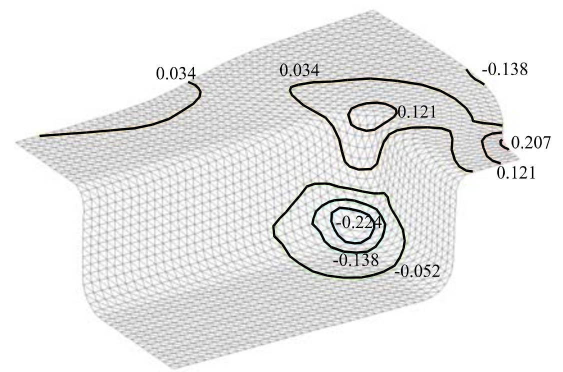

Figure 4. Plastic thickness strain distribution (elastoplastic material model)

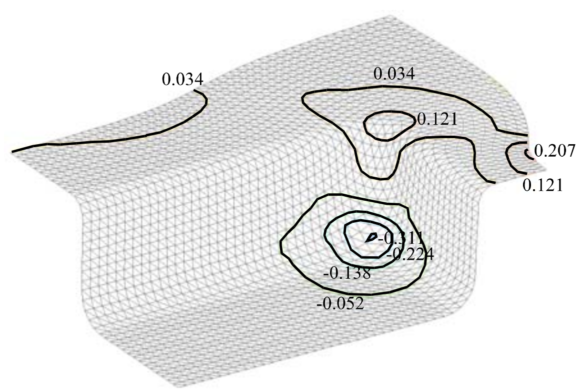

Figure 5. Plastic thickness strain distribution (mixed model)