Prediction of mechanical fatigue caused by multiple random excitations

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Abstract

A simulation method is presented for the fatigue analysis of automotive and other products that are subjected to multiple random excitations. The method is denoted as frequency domain stress-life fatigue analysis and was implemented in the automotive industry at DAF Trucks N.V. in Eindhoven, The Netherlands. As an example case, a chassis part is analysed. The results of the analysis are consistent with fatigue cracks encountered during testing, which illustrates the effectiveness of the adopted method in the automotive industry.

1 Introduction

Due to the relatively large size of trucks on the one hand and the relatively slender build of truck chassis on the other, fatigue life of truck components is largely determined by structural resonances. For this reason and due to increasing requirements for reliability and safety, truck manufacturers and their suppliers spend a lot of time and money optimising and verifying fatigue life of new truck designs. In the commercial vehicle business, in which DAF Trucks N.V. is active, the tendency is to reduce the weight for economic reasons (fuel economy, the possibility for customers to transport more goods). The challenge is to reduce a vehicle's weight without an increase of the truck's sensitivity to fatigue.

Development time becomes shorter and new designs need to be first time right. Time consuming durability tests at the end of the design process are being replaced by CAE analysis up front. The number of required design modification can be reduced when designs are already evaluated on fatigue when the first concepts are set up instead of having to wait for durability tests. This paper describes an approach to numerical fatigue simulation using the Finite Element Method.

Several approaches with respect to fatigue analysis exist. The most well-known are stress-life, strain-life and crack propagation methods [1]. The stress-life method is generally accepted in case of high-cycle fatigue problems, as is the case with trucks excited by road irregularities. Using this method, fatigue failure prediction is based on the magnitude of the stresses occurring in the construction. It is especially well-known for the case that a test specimen is loaded by a fluctuating uni-axial force of constant amplitude. The stress amplitude caused by the loading can be evaluated with an S-N or Wöhler curve, which indicates the number of cycles that can be applied until failure occurs [2].

Trucks are subjected to road excitation through the wheels. The vibrations caused by the road excitations are typically of random nature. Random vibrations are generally described in the frequency domain by means of Power Spectral Densities (PSDs). The method adopted at DAF can be denoted as frequency domain stress life fatigue analysis and is described in detail in the next Section 2. Other sections include the implementation of the method using the Finite Element package NASTRAN and MATLAB (Section

3) and an example of a truck chassis part as an illustration of the use in automotive practice (Section 4). Finally, conclusions are formulated in Section 5.

2 Method for frequency domain stress-life fatigue analysis

The frequency domain stress-life fatigue analysis exists of the following stages:

- Constructing an input PSD matrix describing the multiple random road excitations
- Calculating the response stress PSD matrices throughout the analysed construction
- Determining an equivalent stress PSD from the response stress PSDs
- Performing a uni-axial stress-life fatigue analysis from the equivalent stress PSD

2.1 Constructing the input PSD matrix

Let us start with the description of multiple random road excitations by a three-dimensional excitation PSD matrix $[G_{xx}]$. Assume that the excitation of a truck's left front wheel is given by the auto PSD $G_{11}(f)$. At a certain frequency f_m, the value of $G_{11}(f)$ is $G_{11}(f_m)$. The excitation of the right front wheel at frequency f_m is called $G_{22}(f_m)$ and so on for the many other wheels.

Correlations that exist between the wheels are taken into account using cross PSDs. Wheels at the left rear of a truck are subjected to the same excitation as the left front wheel, only after a time delay, i.e. a phase difference is present in the cross PSDs between front and rear wheels. This phase difference results in complex cross PSDs. Similar correlations exist between left and right wheels.

For each frequency f_m, the input PSD matrix $[G_{xx}]$ can now be defined as:

$$[G_{xx}(f_m)] = \begin{bmatrix} G_{11}(f_m) & \cdots & G_{1n}(f_m) \\ \vdots & \ddots & \vdots \\ G_{1n}^*(f_m) & \cdots & G_{nn}(f_m) \end{bmatrix} \quad m = 1,2\ldots M \tag{1}$$

in which $[G_{1n}^*(f_m)]$ denotes the complex conjugate of $[G_{1n}(f_m)]$. The third dimension is the frequency existing of M frequency bands. Equation (1) is a hermitic matrix, containing the auto PSDs on the diagonal and the cross PSDs on the off-diagonal positions. The size of the matrix is n x n x M, with n the number of multiple excitations. For a truck with 12 wheels and a frequency range of 0.2-20Hz, divided in bands of 0.2Hz, this means that $[G_{xx}]$ is a 12 x 12 x 100 matrix.

A more detailed description of the construction of the input PSD matrix for trucks excited by road irregularities is presented by Anderson [3].

2.2 Calculating the response stress PSD matrix

The response stress PSDs can be determined using a matrix of transfer or Frequency Response functions $[H_\sigma(f_m)]$, which describes the relation between the response stresses somewhere in the construction and the unity (displacement) excitation:

$$\{\sigma(f_m)\} = [H_\sigma(f_m)]\{X(f_m)\} \tag{2}$$

$\{X(f_m)\}$ is related to the input PSD matrix of Equation (1) by:

$$[G_{xx}(f_m)] = E\left(\{X^*(f_m)\}\{X(f_m)\}^T\right) \tag{3}$$

with E the expected value operator. The response stress PSD matrix $[G_{\sigma\sigma}(f_m)]$ is related to $\{\sigma(f_m)\}$ in a similar way. It can be shown [4] that the response stress PSD matrix can be calculated from the input PSD matrix by:

$$[G_{\sigma\sigma}(f_m)] = [H_\sigma^*(f_m)] \cdot [G_{xx}(f_m)] \cdot [H_\sigma(f_m)]^T \tag{4}$$

In case of multi-axial stresses, the stress response PSD matrix $[G_{\sigma\sigma}]$ is a 6 x 6 x M matrix. For fatigue, however, it is often assumed that cracks start at a free surface where a bi-axial (plain) stress state exists [2], in which case $[G_{\sigma\sigma}]$ reduces to a 3 x 3 x M matrix. This is also the case for plate structures. Two normal and one shear stress PSD remain:

$$[G_{\sigma\sigma}(f_m)] = \begin{bmatrix} G_{\sigma xx,\sigma xx}(f_m) & G_{\sigma xx,\sigma yy}(f_m) & G_{\sigma xx,\tau xy}(f_m) \\ G^*_{\sigma xx,\sigma yy}(f_m) & G_{\sigma yy,\sigma yy}(f_m) & G_{\sigma yy,\tau xy}(f_m) \\ G^*_{\sigma xx,\tau xy}(f_m) & G^*_{\sigma yy,\tau xy}(f_m) & G_{\tau xy,\tau xy}(f_m) \end{bmatrix} \tag{5}$$

Again the auto PSDs are real, whereas the cross terms are complex because of phase differences between the bi-axial stresses. These phase differences are a consequence of both phase differences caused by the excitation and damping present in the construction.

2.3 Determining an equivalent stress PSD from the response stress PSDs

As an equivalent stress, the well-known Von Mises stress is used. In the time domain and assuming a bi-axial stress state, it is defined as:

$$\sigma_{VM}^2(t) = \sigma_{xx}^2(t) + \sigma_{yy}^2(t) - \sigma_{xx}(t)\sigma_{yy}(t) + 3\tau_{xy}^2(t) \tag{6}$$

Preumont [5] proposed the following formula for the calculation of an equivalent stress PSD based on the Von Mises stress:

$$G_{\sigma vm}(f_m) = trace([A] \cdot [G_{\sigma\sigma}(f_m)]) \tag{7}$$

The matrix $[A]$ follows from the quadratic formulation of the Von Mises stress and equals in the bi-axial case:

$$[A] = \begin{bmatrix} 1 & -0.5 & 0 \\ -0.5 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} \tag{8}$$

This is correct when all stress components are in-phase. The neglecting of phase differences causes an overestimation of the Von Mises stress. This is illustrated in Figure 1 for sinusoidal stresses where Equation (7) indeed calculates the maximum stress for in-phase stresses as one would expect. The

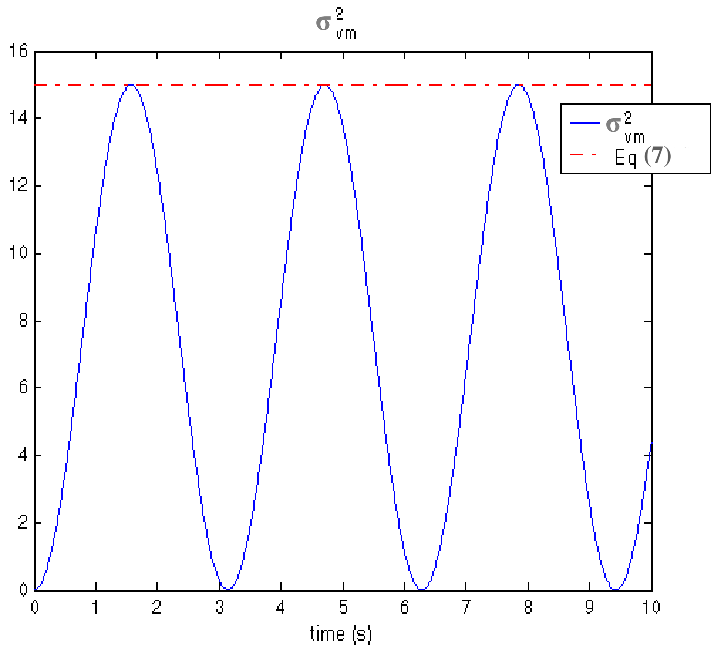

Figure 1: Von Mises stress (Eq. 7) for in-phase sinusoidal stresses

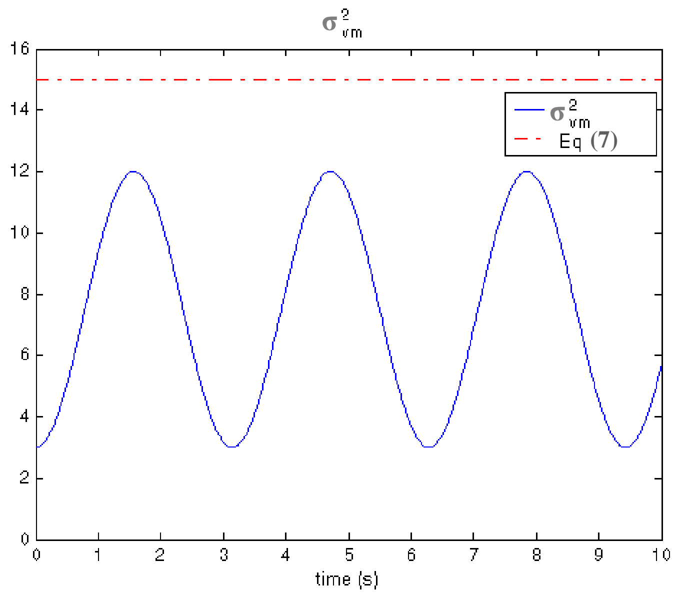

Figure 2: Von Mises stress (Eq. 7) for out-of-phase sinusoidal stresses

overestimation of the Von Mises stress in case of out-of-phase sinusoidal stresses is demonstrated in Figure 2.

Since it is the maximum Von Mises stress that is used for fatigue calculations, it would be beneficial to get rid of the overestimation in case of applying Equation (7). The following equation for the Von Mises PSD [6] takes phase differences into account and can subsequently be used for frequency domain fatigue analysis:

$$G_{\sigma VM}(f_m)=\frac{1}{2}\left(\hat{\sigma}_{xx}^2+\hat{\sigma}_{yy}^2-\hat{\sigma}_{xx}\hat{\sigma}_{yy}\cos(\varphi_{xx}-\varphi_{yy})+3\hat{\tau}_{xy}^2\right)+ \frac{1}{2}abs\left(\hat{\sigma}_{xx}^2+\hat{\sigma}_{yy}^2 e^{2j(\varphi_{yy}-\varphi_{xx})}-\hat{\sigma}_{xx}\hat{\sigma}_{yy}e^{j(\varphi_{yy}-\varphi_{xx})}+3\hat{\tau}_{xy}^2 e^{2j(\varphi_{xy}-\varphi_{xx})}\right) \quad (9)$$

where

$$\begin{aligned}
\hat{\sigma}_{xx}^2(f_m)&=G_{\sigma xx,\sigma xx}(f_m)\\
\hat{\sigma}_{yy}^2(f_m)&=G_{\sigma yy,\sigma yy}(f_m)\\
\hat{\tau}_{xy}^2(f_m)&=G_{\tau xy,\tau xy}(f_m)\\
\hat{\sigma}_{xx}(f_m)\hat{\sigma}_{yy}(f_m)&=\sqrt{\hat{\sigma}_{xx}^2(f_m)\hat{\sigma}_{yy}^2(f_m)}=\sqrt{G_{\sigma xx,\sigma xx}(f_m)G_{\sigma yy,\sigma yy}(f_m)}
\end{aligned} \quad (10)$$

and

$$\begin{aligned}
\varphi_{yy}(f_m)-\varphi_{xx}(f_m)&=angle\left(G_{\sigma xx,\sigma yy}(f_m)\right)\\
\varphi_{xy}(f_m)-\varphi_{xx}(f_m)&=angle\left(G_{\sigma xx,\tau xy}(f_m)\right)
\end{aligned} \quad (11)$$

For in-phase bi-axial stresses, Equation (9) gives the same results as Equation (7). However, for out-of-phase stresses, Equation (9) indeed results in the maximum Von Mises stress for out-of-phase bi-axial stresses as presented in Figure 3. If needed, Equation (9) can easily be extended to multi-axial stress states.

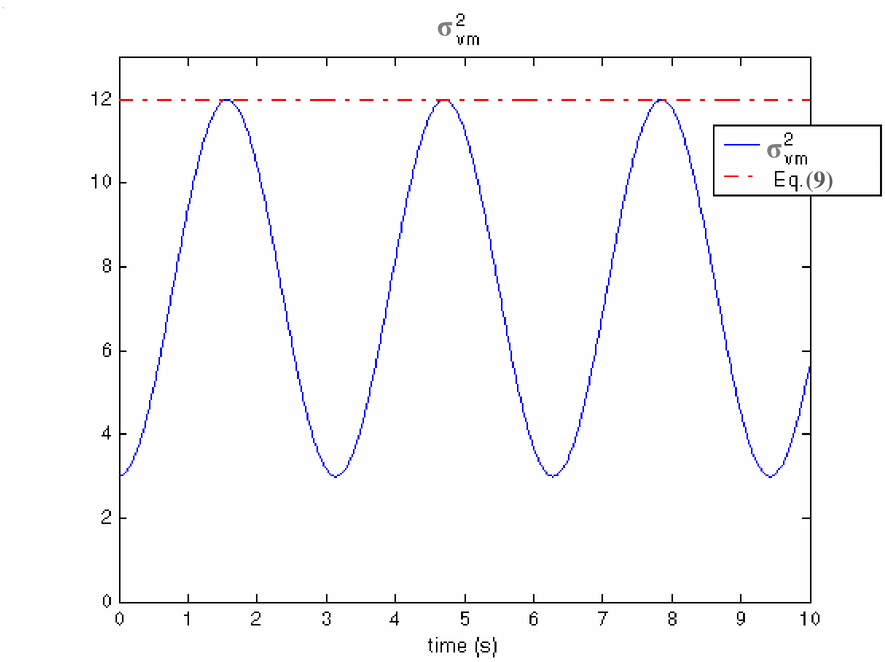

Figure 3: Von Mises stress (Eq. 9) for out-of-phase bi-axial sinusoidal stresses

2.4 Performing a uni-axial frequency domain fatigue analysis

For all frequency bands, Equation (9) provides an equivalent Von Mises stress PSD, which can subsequently be used for uni-axial frequency domain fatigue analysis. The Von Mises stress PSD can be transformed to a probability density function (PDF) by Dirlik rainflow counting [7,8]:

$$p(S) = \frac{\frac{D_1}{Q} e^{-\frac{Z}{Q}} + \frac{D_2 Z}{R^2} e^{-\frac{Z^2}{2R^2}} + D_3 Z e^{-\frac{Z^2}{2}}}{2\sqrt{m_0}} \tag{12}$$

where	$p(S)$	Probability density function
`	S	Stress amplitude
	m_0	The 0th spectral moment
	D_1, D_2, D_3, Q, Z, R	Parameters depending on the 0th, 2nd and 4th order spectral moments

The nth spectral moment of the Von Mises PSD is defined as:

$$m_n = \int_0^\infty f^n G(f) df = \sum_{m=1}^{M} f_m^n G_{\sigma vm}(f_m) \tag{13}$$

The PDF in Equation (12) presents a relation between the magnitude of stress amplitudes and the number of stress amplitudes and can be compared to an S-N curve, which is given by Basquin's equation [2]:

$$S^k N = b \tag{14}$$

with:	S	The stress amplitude
	N	Number of cycles to failure
	k, b	Constant material parameters

The well-known Palmgren-Miner rule can now be applied for obtaining a damage number D:

$$D = \sum_i \frac{n_i}{N_i} \tag{15}$$

where:	n_i	number of cycles that stress amplitude S_i is present
	N_i	number of cycles at which stress amplitude S_i leads to failure

n_i follows from Dirlik's PDF:

$$n_i = N_{tot}\, p(S_i) = E[P] \cdot T \cdot p(S_i) \tag{16}$$

with:	n_i	Number of cycles at stress amplitude interval i

N_{tot} Total number of cycles

$p(S_i)$ Probability that the stress amplitude interval is i

$E[P]$ The expected number of peaks during one second ($=\sqrt{m_4/m_2}$ [7,8])

T Total time

N_i can be obtained simply by rewriting Basquin's equation (Equation (14)):

$$N_i = b \cdot S_i^{-k} \tag{17}$$

Combination of Equations (15) through (17) now yields the fatigue damage number *D:*

$$D = \frac{E[P]T}{b} \sum_{i=0}^{i_{\max}} S_i^k p(S_i) = \frac{E[P]T}{b} \int_{S=0}^{\infty} S^k p(S) dS \tag{18}$$

3 Implementation

The frequency domain stress-life fatigue analysis was implemented in an environment with the Finite Element software NASTRAN and MATLAB. Figure 4 presents an overview of the implementation.

The FEM package NASTRAN is solely used for determining the Frequency Response Functions (FRFs) $[H_\sigma]$ from the excitation displacements to the response stresses in the shell elements of the construction. For this, a frequency response analysis (SOL 108 in NASTRAN) is used. Unit displacements are applied to one of the excited wheels, whereas the displacements of the other wheels are constrained. By evaluating the stresses caused by this single unity excitation, the transfer functions from this excitation to all stress responses are known. The remaining FRFs are calculated by repeating this procedure for all wheels. Since normally the forces are prescribed within the Finite Element Method, special measures need to be taken to prescribe the unit displacements. This is done using the Lagrange multiplier method. It has the benefit that only one decomposition is needed for all unit displacement load cases.

The FRFs are subsequently imported in a MATLAB toolbox developed by DAF, which is called TricaT (Toolbox for RIde and Comfort Analysis of Trucks). After the excitation PSD matrix $[G_{xx}]$ is calculated, the toolbox determines the bi-axial stresses using these FRFs. Subsequently, the equivalent Von Mises stress PSD is calculated using Equation (9) and the fatigue damage is determined with Equation (18). The bi-axial and Von Mises stresses and the fatigue damage can be visualised using standard post-processing software. This procedure gives the opportunity to evaluate multiple concepts quickly, also showing the influence of road type and vehicle speed. No time histories are needed, since all is done in the frequency domain using generalised road descriptions.

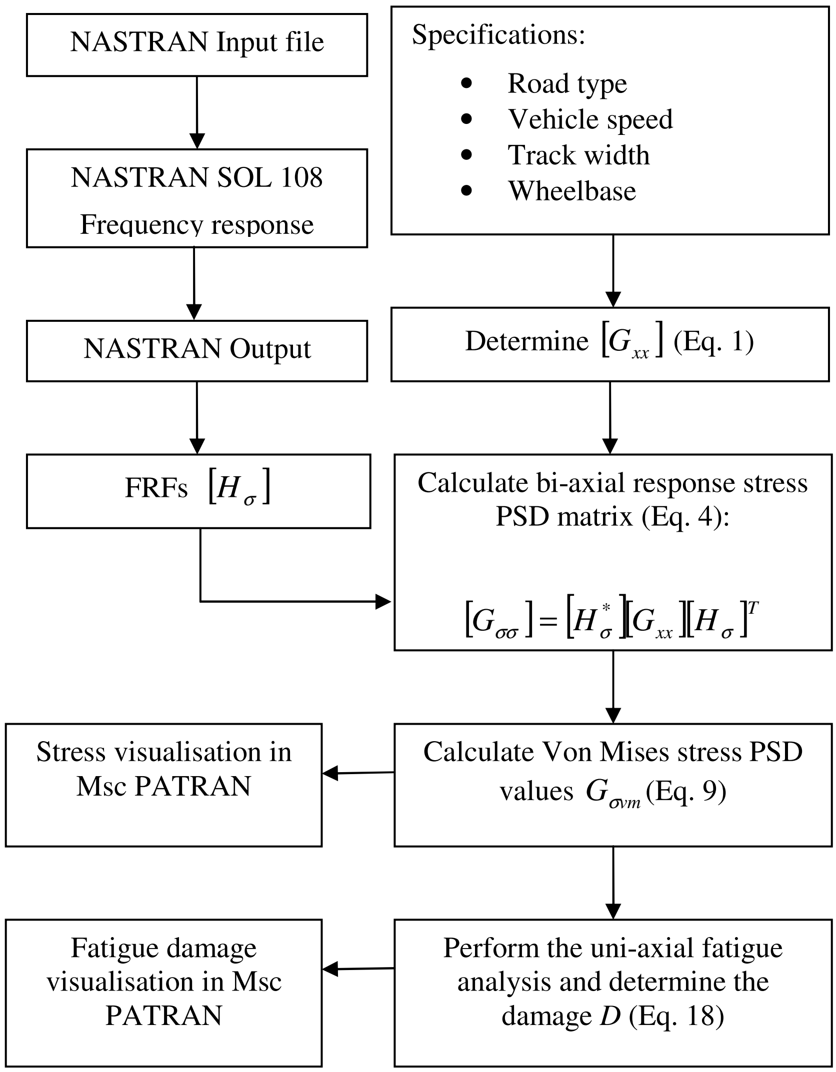

Figure 4: Implementation of the method for fatigue analysis

4 Automotive example

The fatigue life of a cross member is predicted using a FE-model of a 12 wheeled tractor-semi trailer combination as shown in Figure 5. All components are included as superelements and not visible in the figure except for the chassis cross member. The enlarged FE-model of the cross member is also shown in Figure 5.

The following steps are taken to analyse the cross member's fatigue life:

- Perform a NASTRAN frequency response analysis with unity excitation (SOL 108)
- Identify the hot-spots using the NASTRAN/TricaT fatigue analysis software
- Find the cause of the poor fatigue behaviour at the critical locations
- Improve the construction with respect to fatigue
- Simulate again for evaluation purposes

These five steps are described below for the example. Figure 6 shows one of the results after step 2. The figure shows that the Von Mises stress Root Mean Square value (RMS) is maximal in the element within the circle. The Palmgren-Miner damage number results show the same location as being the most

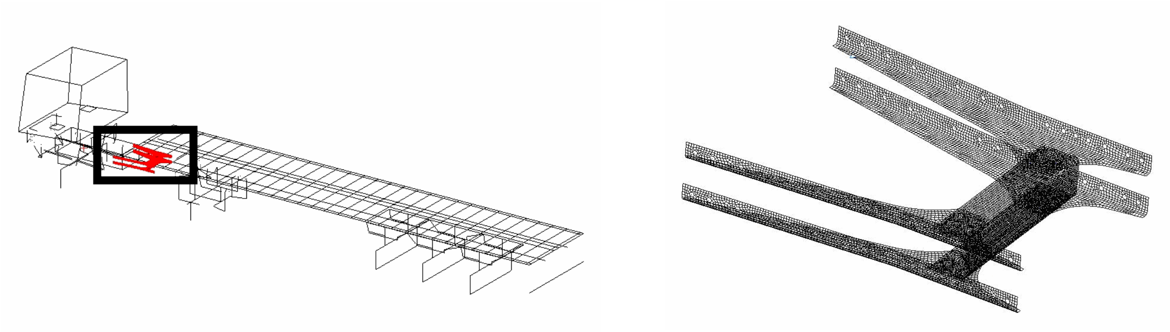

Figure 5: Finite Element model of a full truck and the included chassis cross member

susceptible to fatigue. Tests have indicated that the hot-spot predicted by the fatigue analysis method presented in this paper coincides with hot-spots encountered in practice. Figure 7 shows a picture of a fatigue crack at the same location as predicted by the implemented software.

The next question arising is what causes the fatigue behaviour at this location in the initial design. The software assists in determining this cause by presenting the Von Mises stress PSD at the critical location. In this way, one can identify the frequency bands that contribute most to the fatigue damage and the engineer knows at which frequency to analyse the dynamics. Additionally, the total PSD can be split up into a stress PSD due to the symmetric and asymmetric content of the road excitation. For symmetric road excitation, all wheels are excited in-phase, whereas for asymmetric excitation, the left and right track wheels are excited with 180 degrees phase difference. Figure 8 shows the Von Mises stress PSDs for the identified hot-spot.

Now, it is known at which frequency (10Hz) the problems are mainly caused and if the symmetric or asymmetric road excitation contributes most to this damage (in this case the asymmetric one). DAF's simulation methods additionally allow the engineer to simulate the truck's motion at a specific frequency due to an asymmetric or symmetric road excitation using running modes visualisations. In most of the cases, the experienced engineer can then easily analyse what is the cause of the low fatigue life of the part. In case of the cross member, fatigue failure at the hot-spot was identified to be caused by the combination of an asymmetric motion of the front axle and the fuel tank.

The fourth step for fatigue analysis is to improve the construction. It was found that a modification of the fuel tank mounting would improve the fatigue life of the cross member. The new design proposal should be evaluated again until the cross member meets the demands with respect to fatigue life.

5 Conclusions

This paper describes a method for frequency domain stress-life fatigue analysis. The method comprises the calculation of bi-axial response stress PSDs from multiple random excitations and the determination of an equivalent Von Mises stress PSD. This was subsequently subjected to uni-axial fatigue analysis to result in a Palmgren-Miner damage number. The paper also discussed the implementation of the method, using the Finite Element software NASTRAN and MATLAB. As an illustration, an example case was presented which showed the practical use.

The method gives qualitative good results, especially when relative improvements to the design are analysed. It is not only very effective to use in the design process to evaluate different design concepts but also provides an effective gateway to identify where and how dynamics are responsible for a reduced fatigue life.

Acknowledgements

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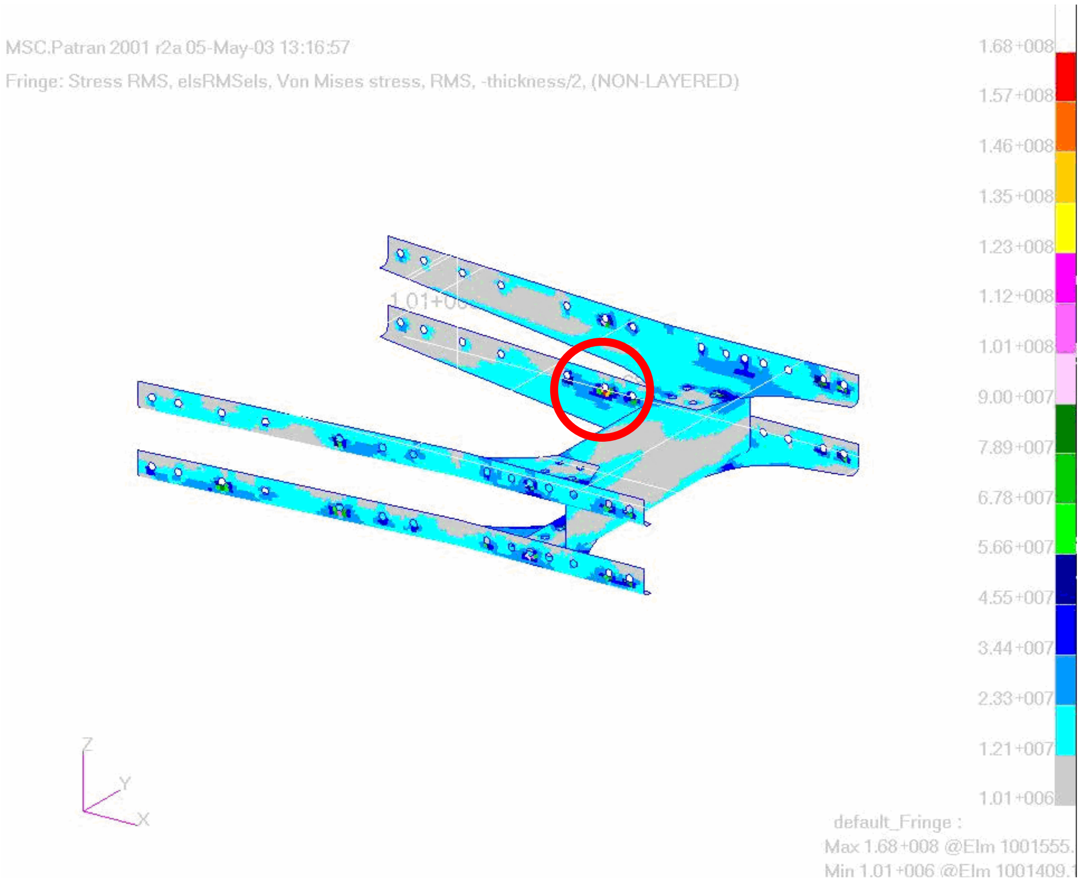

Figure 6: RMS values of the Von Mises stress

Figure 7: Fatigue failure at the hot-spot during testing

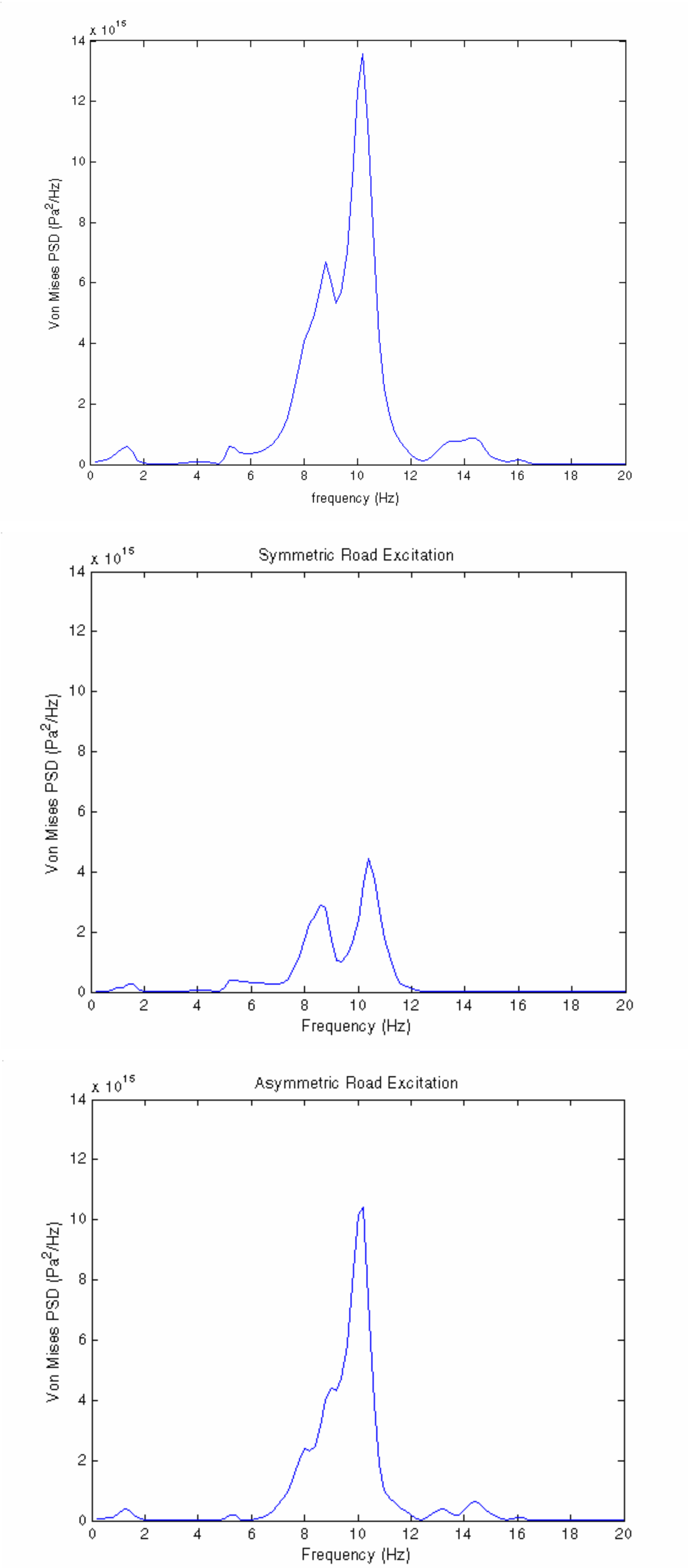

Figure 8: Von Mises stress PSDs at the hot-spot

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