A fast vibro-acoustic response analysis method for double wall structures including a viscothermal air layer*

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Abstract

The damping behaviour of a thin air layer between two flexible panels can be used to reduce sound radiation of structural excited panels. The numerical model of the double wall panels takes into account full acousto-elastic interaction and viscothermal wave propagation in the air layer. This means that the resulting system matrices are complex and frequency dependent which makes it difficult to perform response calculations. In this paper a very efficient calculation method is presented which is based on the superposition of uncoupled structural and acoustic eigenmodes. The frequency dependent behaviour is implemented by updating the reduced acoustic submatrices for each frequency step. The method is successfully implemented in the B2000 processor B2FRF.

1 Introduction

Double wall panels can be effectively damped by the introduction of a very small air layer, see Figure 1. The viscothermal effects in the air layer cause a significant amount of damping. The dissipation of vibrational energy reduces the sound radiation of these kind of structures. Beltman [1] developed a finite element model for viscothermal air layers including acousto-elastic interaction. The elements are based on the *low reduced frequency model* and take into account the effects of inertia, compressibility, viscosity and thermal conductivity. One of the main assumptions of this model is that the acoustic pressure perturbations are constant across the layer thickness. This is allowed when the thickness is much smaller than the acoustic wave length, i.e. when the reduced frequency $k = \frac{\omega h_0}{c_0}$ is much smaller than one.

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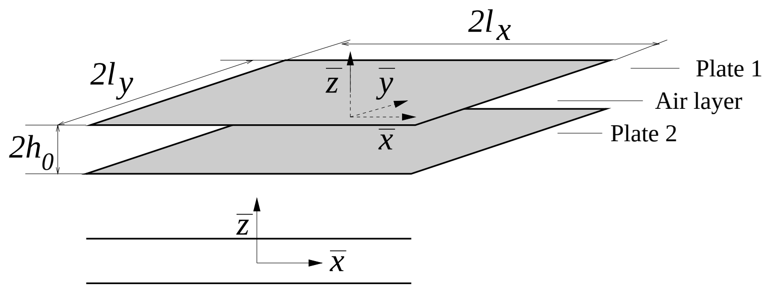

Figure 1: Double wall panel with thin air layer.

2 Viscothermal wave propagation including acousto-elastic interaction

The finite element formulation of the acoustic behaviour of a viscothermal air layer, in contact with a flexible structure results in the following coupled system of equations:

$$-\omega^2 \begin{bmatrix} [M^s] & [0] \\ [M^c(s)] & [M^a(s)] \end{bmatrix} \begin{Bmatrix} \{U\} \\ \{P\} \end{Bmatrix} + \begin{bmatrix} [K^s] & -[K^c] \\ [0] & [K^a] \end{bmatrix} \begin{Bmatrix} \{U\} \\ \{P\} \end{Bmatrix} = \begin{Bmatrix} \{F^{ext}\} \\ \frac{\rho_0}{h_0 B(s)} \{\dot{q}\} \end{Bmatrix} \tag{1}$$

The vector $\{U\}$ contains the structural degrees of freedom and $\{P\}$ contains the acoustic degrees of freedom. This system contains the mass matrices and the stiffness matrices of the structural part and the acoustic part. The acousto-elastic interaction is established by introducing the pressure of the air as a load on the plates and by demanding continuity of normal velocity across the interface. This coupling is established by the two coupling matrices, $[M^c(s)]$ and $[K^c]$, which are related as:

$$[M^c(s)] = \frac{\rho_0}{h_0 B(s)} [K^c]^T \ , \tag{2}$$

with

$$B(s) = \frac{\tanh s\sqrt{\mathrm{i}}}{s\sqrt{\mathrm{i}}} - 1 \ . \tag{3}$$

The acoustic submatrices depend on the shear wave number s which is defined as $s = h_0 \sqrt{\frac{\rho_0 \omega}{\mu}}$ with ω the angular frequency and ρ_0 and μ the mean density and the dynamic viscosity respectively. This dimensionless number is a measure for the ratio of inertial and viscous effects. So when this number is low the viscous effects have to be taken into account. The finite element model was implemented in B2000 by Beltman [1]. He multiplied the second set of equations by the factor $\frac{h_0 B(s)}{\rho_0}$. In this way the resulting set of equations becomes:

$$-\omega^2 \begin{bmatrix} [M^s] & [0] \\ [M^c] & [M^a(s)] \end{bmatrix} \begin{Bmatrix} \{U\} \\ \{P\} \end{Bmatrix} + \begin{bmatrix} [K^s] & -[K^c] \\ [0] & [K^a(s)] \end{bmatrix} \begin{Bmatrix} \{U\} \\ \{P\} \end{Bmatrix} = \begin{Bmatrix} \{F^s\} \\ \{\dot{q}\} \end{Bmatrix} \ . \tag{4}$$

The coupling matrices now have the following relation:

$$[M^c] = [K^c]^T \ . \tag{5}$$

By writing the equations in this way it was not necessary to implement new interface elements for modeling acousto-elastic interaction in B2000. The standard interface elements implemented by Grooteman [2] could be used. The frequency dependent matrices $[M^a(s)]$ and $[K^a(s)]$ only differ from the matrices for adiabatic, inviscid air with a frequency dependent premultiplication factor.
A frequency response calculation can be carried out by solving the coupled set of equations (4) for a given excitation and frequency. Unfortunately, the frequency dependence of the matrices complicates such a frequency response calculation. However, using the special character of the frequency dependency, it is possible to reduce the computational efforts. Furthermore, the size of the matrices can be reduced by modal superposition. The coupled viscothermal response can be expressed in terms of the uncoupled inviscid, adiabatic acoustic modes and the uncoupled structural modes [3].
Consider the acousto-elastic model for the *inviscid*, *adiabatic* case:

$$-\omega^2 \begin{bmatrix} [M^s] & [0] \\ [M^c] & \left[\widetilde{M}^a\right] \end{bmatrix} \left\{ \begin{matrix} \{U\} \\ \{P\} \end{matrix} \right\} + \begin{bmatrix} [K^s] & -[K^c] \\ [0] & \left[\widetilde{K}^a\right] \end{bmatrix} \left\{ \begin{matrix} \{U\} \\ \{P\} \end{matrix} \right\} = \left\{ \begin{matrix} \{F^{ext}\} \\ \{\dot{q}\} \end{matrix} \right\} , \tag{6}$$

where $\left[\widetilde{M}^a\right]$ and $\left[\widetilde{K}^a\right]$ are the matrices for the inviscid, adiabatic case. These matrices have the following relation with the viscothermal acoustic mass and stiffness matrices:

$$[M^a(s)] = \frac{\gamma}{n(s\sigma)} \left[\widetilde{M}^a\right] \quad ; \quad [K^a(s)] = -B(s) \left[\widetilde{K}^a\right] , \tag{7}$$

where:

$$n(s\sigma) = \left[1 + \left(\frac{\gamma - 1}{\gamma}\right) B(s\sigma)\right]^{-1} , \tag{8}$$

with σ the square root of the Prandtl number and γ the ratio of specific heats. The function $B(s)$ takes into account the viscous effects, while the function $n(s\sigma)$ accounts for the thermal effects in the air layer.
The uncoupled acoustic modes for this situation are calculated from:

$$-\omega^2 \left[\widetilde{M}^a(s)\right] \{P\} + \left[\widetilde{K}^a\right] \{P\} = 0 \tag{9}$$

The eigenmodes are stored (columnwise) in the matrix $\left[\widetilde{\Phi}^a\right]$. The modes are normalized so that the largest number in each vector equals 1.0. Using the modal matrix the reduced mass and stiffness matrices are obtained:

$$\left[\widetilde{\Phi}^a\right]^T \left[\widetilde{M}^a\right] \left[\widetilde{\Phi}^a\right] = \left[\widetilde{M}^a_r\right] \quad ; \quad \left[\widetilde{\Phi}^a\right]^T \left[\widetilde{K}^a\right] \left[\widetilde{\Phi}^a\right] = \left[\widetilde{K}^a_r\right] . \tag{10}$$

The reduced matrices are diagonal matrices. Calculation of the acoustic eigenmodes and eigenfrequencies is very straightforward and efficient. All matrices involved are symmetric and real valued. The uncoupled structural modes are obtained from:

$$-\omega^2 [M^s] \{U\} + [K^s] \{U\} = 0 . \tag{11}$$

The structural eigenmodes are stored (columnwise) in the matrix $[\Phi^s]$. The modes are normalized with respect to highest number in each vector. The reduced mass and stiffness matrices of the structural domain become:

$$[\Phi^s]^T [M^s] [\Phi^s] = [M_r^s] \quad ; \quad [\Phi^s]^T [K^s] [\Phi^s] = [K_r^s] \ . \tag{12}$$

The reduced structural matrices are also diagonal matrices. Now consider the viscothermal acousto-elastic problem. The coupled viscothermal response is expressed in terms of uncoupled acoustic modes, $\left[\widetilde{\Phi}^a\right]$, and uncoupled structural modes, $[\Phi^s]$:

$$\begin{aligned} \{U\} &= [\Phi^s] \{\eta^s\} \ ; \\ \{P\} &= \left[\widetilde{\Phi}^a\right] \{\eta^a\} \ . \end{aligned} \tag{13}$$

The response, which is in general complex valued is described by real modal vectors. Therefore the modal participation factors in $\{\eta_s\}$ and $\{\eta_a\}$ will be complex valued. Inserting these expressions into the acousto-elastic system equation (4), premultiplying the structural part by $[\Phi^s]^T$ and the acoustic part by $\left[\widetilde{\Phi}^a\right]^T$, finally gives:

$$-\omega^2 \begin{bmatrix} [M_r^s] & [0] \\ [M_r^c] & [M_r^a(s)] \end{bmatrix} \begin{Bmatrix} \{\eta^s\} \\ \{\eta^a\} \end{Bmatrix} + \begin{bmatrix} [K_r^s] & [K_r^c] \\ [0] & [K_r^a(s)] \end{bmatrix} \begin{Bmatrix} \{\eta^s\} \\ \{\eta^a\} \end{Bmatrix} = \begin{Bmatrix} \{F_r^{ext}\} \\ \{\dot{q}_r\} \end{Bmatrix} , \tag{14}$$

where:

$$\begin{aligned} [M_r^a(s)] &= \frac{\gamma}{n(s\sigma)} \left[\widetilde{\Phi}^a\right]^T \left[\widetilde{M}^a\right] \left[\widetilde{\Phi}^a\right] = \frac{\gamma}{n(s\sigma)} \left[\widetilde{M}_r^a\right] \ ; \\ [K_r^a(s)] &= -B(s) \left[\widetilde{\Phi}^a\right]^T [K^a] \left[\widetilde{\Phi}^a\right] = -B(s) \left[\widetilde{K}_r^a\right] \ ; \\ [M_r^c] &= \left[\widetilde{\Phi}^a\right]^T \left[\widetilde{M}^c\right] [\Phi^s] \ ; && (15) \\ [K_r^c] &= -\, [\Phi^s]^T [K^c] \left[\widetilde{\Phi}^a\right] \ ; && (16) \\ \{F_r^{ext}\} &= [\Phi^s]^T \{F^{ext}\} \ ; \\ \{\dot{q}_r\} &= \left[\widetilde{\Phi}^a\right]^T \{\dot{q}\} \ . \end{aligned}$$

The response of the viscothermal model can thus be expressed in terms of the uncoupled structural modes and the uncoupled inviscid, adiabatic acoustic modes. Once these modes are calculated, the viscothermal response can be determined with this modal superposition approach. The complex and frequency dependent pre-multiplication factors will include viscous and thermal effects in the analysis. The number of participation factors in $\{\eta^s\}$ and $\{\eta^a\}$ is usually much smaller than the number of degrees of freedom in $\{U\}$ and $\{P\}$. The modal superposition technique is thus an attractive approach to calculate the viscothermal acousto-elastic response.

3 Implementation in B2000

The modal superposition method described in the previous section is implemented in the frequency response processor of the finite element package B2000 [4].

3.1 B2FRF

The existing B2000 processor for frequency response functions (B2FRF) created by Grooteman [5] was used as the starting point for the implementation. In this processor Grooteman implemented already five methods for calculating the response of coupled acousto-elastic systems. All these methods are based on the assumption that the mass and stiffness matrices are frequency independent:

- Direct method;
- Mode displacement method;
- Mode acceleration method;
- Ma-Hagiwara method;
- Nastran method.

In the direct method the coupled set of equations is solved directly which is computionally very unattractive. The other four methods are modal methods. The displacement, acceleration and Ma-Hagiwara method [6] use the modes of the coupled system. The Nastran method uses the modes of the uncoupled systems. With some slight modifications the latter method could be used to implement the new method as discussed in section 2.

3.2 Procedure

For the calculation of the response with the new superposition method the following procedure is followed. First the fully coupled acousto-elastic finite element model is constructed. From this model the system matrices are obtained. Then a temporary model of only the acoustic domain with adiabatic and inviscid air is made and the real valued acoustic modes and eigenfrequencies are calculated. The same is done for the structural domain. The real valued mode shapes and eigenfrequencies of the uncoupled structure are calculated. The uncoupled acoustic and structural modes are used to build the reduced mass and stiffness matrix. Full coupling is enabled because the starting point was the fully coupled model. The submatrices which belong to the acoustic subdomain are now for each frequency step multiplied with a frequency dependent factor. For each frequency step the complex valued modal participation factors are solved and with these the response is obtained.

4 Validation

The method was validated using two test problems. The first problem consists of a two dimensional model of a double wall panel including a viscothermal air layer. For this system the numerical results are compared with an analytical solution. The second problem is a more realistic three dimensional problem for which the results obtained with the superposition method will be compared with results obtained with a (full) direct method.

4.1 Comparison with analytical solution

Consider a two dimensional model of a double wall panel subjected to a point force at $\overline{x} = -\frac{l_x}{2}$ on plate 1, see Figure 2. The thickness of plate 1 is 1 mm, the thickness of plate 2 is 2 mm an the thickness of the air layer is 3 mm. The value for l_x is 0.245 m. The panels are simply supported and a pressure release condition at the boundaries of the air layer is applied. Each panel is modeled with 80 Q4.ST elements. The air layer is modeled with 80 Q4.VISC elements. The coupling is established with 160 interface elements (Q8.INT). The numerical results are obtained with 25 structural modes and 20 uncoupled acoustic modes. The following material properties are used:

$$\begin{array}{rclrclrcl} \rho_{p1,2} & = & 2710\ \mathrm{kg/m^3}\ ; & E_{1,2} & = & 70 \cdot 10^9\ \mathrm{N/m^2}\ ; & \nu_{1,2} & = & 0.3\ ; \\ \rho_0 & = & 1.2\ \mathrm{kg/m^3}\ ; & c_0 & = & 340\ \mathrm{m/s}\ ; & \mu & = & 18.2 \cdot 10^{-6}\ \mathrm{Pas}\ ; \\ \lambda & = & 25.6 \cdot 10^{-3}\ \mathrm{W/mK}\ ; & C_p & = & 1004\ \mathrm{J/kgK}\ ; & \gamma & = & 1.4\ . \end{array} \tag{17}$$

Note that the Young's modulus is a real number so no structural damping is involved. The only damping in the double wall panel is introduced via the air layer. The frequency response function (H_R) at $\overline{x}_R = \frac{l_x}{2}$ on plate 2 is compared with the results of an analytical model [7]. The agreement between both results, given in Figure 3 is very good. Even the highly damped out-of-phase modes at low frequencies are very well described by the modal superposition technique.

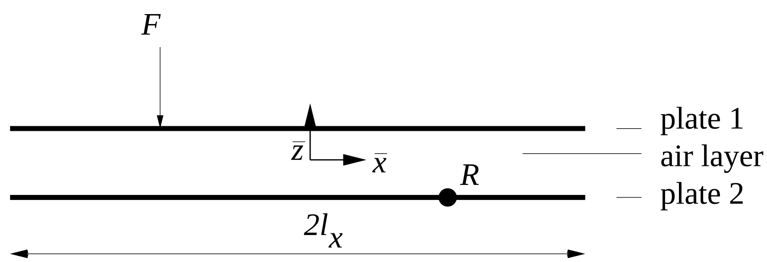

Figure 2: Test problem for validation of the modal superposition technique.

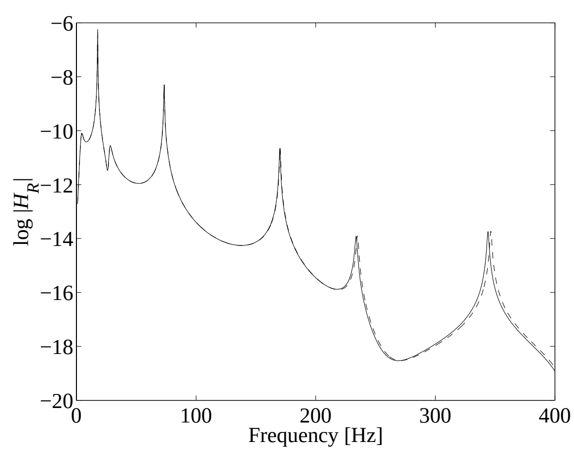

Figure 3: Transfer function of acousto-elastic system, analytical results (—) and numerical results (- -).

4.2 Comparison with direct method

For three dimensonal problems in general an analytical solution cannot be obtained so only numerical techniques can be applied. An alternative, but very inefficient method to obtain the frequency response is to perform a response calculation for each frequency step with the direct method. Then for each frequency step the appropriate full system matrices have to be assembled and one response calculation is performed. The system matrices can only be used for one frequency step because the matrices are frequency dependent. This method requires much time compared with the new modal superposition technique but gives the correct results for equation (4). Therefore the direct method gives a good reference solution with which the new modal superposition method can be validated.

The method was validated using a test problem, consisting of a double wall panel, see Figure 4. The two plates (dimensions: $2l_x = 0.5$ m and $2l_y = 0.25$ m) and the viscous air in

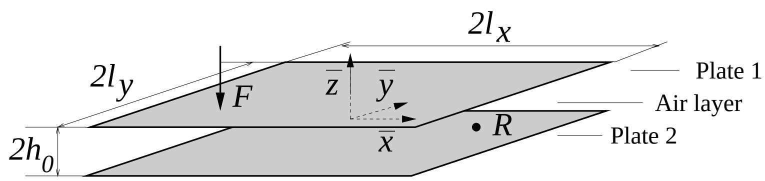

Figure 4: Test problem for validation of the matrix updating technique.

between are modeled with 24×12 elements. The two plates are clamped while for the air layer a pressure release boundary condition is applied. Plate 1 (thickness 1 mm) is excited with unit force at $\overline{x} = -\frac{l_x}{2}$ and $\overline{y} = -\frac{l_y}{2}$. The response of panel 2 (thickness 2 mm) is calculated at $\overline{x} = \frac{l_x}{2}$ and $\overline{y} = \frac{l_y}{2}$. The thickness of the air layer is 1 mm. The elements used are again Q4.ST for the two plates, Q4.VISC for the air layer and Q8.INT for the interface elements. The material properties are the same as in the previous test problem. The reference solution is obtained by using a direct method with the full mass and stiffness matrix corresponding to each frequency step. The solution with the modal superposition method is obtained by applying 25 uncoupled structural modes and 20 uncoupled acoustic modes.

The results given in Figure 5 show that the transfer function is predicted very well. The direct method however takes about 7 hours on a SG-O2 to compute the response for 101 frequency steps. The modal superposition method takes only several minutes.

5 Conclusions

A very efficient method to compute the frequency response of acousto-elastic systems including viscothermal wave propagation was developed and implemented in B2000. The method was validated with an analytical solution and a solution based on full direct evaluation. The results agree very well. The method can be applied for studies on and the development of double wall panels.

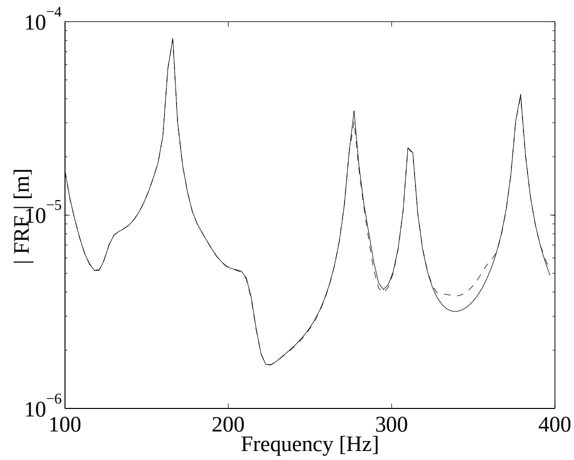

Figure 5: Transfer function of double wall panel. Full model (- -) and reduced model with 25 structural and 20 acoustic modes (—).

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