Reduction of fan noise by means of (circular) side-resonators; theory and experiment

M.J.J. Nijhof, Y.H. Wijnant, A. De Boer
University of Twente, Structural Dynamics and Acoustics group, Section of Applied Mechanics, Faculty of Engineering Technology, P.O. box 217, 7500 AE, Enschede, the Netherlands e-mail: m.j.j.nijhof@ctw.utwente.nl

W.M. Beltman
Intel Corporation, Corporate Technology Group 2111 NE 25th Avenue, OR 97124, Hillsboro, U.S.A.

Abstract

One of the main noise sources in computers are the cooling fans. An important aspect of the noise they generate is tonal noise produced at the rotational frequency of the fan, the blade passing frequency (BPF), and its higher harmonics. Previous research pointed out that so-called side resonators can be applied successfully to reduce this tonal noise. A variety of side resonator geometries is available. A model describing viscothermal wave propagation in a circular side-resonator and the connected duct and the preliminary results of a parameter study of that model where presented previously. The experimental verification of the model is presented here.

1 Introduction

The methods used to cool computer systems that are cheap, failsafe and easy to apply usually involve (axial) cooling fans. The limited space inside the housing of modern PC’s bounds the size of the cooling fans that can be used. To keep up with the demand for adequate cooling, sometimes the rotational velocity of the fans is increased which leads to a noise increase.

1.1 Fan Noise

The aerodynamic noise generated by fans consists of a combination of discrete frequency noise (tonal noise), related to the so-called blade passing frequency (BPF) and its higher harmonics, and broadband noise [3], [4] and [5]. The blade passing frequency of a fan is defined as the number of revolutions the fan makes per second, multiplied with the number of blades of the fan. The noise generated at the BPF and its higher harmonics is associated with harmonic loading developed on the rotor blades surface. Generally, this noise is dominant at high rotational speeds [5]. The broadband noise is generated, amongst others, by separation of boundary layers (trailing edges noise), areas with strong pressure gradients (tip and hub of the blade) and inlet flow turbulence. An example of the noise spectrum of an axial (processor) fan is given in Figure 1(a).

Tonal noise is important for the perceived sound quality. Therefore, the strategy to reduce axial fan noise consists of reducing the intensity or pressure level of the emitted sound spectrum at the BPF and its higher
harmonics. In case the fan speed is not actively controlled during operation, it is obvious that a solution for reducing fan noise must primarily filter out certain narrow frequency bands of the noise spectrum. However, most PC’s today are equipped with a speed controlled fan, which leads to a variable BPF. Therefore, the reduction of noise over a wider frequency band should be a secondary demand. This would also benefit the reduction of the broadband noise.

1.2 Acoustic side-resonators

From previous research ([2] and [6]) it is clear that acoustic resonators can be used effectively to reduce fan noise at specific frequencies or a frequency band without disturbing the mean fluid flow in a duct. The working principle of a side-resonator can be described in terms of acoustic impedance (see Figure 1(b)). A change in impedance, caused by a change in the medium or the geometry through which a sound wave travels, will cause sound waves to be (partially) reflected. Installing a side resonator at a certain location in a duct will change the impedance at that location and the resonator will function as an ‘acoustic mirror’ close to the eigenfrequencies of the resonator.

Figure 1: Typical noise spectrum of a small axial fan (a) and a schematic representation of the working principle of a side-resonator(b)

1.3 Geometries of side-resonators

Two frequently used side-resonators geometries are a (circular or rectangular) prismatic tube (see Figure 2(a)) or a cylindrical air layer (see Figure 2(b)) closed at one end and connected to the duct at the other end. In case multiple tubes of the same dimensions are used, more sound can be reflected back to the source, thus, improving the performance of the resonator setup [7]. A cylindrical layer can be seen as a large number of tubes that are bent alongside the circumference of the duct. Therefore, a cylindrical side-resonator performs better than a single tube resonator of the same length.

Figure 2: Schematic 3D section view of three side-resonator geometries, each installed in a circular duct
Another possible geometry for side-resonators is a circular layer, again closed at one end and connected to the duct at the other end (see Figure 2(c)). Because sound waves fan out in the geometry of a circular side-resonator, its properties differ from a cylindrical side-resonator.

The theory describing the physical behavior of circular resonators was presented previously (see [1], for prismatic tube and cylindrical resonators see [7], [8] and [9]). The results of the experimental verification of the theory are described here.

2 Theory of circular resonators

2.1 Cylindrical sound waves

The basis for the model of a circular resonator is the low reduced frequency (LRF) solution for a sound wave consisting of in- and outgoing cylindrical sound waves (see Figure 2.1). The solution for the pressure perturbation $p$ and radial particle velocity perturbation $v_r$ of cylindrical sound waves travelling in a thin circular layer are found to be, respectively [1]:

$$ p(r) = p_A J_0(-i\Gamma kr) + p_B Y_0(-i\Gamma kr) $$  \hspace{1cm} (1)

$$ v_r(r, z) = -\frac{1}{\gamma} \frac{\Gamma}{A(s, z/h)} [ p_A J_1(-i\Gamma kr) + p_B Y_1(-i\Gamma kr) ] $$  \hspace{1cm} (2)

with $J_0$ and $Y_0$ Bessel functions of zeroth order of the first and second kind respectively. The quantity $\Gamma$ and the wave number $k$ are defined as:

$$ \Gamma = \sqrt{\frac{\gamma}{n(s\sigma)B(s)}} \quad k = \omega/c_0 $$  \hspace{1cm} (3)

with

$$ n(s\sigma) = 
\begin{cases} 
1 + \left[ \frac{\gamma - 1}{\gamma} \right] D(s\sigma) & \text{isothermal walls} \\
D(s\sigma) = B(s\sigma) & \text{adiabatic walls}
\end{cases} $$  \hspace{1cm} (4)

$$ D(s\sigma) = B(s\sigma) $$  \hspace{1cm} (5)

$$ D(s\sigma) = -1 $$  \hspace{1cm} (6)

and the dimensionless groups

$$ s = l\sqrt{\rho_0/\mu} \quad \sigma = \sqrt{\mu C_p/\lambda} \quad \gamma = C_p/C_v $$  \hspace{1cm} (7)

in which $l$ is defined as half the resonator thickness, $\rho_0$ the mean density, $\omega$ the frequency, $\mu$ the dynamic viscosity, $\lambda$ the coefficient of thermal conductivity and $C_p$ and $C_v$ the specific heat at constant pressure and volume, respectively. Other time varying quantities of interest such as density and temperature can be expressed in terms of the functions and coefficient described above [1].
2.2 Coupled acoustic elements

With the general solutions of sound waves travelling in circular layers and prismatic ducts known, it is possible to link the analytical solutions of these models to obtain a solution for larger systems made up of several (different) acoustic elements. Two techniques to device such a coupling are described in the thesis of van Eerden [7]. One of these techniques, the so called FEM-like matrix formulation, is adopted here because of its straightforward implementation. It is based on the principle of conservation of mass at the point where elements are joined, and the continuity of pressure over the element boundaries. The models of the acoustic elements are written in matrix notation and the separate element matrices are then combined into one system matrix (analogous to assembling a system matrix of a FE-model). After the application of all boundary conditions the system can be solved using the same routines that are used in FE-calculations. The matrix formulation was already derived for cylindrical, volume and tube resonators by van Eerden and is used in the models described here. A matrix formulation of a circular resonator based on the solution presented above is given by:

\[
\begin{pmatrix}
Q_1 \\
Q_2
\end{pmatrix} = [K] \begin{pmatrix}
p_1 \\
p_2
\end{pmatrix}
\]

where,

\[
[K] = -\frac{\rho_0}{\gamma} B(s)[N][M]^{-1}
\begin{pmatrix}
A_1 & 0 \\
0 & A_2
\end{pmatrix}
\]

with the matrices \([N]\) and \([M]\) as defined in appendix A. The analogy with the element matrix of a FE-model of elastic materials is trivial. The matrix \([K]\) can be seen as the stiffness matrix of the element, the pressures \(p_1\) and \(p_2\) as displacements and the averaged mass flows \(Q_1\) and \(Q_2\) passing through the element boundaries \(A_1\) and \(A_2\) can be seen as loads. The process of assembling the system matrix, applying the boundary conditions and solving the system are therefore identical to the routines used for structural FE-models and are therefore not described here. A third type of boundary condition, a prescribed impedance (or a prescribed ratio between pressure and particle velocity) at a certain node, is special. The boundary condition can be applied by adding an extra 'stiffness' term \(K_Z\) to the diagonal element \(K_{ii}\) of the system matrix.

\[
K_Z = \frac{A_i \rho}{Z}
\]

with \(Z\) the characteristic impedance of the element under consideration.

3 Test setup

3.1 Design of the resonator setup

The test setup described in this section is developed to validate the LRF model for circular side resonators. A schematic representation of the setup is depicted in Figure 5. It consists of a tube in which a planar wave is induced by a speaker or transducer and a resonator, which is placed half way of the tube. On both sides of the resonator two pressure sensors are installed. The time-signals of these sensors are transformed to the frequency domain and these results are used to calculate the (complex) amplitudes of the back and forward travelling sound waves in both parts of the tube. These amplitudes are the basis for the calculation of the impedance, reflection coefficient or other quantities of interest.

In order to verify a relatively wide variety of resonator dimension with a single setup, the resonator was made out of three parts that can be easily exchanged and reassembled to form a variety of resonators. The resonator consists of a metal ring, clamped between two plastic disks (rigid and acoustically hard). The inner radius and thickness of the metal ring determine the outer radius and the thickness of the air gap inside the resonator. The three parts are simply bolted together in preparation for a measurement (see Figure 6(a)). A range of metal rings with different inner radii and thicknesses was produced to form a set of resonators covering the range of dimensions of interest.

It is also possible to insert a combination of rings between the plastic disks. In this fashion, it is not only possible to measure a larger range of thicknesses with a limited set of disks, but it is also possible to device
two coupled circular resonators, by clamping a ring with a certain inner radius between two rings with a smaller inner radius (see Figure 6(b)).

3.2 Model of the resonator setup

The matrix formulation described in section 2.2 is used to model the setup described above. Figure 7 shows a schematic representation of the setup and the corresponding representation in acoustic elements.

Element I and II are prismatic tube elements, element III is the newly developed circular resonator element. The boundary condition at node 1 is a prescribed pressure perturbation. At node 4 the particle velocity is set to zero, representing an acoustically hard wall. The boundary condition at node 3 is a so called radiation
Impedance. The expression for the radiation impedance of baffled and unflanged tubes is well documented in literature (see [10]). In the test setup, an unflanged end condition would have been disturbed by the support of the impedance tube, so a baffle was used to create an end condition that could be modelled easily.

3.3 Model of a coupled resonator setup

The model of the matrix formulation corresponding to a setup with two serially coupled circular side resonators is depicted in Figure 8. This model resembles the model of the matrix formulation that is used for the setup with a single disk to a great extent. An extra element (IV), representing the second disk is added at node 4. There are no boundary conditions applied at this node. At node 5, a zero velocity boundary condition is imposed, representing the acoustically hard back wall of the second resonator.

4 Results

4.1 Result of a single circular resonator

The developed model provides a scale of acoustic quantities such as for instance the pressure at any given location in the model, the amplitudes of the back and forth travelling waves and the reflection coefficient $R$ which expresses the fraction of the sound that is reflected by the resonator (and the open end of the tube). It is relatively easy to compute transfer functions of arbitrary points in the setup with the available theory, and since the transfer function $H_{21}$ between the pressure sensor pair on the source side of the resonator is obtained directly from the measurement, it is an obvious choice to compare the measurements results with the calculated results on the basis of this transfer functions. However some differences between measurements and calculations are better observed by comparing other quantities. Therefore, the measured and calculated results of both the transfer functions between the pressure sensors and the reflection coefficient and impedance are presented for each examined case.
4.1.1 Effects of viscosity

In the next two cases, it is investigated, whether viscous effects influence the accuracy of the calculated result. First a resonator with a very narrow air layer, and therefore strong viscous effects, is considered. Thereafter, a resonator with an air layer that is four times as thick, and therefore, less subjected to viscous effects is considered.

The measured and calculated results of a resonator with an outer radius $R_{\text{out}} = 125$ mm and a thickness $h = 1$ mm are depicted in Figure 9. It is clear from the plot of the transfer function, that the model predicts the measured data quite well. In Figure 9(b) the scaled impedance and reflection coefficient at the resonator opening on the source side. These measured quantities mach up with their calculated counterparts equally well.

In literature [9] it is shown that the shear wave $s$, number represents the ratio between inertial and viscother- mal forces. In case $s < 10$, the particle velocity of a significant part of the thickness is influenced by the presence of the resonator wall, and the wave propagation is effected considerably by viscothermal effects. Depending on the frequency, the corresponding shear wave number in the resonator for this specific test has a value between: 3.2 and 14.7. This is an indication that for the present case the processes in the resonator were dominated by viscothermal effects for most of the inspected frequency range.

Now, the accuracy of the model of a resonator in which viscous effect are not dominant is considered. Because of the increased thickness of the air layer the wave propagation in the resonator is far less influenced by viscous effects. The measured and calculated results of a resonator with an outer radius $R_{\text{out}} = 125$ mm and a thickness of $h = 4$ mm are depicted in Figure 10. It is clear from the plot of the transfer function, that the model predicts the measurement data very well, and so do the plots in Figure 10(b) representing the scaled impedance and reflection coefficient just at a point just before the resonator. Depending on the frequency that is observed, the corresponding shear wave number in the resonator for this specific case has a value between: 12.9 and 59.0. This indicates that the processes in the resonator are not dominated by viscous effects as was the case for the resonator with a thickness of one. From these result, it is concluded that the predictions of the model are correct both in cases where viscosity is dominant ($s < 10$), and in cases were viscous effects can be neglected ($s \gg 10$).

4.1.2 Effects of the wavelength

In the next two cases, the effect of the outer radius has on the accuracy of the results of the model, is investigated. First, the results of a resonator with an outer radius of 75 mm and a thickness of 2 mm are
Figure 10: Measured and calculated results of a resonator with $R_{\text{out}} = 125$ mm and $h = 4$ mm

Figure 11: Measured and calculated results of a resonator with $R_{\text{out}} = 75$ mm and $h = 2$ mm

The second case that is evaluated is that of a resonator with an outer radius of $R = 219$ mm and a thickness of $2$ mm (Figure 12). Again, a deviation is noticed, however, not in the same region as in the previous case. In the frequency range from 100 Hz to 300 Hz, the 'measured' value for the reflection coefficient is 5 to 50 percent lower than the calculated values. The peaks of the calculated transfer function in this region are also slightly shifted to higher frequencies in comparison with the measured transfer function. The deviations of the calculated results with respect to the measured results in the order of the case as described above, where not observed for smaller radii. However, deviation of similar order were present for a resonator with an outer radius of 219 mm and a thickness between 1 and 4 mm.

In general, the differences between measurements and theory are quite small for the various measurements that were taken. It is therefore assumed that the developed model is sufficiently accurate, and can be used to predict the efficiency of a resonator setup in an attempt to find an optimal resonator configuration for a noise reduction problem.
4.2 Results of coupled circular side resonators

This section deals with layout and results of a setup with two coupled circular resonators. In analogy with the previous section, the model that was used is described first, and afterwards a number of cases are discussed that demonstrate the validity of the developed model over a specific parameter range. Lastly, a case that deals with a resonator setup with three coupled resonators is presented.

4.2.1 Effects of viscosity

In analogy with section 4.1.1, it is verified if the accuracy of the calculated results is dependent of the level at which viscous effects are present. In the first test case two coupled resonators with the following dimensions are considered; outer radii \( R_{1,\text{out}} = 125 \) mm and \( R_{2,\text{out}} = 175 \) mm, and \( h_1 = 2 \) mm and \( h_2 = 1 \) mm. The calculated and measured results are depicted in Figure 13.

The thicknesses of both resonators are relatively small, so viscous effects dominate the acoustical processes. The calculated transfer function, reflection coefficient and impedance agree well with the measurements of these quantities. However, three areas can be identified in which the predicted results are less accurate. Between 1.5 kHz and 1.8 kHz, the measured reflection coefficient is somewhat lower. A similar overestimation
of the calculated reflection coefficient is found in the interval from 600 Hz to 1000 Hz. Furthermore, the measurement data within this area shows an extra dip in the reflection coefficient at 605 Hz and at 320 Hz. Both dips do not show up in the calculated results at all, which could indicate that there is a resonance present in the test setup that is not accounted for in the model.

Figure 14: Measured and calculated results of a coupled resonator with \( R_{1,\text{out}} = 125 \) mm, \( R_{2,\text{out}} = 175 \) mm, \( h_1 = 6 \) mm and \( h_2 = 3 \) mm

For a resonator with two serially coupled disk of much larger thickness, the same dips are observed, as can be seen in Figure 14. However, the depth of the dips are not as much influenced by the change in thickness as the other deviations. The frequencies of the dips do not correspond to any of the characteristic lengths in the setup so their origin remains unexplained.

### 4.2.2 Effects of the wavelength

In analogy with section 4.1.2, it is verified if the accuracy of the calculated results is dependent of the wavelength that will fit in the resonators. Figure 15 shows the results of two serially coupled resonators with the following dimensions: \( R_{1,\text{out}} = 75 \) mm, \( R_{2,\text{out}} = 125 \) mm, \( h_1 = 4 \) mm and \( h_2 = 2 \) mm. Again, theory an measurement results agree with each other quite well, except for a small dip in the measured reflection coefficient close to 600 Hz.

Figure 16 shows the results of two coupled resonators of identical thickneses but with larger outer radii: \( R_{1,\text{out}} = 175 \) mm, \( R_{2,\text{out}} = 219 \) mm, \( h_1 = 2 \) mm and \( h_2 = 1 \) mm. Although the theoretical results correspond to the measured results reasonably well, here too, small deviations in the form of two dips in the measured reflection coefficient are observed.

The results of a test setup with three coupled resonators was also verified. The calculated results are reasonably accurate, however, it was found that the magnitude of the deviations that were observed increase with the number of resonators that are coupled. Therefore, is expected that the calculated results of resonator setups with multiple resonators become increasingly inaccurate when more resonators are coupled. Furthermore, the effects of symmetry of coupled air layers was tested by coupling the air layers in a symmetric and asymmetric way. It was found that the accuracy of the results is not influenced by the way the resonators are coupled.

### 5 Conclusions

It was shown, that the previously presented theoretical model of a single circular resonator predicts the results of the test setup very accurately for the most of the inspected frequency range. The model describing a test
setup with two coupled resonators is accurate in predicting the values of the various acoustical quantities. However, in the measured reflection coefficient of a number of cases with two coupled resonators, one or two dips can be observed that do not appear in the calculated data. The location in the frequency plot of these dips, could not be accounted for by the frequencies corresponding to the wavelengths of the characteristic lengths of the setup. Since the observed deviations are relatively small, the developed model for single and coupled resonators can be considered accurate within the parameter range that is of interest for the application of circular side resonators in PC’s.

References


Matrix formulation of a circular air layer

The coefficients $p_A$ and $p_B$ in the solution for pressure and particle velocity in Equation 1 and 2 are yet to be determined. This can be accomplished by applying appropriate boundary conditions to Equation 1 and 2. In acoustic problems the boundary conditions usually involve applying a certain pressure, velocity or impedance at the boundaries of the geometry.

$$ p_i \rightarrow U_1 \rightarrow U_2 \rightarrow p_{i+1} \text{ - pressure perturbation} $$
$$ v_i \rightarrow \zeta_i \rightarrow \zeta_{i+1} \text{ - scaled acoustic impedance} $$
$$ v_{i+1} \text{ - velocity perturbation} $$

Figure 17: Representation of an acoustic element with possible boundary conditions

To be able to choose between pressure or velocity based boundary conditions at a later stage, the coefficients $p_A$ and $p_B$ are expressed first in terms of the following pressure based boundary conditions.

$$ p = p_1 \text{ at } r = R_{in} \quad ; \quad p = p_2 \text{ at } r = R_{out} \quad (10) $$

where $p_1$ and $p_2$ are yet undetermined functions and $R_{in}$ and $R_{out}$ are the inner and outer radius of the circular layer, respectively. When both boundary conditions are applied to the equation for particle velocity, the following system of equations emerges:

$$ \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = [M] \begin{bmatrix} p_A \\ p_B \end{bmatrix} \quad \text{where, } [M] = \begin{bmatrix} J_0(-i\Gamma k R_{in}) & Y_0(-i\Gamma k R_{in}) \\ J_0(-i\Gamma k R_{out}) & Y_0(-i\Gamma k R_{out}) \end{bmatrix} \quad (11) $$

Next, the particle velocities averaged over the thickness of the layer, on the rims of the disk are taken as boundary conditions:

$$ \frac{1}{2h} \int_{-h}^{h} v_r dz = U_1 \text{ at } r = R_{in} \quad ; \quad -\frac{1}{2h} \int_{-h}^{h} v_r dz = U_2 \text{ at } r = R_{out} \quad (12) $$

where $U_1$ and $U_2$ are yet undetermined, and $R_{in}$ and $R_{out}$ are, again, the inner and outer radius of the circular layer, respectively. The minus sine in the expression for $U_2$, is the result of the convention that, for both nodes of an element, an inward flow is considered to be positive (Figure 17). When both boundary conditions are applied to the equation for particle velocity, the following system of equations emerges:

$$ \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = -\frac{\Gamma}{\gamma} B(s) [N] \begin{bmatrix} p_A \\ p_B \end{bmatrix} \quad \text{where, } [N] = \begin{bmatrix} J_1(-i\Gamma k R_{in}) & Y_1(-i\Gamma k R_{in}) \\ -J_1(-i\Gamma k R_{out}) & -Y_1(-i\Gamma k R_{out}) \end{bmatrix} \quad (13) $$

The expression for $p_A$ and $p_B$ in terms of $p_A$ and $p_2$ is obtained with Equation 11 and is substituted into Equation 13. Both equations of the resulting system are multiplied by $A_i/\rho_0$, with $A_i$ the size of the related boundary surface. The system is now represented in terms of mass fluxes $Q_1$ and $Q_2$ passing through the element boundaries$^1$.

$$ \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = [K] \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} \quad \text{where, } [K] = -\rho_0 \frac{\Gamma}{\gamma} B(s) [N][M]^{-1} \begin{bmatrix} A_1 \\ 0 \\ 0 \\ A_2 \end{bmatrix} \quad (14) $$

$^1$It is assumed that $\rho \ll \rho_0$, so it is valid to use the initial value $\rho_0$ instead of the actual value $\rho$.